Excitation spectrum of the spiral state of a doped antiferromagnet

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The excitation spectrum of the spiral state of a doped quantum antiferromagnetic is discussed. It is shown that the low-lying spin excitations are strongly mixed with the transverse fluctuations of the dipolar polarization field associated with the vacancies. It is argued that for a certain range of phenomenological parameters these transverse fluctuations could lead to the destruction of the long-range order at $T=0$.

I. INTRODUCTION

In this paper we shall discuss in more detail the spectrum of spin excitations for the "spiral" antiferromagnetic state which has been proposed[1–3] as a possible ground state of an antiferromagnet (AFM) with a small concentration of mobile carriers described by the Hubbard-Heisenberg (or t-J) model Hamiltonian

$$H=-t\sum_{\mathbf{r},\hat{\mathbf{a}},\sigma}(c^{\dagger}_{\mathbf{r}+\hat{\mathbf{a}},\sigma}c_{\mathbf{r},\sigma}+\mathrm{H.c.})+J\sum_{\mathbf{r},\hat{\mathbf{a}}}\mathbf{s}_{\mathbf{r}+\hat{\mathbf{a}}}\cdot\mathbf{s}_{\mathbf{r}}\ , \tag{1.1}$$

where $c_{r,\sigma}$ is the electron-annihilation operator constrained to single occupancy and $\mathbf{s}=\frac{1}{2}c^{\dagger}_{\sigma}\hat{\boldsymbol{\tau}}_{\sigma\sigma'}c_{\sigma'}$ is the spin. The sums are over all sites r on one sublattice of a two-dimensional (2D) square lattice and $\hat{\mathbf{a}}=\pm\hat{\mathbf{x}},\pm\hat{\mathbf{y}}$ are the nearest-neighbor vectors.

The spiral AFM state is metallic with the carriers filling up the pockets near $k=(\pm\pi/2,\pm\pi/2)$ in the extended Brillouin zone. It is characterized by the presence of a certain fermionic "polarization" order parameter[1,4] $\langle\mathbf{P}_a\rangle$, which is a vector in both spin and lattice spaces and which is accompanied by the appearance, in the ground state, of a "twist" $\langle\hat{\boldsymbol{\Omega}}\times\partial_a\hat{\boldsymbol{\Omega}}\rangle\neq 0$ (also a vector both in lattice and spin spaces), in addition to the local staggered magnetization $\hat{\boldsymbol{\Omega}}$. Together the two order parameters, which are orthogonal vectors in spin space, define a local triad.[5] Consequently, one expects the excitations of such a state to include in addition to the usual spin-wave modes an extra one which does not exist in the collinear magnet. This mode, which in Ref. 1 was termed the "torsion" mode, corresponds to rotation about the local direction of the staggered magnetization $\hat{\boldsymbol{\Omega}}$, and represents the fluctuations of the spiral plane, i.e., of the direction normal to the plane of the spiral, $\langle\hat{\boldsymbol{\Omega}}\times\partial_\mu\hat{\boldsymbol{\Omega}}\rangle$, in spin space. Alternatively, the additional mode corresponds to the transverse fluctuations of the vacancy "polarization" $\langle\mathbf{P}_a\rangle$, as will be explained below. The "torsion" mode is quite soft, with its energy scale ω_T determined by the inverse pitch of the spiral q, which is proportional to the vacancy density, $n\ll 1$: $\omega_T\lesssim qJ$ (typically 100 meV). This mode therefore would be rather important phenomenologically. Furthermore, we shall argue below that the quantum fluctuations of the transverse component of the vacancy polarization can, in principle, suppress the static expectation value $\langle\mathbf{P}_a\rangle$ (leaving $\langle\mathbf{P}_a^2\rangle\neq 0$) and lead to the destruction of the AFM long-range order on the length scale of the spiral pitch $q^{-1}\sim n^{-1}$. The resulting disordered state is expected to have a spin gap, of order qJ, but should retain the incommensurate correlation in its finite-energy excitation [i.e., the lowest-energy excitations having k lying on a ring near (π,π)].

We shall take as a starting point the semiphenomenological generalized nonlinear σ or CP^1 model describing the interaction of long-wavelength (and low-energy) antiferromagnetic spin modes with fermionic vacancies, which are discussed at some length in Ref. 1 and 4. After formulating the model in Sec. II, we present in Sec. III the analysis of the excitation spectrum, calculate the spin-correlation function, and estimate the effect of the quantum fluctuations. Section IV is a summary and a speculation on the nature of a possible disordered ground state and the relation of the present analysis to the neutron-scattering observations.[6,7]

II. GENERALIZED CP^1 MODEL

Our point of departure is the effective Hamiltonian which generalizes the standard nonlinear σ model (NLσ) to include the coupling of the spin degrees of freedom with mobile vacancies. It is convenient to use the Schwinger-boson representation in terms of which the staggered magnetization unit vector is $\hat{\boldsymbol{\Omega}}=\bar{z}\hat{\boldsymbol{\tau}}z$, with z_r a two-component spinor $\bar{z}_\sigma z_\sigma=1$. The net magnetization field $\mathbf{m}$ is represented as $\mathbf{m}=\mathrm{Re}(\bar{\eta}\hat{\boldsymbol{\tau}}z)$, in terms of a conjugate spinor[4] η_σ ($\mathrm{Re}\bar{\eta}z=0$), obeying the commutation relations $[z_\nu,\bar{\eta}_\sigma]=\delta_{\sigma\nu}-z_\nu\bar{z}_\sigma$ and $[\bar{\eta}_\sigma,\eta_\nu]=z_\sigma\bar{\eta}_\nu+\eta_\sigma\bar{z}_\nu$, which reproduce

$$[m^\alpha(r),m^\beta(r')]=i\varepsilon^{\alpha\beta\gamma}m^\gamma(r)\delta^{(2)}(r-r')$$

