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Effects of field-sweep rate on the magnetization of melt-textured $YBa_2Cu_3O_{7-\delta}$

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The effects of field-sweep rate $K=\partial H/\partial t$ on magnetization hysteresis loops $M(H)$ and on flux-creep studies $M(t)$ in high-temperature superconductors have been investigated both theoretically and experimentally. We find the basic relation between M and K is, to first order, the following: $M=\text{const}-\{[dM/d\ln(t)]\ln(K)\}-[Kt_{\text{eff}}/10]$, where $dM/d\ln(t)=aC/30$ is the flux-creep rate in a cylindrical sample of radius a, and t_{eff} is an effective attempt time for vortex hopping. The largest possible M, which corresponds to the critical current density J_{c0} in the absence of thermal activation, develops when $K \geq K_{\max}=aC/[(1+a\alpha)t_{\text{eff}}]$ with $\alpha=\partial J/\partial H$. The time origin of flux creep, which is essential in studying the initial stages of relaxation, is given by $t^*=aC/K(1+a\alpha)$. The model agrees well with experiments on a melt-textured-growth sample of $Y_1Ba_2Cu_3O_{7-\delta}$, yielding $t_{\text{eff}}\sim 0.24\pm0.03$ s at 27 K. By incorporating the calculated time origin into flux-creep studies of $M(t)$, we obtain a very good description in terms of the interpolation formula from vortex-glass–collective pinning theory.

I. INTRODUCTION

Ever since the recognition of giant flux creep in high-temperature superconductors (HTS's),[1] tremendous work has been done to understand this peculiar phenomenon;[2-4] nevertheless, many aspects of the experiments and interpretation remain highly controversial.[4] Recent discussions on the time scale t_{eff}, for instance, attract special attention. Here t_{eff} is an effective attempt time for a vortex bundle to jump over a potential barrier; it acts as a scaling quantity in the standard Anderson-Kim[5,6] expression for thermally assisted flux creep (TAFC):

$$J(t)=J_{c0}\left[1-\frac{k_BT}{U_0}\ln\left[\frac{t}{t_{\text{eff}}}+1\right]\right], \tag{1.1a}$$

where J_{c0} is the critical current density in the absence of thermal activation and U_0 is an effective pinning energy. In previous work, t_{eff} was considered a microscopic time scale of order 10^{-10} s.[5,7] Recent studies argue that t_{eff} is a macroscopic quantity related to the flux flow resistivity and sample geometry.[8-10] On the other hand, to our knowledge, most experimental efforts to measure values for t_{eff} still have not been successful. In part, the difficulty lies in the fact that, if $t \gg t_{\text{eff}}$, which the condition usually met in flux-creep measurements, then Eq. (1) reduces to the form

$$J(t)=J'_{c0}-C\ln(t), \tag{1.1b}$$

where $C=J_{c0}kT/U_0$ and $J'_{c0}=J_{c0}-C\ln(t_{\text{eff}})$. Since the accessible quantities in experiments are C and J'_{c0}, then J_{c0}, U_0, and t_{eff} cannot be separated. To cleanly measure values for J_{c0}, U_0, and T_{eff}, we need to investigate the initial stage of magnetic relaxation, which poses another difficulty: How can one define the time origin of flux creep? In this work we propose a way to determine these quantities by studying the effects of field-sweep rate on the magnetization.

