Dynamical activation processes described by generalized random walks

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The dynamical activation processes of a complex system under an external force are studied. The system is composed of clusters, and each cluster is an aggregation of strongly correlated units. The activation processes of the clusters are assumed to be described by generalized random walks, and their properties are studied by one of a response function $\Psi_c(t)$. In the analysis, effects of the other clusters are taken into account by an average over $\Psi_c(t)$, that is $\langle \Psi_c(t) \rangle$. The temporal behaviors of $\Psi_c(t)$ and $\langle \Psi_c(t) \rangle$ have been expressed as a power law. The indices of the power law are parametrized. The parameters are introduced to characterize the structures of the units and the distributions of the clusters in the system. In this sense, the system is a multifractal.

I. INTRODUCTION

Complex systems have no characteristic length,[1] except for irregularities and fragmentations. This feature gives rise to the development of such interesting concepts as fractals, self-similarity, and chaos, etc. Self-similarity is an invariant property with respect to a scaling operation or operations. To characterize the spatial or the temporal patterns of complex systems,[2,3] the scaling law giving the fractal dimension has been extensively used. The fractal dimension is expressed as an index of the power law for these patterns. In this sense, the scaling law can be regarded as a fundamental attribute specifying the degree of complexity or irregularities.

Originally the inverse power law has been known as the so-called Zipf law and the Bradford law.[4] The frequency size distributions have been expressed as an inverse power law. Since the pioneering study of fractal geometry by Mandelbrot,[1] many hierarchical spatial or temporal patterns have been reproduced by computer simulations, and the geometrical structures are classified according to their fractal dimensions. As for the temporal patterns, however, the analysis has been restricted to some simple systems.

Shlesinger, Hughes, and Montroll[5] propose an interesting process exhibiting a long-time-tail behavior. Similar phenomena have been observed in ultrametric space,[6] and in charge transfer in amorphous materials.[7] The long-time tail can be expressed as an inverse-power dependence, $t^{-1-\gamma}$ (t is the time with γ a constant). On a log-log plot, the long-time tail is a linear curve, and its slope yields an index giving the fractal dimension. Using this approach, Shimada[8] found fractal behavior of *Drosophila;* the frequency-of-staying (FS) time distribution at feeding spots can be expressed in a power-law form. Physiologically, the animal's behaviors are understood as an activation process (response) of the biological system to stimuli in the environment. A similar understanding is possible for the temporal patterns observed in the elastic shocks.[9] The response behavior seems to have arisen from dynamical activation processes under external forces.[10] Unfortunately, we have no suitable model at present for these activation processes.

In this paper, we propose an activation process for a model system. The system is composed of clusters, and each cluster is an aggregation of strongly correlated units.[11,12] The units are assumed to satisfy scaling rules. The activation process of clusters may be described by a response function $\Psi_c(t)$ based on generalized random walks (GRW).[13] In the analysis of $\Psi_c(t)$, we pay special attention to effects due to other clusters, and define a quantity $\langle \Psi_c(t) \rangle$ averaged over the clusters. The temporal behaviors of $\Psi_c(t)$ and $\langle \Psi_c(t) \rangle$ are found to follow a power law specified by indices. The indices are given in parametrized forms, which are determined by scaling factors for the clusters and the distribution of the clusters.

II. GENERALIZED RANDOM WALKS

We consider an activation process in a system composed of clusters [Fig. 1(a)]. A cluster is an aggregation of strongly correlated units [Fig. 1(b)]. The units are assumed to satisfy scaling rules to be specified later. We describe the activation process as a stochastic process by a recursion relation of generalized random walks (GRW).[13] The recursion relation of a probability $W(m,N)$, in which a cluster (walker) is in a state (site) m after N steps reads

$$W(m,N)=\sum_{k=1}^{M}\sum_{\alpha=-1}^{1}P^{\alpha}_{N,N-k}(m\,|\,m-\alpha)W(m-\alpha,N-k), \tag{2.1}$$

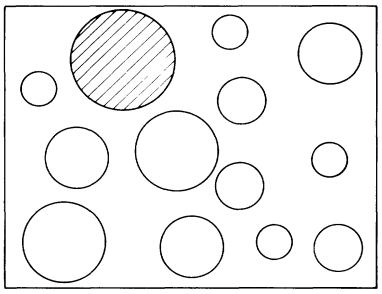
where $\alpha=+1$, 0, or -1, and the $P^{\alpha}_{N,N-k}(m\,|\,m-\alpha)$'s represent jumping probabilities between states $m-\alpha$ at step $N-k$ and m at step N; see Fig. 2.