and

$$[m^\alpha(r),\Omega^\beta(r')]=i\varepsilon^{\alpha\beta\gamma}\Omega^\gamma(r)\delta^{(2)}(r-r')\ .$$

of the NLσ model. The NLσ Hamiltonian $H_{\mathrm{NL}\sigma}=\mathbf{m}^2+c^2(\partial_a\hat{\mathbf{\Omega}})^2$ is rewritten[4] as

$$H_{\mathrm{NL}\sigma}=\bar{\eta}_\sigma(\delta_{\sigma\nu}-z_\sigma\bar{z}_\nu)\eta_\nu+c^2\partial_a\bar{z}_\sigma(\delta_{\sigma\nu}-z_\sigma\bar{z}_\nu)\partial_a z_\nu \;, \tag{2.1}$$

where c is the spin-wave velocity and the energy is in units of inverse susceptibility, i.e., is scaled with J.

The generalized hamiltonian that we consider has the form

$$H_{\mathrm{eff}}=H_{\mathrm{NL}\sigma}-[g\bar{P}_a\varepsilon_{\sigma\nu}z_\sigma\partial_a z_\nu+\mathrm{H.c}]+\kappa|D_aP_b|^2+V(P) \;, \tag{2.2}$$

where the second term represents the coupling [$g\sim O(1)$ in units of J] of the magnetization current to the dipole polarization field of the vacancies, P_a; the third term is the stiffness[8] for the P_a field with covariant derivative D_a defined below, and $V(P)$ is an effective potential. Vacancy polarization in terms of sublattice fermion fields $\psi^{A,B}$ is expressed in momentum space by

$$P_a(k)\equiv i\sum_{k'}\sin k'_a\bar{\psi}^A_{k'-k/2}\psi^B_{k'+k/2} \;. \tag{2.3}$$

Each component of P_a is a complex number which defines a spin space vector perpendicular to local $\hat{\mathbf{\Omega}}$ via $\mathbf{P}_a\equiv\mathrm{Tr}[R^\dagger\hat{\tau}R(P_a\tau^++\bar{P}_a\tau^-)]$, where

$$R\equiv\begin{bmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{bmatrix}$$

is an SU(2) rotation matrix relating local $\hat{\mathbf{\Omega}}$ to a fixed basis.

We note that H_{eff} of Eq. (2.2) possesses the gauge symmetry, $z\to e^{i\zeta}z$ implicit in a Schwinger-boson representation. Under this gauge transformation, $P_a\to e^{i2\zeta}P_a$ (corresponding to $\psi^A\to e^{-i\zeta}\psi^A$, $\psi^B\to e^{i\zeta}\psi^B$ transformations of sublattice fermions), so that the covariant derivative is $D_a\equiv\partial_a+i2A_a$, with $iA_a\equiv\bar{z}\partial_a z$. The freedom in the choice of the phase of P_a corresponds to the freedom in the choice of the origin for the polar angle parameterizing a vector perpendicular to $\hat{\mathbf{\Omega}}$.

Since for low hole concentration the vacancy Fermi surface consists of two inequivalent valleys,

$$\mathbf{k}\approx\mathbf{k}_{1,2}=\left[\frac{\pi}{2},\pm\frac{\pi}{2}\right] \;,$$

it is useful to define the valley polarization vectors

$$P_{(1,2)}\approx\sum_{k'\approx k_{1,2}}\bar{\psi}^A_{k'-k/2}\bar{\psi}^B_{k'+k/2}$$

in terms of which $P_{x,y}=P_1\pm P_2$. We also introduce

$$P^z_{(1,2)}=\frac{1}{2}\sum_{k'\approx k_{1,2}}[\bar{\psi}^A_{k'-k/2}\psi^A_{k'+k/2}-\bar{\psi}^B_{k'-k/2}\psi^B_{k'+k/2}] \;,$$

which allows one to derive the approximate commutators

$$\begin{aligned}&[P^z_\mu(r),P_{\mu'}(r')]=P_\mu\delta_{\mu\mu'}\delta^{(2)}(r-r') \;,\\ &[P_\mu(r),\bar{P}_{\mu'}(r)]=2P^z_\mu\delta_{\mu\mu'}\delta^{(2)}(r-r') \;,\end{aligned} \tag{2.4}$$

where μ labels the valleys. These commutators combined with H_{eff} describe the long-wavelength, low-frequency dynamics of the polarization field P_μ. Such a coarse-grained description is plausible on length scales larger than the intervacancy distance, $l\gg k_F^{-1}\sim n^{-1/2}$, with P_μ interpreted as the total pseudospin magnetization of a region containing $nl^2\gg 1$ holes.

The effective potential $V(P)$ can be determined by estimating energy cost of polarizing the μth Fermi sea; e.g., for the simplest case of a circular valley with n pseudo spin up (down) populations, $P^z_\mu=\frac{1}{2}(n_+-n_-)$, the total energy is

$$E_\mu=\pi/m(n_+^2+n_-^2)=2\pi/m|\mathbf{P}_\mu|^2+\pi/2mN_\mu^2 \;,$$

where $m\sim O(1)$ (in units of J^{-1}) is the effective mass of the carrier and $N_\mu\equiv n_++n_-$ is the total occupancy of μth valley. [The energy here was written in terms of the polarization vector $|\mathbf{P}_\mu|^2=(P^z_\mu)^2+\frac{1}{2}\{\bar{P}_\mu,P_\mu\}$ under the assumption of isotropy.] Clearly, the magnitude of the polarization is bounded by the density of holes in the valley, $|P_\mu|\le\frac{1}{2}N_\mu$. This constraint can be built into the effective polarization potential by defining