The influence of the field-sweep rate on magnetization curves has been studied by several authors.[11-13] Pust *et al.*[11] first observed in measurements of hysteresis loops $M(H)$ an increase of M with field-sweep rate. They attributed this phenomenon to a competition between field-sweep rate and magnetic relaxation and found that M increased with sweep speed as $M\sim C\ln(K)$, where $K=\partial H/\partial t$. The same result was found by Griessen,[12] who first indicated a method to calculate the time origin for magnetic relaxation from a subcritical state. Very recently, in investigating the initial stages of flux creep in HTS materials, Gurevich *et al.*[13] obtained the same relation between M and $\ln(K)$ by more strict theoretical considerations. In addition, they concluded that the value of t_{eff} may depend on the initial experimental conditions and found that with a low sweep rate, e.g., 5×10^{-6} T/s, then t_{eff} may be as large as 5×10^{3} s. However, in the present paper, we consider t_{eff} to be independent of sweep rate and report an alternative way of studying the effects of competition between sweep rate and relaxation in measurements of magnetic hysteresis loops $M(H)$ and flux creep $M(t)$. The theory is based on Bean's critical state model[14] and the TAFC model. To first order, our results reproduce the $M\sim\ln(K)$ relation, but additionally provide a way to determine t_{eff}. Furthermore, the time origin in flux-creep measurements is well defined in this model. The largest possible magnetization, produced for rates $K\geq K_{\max}$, provides information on J_{c0}. Experiments on a melt-textured-growth sample of $Y_1Ba_2Cu_3O_{7-\delta}$ agreed well with the model.

II. THEORETICAL MODEL

Consider a type-II superconductor subjected to the magnetic field that increases with time. According to the Bean model, for $H>H_{c1}$, the lower critical field, an induced current density J_c flows around the sample perimeter such as to exclude the external field. Meanwhile, magnetic relaxation occurs due to the Lorentz driving force and thermally activated flux motion, thereby reducing the induced current. In this scenario, the field configuration changes instantly to follow the rising field. Figure 1 shows schematically the evolution of the local flux density $h(r,t)$ in a cylindrical sample with radius a. Here we neglect the influence of H_{c1} and any variation of critical current density with local field in the sample, since the sample radius is small. Considering first a case in which the applied field $H(t)$ is small, then Fig. 1(a) shows the corresponding $h(r)$. Its slope is proportional to the current density according to $\nabla\times h=J$. Let us suppose that the field is applied step by step at times $t_0,t_1,\ldots$ with time intervals $\Delta t\sim t_{\mathrm{eff}}$ and with equal field steps ΔH. It is easy to see that for a small field, the field distribution corresponds almost exclusively to the current density J_{c0}; in other words, the influence of current decay is not so important. Figure 1(b) graphically defines a field H'; for applied fields $H>H'$, the internal gradient is nonlinear, since part of the internal field corresponds to now decayed current density that was induced at earlier times. Using Fig. 1(b), we find for H',

$$H/J(t)=H'(t+\Delta t)/J_{c0}$$

or

$$H'=J_{c0}(\partial H/\partial t)/(\partial J/\partial t)\ . \tag{2.1}$$

We can estimate the magnitude of H'. Using

$$\partial J/\partial t=J_{c0}(k_BT/U_0)(1/t_{\mathrm{eff}})\sim C/t_{\mathrm{eff}}$$

and assuming typical values $J_{c0}/C\sim20$, $\partial H/\partial t\sim100$ G/s, and $t_{\mathrm{eff}}\sim10^{-2}$ s, we obtain $H'\sim20$ G. This is comparable to H_{c1}. Thus, generally speaking, H' is relatively small. For $H>H'$, the magnetic moment observed at time t can be calculated using Fig. 2 to be the following (per unit length along the cylindrical axis):