Under an external force, the jumping probabilities are influenced by the activation processes of the other clusters, Fig. 3(a). Normalization of the jumping probabilities is given by

$$\sum_{k=1}^{M}\sum_{\alpha}P^{\alpha}_{N+k,N}(m+\alpha|m)=1\ . \tag{2.2}$$

In generalized walks, as a subsidiary condition the jumping probabilities are related to the corresponding ones by a map F,

$$F:\ P^{\alpha}_{N,N-k+1}(\cdot|\cdot)\rightarrow P^{\alpha}_{N,N-k}(\cdot|\cdot),\quad \text{for } k=1,2,\ldots,M\ . \tag{2.3}$$



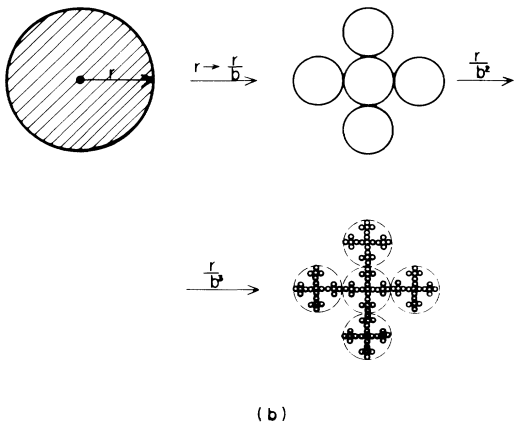


FIG. 1. (a) System composed of clusters. (b) Clusters are correlated and units of the cluster are distributed so as to satisfy scaling rules.

The map F symbolically denotes memory effects, or nonlinear effects between the jumping probabilities at successive steps. Fig. 3(b) shows scaling rules to be given later for the subunits of the cluster.

Note that translational invariance of $P^{\alpha}_{N,N-k}(m\,|\,m-\alpha)$ with respect to m and N leads to conservation of probability according to Eq. (2.1):

$$\sum_{m}W(m,N)=1\ . \tag{2.4}$$

In what follows, for simplicity, we limit ourselves to the activation process in which we can neglect the m dependence of $P^{\alpha}_{N,N-k}(m\,|\,m-\alpha)$,

$$P^{\alpha}_{N,N-k}(m\,|\,m-\alpha)=P^{\alpha}_{N,N-k} \tag{2.5}$$

and introduce generating functions for W and P, respectively, defined by

$$\tilde{W}(N;z)=\sum_{m}W(m,N)z^{m}\ , \tag{2.6}$$

$$\tilde{P}(k;z)=\sum_{\alpha}P^{\alpha}_{N,N-k}z^{\alpha}\ , \tag{2.7}$$

where we have assumed $0<z<1$, and have shifted the N dependence of $P^{\alpha}_{N,N-k}$ into the difference k $[=N-(N-k)]$, for simplicity. Then we can rewrite the recursion relation, and it becomes

$$\tilde{W}(N;z)=\sum_{k=1}^{M}\tilde{P}(k;z)\tilde{W}(N-k;z)\ , \tag{2.8}$$

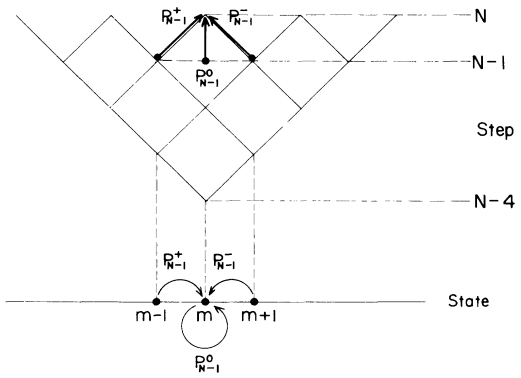


FIG. 2. (a) Recursion relation for $W(m,n)$. (b) Normalization of the jumping probabilities.

