Nonequilibrium phase transitions in lattice systems with random-field competing kinetics

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We study a class of lattice interacting-spin systems evolving stochastically under the simultaneous operation of several spin-flip mechanisms, each acting independently and responding to a different applied magnetic field. This induces an extra randomness which may occur in real systems, e.g., a magnetic system under the action of a field varying with a much shorter period than the mean time between successive transitions. Such a situation—in which one may say in some sense that frustration has a dynamical origin— may also be viewed as a nonequilibrium version of the random-field Ising model. By following a method of investigating stationary probability distributions in systems with competing kinetics [P. L. Garrido and J. Marro, Phys. Rev. Lett. **62**, 1929 (1989)], we solve one-dimensional lattices supporting different field distributions and transition rates for the elementary kinetical processes, thus revealing a rich variety of phase transitions and critical phenomena. Some exact results for lattices of arbitrary dimension, and comparisons with the standard quenched and annealed random-field models, and with a nonequilibrium diluted antiferromagnetic system, are also reported.

I. INTRODUCTION

The study of nonequilibrium steady states, phase transitions, and critical phenomena in well-defined interacting-particle or -spin lattice systems attracts considerable interest nowadays. This is partly due to the fact that systems in which non-Hamiltonian constraints prevent the realization of the thermodynamic equilibrium state are good models for many situations in physics and other fields. For instance, driven diffusive lattice gases may model solid electrolytes,[1] and reaction-diffusion Ising systems are relevant to population genetics, spin diffusion in magnets, and chemically reacting systems.[2] Nonequilibrium systems are also interesting because they undergo a rich variety of phase transitions and exhibit critical phenomena where one may explore the extension of the established concepts and techniques of equilibrium theory. As an example, a claim which deserves scrutiny and confirmation is that, unlike the practical situation for equilibrium phase transitions, *relevant* and *marginal* parameters (using renormalization-group language) may rather frequently exist in systems far from equilibrium,[3–5] thus making the comparison between related models and the definition of universality classes more intriguing. By and large, one also hopes that the investigation of general questions in specific nonequilibrium systems will provide hints for the extension of the Gibbs ensemble theory to a variety of fascinating phenomena. The study of nonequilibrium steady states and phase transitions is even more appealing when the system under analysis involves microscopic disorder inducing randomness and frustration. In fact, a notable outcome of the comparison between the behavior of standard (equilibrium) disordered model systems and existing related experimental data, often involving observations reported as being *unusual,* is the recognition that the macroscopic behavior of real systems may be dominated by kinetics and certain nonequilibrium features.[6]

That situation is the main motivation of the work reported in this paper, where certain techniques, such as those in Refs. 7–9, are applied to a class of interacting-particle or -spin, Ising-like models with competing kinetics that involve random external magnetic fields, thus inducing disorder and *dynamical frustration.* In a sense this study is parallel to previous ones on two different lattice systems with competing kinetics,[4,5] though the respective physical situations and resulting macroscopic behaviors differ. Also, the existence of a distribution of applied magnetic fields makes the present model mathematically more involved. The latter fact notwithstanding, we report here the exact solution of a class of one-dimensional model systems endowed with that kind of disorder and frustration, and some partial exact results for lattices of arbitrary dimension. In addition to the fact that our model represents a nonequilibrium situation, which may, in principle, be implemented in the laboratory, as discussed later on, the study in this paper may bear some relevance to the theory of disordered systems, e.g., in relation to some of the *peculiarities* detected in the behavior of random-field systems (and dilute antiferromagnetic systems under a uniform field), a topic where exact results are scarce. In fact, even though familiar models for that kind of situation only involve quenched disorder, one may argue that some of the reported unusual observations might also be related to the possible diffusion of disorder, e.g., caused by a thermally activated, random atomic migration. That is, one may conceive a kind of *dynamical frustration* in real systems, which is in some way contained in our model system with competing kinetics. The present study reveals some interesting features of nonequilibrium steady states, phase transitions, and critical phenomena, and provides additional motivation for investigating those versions of our model whose solution cannot be accomplished by the main method employed here; in fact, we expect the latter cases to yield the most interesting behavior and perhaps to elucidate some features in real systems. A short, partial account of results from the work described in this paper has been reported elsewhere.[10]