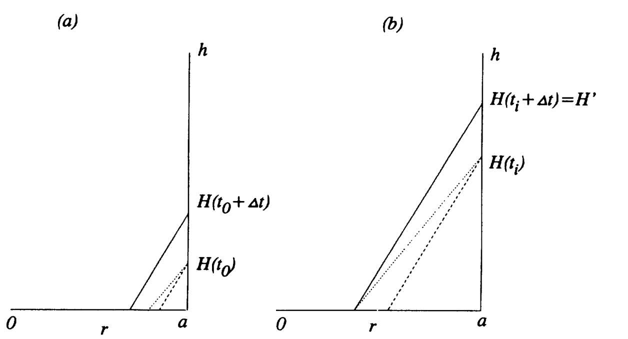

FIG. 1. Field configurations. (a) For small field, the slope $\partial H/\partial r$ corresponds to the critical current density J_c. The solid line represents the internal field $h(r,t_0+\Delta t)$ induced by the external field H applied at $t=t_0+\Delta t$; the dashed line is $h(r,t_0)$ induced by H applied at $t=t_0$. The dotted line described the relaxation of $h(r,t_0)$ that would occur at $t_0+\Delta t$, if the field were kept at constant value of $H(t_0)$. (b) When H is stepped to value of $H'=J_{c0}(\partial H/\partial t)/(\partial J/\partial t)$ at $t=t_i+\Delta t$, the corresponding solid line of $h(r,t_i+\Delta t)$ just meets the dotted line of the relaxed field originally induced at t_i.

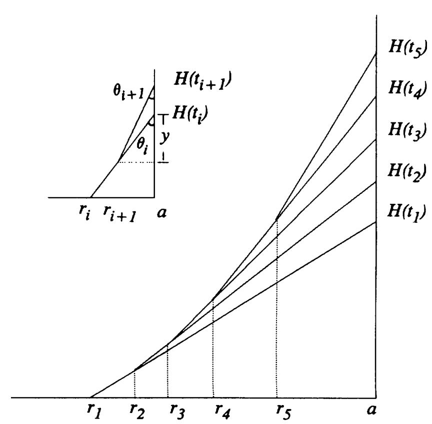

FIG. 2. Distribution of $h(r,t_5)$ induced by external field H applied step by step at $t_1,t_2,\ldots,t_5$ with equal step size ΔH and equal interval $\Delta t=t_{i+1}-t_i$. r_i represents the radius at which the fields originally induced at t_i and t_{i+1} have the same value at time. Inset: the construction used to calculate r_i.

$$\begin{aligned} m(t) &= \tfrac{1}{2}\int |\mathbf{r}\times\mathbf{J}|\, d^3r \\ &= \pi\left[\int_{r_1}^{r_2} J(t,t_1) r^2 dr + \int_{r_2}^{r_3} J(t,t_2) r^2 dr + \cdots + \int_{r_n}^{r_{n+1}} J(t,t_n) r^2 dr + \cdots\right] \\ &= \frac{\pi}{3}\sum_{i=1} [J(t,t_i)(r_{i+1}^3 - r_i^3) + J(t,t_H)(a^3 - r_H^3)] \; . \end{aligned} \tag{2.2}$$

Here $H(t,t')$ is defined as the current density induced at time $t' \le t$ and observed at an arbitrary time t. The time t_H is the time at which the field reaches its "final" value. In a swept-field experiment to measure magnetization loops, it is the time at which $m(H)$ is measured and so $t = t_H$; in a magnetic relaxation study, t_H is the time at which the application of H is completed, so we have $t \ge t_H$. Transforming to a situation in which the field continuously changes, we replace the summation by an integration:

$$m(t) = \frac{\pi}{3}\int_0^{t_H} J(t,t')\frac{\partial}{\partial t'} r^3(t,t') dt' \; , \tag{2.3a}$$

with

$$r(t,t_H) \equiv a \; . \tag{2.3b}$$

The quantity $r(t,t')$ is given by the relation (see inset of Fig. 2)

$$(a - r_{i+1})/y = \tan\theta_i \; ,$$

$$(a - r_{i+1})/(y + \delta H) = \tan\theta_{i+1} \; .$$

Noting $\cot\theta = -J(t,t')$, we find that

$$r(t,t') = \left[a + \frac{\partial H/\partial t'}{\partial J(tt')/\partial t'}\right] S\left[a + \frac{\partial H/\partial t'}{\partial J(t,t')/\partial t'}\right] , \tag{2.4}$$

where the step function $S(x)$ avoids a negative r. In practice, the correct procedure is first to find t^*, the last time at which $r(t,t^*)=0$. The integration in Eq. (2.3a) then starts from t^*. Equations (2.3) and (2.4) form the basis for our model. We use these expressions to determine the moment m in cases when (1) the field is swept continuously, and (2) when the field is latched at a fixed value for a magnetic relaxation (flux-creep) experiment.