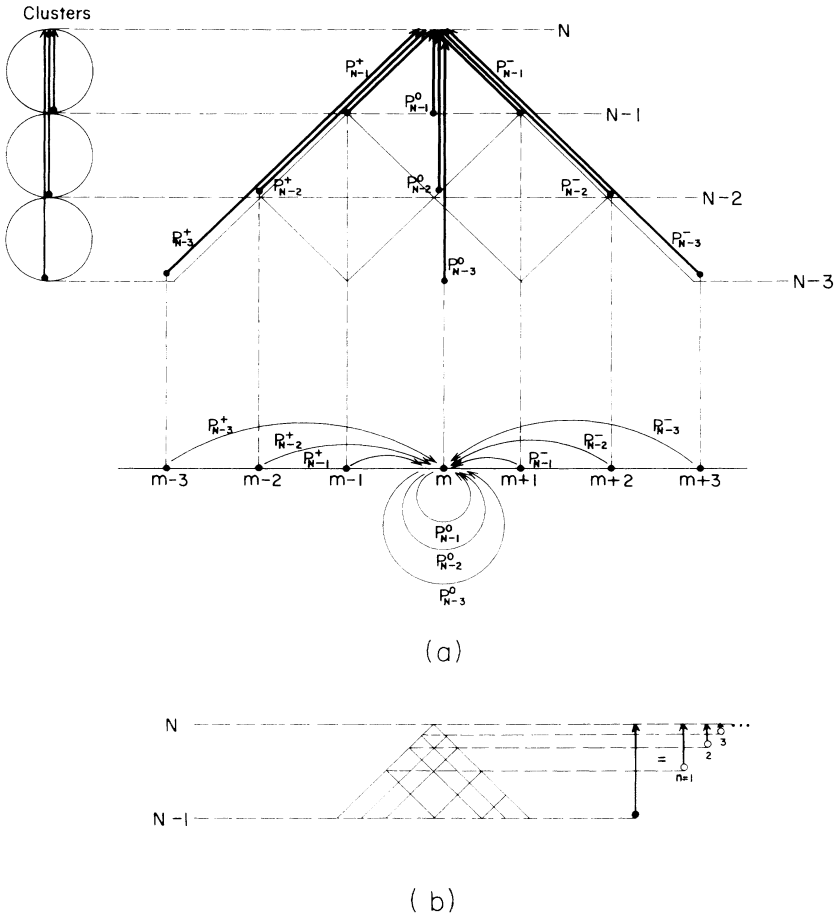


FIG. 3. (a) Jumping probabilities influenced by the activation process of other clusters. (b) Jumping probabilities between the effective time units satisfying scaling rules (3.7) and (3.8).

after a summation over m in (2.1). The normalization of $\tilde{P}(k;z)$ is given by

$$\sum_{k=1}^{M} \tilde{P}(k;1)=1, \tag{2.9}$$

and the subsidiary condition (2.3) reads

$$F:\ \tilde{P}(k-1;z)\rightarrow\tilde{P}(k;z)\ , \tag{2.10}$$

for any k.

III. SPECIFICATION OF ACTIVATION PROCESSES

In the following we restrict the activation process to the case in which $\alpha=0$ [$P^{\alpha}_{N,N-k}$ in (2.7)] and we assume that $P^{\alpha}_{N,N-k}$ is a function of k [$=N-(N-k)$];

$$P^{0}_{N,N-k}\equiv p^{0}(k\ \Delta t)\ . \tag{3.1}$$

Here another argument $k\ \Delta t$ is introduced to clarify the functional dependence of $P^{\alpha}_{N,N-k}$, and we consider $k\ \Delta t$ as k effective time units. The effective time unit Δt characterizes variations of the states (events) when we regard the clusters as points; see Fig. 3(a).

The probability $p^{0}(\tau)$ ($k\ \Delta t=\tau$) corresponds to a waiting-time probability between the events. In the continuum limit $\Delta t\rightarrow 0$, the activation process corresponding to $\tilde{W}_{0}(t;z)$ (with $N\ \Delta t=t$) is expressed by

$$\tilde{W}_{0}(t;z)=\int_{t_{\min}}^{t_{\max}} p^{0}(\tau)\tilde{W}_{0}(t-\tau;z)d\tau\ , \tag{3.2}$$

cf. (2.8), where $p^{0}(\tau)$ is a continuous function defined by

$$p^{0}(\tau)d\tau=p^{0}(k\ \Delta t)\Delta t\ , \tag{3.3}$$

and $t_{\max}=M\ \Delta t$ and $t_{\min}=\Delta t$. Normalization of $p^{0}(\tau)$ is expressed by

$$\int_{t_{\min}}^{t_{\max}} p^{0}(\tau)d\tau=1\ , \tag{3.4}$$

corresponding to (2.9).

In what follows, we consider an activation process described by a function defined by a sum of contributions $p^{0}(k\ \Delta t)$ [$=p^{0}(\tau)$];

$$\tilde{p}^{0}(t)\equiv\sum_{k=1} p^{0}(k\ \Delta t)\ . \tag{3.5}$$

Note that the function $\tilde{p}^{0}(t)$ represents a renormalized transition probability in the recursion relation (2.8). In

Fig. 3(a), we show processes corresponding to $\alpha=-1$, 0, and $+1$. The process denoted by $\tilde{p}^0(t)$ represents the contributions specified by some of the constituent clusters.

Since the cluster is an aggregation of the units [see Figs. 1(b) and 3(b)], the process between the effective time units is decomposed into a sum of the contributions arisen from the units having a new index n, $\tilde{p}_n^0(k\,\Delta t)$;

$$p^0(k\,\Delta t)=\sum_{n=1} p_n^0(k\,\Delta t) \tag{3.6}$$

[see Fig. 3(b)].