The paper is organized as follows. In Sec. II, we define a class of model systems with random-field competing kinetics and discuss their possible physical relevance. In Sec. III, we distinguish a subclass of (actually, one-dimensional) systems that may, in fact, be solved by finding explicitly their corresponding effective Hamiltonians, and familiar transition rates are classified according to their implications on the resulting expression for the latter. The study of the macroscopic behavior of that class of systems is initiated in Sec. IV, and Sec. V reports on the resulting thermodynamics, including critical behavior, for some specific field distributions of interest; we also consider, in particular, a distribution that involves strong fields freezing the spin configuration with a given probability. Section VI contains a comparison with some related systems. Section VII is devoted to the case of arbitrary dimension. Some concluding remarks are drawn in Sec. VIII, which contains a summary of our main results.

II. DESCRIPTION OF THE MODEL

Consider a regular d-dimensional lattice Ω, and denote by $\mathbf{s}\equiv\{s_r=\pm 1;\mathbf{r}\in\Omega\}$ any spin (equivalently, particle) configuration that is in contact with a heat bath, by $\mathbf{S}\equiv\{\mathbf{s}\}$ the set of possible configurations, and by $P(\mathbf{s};t)$ the probability of $\mathbf{s}$ at time t. The system evolves in time according to a homogeneous Markov process, as implied by the master equation,[9,11,12]

$$\partial P(\mathbf{s};t)/\partial t=\sum_{\mathbf{s}'\in\mathbf{S}}[c(\mathbf{s}|\mathbf{s}')P(\mathbf{s}';t)-c(\mathbf{s}'|\mathbf{s})P(\mathbf{s};t)]\ , \qquad (2.1)$$

where $c(\mathbf{s}|\mathbf{s}')$ are positive-definite rates per unit time for transitions from $\mathbf{s}'$ to $\mathbf{s}$. A main distinguishing feature of the model of interest is that $c(\mathbf{s}|\mathbf{s}')$ involves a simultaneous competition of independent (random) spin-flip (or creation-annihilation) mechanisms, each of which producing, as in the so-called Glauber dynamics,[11] the change $s_r\to -s_r$ of the variable at site $\mathbf{r}$. This generates a new configuration, $\mathbf{s}'$ or $\mathbf{s}^r$, from $\mathbf{s}$, with a probability per unit time which may be written as

$$c(\mathbf{s}^r|\mathbf{s})=\langle\langle c(\mathbf{s}^r|\mathbf{s};h)\rangle\rangle\equiv\int_{-\infty}^{+\infty}dh\,p(h)c(\mathbf{s}^r|\mathbf{s};h)\ . \qquad (2.2)$$

Here, h represents the random applied magnetic field (or chemical potential) having a normalized distribution $p(h)$, and each elementary Glauber mechanism driven by $c(\mathbf{s}^r|\mathbf{s};h)$ is assumed to satisfy individually a detailed-balance condition, i.e.,

$$c(\mathbf{s}^r|\mathbf{s};h)=c(\mathbf{s}|\mathbf{s}^r;h)\exp[-\beta\Delta H_h]\ , \qquad (2.3)$$

with $\beta=1/k_BT$ and $\Delta H_h\equiv H(\mathbf{s}^r;h)-H(\mathbf{s};h)$, with respect to some specific *Hamiltonian,* which we shall take to be

$$H(\mathbf{s};h)=-J\sum_{\mathrm{NN}}S_rS_{r'}-h\sum_r s_r \quad (\text{for all } h)\ , \qquad (2.4)$$

where the first sum is to be taken over nearest-neighbor (NN) pairs of sites. Note that some of the qualities of the model, particularly the choices (2.3) and (2.4), are included here only for the sake of simplicity and concreteness and that one may easily devise more general conditions.