A. Field changing with constant sweep rate

First we consider the case where H rises at a constant rate K. This is the usual situation in measurements performed in a vibrating sample magnetometer (VSM). We assume that the field has already penetrated to the center of the sample. Using Eq. (2.3), the magnetic moment is given by Eq. (2.5):

$$\begin{aligned} m(t) &= \frac{\pi}{3}\left[J(t,t)a^3 - \int_{t^*}^{t} r^3(t,t')\frac{\partial}{\partial t'}J(t,t')dt'\right] \\ &= \frac{\pi a^3}{3}\left[J(t,t^*) - \frac{3K}{a}(t-t^*) \right. \\ &\quad - \frac{3K^2}{a^2}\int_{t^*}^{t}(\partial J/\partial t')^{-1}dt' \\ &\quad \left. - \frac{K^3}{a^3}\int_{t^*}^{t}(\partial J/\partial t')^{-2}dt'\right] . \end{aligned} \tag{2.5}$$

In order to make an explicit calculation, we need to know $J(t,t')$. Several different models for the relaxation of currents in HTS materials have been proposed;[9,15–17] here we use the straightforward Anderson-Kim relation. This not only simplifies the calculations but it also is the asymptotic form of the other models, when $t - t^*$ is close to $t_{\rm eff}$, which is the time domain of interest here. From Eq. (1.1a), we have for $J(t,t')$ and its derivative

$$J(tt') = -\left[J_{c0} - C\ln\left[\frac{t-t'}{t_{\rm eff}} + 1\right]\right] , \tag{2.6}$$

$$\frac{\partial J}{\partial t'} = \frac{-C}{t - t' + t_{\rm eff}} + \alpha K \; . \tag{2.7}$$

Here $K = \partial H/\partial t'$ is the field-sweep rate and $\alpha = \partial J/\partial H$. We will neglect the t' dependence of α (α is an implicit function of t' through H) for simplification. Substituting Eqs. (2.6) and (2.7) into Eq. (2.4), we find for t^*, the root of $r(t,t^*)=0$:

$$t - t^* = \frac{aC/K}{1 + a\alpha} - t_{\rm eff} \; . \tag{2.8}$$

Substituting this expression for $(t - t^*)$ into Eq. (2.5) and retaining the first two (largest) terms, we obtain the following result for $m(t)$:

$$m(t) = \frac{\pi a^3}{3}\left[\text{const} - C\ln K + \frac{3t_{\rm eff}}{a}K + \cdots\right] , \tag{2.9}$$

where the constant term is

$$\text{const} = -J_{c0} + C\ln\frac{aC}{t_{\rm eff}(1 + a\alpha)} - \frac{3C}{1 + a\alpha} \; .$$

From Eq. (2.9) we see that the magnitude of m increases with $\ln K$ to first order, which agrees with the earlier work.[11–13] There is, however, an additional term that is proportional to K. Measuring the coefficient of this linear term $3t_{\rm eff}/a$ provides a means of determining the important quantity $t_{\rm eff}$. Since the relaxation rate C is of order $10^{-2}J_{c0}$, and $t_{\rm eff}$, whose magnitude is rather controversial, may be quite small, the effect from this term may be observed only for high sweep rates K. As a generality, note that this model does not necessarily assume that $\partial J/\partial H=0$.[11]

Another important feature is related to the time t^*. In particular, the model provides relation Eq. (2.9) only if K is less than a certain value $K_{\max}$, which is the solution of Eq. (2.8) when $t-t^*=0$:

$$K_{\max}=\frac{aC}{(1+a\alpha)t_{\rm eff}} . \tag{2.10}$$

The physical explanation is when $K>K_{\max}$, the relaxation of shielding current is relatively slow compared with the redistribution of internal field produced by the changing external field. Then the field gradient is just J_{c0}, which gives the larges moment. Equation (2.10) shows that this condition can be met most easily if the radius a is small.