The index n represents the scaling stage for the units. To simplify the notation, we shall use t for Δt [$\tilde{p}(t)\equiv p^0(\Delta t)$ and $\tilde{p}_n(t)\equiv p_n^0(\Delta t)$]. We consider the scaling stage characterized by the two scaling factors a and b. Furthermore, as an explicit specification for F in (2.10), we impose two conditions on $\tilde{p}_n(t)$ [$=p_n^0(\Delta t), n\geq 1$]

$$F1:\ \tilde{p}_1(t+\delta t)=[1-\Phi(\varepsilon t)\delta t]\tilde{p}_1(t)\ \ (\varepsilon\ll 1)\ , \tag{3.7}$$

$$F2:\ \tilde{p}_n(t)=ab\tilde{p}_{n-1}(bt)\ \ (n\geq 2)\ , \tag{3.8}$$

where $t\equiv\Delta t$, and δt characterizes variations of the states between the effective time units, "local states" of the cluster $\delta t\ll\Delta t=t$.

The condition $F1$ specifies the time evolution of the activation process of the units, and $\Phi(\varepsilon t)$ is a slow varying function due to the smallness of factor $\varepsilon\ll 1$. The condition $F2$ characterizes the scaling rules between the successive scaling stages. Here, note that processes specified by (3.7) and (3.8) lead to the basic rule for the temporal scaling given by Shlesinger and Hughes.[5] It is important to see that condition (3.7) is satisfied by the differential equation, and that the factor b contains a t dependence as will be shown in the next section. Conditions (3.7) and (3.8) have arisen from the subsidiary condition of the GRW [see (2.3) or (2.10)]. Note also that $\tilde{p}^0(t)$ [see (3.5)] is to be interpreted as the staying probability density of the GRW, that is a waiting-time probability density for the activation process of the cluster.

IV. SCALING PROCESSES

The process that we considered is originally expressed by Eq. (2.1), with normalization condition (2.2) and subsidiary condition (2.3). We depict the process on the discrete site step space (see Figs. 2 and 3). In Fig. 3(b), we show an enlarged triangle and partition the triangle into discrete ones as shown, characterized by a set of time intervals scaled from t to $t'=bt$, $t''=b^2t$, . . . , $t^{(n)}=b^nt$, and so on.

By solving Eq. (3.7), we obtain expressions for $p_n(t)$ satisfying conditions (3.7) and (3.8):

$$\tilde{p}_1(t)[=p_1^0(\Delta t)]=ae^{-bt}\ , \tag{4.1}$$

$$\tilde{p}_2(t)=ab\tilde{p}_1(bt)\ \ [\text{cf. (3.8)}]\ , \tag{4.2}$$

where

$$b=b(\varepsilon)=\frac{1}{t}\int_0^t\Phi(\varepsilon y)dy\ , \tag{4.3}$$

a is an initial value for $\tilde{p}_1(t)$ [$\tilde{p}_1(0)=a<1$], and ε is a very small parameter.

Note that $\Phi(\varepsilon t)$ is slow varying because of ε, and that $b(\varepsilon)$ is approximately a constant parameter. The $\tilde{p}_n(t)$ is expressed by

$$\tilde{p}_n(t)\ [=\tilde{p}_n^0(t)]=ab\tilde{p}_{n-1}(bt)$$
$$=a^nb^{n-1}e^{-b^nt}\ . \tag{4.4}$$

Here it is important to note that one can include effects of the other clusters through the function $\Phi(t)$ because $\Phi(t)$ plays the role of a transition probability in (3.7). To characterize these effects, suppose that the slow varying of function $\Phi(t)$ satisfies a scaling relation,

$$\Phi(t)=a^{(c)}\Phi(t/b^{(c)})\ , \tag{4.5}$$

where $a^{(c)}$ and $b^{(c)}$ are new scaling factors characterizing distributions of the other clusters around the cluster (walker); see (2.1).

By solving the functional equation (4.5), we obtain

$$\Phi(t)=At^{\mu}\ \ \left[\mu=\frac{\ln a^{(c)}}{\ln b^{(c)}}\right]\ , \tag{4.6}$$

where A is a constant.[14] Substituting (4.6) into (4.1), we get

$$\tilde{p}_1(t)=\begin{cases} a\exp\left[\dfrac{-A}{1+\mu}t^{\mu+1}\right], & \mu+1\neq 0 \\ at^{-A}, & \mu+1=0\ . \end{cases} \tag{4.7}$$

Based on results (4.7), we can reproduce the usual processes as specialized cases: exponential dependence for $\mu=0$, Gaussian dependence for $\mu=1$ and power-law dependence for $\mu=-1$.

V. LOCAL RESPONSE FUNCTION

To investigate the activation process, we introduce a local response function defined by the sum of jumping probabilities (4.4) [cf. (3.6) for $k=1$]. Here again note that the respective jumping probabilities are given as the subsidiary conditions of the GRW.