The situation depicted by Eqs. (2.1)–(2.4) deserves a comment. The spin system has an *effective kinetics* that may be interpreted as consisting of a simultaneous competition of independent (Glauber) canonical mechanisms, each acting with probability $p(h)$ as if the strength of the applied magnetic field had a given random value h over the whole system. In other words, the model is precisely the Glauber or kinetic Ising model with nonconserved magnetization,[11] except that the applied magnetic field is here assumed to change randomly at each kinetic step according to distribution $p(h)$. Clearly, this may represent, for instance, the case of a magnetic system under the action of a magnetic field varying according to $p(h)$ with a period shorter than the mean time between successive transitions modifying the spin configuration. Even though one may expect this time interval to be relatively short in general—e.g., the Larmor precession of a nucleus in the field of its neighbor, which may be taken as an order-of-magnitude estimate of the relevant time scales, is typically around 10^{-5} sec—chances are that such a model situation can actually be implemented in the laboratory. (Note, however, that the condition here essentially differs from a more familiar case[13] in which a system is periodically driven by the action of a field between two ordered phases.)

The model also admits a different interpretation; that is, given that the elementary Glauber processes are local—i.e., transitions just involve in practice a local, small domain of the lattice—so is the resulting effective rate (2.2). Consequently, one may presume that only the field acting on the sites in a neighborhood of the spin whose flip is implicated by each transition [in fact, only the field on the involved spin when one is restricted to Hamiltonian (2.4)] is randomly changed at each kinetic step to have a value h chosen from $p(h)$. Thus, starting from an arbitrary spatial distribution for the fields, say $p_0^s(h)$, kinetics will soon establish a random spatial distribution $p_t^s(h)$, which is a realization of the given $p(h)$. Consequently, under that interpretation, the system may be described (at each time) by the single Hamiltonian

$$H(\mathbf{s};\mathbf{h})=-J\sum_{\mathrm{NN}}s_rs_{r'}-\sum_r h_rs_r\ , \qquad (2.5)$$

$\mathbf{h}\equiv\{h_r\}$, where h_r is *spatially* distributed according to $p_t^s(h)$. This is the familiar random-field Ising model,[14] except for the fact that $p_t^s(h)$ keeps continuously changing by kinetics in such a way that it always maintains itself as a realization of $p(h)$. This system may be viewed as a *nonequilibrium random-field model* (NERFM). In fact, as argued before, chances are that the involved (dynamical) frustration, which essentially differs from the ones occurring in the (equilibrium) quenched and annealed random-field cases (cf. Sec. VI), may bear some relevance in relation to the macroscopic behavior of the familiar random-field class of (natural) systems.

The *a priori* similarities and differences between the NERFM and the former interpretation, or *magnetic sys-*

tem under random field (MSURF), deserve further comment. This may be first illustrated by considering the method one would employ to simulate the dynamical processes by the Monte Carlo method. That is, given that one is, in general, dealing with nonequilibrium states, one should bear in mind that these may depend even quite strongly on details of the kinetics. Consequently, it is not equivalent *a priori* to choosing at random a rate $c(\mathbf{s}^r|\mathbf{s};h)$ with probability $p(h)$ than to compute first the effective rate $c(\mathbf{s}^r|s)$ according to (2.2). The latter procedure is the one corresponding to the MSURF model definition. On the contrary, when simulating the NERFM, once a site **r** is selected at random, one would compute the probability of flipping s_r, which only depends on T and on

$$2Js_r\left[\sum_{|\mathbf{r}-\mathbf{r}'|=1} s_{r'}+\tfrac{1}{2}h\right],$$

where h is to be sampled from $p(h)$.

In practice, however, it seems that only the energy is essentially biased by any changes concerning the applied magnetic field, and the only significant differences between the NERFM and the MSURF refer to the amplitude of the energy fluctuations, which are anomalously large in the latter interpretation given that any field change then affects the whole system. That is, the energy is naturally defined for the MSURF as a double average of (2.4), namely,

$$U\equiv[\langle\langle H(\mathbf{s};h)\rangle\rangle]_{\rm av}\equiv\sum_{\mathbf{s}\in\mathbf{S}}P^{\rm st}(\mathbf{s})\int_{-\infty}^{+\infty}dh\,p(h)H(\mathbf{s};h)\ ,\tag{2.6}$$

where $H(\mathbf{s};h)$ depends on two random variables and $P^{\rm st}(\mathbf{s})$ is the stationary solution of (2.1), and the corresponding mean-square fluctuations are

$$\sigma_U^2\equiv[\langle\langle\{H(\mathbf{s};h)-U\}^2\rangle\rangle]_{\rm av}\ .$$