B. Time origin in flux-creep measurements

In flux-creep measurements, a sample is first cooled to a certain temperature in zero field (ZFC), then the field is applied at a constant sweep rate so that it approaches a target value, either from lower or from higher fields. The measurements commence after H stops at the desired value at time t_H. The same calculations apply in this case, except that now we have $t\ge t_H$. This gives

$$m(t)=\frac{\pi a^3}{3}\left[J(t,t^*)-\frac{3K}{a}(t_H-t^*)+\cdots\right]$$
$$=\frac{\pi a^3}{3}\left[-\tilde{J}_{c0}+C\ln\left[\frac{t-t^*}{t_{\rm eff}}+1\right]+\cdots\right],$$

where

$$\tilde{J}_{c0}=J_{c0}+(3K/a)(t_H-t^*) .$$

Therefore the time origin in flux-creep measurements is not t_H, but rather t^*. According to Eq. (2.8), we have

$$\Delta t=t_H-t^*=\frac{aC/K}{1+a\alpha}-(t-t_H)-t_{\rm eff} . \tag{2.11}$$

For example, immediately after application of the external field when $t=t_H$, we have $\Delta t\sim 10$ s, assuming that $a\sim 0.1$ cm, $K\sim 100$ G/s, $\alpha\sim 100$ A G^{-1}cm^{-2}, and $C\sim 10^5$ A cm^{-2}. This is not a short time, and so significant magnetic relaxation takes place without being observed. Alternatively, we see from Eq. (2.11) that $\Delta t=0$ when $t-t_H\approx aC/K(1+a\alpha)$. So if we neglect relaxation from t_H to $t=\Delta t$, then t_H behaves as a true time origin. This agrees with the approach of Griessen[12] and Gurevich *et al.*[13] Actually both of them independently derived expressions similar to Eq. (2.8). However, Gurevich defined Δt as $t_{\rm eff}$ and interpreted it as the time interval needed to begin steady magnetic relaxation, following the abrupt change of external field at $t=t_H$. As shown clearly in the above discussion, our view of Δt is that, with any finite field sweep rate, flux creep starts before the field reaches the desired value. This means that flux-creep measurements generally start from a subcritical state unless the field-sweep rate is higher than $K_{\max}$. This interpretation is the same as that of Griessen.

To facilitate comparison with experimental results, it is worth rewriting Eq. (2.9) in practical units (cm, gauss, and A/cm^2). Then the factor $(1+a\alpha)$ in the above equations is changed to $(0.79+a\alpha)$, where the numerical factor 0.79 is just $(4\pi/c)^{-1}$ with c corresponding to 10 in practical units. For the magnetization M, we have

$$M(t)=\frac{a\times\text{const}}{30}-\frac{aC}{30}\ln K+\frac{t_{\rm eff}}{10}K+\cdots . \tag{2.12}$$

Recalling the Bean formula $M=aJ_c/30$ shows that the coefficient of $\ln(K)$ is just the flux-creep rate $dM/d\ln(t)$ in magnetic relaxation measurements.

III. EXPERIMENTS

A melt-textured-growth sample of $YBa_2Cu_3O_7$ with mass of 0.0747 g was used in the experiments. Its approximate dimensions were $0.21\times0.30\times0.20$ cm^3. The T_c measured in a small field $H=10$ G applied parallel to the c axis, was 90 K. Prior measurements of the dc magnetization were made using a commercial superconducting quantum interference device (SQUID) magnetometer (Quantum Design MPMS). For example, an analysis with the Bean model yielded J_c values at $H=1$ T of 2.5×10^5 A/cm^2 and 3.4×10^4 A/cm^2 for $T=25$ and 60 K, respectively. Experiments to test the effects of field-sweep rate on magnetization were performed in a laboratory-constructed VSM. Hysteresis loops $M(H)$ with various sweep rates were recorded at temperatures of 27, 28, and 60 K in fields up to H=3.4, 5.0, and 3.7 T, respectively. Temperature stabilization was of crucial importance for the measurements, since modest temperature fluctuations, e.g., 0.1 K, could strongly affect the relative size of the signal. Figure 3, a plot of magnetic moment $m(H)$ at 60 K, illustrates the impact of field-sweep rate. Measurements at progressively slower rates, shown as the set of line segments in the range 2.9–3.5 T, yielded ever smaller values for the magnetic moment m. To examine the explicit rate dependence, we consider magneti-