$$V(P_a)\equiv\chi_d^{-1}|\mathbf{P}_a|^2+\pi/2m(N_1^2+N_2^2)$$

for partially polarized valleys $|\mathbf{P}_x\pm\mathbf{P}_y|\le\frac{1}{4}N_{1,2}$ and *infinite* otherwise. In the above formula we have identified the coefficient of the quadratic term as the uniform dipolar susceptibility of the vacancies, $\chi_d=m/\pi$. [Note that $V(P)$ can be readily generalized to include other nonlinear effects such as interaction between different valleys as well as anisotropy, e.g., $(P^z_a)^2$ term.] In addition to $V(P)$, H_{eff} of Eq. (2.2) includes the polarization "stiffness" κ, which assigns energetic cost to a spatially nonuniform distribution of P_a. On dimensional grounds we expect $\kappa\sim k_F^{-2}m^{-1}$ with the Fermi wave number $k_F\sim\sqrt{n}$. While this "stiffness" term incorporates the wave-number dependence of the *static* dipole susceptibility, more generally one may attempt to include the correct frequency dependence of $\chi_d(k,\omega)$.

The mean-field ground state of H_{eff} is found by using the identity

$$\partial_a\bar{z}_\sigma(\delta_{\sigma\nu}-z_\sigma\bar{z}_\nu)\partial_a z_\nu=|\varepsilon_{\sigma\nu}z_\sigma\partial_a z_\nu|^2 \;, \tag{2.5}$$

to rewrite Eqs. (2.1) and (2.2) as

$$\begin{aligned}H_{\mathrm{eff}}=&\bar{\eta}_\sigma(\delta_{\sigma\nu}-z_\sigma\bar{z}_\nu)\eta_\nu+c^2|\varepsilon_{\sigma\nu}z_\sigma\partial_a z_\nu-c^{-2}gP_a|^2\\ &-g^2c^{-2}\bar{P}_aP_a+\kappa|D_aP_b|^2+V(P) \;.\end{aligned} \tag{2.6}$$

The form of Eq. (2.6) suggests a mean-field solution with nonvanishing $Q_a=\langle\varepsilon_{\sigma\nu}z_\sigma\partial_a z_\nu\rangle$ and $\langle P_a\rangle$:

$$Q_a=c^{-2}g\langle P_a\rangle \;, \tag{2.7}$$

which is a classical ground state provided that $g^2>c^2\chi_d^{-1}$. The magnitude $\langle P_a\rangle$ is determined by $V(P)$ and, in the simplest case of fully polarized valleys, is fixed by density of holes: e.g., $p\equiv|\langle P_1\rangle|=|\langle P_2\rangle|=n/4$, corresponding to the spirals in (1,0) or (0,1) directions on the lattice. The amplitude of this order parameter determines the wave number of the incommensuration,

$q_a = 2|Q_a|$, and the phase fixes the direction of the vector normal to the spiral plane in spin space. Thus, for example, $Q_a = i\mathbf{x}_a q/2$ defines a spiral in the (xy) spin plane with inverse pitch q:

$$\langle z(r)\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{(i/2)qx} \\ e^{-(i/2)qx}\end{bmatrix} , \tag{2.8}$$

which corresponds to $\hat{\mathbf{\Omega}}(r) = (\cos(qx), \sin(qx), 0)$.

III. EXCITATION SPECTRUM AND SPIN-CORRELATION FUNCTION

To facilitate the mode analysis, we use the decomposition

$$z_\sigma = (1-\tfrac{1}{2}|u|^2)w_\sigma + u\,\varepsilon_{\sigma\nu}w_\nu^* , \tag{3.1}$$

with the static spinor field w_σ, $\bar{w}_\sigma w_\sigma = 1$, $\varepsilon_{\sigma\nu}w_\sigma \partial_a w_\nu = Q_a$ describing the spiral state, and the complex field u parameterizing deviation away from it. The gauge transformation (GT) $z_\sigma \to e^{i\zeta}z_\sigma$ is implemented by $w_\sigma \to e^{i\zeta}w_\sigma$, $u \to e^{-i2\zeta}u$. Note that the twist order parameter Q_a transforms into $e^{i2\zeta}Q_a$ under a GT. We have explicitly

$$A_a = -i\bar{z}\partial_a z = a_a + 2\,\mathrm{Im}[\bar{Q}_a u] , \tag{3.2}$$

where $a_a \equiv -i\bar{w}\partial_a w$ is the background gauge field (which vanishes for the planar spiral state) and

$$\varepsilon_{\sigma\nu}z_\sigma \partial_a z_\nu = Q_a - [\partial_a - 2ia_a - 2i\,\mathrm{Im}(\bar{Q}_a u)]u . \tag{3.3}$$

The fluctuations of P_a are conveniently parametrized by exploiting an apparent anisotropy which suggests the use of Villain-like representation[9] for valley polarization

$$P_\mu \approx (p^2 - \pi_\mu^2)^{1/2} e^{i\theta_\mu} , \tag{3.4a}$$

$$P_\mu^z = \pi_\mu , \tag{3.4b}$$

with $[\pi_\mu, \theta_{\mu'}] = i\delta_{\mu\mu'}$ (where μ is the valley index). Expanding P_μ in the coupling term $gP_\mu \varepsilon_{\sigma\nu}z_\sigma \partial_\mu z_\nu$ for $\pi_\mu^2 \ll p^2$ about the mean-field value [i.e., $\langle\theta_{1,2}\rangle = \langle\pi_{1,2}\rangle = 0$ for the x spiral] generates a quadratic term $\alpha\pi_\mu^2$ with $\alpha = gqp^{-1} = g^2c^{-2}/2 \sim O(1)$. Since π_μ is conjugate to the angle θ_μ, the latter term gives rise to the kinetic-energy term in the Lagrangian: $\alpha^{-1}(D_t\theta_\mu)^2$, where D_t denotes the *covariant* time derivative. It is convenient to introduce $\theta \equiv \frac{1}{2}(\theta_1+\theta_2-\pi)$ and $\phi \equiv \frac{1}{2}(\theta_1-\theta_2)$ in terms of which the *Lagrangian* describing transverse fluctuations of the polarization becomes

$$L_P \approx \alpha^{-1}|\partial_t\theta - 2A_t|^2 + \alpha^{-1}(\partial_t\phi)^2 + \kappa p^2|\partial_a\theta - 2A_a|^2 + \kappa p^2(\partial_a\phi)^2 . \tag{3.5}$$

