Magnetic-flux patterns on the surface of a type-II superconductor

David A. Huse
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
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Various regimes exist where the pattern of magnetic flux through the surface of a type-II superconductor is predominantly determined by surface effects and not by the pattern of flux in the bulk. In particular, for zero pinning and small enough flux density, the flux lattice at a flat surface is isomorphic to the classical two-dimensional electron lattice and crystallizes at a much higher temperature than does the bulk flux lattice. Within uniaxially anisotropic London theory, the flux-line energy recently calculated by Sudbø and Brandt shows that for sufficiently strong anisotropy of the sign appropriate for layered superconductors an isolated straight flux line in the bulk is only stable when it is either nearly parallel or nearly perpendicular to the layers. The "flux chains" observed on BSCCO by Bolle *et al.* occur in a regime where both orientations of stable flux lines should be present and thus reflect the interactions between parallel and nearly perpendicular flux lines.

The patterns of magnetic flux emerging from surfaces of high-temperature superconductors in a magnetic field have been studied in a number of recent experiments.[1-5] These patterns have generally been interpreted as being representative of the magnetic-flux patterns that occur in a bulk sample well away from a surface. However, surface effects can play an important or even dominant role in determing the flux pattern seen at a surface. Two types of surface effects are discussed in the following: first, as was shown by Pearl,[6] the ends of flux lines farther apart than the superconductor's magnetic penetration length, λ, act as magnetic monopoles and thus interact with a Coulomb repulsion falling off with distance r only as $1/r$, while in the bulk the interaction between flux lines falls off as[7] $e^{-r/\lambda}$. Thus the surface interaction can dominate for $r \gg \lambda$ (low magnetic field). Also, because of the large energy per unit length of a flux line, roughness of the surface can produce a strong pinning potential for the ends of the flux lines. This note concludes with a suggestion for what might be causing the "flux chains" observed by Bolle *et al.*[4] on the $\hat{c}$ faces of BSCCO crystals with the applied field rotated slightly (e.g., $\simeq 20°$) out of the *ab* plane.

Let us first consider the simplest case of a semi-infinite piece of an ideal type-II superconductor with no pinning, a perfectly flat surface and a small magnetic field oriented normal to the surface. If the superconductor is anisotropic, let us assume the surface is normal to one of its principal axes of symmetry (e.g., the $\hat{c}$ axis). The field is assumed to be small enough that the flux lines in the bulk of the material are in the fluid state, due to thermal fluctuations.[8-10] This is predicted[8-10] to occur only when the average distance between neighboring flux lines is well in excess of the penetration length λ. On length scales larger than λ, the magnetic field outside of the sample[6] is simply that due to the ends of the flux lines at the surface, which act as magnetic monopoles (a la Dirac); each flux-line end emits $\phi_0 = hc/2e$ of magnetic flux into this half-space. Let us fix the position of the flux line ends on the surface, and consider the free energy of this constrained system. The dependence of the energy of the magnetic field outside of the sample on the positions of the ends of the flux lines is given by the sum over all pairs of flux-line ends of the repulsive monopole-monopole interaction energies:

$$V(r) = \frac{e^2}{8\alpha^2 r} \simeq \frac{3.9\times 10^4 \text{ K}\,\mu\text{m}}{r} , \tag{1}$$

where $\alpha = e^2/\hbar c = \pi e/\phi_0 \simeq \frac{1}{137}$ is the fine-structure constant and r is the distance between the two flux-line ends. Thus, if we first ignore the free energy due to the interactions and fluctuations *in* the sample this is a classical two-dimensional Coulomb system,[11] as studied, e.g., by Grimes and Adams,[12] but with an interaction energy $1/(8\alpha^2) \simeq 2350$ times larger than that of electrons. The melting temperature T_M of this system is[12]

$$k_B T_M \simeq 0.0017 n^{1/2} e^2/\alpha^2 , \tag{2}$$

where n is the areal density of flux lines. This is $T_M \simeq 120\sqrt{\bar{B}}$ if T_M is measured in degrees Kelvin (K) and the average normal magnetic field $\bar{B}$ is measured in Gauss.

The free energy of the flux lines *in* the sample will also depend on the positions of the ends of the flux lines at the surface. The energetic and entropic interactions between flux lines in the sample are both repulsive, so the resulting additional interaction between flux-line ends is also repulsive, adding to the repulsive Coulomb interaction (1) and only increasing the melting temperature. Thus the above estimate of T_M is a lower bound. (If the bulk is anisotropic these interactions from within the sample may also distort the surface crystal; see below.) Thus for known type-II superconductors the vortex or flux line ends at a perfectly flat surface of a thick sample in the absence of pinning should always be crystallized for fields $\bar{B}$ greater than a Gauss and less than the upper field B_M, where the bulk vortex lattice melts (see, e.g., Ref. 10).