We start with the expression (4.4),

$$\tilde{p}_n(t)=a^nb^{n-1}e^{-b^nt} \tag{5.1}$$

with (4.5) or (4.6).

The activation process of the cluster c is described by

$$p_c(t)=\sum_{n=1}\tilde{p}_n(t) \tag{5.2}$$

[see Fig. 3(b)], or cf. (3.6) for $k=1$, and Δt is replaced by t. A normalized expression proportional to $p_c(t)$ is then given by

$$\Psi_c(t)=\frac{1}{\Omega}p_c(t)\ , \tag{5.3}$$

$$\Omega=\sum_{c=1}p_c(t)\ . \tag{5.4}$$

From now on we call $\Psi_c(t)$ a local response function.

Multiplication of b in the numerator and the denominator of $\Psi_c(t)$ leads us to

$$\Psi_c(\tilde{t})=(b\Omega)^{-1}\sum_{n=1} e^{F(n;a,b)}\ , \tag{5.5}$$

$$F(n;a,b)=n\ln(ab)-b^n\tilde{t}\quad (a,b<1,\ \tilde{t}=t/\delta t)\ , \tag{5.6}$$

where δt is the unit time in (3.7).

With the aid of (4.3) and (4.6), we can taken into account the effects that arise in the local response function $\Psi_c(\tilde{t})$ from the other clusters. The first factor $\exp[n\ln(ab)]$ $(a,b<1)$ in e^F decreases, while the second factor $e^{-b^n\tilde{t}}$ increases, as $n\to\infty$. Therefore we can expect a value n such that $F(n;a,b)$ has a maximum for the given a and b. Figure 4 shows some of typical behaviors of $F(n;a,b)$.

An asymptotic form of the local response function $\Psi_c(\tilde{t})$ is obtained by evaluating (5.5) in terms of the maximum of F. The value n_0 giving the maximum term is given by $\partial F/\partial n=0$, that is

$$\frac{1}{t}=\frac{1}{\delta t}\frac{e^{n_0\ln b}}{1+\gamma}\quad (\gamma=\ln a/\ln b)\ . \tag{5.7}$$

Note that $t\to\infty$ as $n_0\to\infty$ (cf. $b<1$) and vice versa. Therefore n_0 is related to t by a function q