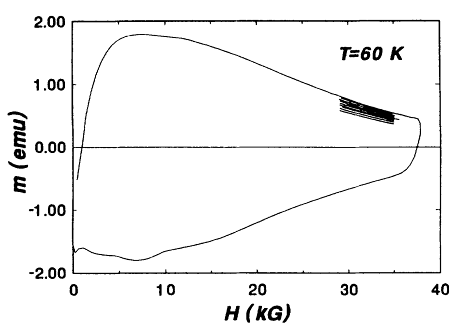

FIG. 3. At $T=60$ K, the magnetic moment m vs field H for different field-sweep rates K. The complete loop was measured at the highest rate, $K=250$ G/s. Measurements at other rates, 200, 170, 129, 96, 75, 53, 42, 31, and 21 G/s, are shown for fields in the range 2.9–3.5 T.

zation M versus rate K at fixed temperature and fixed field. Figure 4(a) presents the results at $T=28$ K, $H=4.8$ T; similar results at 60 K, 3.2 T are shown in Fig. 5(a). In these two experiments, the sweep rate ranged from 20 to 250 G/s. Figure 6 shows additional results at 27 K in a field of 3.2 T. In this case, we were able to employ an expanded interval of 5–800 G/s. These three cases are denoted as $M(K;T=28\text{ K})$, $M(K;T=60\text{ K})$, and $M(K;T=27\text{ K})$, respectively.

To compare the quantities $dM/d\ln(K)$ and $dM/d\ln(t)$ experimentally, short magnetic relaxation measurements were made at the same temperatures and fields (28 K, 4.8 T and 60 K, 3.2 T), using the VSM. These time-dependent measurements of $M(t)$ are shown in Figs. 4(b) and 5(b), and are denoted as $M(t;T=28\text{ K})$ and $M(t;T=60\text{ K})$. The magnetic field was applied at rates of 150 and 260 G/s, respectively.

In all the sweep-rate studies, an *approximately* logarithmic dependence of M on the rate K was observed, as given by Eq. (2.9). Careful examination of Figs. 4(a), 5(a), and 6 reveals, however, slight deviations from a strictly logarithmic relation at both high-K and low-K sweep rates. In the low-K region, the departure from linearity (on the semilogarithmic plot) comes from the fact that the Anderson-Kim expression, Eq. (1), is only approximately correct for high-T_c superconductors.[16] With still slower sweep rates, the deviation would be even more severe. The central interest in this work, however, is the high-K region: Here the decreasing slope of the curves is a consequence of the linear term in K in Eq. (2.12). We can assess the importance of the correction term by considering the ratio of its size relative to the logarithmic term, $3Kt_{\text{eff}}/[aC\ln(K)]$. Assuming that t_{eff} is ~ 0.1 s, $aC/30\sim 60$, and that $K=150$ G/s, the ratio is about 10^3 for $M(K;T=28\text{ K})$. Under these conditions, the effect of the linear K term is rather small. For $M(K;t=60\text{ K})$, however, the ratio may be $\sim 10^{-2}$, due to the smaller value of $aC/30$. Even larger ratios might develop, if t_{eff} increased at higher temperatures.[18] Unfortunately, long-term fluctuations in sample temperature were larger in this case [$\Delta T\sim 0.2$ K for $M(K;T=60\text{ K})$, compared with ~ 0.05 K for $M[(K;T=28K)]$. Consequently, it was difficult to analyze the linear term effect very accurately. By compensating for the temperature variations, we found that t_{eff} was the order of 10^{-1} s for both cases.