Such a field range exists except *very* near T_c where the bulk vortex lattice never freezes.[10] Note that the interaction energy between flux lines in the bulk[7] falls off with r as $\exp(-r/\lambda)$; the resulting bulk flux lattice melting temperature vanishes exponentially as $\bar{B}$ vanishes[10] and will be well below the surface melting temperature for $\bar{B}\lambda^2 \ll \phi_0$. Thus a substantial field and temperature range exists where the flux lines in the bulk are in the fluid phase, while they crystallize at the surface for the ideal, pinning-free semi-infinite sample considered here.

Why do decorations of high-T_c superconductors[1-5] at low field (i.e., 1–100 G) not therefore see crystalline long-range order of the flux-line ends? Presumably because of pinning. One very potent type of pinning is surface roughness. The energy of an isolated vortex or flux line per unit length in the superconductor is[7]

$$\varepsilon_1 \approx \left[\frac{\phi_0}{4\pi\lambda}\right]^2 \ln\kappa = \frac{e^2}{16\alpha^2\lambda^2}\ln\kappa \tag{3}$$

for large κ, where $\kappa = \lambda/\xi$ and ξ is the Ginzburg-Landau coherence length. For BSCCO parameters $\lambda \simeq 3000$ Å and $\kappa \simeq 200$ this is $\varepsilon_1 \simeq 100$ K/Å or well over 1000 K per vortex per CuO_2 layer; it is still larger for YBCO. Thus surface roughness on the vertical scale of one or more layers (and horizontal scales greater than the coherence length) generates a very strong pinning potential for the end of the vortex, even if the bulk is free of pinning defects. This surface pinning could dominate over bulk pinning in some decoration experiments. However, the nature and strength of the bulk pinning is quite uncertain so it seems difficult to make a quantitative comparison, even if we knew how rough the surface is. Also samples may have surface "dead" layers that are not superconducting so the actual surface of the sample may not represent the surface of the superconductor.

Since the surface contribution to the repulsive interaction between flux lines (1) does not contain any material parameters, for a given observed pattern of flux-line ends one may calculate the excess energy of the magnetic field outside of the sample over that of the ideal ground state, which is a perfect flux-line lattice with the same $\bar{B}$. If this excess energy is less than or roughly $k_B T$ per flux-line end, then the distortions may be primarily due to thermal fluctuations. If it is many times $k_B T$ it must be due to pinning and gives an estimate of the pinning energy. Of course, this comparison on an energy scale of $k_B T$ will not be possible if the positions of the flux lines are not known to sufficient accuracy.

What happens if the bulk crystallizes? If the bulk is isotropic then it will be in perfect registry with the surface crystal. However, when the bulk flux crystal is anisotropic, as is expected in both BSCCO[3] and twin-free YBCO[2] (even for field along the $\hat{c}$ axis, because of a/b in-plane anisotropy), the match will not be perfect, since the surface interactions via the magnetic field are isotropic. Connecting an anisotropic bulk flux crystal with an isotropic surface crystal requires bending and stretching many flux lines. If this costs too much energy, the surface and bulk will remain in registry. When the sample is of a finite thickness (and free of stacking faults) the actual anisotropy of the flux crystal will then be determined by a competition between surface and bulk energies, with very thin samples being more isotropic. Murray *et al.* and Grier *et al.*[3] find varying anisotropies in decorations of the c face of BSCCO crystals with up to 10% distortions seen, while Dolan *et al.*,[2] observed varied distortions of up to 15% on twin-free YBCO crystals. It would be interesting to test whether these anisotropies change systematically with the sample thickness along the flux lines as would be expected; however, this requires crystals with uniform orientation of the a and b axes (no stacking faults, twins, or similar defects).

It has been observed that surface flux patterns tend to align with structural defects in the material.[2,3] Could they also align with other defects, e.g., vortices or flux lines, running parallel to the surface? This may be occurring in the "flux chains" observed by Bolle *et al.*[4] on BSCCO crystals for fields oriented close to, but not in the a-b plane. Consider first the decorations of YBCO crystals by Dolan *et al.*[2] with the field in the a-b plane, as illustrated in Fig. 1(a). There, "stacks" of flux-line ends