Note that angle ϕ here describes the fluctuations of the *spatial* orientation of polarization field, $P_{x,y} = 2pe^{i\theta}(i\cos\phi, \sin\phi)$, while angle θ, which is *not* gauge invariant, rotates $\mathbf{P}_a$ about $\hat{\mathbf{\Omega}}$ in spin space. The fluctuation Lagrangian can now be derived from Eq. (2.2) by observing[4] that the η term leads to the kinetic energy $|D_t z|^2$ and combining Eqs. (2.2), (2.5), and (3.1)–(3.5). After some algebra one arrives at

$$\begin{aligned} L = {} & |\partial_t u|^2 + \alpha^{-1}(\partial_t\theta)^2 + \alpha^{-1}(\partial_t\phi)^2 \\ & + \kappa p^2(\partial_a\phi)^2 + \kappa p^2|\partial_a\theta - 2a_a - 4\,\mathrm{Im}(\bar{Q}_a u)|^2 \\ & + c^2|D_x u + i2gc^{-2}pe^{i\theta}\cos\phi - Q_x|^2 \\ & + c^2|D_y u + 2gc^{-2}pe^{i\theta}\sin\phi - Q_y|^2 . \end{aligned} \tag{3.6}$$

For a spiral in the x direction, we identify $Q_x = i(2g/c^2)p = i(q/2)$ and $Q_y = 0$; let $a_a = 0$ and expand assuming $\phi, \theta \ll 1$. The quadratic Lagrangian describing fluctuations about this spiral state becomes[10]

$$\begin{aligned} \delta L = {} & |\partial_t u|^2 + c^2|\partial_x u - \tfrac{1}{2}q\theta|^2 + c^2|\partial_y u + \tfrac{1}{2}q\phi|^2 \\ & + \alpha^{-1}(\partial_t\theta)^2 + \alpha^{-1}(\partial_t\phi)^2 \\ & + \kappa p^2\{|\partial_x\theta + 2q\,\mathrm{Re}u|^2 + (\partial_y\theta)^2 + (\partial_x\phi)^2 + (\partial_y\phi)^2\} \end{aligned} \tag{3.7}$$

[n.b. $u(r,t)$ a complex field], which may be rewritten as a quadratic form (in Fourier space): $\delta L = \Phi^\dagger M\Phi$, with $\Phi^\dagger(k,\omega) = [(\mathrm{Im}u)_{k,\omega}, (\mathrm{Re}u)_{k,\omega}, \theta_{k,\omega}, \phi_{k,\omega}]$, and M being

$$M = \begin{bmatrix} -\omega^2 + c^2k^2 & 0 & 0 & 0 \\ 0 & -\omega^2 + c^2k^2 + \bar{\kappa}q^2 & -\frac{i}{2}(c^2+\bar{\kappa})qk_x & -\frac{i}{2}c^2qk_y \\ 0 & \frac{i}{2}(c^2+\bar{\kappa})qk_x & -\alpha^{-1}\omega^2 + \frac{1}{4}(\bar{\kappa}k^2 + c^2q^2) & 0 \\ 0 & \frac{i}{2}c^2qk_y & 0 & -\alpha^{-1}\omega^2 + \frac{1}{4}(\bar{\kappa}k^2 + c^2q^2) \end{bmatrix} , \tag{3.8}$$

where we have defined $\bar{\kappa} \equiv 4\kappa p^2 \sim O(q)$. We observe that the Im$(u)$ mode is trivial and has the spin-wave spectrum $\omega_1 = ck$. Going back to Eq. (3.1), one may check that it corresponds to rotating staggered magnetization *in the plane* of the spiral. The rest of the spectrum involves the *out-of-plane* deviation Re(u) coupled to the phase fluctuations of the polarization field (θ,ϕ). The exact form of the spectrum is messy, but is simplified for the $k_y = 0$ line, where one has, to lowest order in $k \sim q \ll 1$ (setting $\alpha = c = 1$),

$$\omega^2 = \tfrac{1}{2}(q^2 + 4k_x^2)^{1/2} , \tag{3.9a}$$

$$\omega_3=\bar{\kappa}^{1/2}\frac{|q^2-k_x^2|}{(q^2+4k_x^2)^{1/2}}\,, \tag{3.9b}$$

$$\omega_4=\tfrac{1}{2}(q^2+\bar{\kappa}k_x^2)^{1/2}\,, \tag{3.9c}$$

and, for the $k_x=0$ line,

$$\omega_2=\tfrac{1}{2}(q^2+4k_y^2)^{1/2}\,, \tag{3.10a}$$

$$\omega_3=\bar{\kappa}^{1/2}\frac{(q^4+k_y^4)^{1/2}}{(q^2+4k_y^2)^{1/2}}\,, \tag{3.10b}$$