Energy and fluctuations for the NERFM, on the other hand, are defined by the same double average except that $dh\,p(h)$ and (2.4) are, respectively, replaced by $d\mathbf{h}\,p_t^s(\mathbf{h})$ [or $\prod_k dh_k\,p(h_k)$] and (2.5), the latter depending on $N+1$ random variables, with N denoting the number of lattice sites. It then follows, in particular, that a term $[\langle\langle h^2(\sum_r s_r)^2\rangle\rangle]_{\rm av}$ enters σ_U^2 for the MSURF, while this is replaced for the NERFM by

$$\left[\left\langle\!\!\left\langle\left[\sum_r h_r s_r\right]^2\right\rangle\!\!\right\rangle\right]_{\rm av}=\left[\left\langle\!\!\left\langle h^2\left[\sum_r s_r\right]^2\right\rangle\!\!\right\rangle\right]_{\rm av}+2(\mu^2-\langle\langle h^2\rangle\rangle)\left\langle\sum_{r\neq r'}s_r s_{r'}\right\rangle\ ,\tag{2.7}$$

where $\langle\cdots\rangle$ represents the stationary average and $\mu=\langle\langle h\rangle\rangle$. More generally, the two interpretations will differ with respect to any function that is nonlinear in h (cf. Sec. V). Otherwise, phase diagrams and critical behavior, which are our main concern here, seem to be the same. Consequently, most results below refer to both cases.

Concerning the nature of the model, one should also remark that both interpretations of the model, NERFM and MSURF, have two simple well-known limits for $p(h)=\delta(h\pm h_0)$, respectively, where δ is the Dirac δ function and h_0 represents a positive constant. That is, within any of those two limits, any spin-flip rate satisfying (2.3) will drive the system to the (unique) Gibbs equilibrium state corresponding to temperature T and energy $H(\mathbf{s};\pm h_0)$. For more general distributions $p(h)$, however, the situation will be more involved. In fact, the competition between several field values will, in general, cause the system to tend asymptotically towards a nonequilibrium steady state, as if it were acted on by some external non-Hamiltonian agent, whose explicit dependence on $p(h)$, T, J, and $c(\mathbf{s}^r|\mathbf{s})$ is unknown. This is the case in general, even when the interest is in the simplest field distribution describing a crossover between those two limiting conditions, say when

$$p(h)=q\delta(h-h_0)+(1-q)\delta(h+h_0)\ .$$

The model system may then, in principle, allow one to analyze a variety of nonequilibrium phase transitions and critical phenomena, and, as indicated above, chances are that some version of it may be pertinent to understanding some of the reported peculiarities of frustrated systems. In particular, our selection of distributions $p(h)$ is dictated by a search for both simplicity and some relevance in relation to the study of random-field and other *impure* systems.

III. CLASSIFICATION OF KINETICS

The class of model systems introduced in the preceding section may be investigated by applying a systematic method developed previously for finding stationary states for certain systems with competing kinetics.[7,8] This proceeds by assuming the existence of a strictly positive (for all **s**) stationary solution of (2.1), $P^{\rm st}(\mathbf{s})$, and defining an analytic object $E(\mathbf{s})$ according to

$$P^{\rm st}(\mathbf{s})\equiv\exp[-E(\mathbf{s})]\left[\sum_{\mathbf{s}\in\mathbf{S}}\exp[-E(\mathbf{s})]\right]^{-1}\ .\tag{3.1}$$

It follows quite generally that

$$E(\mathbf{s})=\sum_{k=1}^{N}\ \sum_{\{(r_1,\ldots,r_k)\}}{}^{\prime}\ J^{(k)}_{r_1\cdots r_k}s_{r_1}\cdots s_{r_k}\ ,\tag{3.2}$$

where the summation $\sum'$ is taken over every set of k lattice sites in the system. To be useful, however, $E(\mathbf{s})$ needs to have some appropriate short-range behavior, namely, it needs to involve only a finite number of coefficients $J^{(k)}$, even when it refers to a macroscopic ($N\to\infty$) system. Consequently one requires, for example,

$$J^{(k)}_{r_1 \cdots r_k}=0 \quad \text{for all } k \geq k_0 \;, \tag{3.3}$$

where k_0 is independent of N, at least for $N > N_0$. Clearly, one may approach the understanding of a system with competing kinetics by finding $E(\mathbf{s})$ with property (3.3). For instance, a one-dimensional system having a well-defined, short-ranged $E(\mathbf{s})$ may be solved by using the relatively simple, standard tools of equilibrium theory.