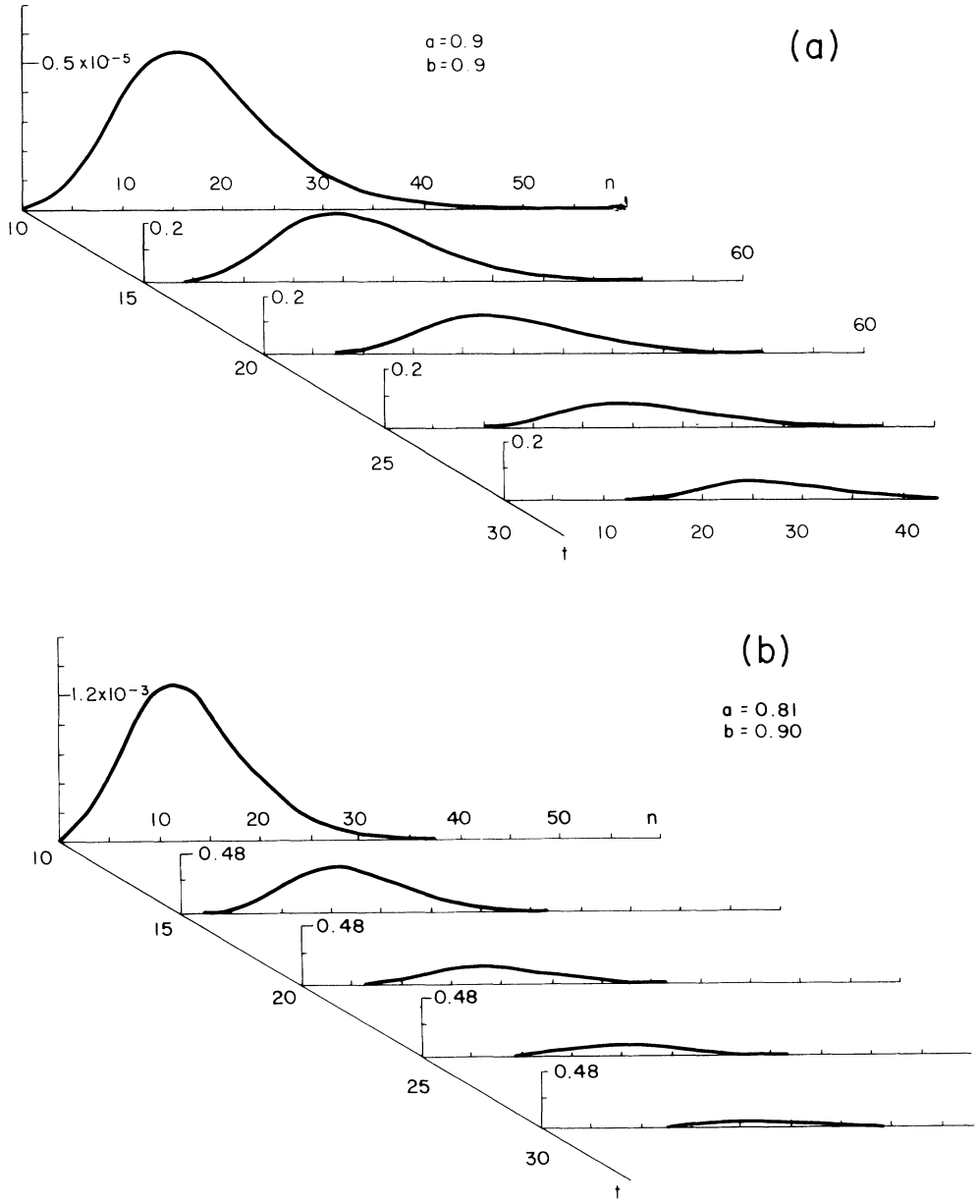


FIG. 4. (a) and (b) behaviors of $F(n,a,b)$.

$$n_0 = q(\tilde{t}, a, b) \quad (\tilde{t} = t/\delta t) \ . \tag{5.8}$$

In this procedure, the asymptotic form of $\Psi_c(t)$ is obtained as follows:

$$[\Psi_c(t)]_{\text{asym}} \sim \Gamma_0 t^{-1-\gamma} \equiv G^{(c)}(n_0) \ ,$$
$$G^{(c)}(n_0) = a^{n_0} b^{n_0} e^{-b^{n_0}\tilde{t}}/b\Omega \ , \tag{5.9}$$
$$[\Gamma_0 = (1+\gamma)^{1+\gamma} e^{-1-\gamma}/b\Omega, \delta t = 1] \ ,$$

where γ is parametrized by ε contained in $b(\varepsilon)$ [see (4.3)].

If we approximate the sum in (5.5) by the integral and use the saddle-point method, we obtain the asymptotic form of $\Psi_c(t)$:

$$[\Psi_c(t)]_{\text{asym}} \sim \frac{\Gamma_1}{|\ln b|} t^{-1-\gamma} + O\left[\frac{1}{|\ln b|^3}\right] \ ,$$
$$[\Gamma_1 = (\pi/2)^{1/2}(1+\gamma)^{1/2+\gamma}/b\Omega, \delta t = 1] \ , \tag{5.10}$$

where the factor $|\ln b|$ arises from the integration.

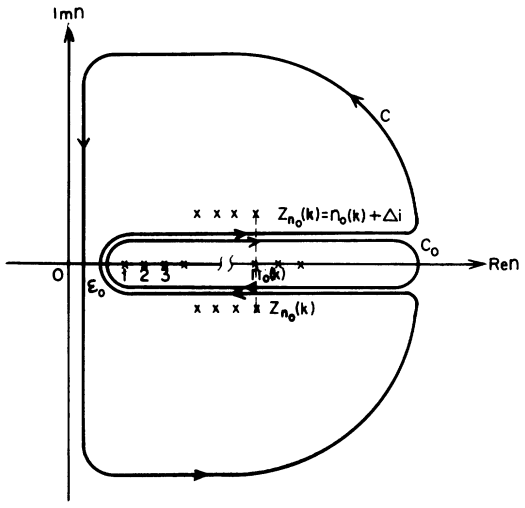


FIG. 5. Complex n plane: Contours C and C_0. The contour C_0 contains the simple poles of $\cot(\pi n)$, while the contour C contains the simple poles of $\rho_\Delta(n)$.

VI. AVERAGE RESPONSE FUNCTION

Properties of the response function are studied by taking an average for the respective clusters. To this end, we introduce an average response function defined by

$$\langle \Psi_c(t) \rangle = \int \sum_{n=1}^{\infty} G^{(c)}(n) P(c) dc \ ,$$
$$\left[G^{(c)}(n) = \frac{1}{b\Omega} e^{F(n;a,b)}, \ \delta t = 1 \right] \ , \tag{6.1}$$

from (5.5), where $P(c)$ is a probability specifying distributions of the clusters. When $P(c)$ is given by $\delta(c - c_0)$, and the contribution is approximated by the maximum term (n_0), we can reproduce the previous result (5.9) from Eq. (6.1):

$$\langle \Psi_c(t) \rangle = \sum_{n=1}^{\infty} G^{(c_0)}(n) \sim G^{(c_0)}(n_0) \ . \tag{6.2}$$