In order to obtain more accurate and reliable results for t_{eff}, some experimental modifications were made to

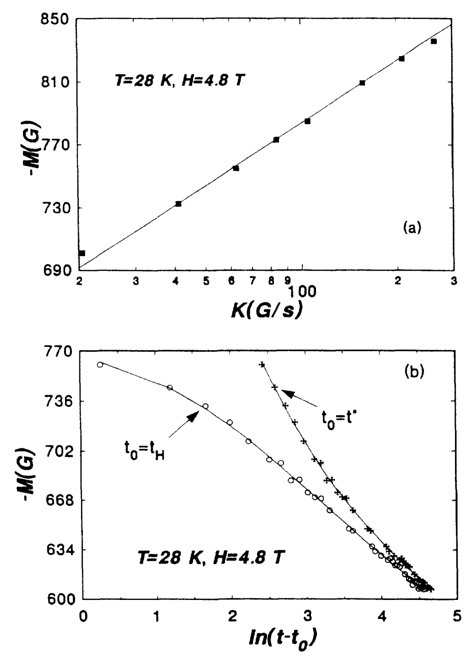

FIG. 4. (a) Magnetization M (expressed in units of [emu/cm^3]=[G cm^3]/cm^3=G) vs sweep-field rate K, for $T=28$ K and $H=4.8$ T. (b) Time-dependent M vs $\ln(t-t_0)$ with time origin t_0 defined as either t_H or t^*, where the units of time are seconds.

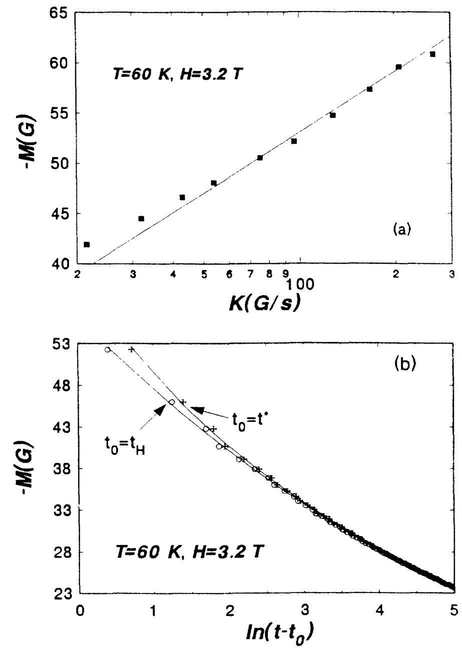

FIG. 5. Same quantities as those in Fig. 4, but with $T=60$ K and $H=3.2$ T.

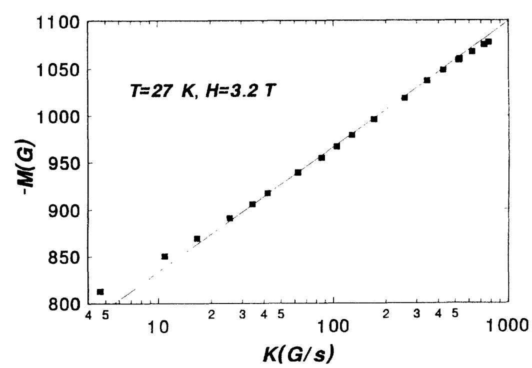

FIG. 6. Magnetization M vs sweep rate K at $T=27$ K and $H=3.2$ T. Note particularly the departure from linearity at very high sweep rates.

improve the temperature stability and to increase the range of sweep rate. First, all experimental conditions were stabilized, including the liquid-helium level in the cryostat Dewar. This reduced the temperature fluctuations to ~ 0.03 K for measurements at $T = 27$ K. Second, the magnet power supply was replaced with a unit (Lake Shore Superconducting Magnet Power Supply model 612) providing higher charging voltages (up to 30 V) and higher output power (up to 10^3 V A). Sweep rates from ~ 5 to 800 G/s were obtained. These measures yielded much smoother and higher-resolution data, as seen in Fig. 6. Consequently the flattening in both the high-K and low-K regions is much more evident. Analysis of the data at high sweep rates yielded the value $t_{\text{eff}} = 0.24 \pm 0.03$ s. This result agrees with predictions in the literature[9] and with our early results.[18]