In Secs. III–V, we shall restrict ourselves to systems satisfying a kind of global detailed-balance (GDB) property; such a restriction has an obvious physical meaning, and it allows us the practical computation of $E(\mathbf{s})$ in some interesting cases. That is, we shall require that

$$c(\mathbf{s}^r|\mathbf{s})\exp[-E(\mathbf{s})]=c(\mathbf{s}|\mathbf{s}^r)\exp[-E(\mathbf{s}^r)] \tag{3.4}$$

for any $\mathbf{s}$ and $\mathbf{s}^r$, where $E(\mathbf{s})$ is defined by (3.1). This amounts, in practice, to finding a subclass of our systems that is characterized by some families of functions $c(\mathbf{s}^r|\mathbf{s};h)$ and $p(h)$ in (2.2) implying condition (3.4). Although (3.4) involves a drastic simplification of dynamics, as discussed below, it still produces an interesting subclass; in particular, one thus finds simple explicit expressions for a short-ranged effective Hamiltonian $E(\mathbf{s})$ in the case of several familiar transition rates and some relevant field distributions.

The physical situations of interest provide no specific criteria to determine what transition rates should be used. Thus, we shall only require that the elementary processes satisfy (2.3). That is,

$$c(\mathbf{s}^r|\mathbf{s};h)=f_r(\mathbf{s};h)\exp[-\tfrac{1}{2}\beta\,\Delta H_h] \;, \tag{3.5}$$

where the function $f_r(\mathbf{s};h)$ is analytical, positive definite, and independent of the variable s_r. This is satisfied, for instance, by the following cases:

$$f_r(\mathbf{s};h)=\alpha=\text{const} \;, \tag{3.6a}$$

$$f_r(\mathbf{s};h)=\alpha\left[\cosh(\beta h)\prod_{\mathbf{i}=1}^{d}\cosh K(s_{r+i}+s_{r-i})\right]^{-1} \;, \tag{3.6b}$$

where $K\equiv\beta J$ and $\mathbf{i}$ represents unity vectors along each principal direction in the lattice,

$$f_r(\mathbf{s};h)=\alpha[\cosh(K)]^{-2d}[\cosh(\beta h)]^{-1} \;, \tag{3.6c}$$

$$f_r(\mathbf{s};h)=\tfrac{1}{2}\alpha[\cosh(\tfrac{1}{2}\beta\,\Delta H_h)]^{-1} \;, \tag{3.6d}$$

$$f_r(\mathbf{s};h)=\exp[-\tfrac{1}{2}|\beta\,\Delta H_h|] \;, \tag{3.6e}$$

and

$$f_r(\mathbf{s};h)=\exp[-|\beta\,\Delta H_h|] \;. \tag{3.6f}$$

The choices (3.6a)–(3.6e) have been used before in different problems by van Beijeren and Schulman,[15] Ligget,[9] Glauber,[11] de Masi, Ferrari, and Lebowitz,[16] Kawasaki,[17] and Metropolis *et al.*,[18] respectively; (3.6f) is a trivial modification of the latter, which induces, however, a distinct macroscopic behavior.

On the other hand, the application to a one-dimensional system of the method outlined above makes it convenient to define the quantities

$$A\equiv\ln[\langle\langle\varphi_0\exp(\beta h)\rangle\rangle\langle\langle\varphi_0\exp(-\beta h)\rangle\rangle^{-1}] \;, \tag{3.7}$$

$$B\equiv\ln[\langle\langle\varphi_+\exp(\beta h+2K)\rangle\rangle\langle\langle\varphi_+\exp(-\beta h-2K)\rangle\rangle^{-1}] \;, \tag{3.8}$$

and

$$C\equiv\ln[\langle\langle\varphi_-\exp(\beta h-2K)\rangle\rangle\langle\langle\varphi_-\exp(-\beta h+2K)\rangle\rangle^{-1}] \;, \tag{3.9}$$

where φ_0 and $\varphi_\pm$ stand for the function $f_r(\mathbf{s};h)$ for $s_{r+i}+s_{r-i}=0$ and ±2, respectively. In fact, one may then prove after some algebraic manipulations that a necessary and sufficient condition for the GDB condition (3.4) to hold when the system evolves according to rates (3.6) is that

$$2A=B+C \;. \tag{3.10}$$

Moreover, it also follows[8] that, under the conditions enumerated before in this section, an effective Hamiltonian indeed exists, as given by

$$E(\mathbf{s})=-K_e\sum_{\text{NN}}S_rS_{r'}-\beta h_e\sum_r s_r \;. \tag{3.11}$$