$$\omega_4=\tfrac{1}{2}(q^2+\bar{\kappa}k_y^2)^{1/2}\,. \tag{3.10c}$$

The spectrum is illustrated in Fig. 1. We observe from Eq. (3.9b) that ω_3 $(k_x=\pm q,\ k_y=0)=0$ corresponding to Goldstone modes associated with the rigid rotation of the plane of the spiral in spin space. Equation (3.9b) yields near $k_x^2=q^2$ points the characteristic velocity $2\bar{\kappa}^{1/2}/\sqrt{5}\sim O(\sqrt{q}\,)$. Clearly, the two low-lying modes in Fig. 1 are predominantly the fluctuations of (θ,ϕ) and have the characteristic[11] energy $\sim O(q)$. For $k\gg q$, $\omega_2(k)\approx k$, yielding the standard spin-wave dispersion. Curiously, in the absence of the stiffness term for the polarization, i.e., $\bar{\kappa}=0$, we would have $\omega_3(k_x,0)=0$ identically, which physically stems from the existence of a more general mean-field solution of Eq. (2.6), where the polarization is an arbitrary function of one variable (i.e., x) and thus the plane of the spiral varies as a function of x. While the degeneracy is lifted by the stiffness $\bar{\kappa}\neq 0$, the "softness" of the $\omega_3(k)$ mode remains.

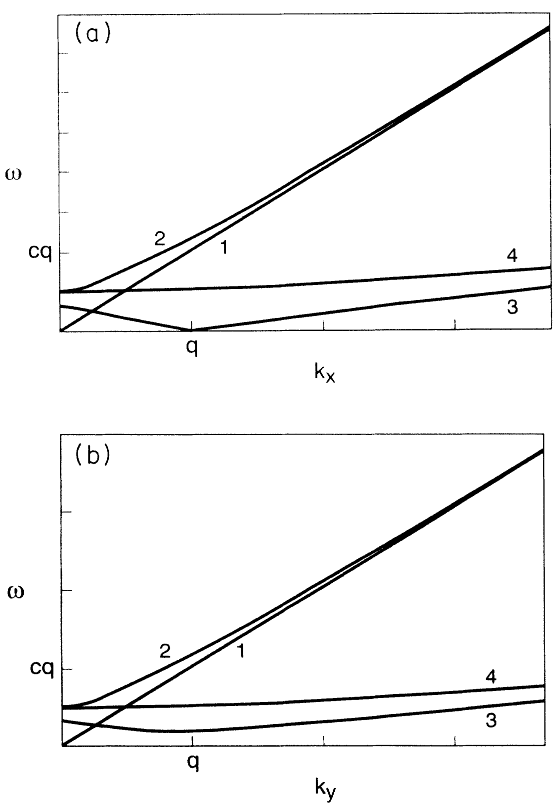

FIG. 1. Excitation spectrum: (a) $\omega_l(k_x)$ and (b) $\omega_l(k_y)$ (for $\bar{\kappa}=q$, $\alpha=1$).

Let us now compute the spin-correlation function $S(r,t)\equiv\langle \mathbf{s}(r,t)\cdot\mathbf{s}(0,0)\rangle$. In terms of Schwinger bosons, we have

$$\langle \mathbf{s}(r,t)\cdot\mathbf{s}(0,0)\rangle=\tfrac{1}{2}\langle|\bar{z}(r,t)z(0,0)|^2\rangle-\tfrac{1}{4}\,, \tag{3.11}$$

using the parametrization of Eq. (3.1) and taking care to treat separately even and odd distances r, which involve spins from the same or opposite sublattices.[12] We find

$$\begin{aligned}S(r,t)=&(-1)^{|r|}\cos(\mathbf{q}\cdot\mathbf{r})[\tfrac{1}{4}-\langle|u(0,0)|^2\rangle]\\&+(-1)^{|r|}\cos(\mathbf{q}\cdot\mathbf{r}\langle u_I(r,t)u_I(0,0)\rangle\\&+(-1)^{|r|}\langle u_R(r,t)u_R(0,0)\rangle\,,\end{aligned} \tag{3.12}$$

where u_R and u_I stand for Re u and Im u. The dynamic structure factor is thus

$$\begin{aligned}S((\pi,\pi)&+\mathbf{k},\omega)\\&=\tfrac{1}{2}[1-4\langle|u|^2\rangle][\delta(\mathbf{k}-\mathbf{q})+\delta(\mathbf{k}+\mathbf{q})]\delta(\omega)\\&\quad+\tfrac{1}{2}[\langle\bar{u}_I(\mathbf{k}-\mathbf{q},\omega)u_I(k-q,\omega)\rangle+(\mathbf{q}\to-q)]\\&\quad+\langle\bar{u}_R(\mathbf{k},\omega)u_R(\mathbf{k},\omega)\rangle\,.\end{aligned} \tag{3.13}$$

The correlation functions in this expression are found in terms of the eigenvalues ω_l and eigenvectors $\chi_i^{(l)}$ of the matrix M [Eq. (3.8)], $M\chi_i^{(l)}=\omega_l\chi_i^{(l)}$,

$$\langle|u_I(k,\omega)|^2\rangle=\frac{1}{2\omega_1(k)}\delta(\omega-\omega_1(k))\,, \tag{3.14a}$$

$$\langle|u_R(k,\omega)|^2\rangle=\tfrac{1}{2}\sum_{l=2,3,4}\frac{|\chi_2^{(l)}(k)|^2}{\omega_l(k)}\delta(\omega-\omega_l(k))\,. \tag{3.14b}$$

The first term in Eq (3.13) yields Bragg peaks with intensity renormalized by the zero-point motion $\langle|u|^2\rangle$ and shifted away from (π,π) by $\mathbf{q}$ in the (0,1) or (1,0) directions. The presence of these peaks is the consequence of our assumption of the long-range order in the spiral states at zero temperature. The integrated intensity, or equal-time correlation function obtained by integrating $S((\pi,\pi)+\mathbf{k},\omega)$ over ω and shown in Fig. 2, exhibits in addition to incommensurate peaks the "background" structure centered on the commensurate wave number, $k=0$. This commensurate scattering arises from the inelastic scattering (see Fig. 3) with ω near $\omega_3(0)\lesssim O(q^{3/2})$.