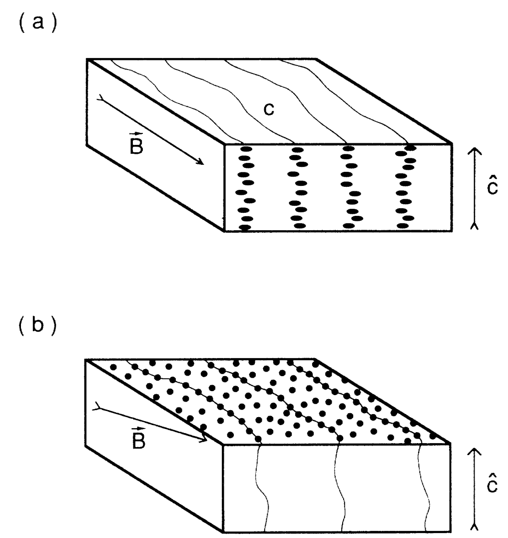

FIG. 1. (a) Schematic illustration of the decoration experiment of Dolan *et al*, Ref. 2, for a YBCO crystal. The field **B** is perpendicular to the $\hat{c}$ axis and the face decorated is perpendicular to **B**. The pattern seen reflects the penetration depth anisotropy by having oval-shaped flux lines arranged in stacks, as shown. The fine lines show the projections of the vortices onto the upper surface, the $\hat{c}$ face. (b) Schematic illustration of the decoration experiment of Bolle *et al.*, Ref. 4, for a BSCCO crystal. The field **B** is oriented at an angle of 70° away from the $\hat{c}$ axis, and the face decorated is perpendicular to $\hat{c}$. It is proposed that there are stacks of interplane or Josephson vortices as in (a), along the fine lines. These interplane vortices, however, are much more closely spaced than in (a) because of the higher field and anisotropy; the corresponding face was not decorated, so the individual vortices are not shown here. The component of **B** parallel to $\hat{c}$ is seen in the decoration and penetrates the crystal in isolated flux lines that run nearly parallel to $\hat{c}$. These flux lines have a higher density along the stacks, indicated by the fine lines, than between stacks. This produces the flux chains seen on the $\hat{c}$ face in Ref. 4.

were seen on a face normal to the field with spacing between stacks of $\simeq 6.7$ μm normal to the field in the a-b plane and spacings of $\simeq 1.2$ μm between flux lines within each stack along the c axis. Such a pattern is essentially what is expected within London[13] theory with, as usual, a vortex on each flux line. This density of flux lines corresponds to $\bar{B} \simeq 2.5$ G. Let us assume that the vortex pattern is qualitatively the same for BSCCO, which has at least 10 times greater penetration depth anisotropy, at the fields (roughly 10 times higher) studied by Bolle *et al.*[4] Then the spacing between stacks (of interplane or Josephson vortices) would still be $\simeq 7$ μm along the a-b plane, but the in-stack spacing along the c axis between vortices would be only $\simeq 0.1$ μm. Let us then ask what happens when one adds a field parallel to the c axis, thus rotating the total field a little, attaining the geometry studied by Bolle *et al.*[4] and illustrated in Fig. 1(b).

For a sufficiently isotropic type-II superconductor, the low-field state for any field orientation consists of flux lines running parallel to each other and to the average field in the sample and arranged in a pattern that minimizes the interaction energy. This appears to be the case in a recent decoration study of YBCO where the orientation of the field was varied.[5] However, for somewhat larger anisotropies (penetration length anisotropy ratio, Γ, in excess of about 10) a single isolated flux line is actually unstable for some orientations. This can be seen within anisotropic London theory via the energy of an isolated flux line, as calculated by Sudbø and Brandt.[14] For the anisotropy ratio of $\Gamma > 55$ of BSCCO, only flux lines running nearly parallel or nearly perpendicular to the layers are stable; flux lines with other average orientations can lower their free energy by separating into segments running in the two stable directions. Thus, if, as in the experiment of Bolle *et al.*,[4] one has a small average field in the material that is not in one of the stable directions, the system will have flux lines running in both stable directions present in a proportion determined by the average field. How these flux lines are arranged will then be determined by their interactions. Parallel flux lines generally repel one another,[15] but within London theory perpendicular *straight* flux lines do not interact. Due to the core interactions neglected in London theory, and due to distortions of the lines away from straight,[14] the nearly perpendicular flux lines that should be present in the experiment of Bolle *et al.*[4] may actually attract one another.

This suggests the following proposed description of the flux patterns observed by Bolle *et al.*:[4] Two types of vortex or flux lines are present in the material; some running nearly parallel to the layers and others running nearly normal to the layers. Those running nearly parallel to the layers are arranged in stacks as they would be for a field orientation parallel to the layers as illustrated in Fig. 1(a). The flux lines running nearly normal to the layers are weakly attracted to those parallel to the layers. This causes their density to be higher along the stacks than it is between the stacks, producing linear chains of closer-spaced flux lines on the $\hat{c}$ face of the sample, as illustrated in Fig. 1(b). The spacing between the chains and its dependence on the magnitude and angle of the field as measured in Ref. 4 is consistent with this description. This proposal might be tested by imaging the flux pattern on other faces of the crystal; one possibility would be to simultaneously decorate opposite c faces of a thin sample in hopes of determining the actual orientations of the flux lines passing through the sample.

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[15] As noted by A. M. Grishin, A. Yu. Martynovich, and S. V. Yampol'skii, Zh. Eksp. Teor. Fiz. **97**, 1930 (1990) [Sov. Phys. JETP **70**, 1089 (1990)]; A. I. Buzdin and A. Yu. Simonov, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 168 (1990) [JETP Lett. **51**, 191 (1990)], parallel flux lines do attract one another in an anisotropic superconductor when they are not aligned with any of the material's principal axes and their separation is in the correct direction. However, for the flux lines considered here which are nearly parallel to the principal axes, this attraction is very weak and only occurs at large distances. This attraction is a strong effect only for flux lines oriented in directions where they are unstable.