For a more general case of the average response function, we have to evaluate $P(c)$ correctly. To this end, we sum up the contributions of the respective clusters activated at times $t_1, t_2, \ldots, t_k$. Namely, we consider a set of the clusters activated at $\{t_k\}$ $(k = 1, 2, \ldots)$. Furthermore, we suppose that the total system is described by a distribution of the clusters specified by $\{n_0^{(k)}\}$, that is $p(c) \to \rho_\Delta(n)$,

$$\rho_\Delta(n) = \frac{1}{\pi} \sum_{k=1} C_k \frac{\Delta}{[n - n_0^{(k)}]^2 + \Delta^2} \ , \tag{6.3}$$

where C_k is a weight factor for the cluster k, and $n_0^{(k)}$ is a maximum value concerned with the cluster k [see (5.7) or (5.8)], and Δ denotes a variance of the clusters common to $n_0^{(k)}$.

In the above case, we can express the asymptotic form of the total response function by

$$\langle \Psi_c(t) \rangle_{\text{asym}} = \sum_{n=1} G^{(c)}(n) \rho_\Delta(n) \ . \tag{6.4}$$

In the limit $\Delta \to 0$ of (6.4), we can get the result obtained by (6.2) or (5.9). The right-hand side of (6.4) is rewritten as a sum of the residues of the poles within the contour c_0 in Fig. 5

$$\langle \Psi_c(t) \rangle_{\text{asym}} = \frac{i}{2} \oint_{C_0} \cot(\pi n) G^{(c)}(n) \rho_\Delta(n) dn$$
$$= \frac{i}{2} \sum_{k=1} C_k [\cot(\pi z_{n_0^{(k)}}) G^{(c)}(z_{n_0^{(k)}}) - \text{c.c.}] + \frac{i}{2} \int_{\varepsilon_0^{-i\infty}}^{\varepsilon_0^{+i\infty}} \cot(\pi n) G^{(c)}(n) \rho_\Delta(n) dn \ , \tag{6.5}$$

where

$$z_{n_0^{(k)}} = n_0^{(k)} + \Delta i \ , \tag{6.6}$$

and c.c. stands for complex conjugates of the corresponding expressions. In (6.5), we have deformed the contour c_0 into the contour c by noting that $G^{(c)}(n)$ goes to zero rapidly compared with the divergent terms of $\cot(\pi n)$ as $n_0 \to \infty$. When Δ tends to zero, dominant contributions arise from the terms in the [ ] of (6.5). The quantities in the terms are evaluated by putting $y = \pm\Delta$ as follows:

$$\cot[\pi(n_0^{(k)} + iy)] = \begin{cases} e^{-i\pi/2}\coth(\pi y), & y > 0 \\ e^{i\pi/2}\coth(\pi y), & y < 0 \ , \end{cases} \tag{6.7}$$

$$G^{(c)}(n_0^{(k)} + iy) = G^{(c)}(n_0^{(k)})(ab)^{iy} b\Omega \ . \tag{6.8}$$

As for $G^{(c)}(z_{n_0^{(k)}})$, we have used an approximated relation $b^{iy}t_k \to t_k$ as $n_0 \to \infty$. The factors a, b are parameters in the relation

$$n_0^{(k)} = q_k(t_k, a, b) \quad (\delta t = 1) , \tag{6.9}$$

cf. (5.8), where $\tilde{t}_k$ is replaced by t_k for $\delta t = 1$.

The first term of (6.5) is then rewritten into

$$\begin{aligned} &\frac{i}{2}\sum_{k=1} C_k[\cot(\pi z_{n_0^{(k)}})G^{(c)}(z_{n_0^{(k)}}) - \text{c.c.}] \\ &\quad = \sum_{k=1} C_k G^{(c)}(z_{n_0^{(k)}})\coth(\pi\Delta) \\ &\qquad \times\{\sin[\pi\Delta(\ln b_k)(1+\gamma_k)+\pi/2]\} \\ &\quad \approx \tilde{G}^{(c)}(t)\coth(\pi\Delta)\{\cos[\pi\Delta|\ln b|(1+\gamma)]\} , \end{aligned} \tag{6.10}$$

where

$$\tilde{G}^{(c)}(t) = \sum_{k=1} C_k G^{(c)}(z_{n_0^{(k)}}) . \tag{6.11}$$

The last member of (6.10) was obtained by neglecting the k dependence in sin[ ]. This was done because of the smallness of parameter Δ. Specifically, when the activations of the clusters are described by a single mode, that is $C_k = \delta(k - k_0)$, the contribution of $\langle \Psi_c(t) \rangle_{\text{asym}}$ is expressed by