Let us now examine the effect of quantum fluctuations and the validity of the assumption of long-range order. Long-range order (LRO) would disappear if the fluctuations

$$\Delta(q)=4\langle|u_{(0)}^2|\rangle=4\sum_k\int\frac{d\omega}{2\pi}\langle|u(k,\omega)|^2\rangle \tag{3.15}$$

were large enough. Since the collinear limit $q=0$ we know that the AFM state is ordered (at $T=0$), we are interested in the part of $\Delta(q)$ due explicitly to the presence of incommensuration. Observing that for $q=0$ we would have $\omega_2(k)=\omega_1(k)$ and $\chi_2^{(3)}=\chi_2^{(4)}=0$, $\chi_1^{(1)}=\chi_2^{(2)}=1$, we obtain

$$\Delta(q)-\Delta(0)=\sum_k\left\{\frac{\omega_1(k)|\chi_2^{(2)}(k)|^2-\omega_2(k)}{\omega_1(k)\omega_2(k)}+\sum_{l=3,4}\frac{|\chi_2^{(l)}(k)|^2}{\omega_l(k)}\right\}. \tag{3.16}$$

The first term clearly vanishes as $q\rightarrow 0$. The second contributes in as much as there is mixing of the modes, which is confined to small k. In particular, mixing is strong for $|k|\lesssim q$, in which case $\chi_2^{(3)}(k)\sim O(1)$, $\omega_3(k)\sim O(q^{3/2})$, so that

$$\sum_{|k|\le q}\omega_3^{-1}(k)|\chi_2^{(3)}(k)|^2\sim O(q^{1/2}) ,$$

while, for the ordinary spin-wave dispersion,

$$\omega_1(k)\sim k,\ \sum_{|k|<q}\omega_1^{-1}(k)\sim O(q) .$$

Hence we expect $\Delta(q)-\Delta(0)\sim O(\sqrt{q}\,)$ and conclude that the presence of the incommensurate structure leads to an additive contribution to zero-point motion scaling with $q^{1/2}$.

Similarly, one needs to estimate the fluctuation correction to the mean-field expectation value of the polarization In analogy with Eq. (3.15),

$$\langle|\theta|^2\rangle=\tfrac{1}{2}\sum_k\sum_l\frac{|\chi_3^{(l)}|^2}{\omega_l(k)}\sim O(1) , \tag{3.17}$$

with the rough estimate obtained by taking

$$\sum_{|k|<k_F}(\sqrt{\bar\kappa}k)^{-1}\sim O\left[\frac{k_F}{\sqrt{q}}\right]\sim O(1) .$$

We arrive at the conclusion that, at least for sufficiently small q, the important fluctuations are those of the polarization field. Depending on the parameters of $H_{\rm eff}$, in particular κ, the zero-point motion of the phase of P_a can become large enough to suppress $\langle P_a\rangle$, thus destroying the LRO even at $T=0$.

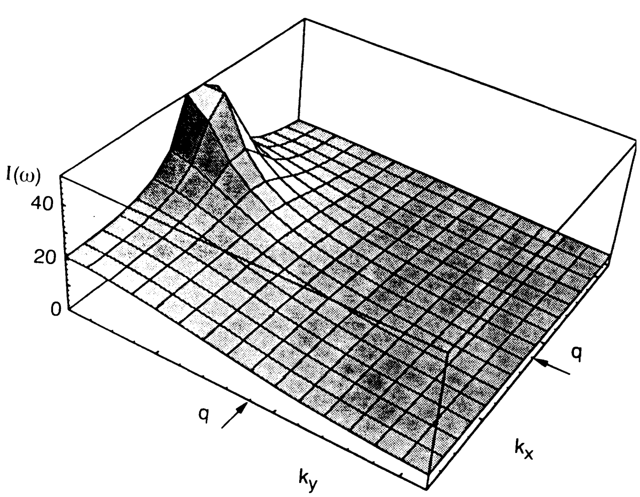

FIG. 2. Total scattering intensity (in arbitrary units) as a function of **k**. The divergence corresponding to Bragg peaks at $\mathbf{k}=\pm q\hat{\mathbf{x}}$ has been cut off.

One possible consequence of large transverse fluctuations of the polarization would be the complete suppression of static spin correlations, i.e., $\langle\hat{\Omega}\rangle=0$, as well as $\langle P_a\rangle=0$, which could occur for sufficiently small polarization stiffness κ. We expect this phase to exhibit finite-frequency short-range incommensurate correlations arising from

$$\langle\bar{P}_a(r,t)P_a(0,0)\rangle\sim n^2e^{-r/\xi}e^{-t/\tau} ,$$

with $\xi^{-1}\sim O(n)$ and $\tau^{-1}\sim O(nJ)$. This intuitively corresponds to fairly long-lived regions of incommensurate "spiral" correlations with the orientation of the spiral plane varying on the scale of the correlation length determined by the transverse fluctuations of the polarization (and therefore controlled by κ). This length has to be compared to the pitch of the local incommensuration, controlled by n, with the expectation that when the latter becomes sufficiently small, the "twist" within the polarization correlation length also becomes small, allowing a possibility of reentry into the Nèel-ordered AFM phase. The disordered phase with local incommensuration can be investigated using the effective Hamiltonian of Eq (2.2)