That is, the system may be represented by an effective Hamiltonian with a simple structure, namely, the structure of the NN Ising Hamiltonian (2.4) appearing in the definition of the model, while it involves effective parameters that contain a complex interplay of T, $p(h)$, J, and kinetics. More precisely, one finds

$$h_e=\tfrac{1}{8}(2A+B+C)\beta^{-1} \;, \quad K_e=\tfrac{1}{8}(C-B) \;, \tag{3.12}$$

for the parameters in (3.11).

The nature of the resulting description, i.e., (3.1) with (3.11) and (3.12), which is implied by condition (3.4), deserves a comment. As indicated by (3.1) and (3.4), the description has a canonical structure; the effective parameters (3.12), however, involve details of kinetics, such as the transition rates and the disorder distribution. Consequently, as is made explicit later on, the macroscopic properties of the steady state are influenced, even dominated, by those, let us say, noncanonical details. In order to interpret this kind of *quasicanonical* situation, one may imagine the existence of an external constraint on the spin system that modifies the parameters of the original Hamiltonian, (2.4), and replaces the effect of the competing kinetics. That is, even though the model is relatively simple when (3.4) holds, the spin system is subjected to a constraint. Note also that, as indicated in Sec. VII, the GDB condition (3.4) for the effective transition rate only holds in some exceptional cases, perhaps only for some one-dimensional cases, so that the model system introduced in this paper will, in general, present even a more complicated (nonequilibrium) behavior than the *quasicanonical* version described in Secs. II–V.

We may now analyze the consequences of (3.10) for the choices (3.6), and the resulting explicit expressions for h_e and K_e. The cases (3.6a), (3.6b), and (3.6c) imply a similar behavior to each other. Namely, a system driven by a competing kinetics of that class, to be identified in the following as *soft kinetics,* satisfies GDB for any distribution $p(h)$, and one always has $K_e = K$. The competing nature of kinetics and some differences between the rates in that class are only reflected in the *effective field,* which is given by

$$\tanh(\beta h_e) = \langle\langle \sinh(\beta h) \rangle\rangle \langle\langle \cosh(\beta h) \rangle\rangle^{-1} \tag{3.13}$$

for (3.6a) and by

$$\tanh(\beta h_e) = \langle\langle \tanh(\beta h) \rangle\rangle \tag{3.14}$$

for both (3.6b) and (3.6c).

The case consisting of elementary kinetical process driven by rates (3.6d), (3.6e), or (3.6f) will be identified in the following as *hard kinetics.* This induces a more intricate situation than the *soft* case. That is, for $J=0$, GDB is satisfied for any distribution $p(h)$, and it follows that $K_e = 0$, and that h_e is given either by (3.14) or by

$$h_e = \tfrac{1}{2}\beta^{-1}\ln\{\langle\langle \exp[\beta(h-2|h|)] \rangle\rangle \times \langle\langle \exp[-\beta(h+2|h|)] \rangle\rangle^{-1}\} , \tag{3.15}$$

respectively, for (3.6d) and (3.6f). On the other hand, the more interesting case with $J \neq 0$ only satisfies the GDB property for field distributions such that $p(h) = p(-h)$, and one gets $h_e = 0$ and either

$$\tanh(2K_e) = \langle\langle \tanh[2K + \beta h] \rangle\rangle \tag{3.16}$$

for (3.6d) or

$$\tanh(2K_e) = \langle\langle \exp[-n|2K+\beta h|]\sinh(2K+\beta h) \rangle\rangle \times \langle\langle \exp[-n|2K+\beta h|]\cosh(2K+\beta h) \rangle\rangle^{-1} \tag{3.17}$$

with $n = 1$ and 2 for (3.6e) and (3.6f), respectively.