$$\langle \Psi_c(t) \rangle_{\text{asym}} = \frac{\Gamma_0 t^{-1-\gamma}}{\Delta\pi}\cos[\pi\Delta|\ln b|(1+\gamma)] + o(\Delta), \tag{6.12}$$

where we have approximated $\coth(\Delta\pi)$ by $1/\Delta\pi$ as $\Delta \to 0$, and we have replaced t_{k_0}, b_{k_0}, and γ_{k_0} by t, b, and γ, respectively. The above results show that the asymptotic behavior is enhanced by Δ, and is modified by a slowly varying oscillation. As a more general case, we consider a process that C_k and t_k are specified by

$$C_k \sim a^{\alpha n_k} (\alpha > 0,\ a < 1) \tag{6.13}$$

and

$$t_k \sim b^{n_k} t , \tag{6.14}$$

respectively.

Equation (6.13) states that the magnitude of the weight factor C_k decreases as n_k increases, whereas (6.14) shows that the activation times are described by the scaling times. Under these conditions, the quantity $\tilde{G}^{(c)}(t)$ [see (6.11) or (6.9)] is expressed by

$$\tilde{G}^{(c)}(t) = \frac{1}{b\Omega}\sum_{k=1} \tilde{a}^{n_k} b^{n_k} e^{-b^{2n_k}t} \qquad (\tilde{a} = a^{1+\alpha},\ \delta t = 1) . \tag{6.15}$$

If we assume $\partial n_k / \partial k > 0$ and approximate the k summation of $\tilde{G}^{(c)}(t)$ by a maximum term $\tilde{n}_0$ similar to (5.8), the result reads

$$\left[\tilde{G}^{(c)}(t)\right]_{\text{asym}} \sim \tilde{\Gamma}_0 t^{-(1-\tilde{\gamma})/2}$$

$$\left[\tilde{\Gamma}_0 = \frac{1}{2}(1+\tilde{\gamma})^{(1+\tilde{\gamma})/2} e^{(-1+\tilde{\gamma})/2} b\Omega, \quad \text{cf. (5.9)}, \quad \tilde{\gamma} = \frac{\ln\tilde{a}}{\ln b}\right] . \tag{6.16}$$

Finally, substituting (6.16) into (6.10), we obtain an expression for $\langle \Psi_c(t) \rangle_{\text{asym}}$,

$$\langle \Psi_c(t) \rangle_{\text{asym}} = \frac{\tilde{\Gamma}_0 t^{-(1-\tilde{\gamma})/2}}{\Delta\pi}\cos[\pi\Delta|\ln b|(1+\gamma)] + o(\Delta) . \tag{6.17}$$

Compare the results of (6.12) and (6.17) with the factors given in (5.9) or (5.10). We note that the factors are modified in (6.12) and (6.17). The modifications arise from the presence of the other clusters specified by the set of activation times (t_k), that is $\{n_0^{(k)}\}$, $(k = 1, 2, \ldots)$; see (6.3) and (6.9).

VII. CONCLUDING REMARKS

We have proposed a model system composed of clusters as a complex system. Based on this idealized model system, we have studied a dynamic activation process. The activation process was assumed to be described by a response function of the clusters. The clusters are constituent elements of the system, and each cluster is an aggregation of strongly correlated units. The units were assumed to satisfy the time evolution (3.7) and the scaling rules in (3.8). The two conditions (3.7) and (3.8) are given as subsidiary conditions of generalized random walks (GRW). The model system was specified by two types of scaling factors: $\{a, b\}$ in (3.8), and $\{a^{(c)}, b^{(c)}\}$ in (4.5). The first pair $\{a, b\}$ determines the main behavior of the response function, while other pair $\{a^{(c)}, b^{(c)}\}$ characterizes the activation process in terms of the function $\Phi(t)$ for the response function. For the cluster, a local response function was introduced by the transition probability of GRW. Furthermore, to specify the states of the system, an average response function was introduced to specify distribution functions of the clusters. By evaluating the average response function $\langle \Psi_c(t) \rangle$ approximately, we obtained the asymptotic behavior of $\langle \Psi_c(t) \rangle$ for some special cases. Based on this analysis, we found that effects due to other clusters modify the behavior of the local response function in two ways: The first is a modification of the index [as given by (4.3)] of the response function, that is a parametrized index with (4.5) or (4.6) [see also (5.9) or (5.10)]. The second is a modification of the behavior into an oscillating power form [see (6.12) and (6.17)].

[1]In some cases, we encounter systems in which infinitely many time scales coexist. See, e.g., B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1982); M. Schroeder, *Fractal, Chaos, Power Laws: Minutes from an Infinite Paradise* (Freeman, New York, 1990).

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[14]In general, A is a t-dependent function with a periodicity in the variables $\ln t$: $A(\ln t)=A(\ln t-\ln b^{(c)})$; see Ref. 3 and Ref. 5.