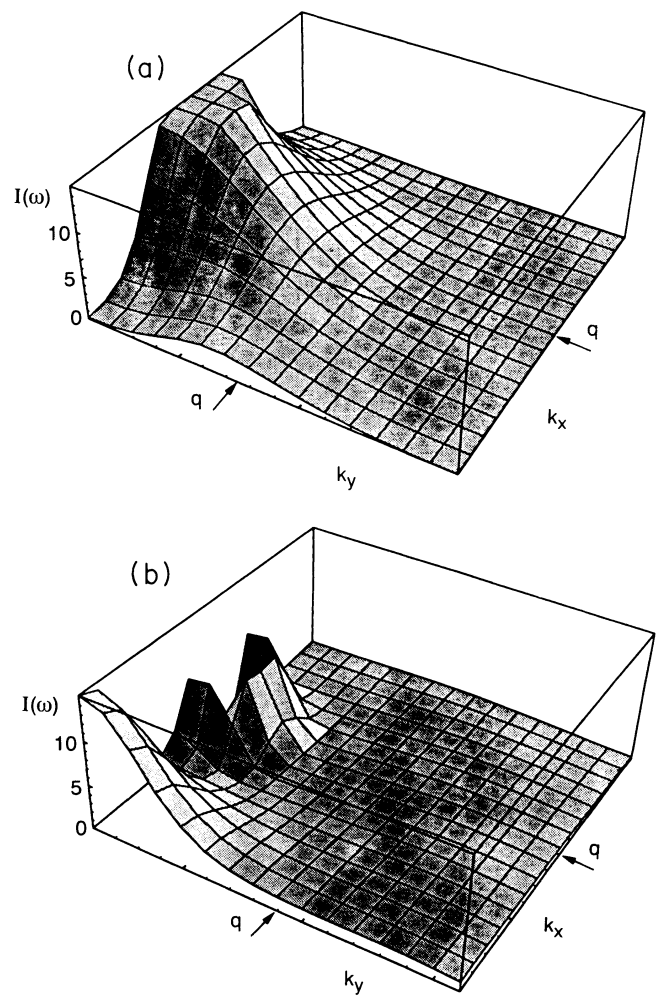

FIG. 3. Dynamic structure factor $S((\pi,\pi)+\mathbf{k}\omega)$ for several values of ω: (a) $\omega=0.1cq$, (b) $\omega=\sqrt{\bar\kappa}cq$ (for $\bar\kappa=q=0.1$, $c=1$) measured with "finite-energy resolutions," i.e., convolved with the Gaussian of width $0.1cq$.

and polarization field dynamics of Eq. (3.4) with further simplification of the effective potential $V(P)$ replaced by the magnitude constraint. The magnitude of P together with its stiffness would then control the phases. This study, however, is beyond the scope of the present analysis.

IV. SUMMARY AND CONCLUSION

In the earlier sections we have discussed the generalized CP^1 model suggested by the analysis of a doped quantum AFM. The essential ingredient of the model is the introduction of a dipole polarization field P_a, which couples to the "twist" $\varepsilon_{\sigma\nu}z_\sigma \partial_a z_\nu$. This model, at least on a semiclassical level, is equivalent to a NLσ model generalized to include the coupling of the magnetization current $\hat{\Omega}\times\partial_a\hat{\Omega}$ to the spin vector polarization field $\mathbf{P}_a$. We found that the spiral states, which appear as mean-field solutions, are stable with respect to small deviations. The excitation spectrum is found to contain three Goldstone modes, as expected on general grounds for a noncollinear state with the spin space triad order parameter.[5] The "extra" mode is associated with the transverse fluctuation of the P_a field, corresponding to rotations about local $\hat{\Omega}$ axis, and fluctuations of the normal to the plane of the spiral in spin space. The fluctuation spectrum exhibits strong spatial an isotropy for $k\sim O(q)$ with the "soft" direction along the incommensuration vector $\mathbf{q}$. Estimating the order of magnitude of the quantum fluctuation corrections to $\langle\hat{\Omega}\rangle$ and $\langle P_a\rangle$ suggests that the long-range-ordered spiral state is possible, provided that incommensuration is small, e.g., $q \ll 1$ (so that additional contribution to the zero-point fluctuation of staggered magnetization, which scales with $q^{1/2}$, is controlled) and that the polarization stiffness is large enough, e.g, $q\kappa \gg 1$. The latter condition is required to prevent the transverse fluctuations of P_a from suppressing the $\langle P_a\rangle$ order parameter. This analysis suggests the existence of two distinct disordered phases in the vicinity of the ordered spiral state. The first possibility is the case of strong incommensuration, i.e., $q\sim|\langle P_a\rangle|\sim O(1)$, with $\langle\hat{\Omega}\rangle=0$ because of quantum fluctuations. Note that the presence of the $\langle P_a\rangle\neq 0$ order parameter breaks the lattice symmetry (as well as gauge invariance). Another and physically more appealing disordered state could appear for sufficiently small κ and would have $\langle P_a\rangle=0$ as well as $\langle\hat{\Omega}\rangle=0$, as a result of transverse fluctuations of P_a. We expect this phase to exhibit finite-frequency short-range incommensurate correlations with the correlation length $\xi\sim O(n^{-1})$ and time $O^{-1}\sim P(nJ)$. The possibility of such a locally incommensurate[13] quantum paramagnetic phase is most interesting physically in connection with the observation[6,7] of incommensurate inelastic scattering in the disordered AFM phase of $La_{2-x}Sr_xCuO_4$.