Note that the essential formal distinction between *soft* and *hard kinetics,* which is responsible for the reported differences in the parameters of the effective Hamiltonian, is that the function characterizing the former factorizes, i.e.,

$$f_r(\mathbf{s};h) = f_r^{(1)}(s_{r+1} + s_{r-1}) f_r^{(2)}(h) ,$$

where $f_r^{(2)}(h)$ has no dependence on $s_{r\pm1}$, while this is not an attribute of *hard kinetics.* The latter case thus requires a condition on $p(h)$ in order to accomplish with the (strong) GDB property.

Let us now evaluate the effective parameters h_e and K_e for some specific field distributions of interest. We first note that a system driven by *soft kinetics* in a sense reduces (only) when the field distribution is even, $p(h) = p(-h)$, to the canonical Ising model in the absence of a field, i.e., $K_e = K$ and it gives $h_e = 0$ from both (3.13) and (3.14) (there are, however, some essential differences between the energy fluctuations in those two cases; cf. Sec. V). Thus, looking for a more complex behavior when the competition is soft, one may consider instead, for instance, a symmetric distribution with nonzero mean; i.e., any distribution such that

$$p(\mu+h) = p(\mu-h) \quad \text{with } \mu \neq 0 .$$

Interestingly enough, the latter still produces the "canonical" behavior $h_e = \mu$ and $K_e = K$ for rate (3.6a), while one has $K_e = K$ and a more involved expression for h_e when the rate is (3.6b) or (3.6c).

To make that fact explicit, we have studied the distribution

$$p(h) = \tfrac{1}{2}q\delta(h - [\mu+\kappa]) + \tfrac{1}{2}q\delta(h - [\mu-\kappa]) + (1-q)\delta(h-\mu) .$$

This transforms the formula (3.14) into

$$h_e = \tfrac{1}{2}\beta^{-1}\ln([c_\mu^2(1+t_\mu) + s_\kappa^2(1+\{1-q\}t_\mu)] \times [c_\mu^2(1-t_\mu) + s_\kappa^2(1-\{1-q\}t_\mu)]^{-1}) , \tag{3.18}$$

where $s_x \equiv \sinh(\beta x)$, $c_x \equiv \cosh(\beta x)$, and $t_x \equiv \tanh(\beta x)$. Within the limit $\beta \to \infty$ $(T \to 0)$, the predominant behavior implied by (3.18) is such that $\tanh(\beta h_e)$ goes to 1, $1-\frac{1}{2}q$, or $1-q$, respectively, according to whether μ is larger than, equal to, or smaller than κ. That is,

$$h_e \approx \begin{cases} \mu - \kappa + (2\beta)^{-1}\ln(2/q) & \text{when } \mu > \kappa \\ (2\beta)^{-1}\ln[(4-q)/q)] & \text{when } \mu = \kappa \\ (2\beta)^{-1}\ln[(2-q)/q] & \text{when } \mu < \kappa \end{cases} \tag{3.19}$$

as $T \to 0$. This zero-temperature limit, which reveals the most intriguing behavior of the system, will be discussed later on in more detail.

Still concerning *soft kinetics,* it also seems interesting *a priori* to consider the uneven field distribution

$$p(h) = q\delta(h-\mu_1) + (1-q)\delta(h-\mu_2) \quad \text{with } \mu_2 > \mu_1 .$$

We now get from (3.13) that the rates (3.6a) produce are

$$\tanh(\beta h_e) = [t_a + (1-2q)t_b][1-(1-2q)t_a t_b]^{-1} , \tag{3.20}$$

where $a \equiv \frac{1}{2}(\mu_1+\mu_2)$ and $b \equiv \frac{1}{2}(\mu_2-\mu_1)$, in addition to $K_e = K$. The right-hand side (rhs) of expression (3.20) goes to unity as $T \to 0$, except for $\mu_1 = -\mu_2$ when it goes to $1-2q$, which may be positive, negative, or zero. On the other hand, the rates (3.6b) and (3.6c), which also imply $K_e = K$, produce an effective field essentially differing from (3.20), i.e., it then follows from (3.14) that

$$\tanh(\beta h_e) = q\tanh(\beta\mu_1) + (1-q)\tanh(\beta\mu_2) . \tag{3.21}$$

As $T \to 0$, $\tanh(\beta h_e) \to 1$ for $\mu_2 > \mu_1 > 0$, while $\tanh(\beta h_e)$ again is reduced within that limit to $1-2q$ for $\mu_2 > 0 > \mu_1$. Those changes of the *effective field* as one varies the system parameters, even in the relatively simple one-dimensional case with *soft kinetics,* already illustrate the great richness of the model behavior.