Next we consider effects associated with the time origin in a flux-creep experiment. Since the magnetic relaxation is a nonlinear function of t, the proper definition of the time origin is essential for studying the initial stages of magnetic relaxation. This is clearly demonstrated in Figs. 4(b) and 5(b), where M is plotted versus $\ln(t - t_0)$, using either t_H and t^* as the time origin t_0. The figures show time-dependent magnetization results $M(t; T = 28$ K) and $M(t; T = 60$ K), respectively. Comparison of Figs. 4(b) and 5(b) indicates that t_H is *not* the correct time origin, since the two curves appear quite different while one should expect more uniformity in the results. The origin of this difference can be explained as follows: At high temperature, the magnetic relaxation rate is smaller, which causes $t_H - t^*$ to be small. In other words, the field application resembles more closely a step function. According to Eq. (2.11), the time offset $(t_H - t^*)$ is only ~ 0.56 s for $M(t; T = 60$ K); in contrast, it is ~ 10 s for $M(t; T = 28$ K). Therefore at 28 K, much relaxation had already taken place when the measurements commenced; by comparison, the measurements of $M(t)$ at 60 K effectively commenced much earlier, allowing us to see the giant magnetic relaxation at short times. If instead we use calculated values of t^* for the time origin t_0, then both the relaxation curves at 28 and 60 K have similar functional dependencies $M(t)$. The results are described extremely well by the theoretical power-law "interpolation formula"[15]

$$M = M_{c0} / [1 + (\mu k_B T / U) \ln(t / t_{\text{eff}} + 1)]^{1/\mu} ,$$

as well as our empirical double-logarithmic formula:[16]

$$M = M_0 + C' \ln[\ln(t / t_{\text{eff}} + 1)] .$$

These expressions have been shown to provide a precise description of $M(t)$ in long-term magnetic relaxation studies.[16]

Incorporating t^* as the time origin, we can then compare values of $dM/d\ln(K)$ and $dM/d\ln(t)$. The relationship between these quantities was treated theoretically[11] in 1989, but there have been no experimental studies of this feature, to our knowledge. For a meaningful comparison, we must choose an appropriate time window to obtain an average value of $dM/d\ln(t)$, since M is not a precisely linear function of $\ln(t)$. The appropriate time t for evaluating the logarithmic time derivative is given by Eq. (2.8), which depends on the rates of flux creep and field sweep. Measured from t_H, the appropriate average is taken after 80 s for $M(t; T = 28$ K) and after 7 s for $M(t; T = 60$ K). The resulting values of $dM/d\ln(t)$ are 66 and 9.1 G, respectively. These results compare very favorably with the respective average slopes $dM/d\ln(K)$, which are 62 and 7.9 G. The respective values differ by $\sim 3\%$ and $\sim 13\%$. The larger discrepancy in the latter case may be due in part to the limited time resolution (~ 1.5 s) in the time-dependent study. Overall, we judge the agreement to be quite acceptable.

To conclude, the effects of magnetic-field sweep rate on magnetization measurements in high-temperature superconductors have been studied using the Bean critical-state model and TAFC model. To first order, a logarithmic dependence of magnetization M on field-sweep rate K is found, with a slope $dM/d\ln(K)$ equal to the flux-creep rate $dM/d\ln(t)$. A linear correction term to the $\ln(K)$ dependence allows the time scale t_{eff} to be determined experimentally at high sweep rates. At very low sweep rates, there are deviations from a strictly logarithmic K dependence arising from inadequacies in the Anderson-Kim model. Future work will attempt to treat this problem using more recent and realistic theoretical models for flux motion in high-T_c superconductors.

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