Finally, we note that the possible locally incommensurate disordered states were recently discussed by Sachdev and Read,[14] who have arrived at an effective Hamiltonian similar to Eq. (2.2) from the large-N analysis of a frustrated quantum AFM. Also, the excitation spectrum analysis for the "spiral state" similar to ours has been recently performed by Gan, Andrei, and Coleman.[15]

ACKNOWLEDGMENTS

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APPENDIX

Here we give a more general expression for the spin-correlation function [Eq. (3.11)] in terms of the "slow" and "fast" fields w_σ and u, respectively, as defined by the parametrization of Eq. (3.1):

$$\begin{aligned}2\langle \mathbf{s}(r,t)\cdot\mathbf{s}(0,0)\rangle+\tfrac{1}{4}=&\tfrac{1}{2}[1+(-1)^{|r|}-4(-1)^{|r|}\langle|u(0,0)|^2\rangle]\langle|\bar{w}_\sigma(r,t)w_\sigma(0,0)|^2\rangle\\&+\tfrac{1}{2}[1-(-1)^{|r|}+4(-1)^{|r|}\langle|u(0,0)|^2\rangle]\langle|w_\sigma(r,t)\varepsilon_{\sigma\nu}w_\nu(0,0)|^2\rangle\\&+(-1)^{|r|}\{\langle\bar{u}(r,t)u(0,0)\rangle\langle[\bar{w}_\sigma(r,t)w_\sigma(0,0)]^2\rangle+\text{H.c.}\}\\&-(-1)^{|r|}\{\langle\bar{u}(r,t)\bar{u}(0,0)\rangle\langle[w_\sigma(r,t)\varepsilon_{\sigma\nu}w_\nu(0,0)]^2\rangle+\text{H.c.}\}\ .\end{aligned}$$

For the spiral state, e.g. $w^T=(1/\sqrt{2})(e^{i/2qx},e^{-i/2qx})$, this leads to Eq. (3.12).

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difference in the estimate of the energy scale of the low-lying "torsion" mode is entailed.

[11]Since our description of the fluctuations of the polarization field only applies at the long wavelength, the above spectral analysis holds only for $k \lesssim k_F$.

[12]Note the relation of z's on different sublattices: if $z^A = z$, then $z^B = \varepsilon z^*$.

[13]One may note the overall similarity of the phase diagram envisioned here with that in the vicinity of the Lifshitz point in the classical Ising system with competing interactions. See, for example, R. M. Hornreich, M. Luban, and S. Shtrikman, Phys. Rev. B **19**, 3799 (1979).

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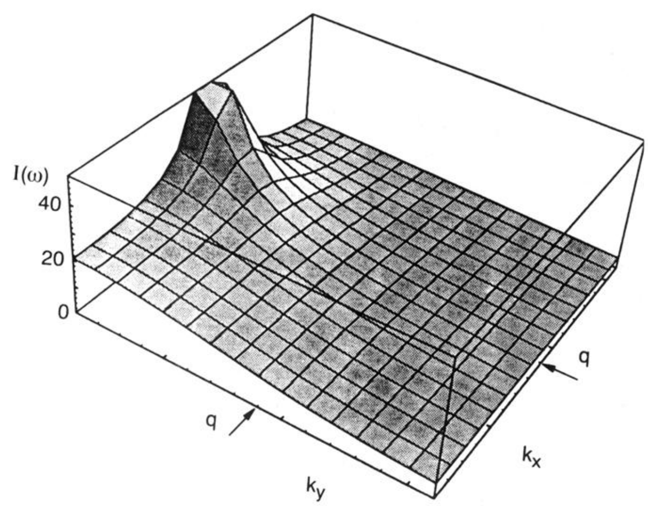

FIG. 2. Total scattering intensity (in arbitrary units) as a function of $\mathbf{k}$. The divergence corresponding to Bragg peaks at $\mathbf{k} = \pm q\hat{\mathbf{x}}$ has been cut off.

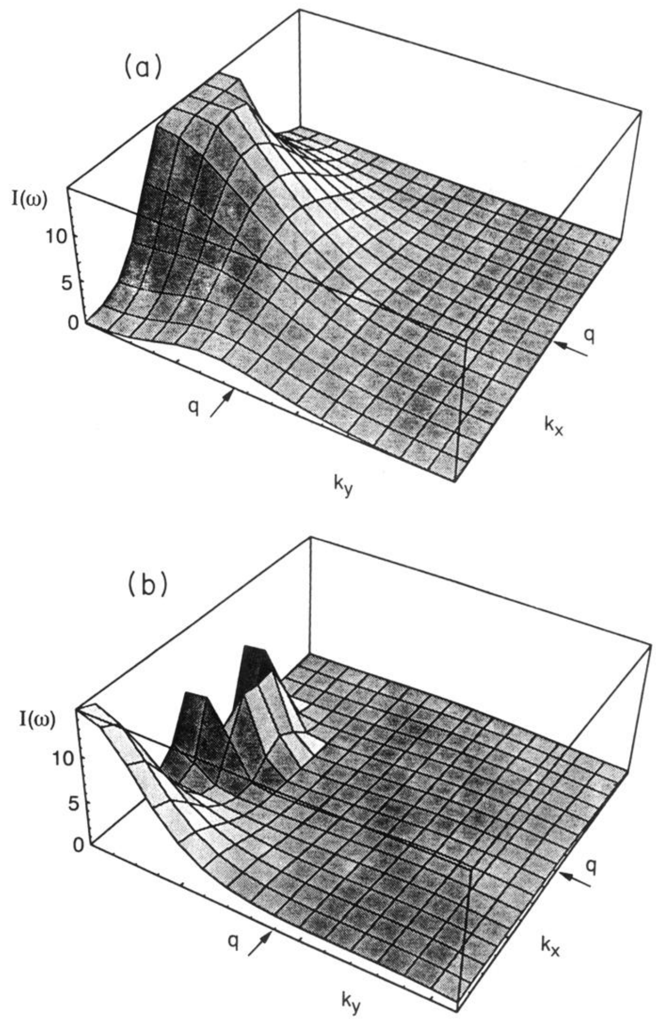

FIG. 3. Dynamic structure factor $S((\pi,\pi)+\mathbf{k}\omega)$ for several values of ω: (a) $\omega=0.1cq$, (b) $\omega=\sqrt{\bar{\kappa}cq}$ (for $\bar{\kappa}=q=0.1$, $c=1$) measured with "finite-energy resolutions," i.e., convolved with the Gaussian of width $0.1cq$.