The system behaves in a qualitatively different way when it evolves according to *hard kinetics.* As a matter of fact, only an even field distribution $p(h) = p(-h)$ will then make the GDB condition hold in nontrivial cases, as stated before. In order to make this more explicit, while still trying to extract the relevant general behavior, we have analyzed the case

$$p(h)=\tfrac{1}{2}q\delta(h-\kappa)+\tfrac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h)$$

which is also characterized by $h_e=0$. The situation may be summarized as follows. For rates (3.6d), we get from (3.16) that

$$K_e=K+\tfrac{1}{4}\ln\{[1-(1-q)t_{2J}^2t_\kappa^2-qt_{2J}t_\kappa^2]\times[1-(1-q)t_{2J}^2t_\kappa^2+qt_{2J}t_\kappa^2]^{-1}\} . \quad (3.22)$$

Consequently, one has three different behaviors as $T\to 0$, namely,

$$K_e\approx\begin{cases}K(1-\kappa/2J)+\tfrac{1}{4}\ln(2/q) & \text{when } 2J>\kappa\\ \tfrac{1}{4}\ln[(4-q)q^{-1}] & \text{when } 2J=\kappa\\ \tfrac{1}{4}\ln[(2-q)q^{-1}] & \text{when } 2J<\kappa .\end{cases} \quad (3.23)$$

For rates (3.6f), the situation is different, i.e., (3.17) variously leads to

$$K_e=K+\tfrac{1}{4}\ln\{[qc_\kappa+(1-q)][qc_{3\kappa}+(1-q)]^{-1}\} \quad \text{for } 2J>\kappa>0 , \quad (3.24a)$$

which leads to $K_e\approx K(1-\kappa/2J)$ as $T\to 0$,

$$K_e=K+\tfrac{1}{4}\ln\{[qc_{2J}+(1-q)][qc_{6J}+(1-q)]^{-1}\} \quad \text{for } 2J=\kappa , \quad (3.24b)$$

which leads to $K_e\approx 0$ as $T\to 0$, and

$$K_e=K+\tfrac{1}{4}\ln\{[qe^{-\beta\kappa}+qe^{-3\beta\kappa+8\beta J}+2(1-q)]\times[qe^{-3\beta\kappa}+qe^{-\beta\kappa+8\beta J}+2(1-q)]^{-1}\} \quad (3.24c)$$

for $2J<\kappa$; the latter case may present several distinct sorts of behavior when $T\to 0$, as we shall describe later on.

IV. THERMODYNAMICS

In this section we initiate an explicit study of the resulting macroscopic behavior. Natural definitions for the energy U and for the corresponding fluctuations, which involve $p(h)$ and $P^{\mathrm{st}}(\mathbf{s})$, were discussed in Sec. II. In addition to those quantities, we shall study the behavior of a specific heat defined

$$C_v\equiv(\partial U/\partial T)_{v,\ldots} ,$$

the magnetization M and its square mean fluctuations defined in the usual way, i.e.,

$$\sigma_M^2\equiv\langle\{(\Sigma_r s_r)-M\}^2\rangle , \quad \text{where } \langle\cdots\rangle\equiv\sum_{s\in S}P^{\mathrm{st}}(s)\cdots$$

is the first average indicated in (2.6), and the magnetic susceptibility defined as

$$\chi_T\equiv(\partial M/\partial\mu)_{v,\ldots} ,$$

where μ is the mean of distribution $p(h)$. It is then a simple exercise to write those quantities as a function of K_e and h_e. It follows, for instance, that

$$U\equiv-J(\partial\ln Z/\partial K_e)-\mu\{\partial\ln Z/\partial(\beta h_e)\} \quad (4.1)$$

and

$$M\equiv Z^{-1}\{\partial Z/\partial(\beta h_e)\} . \quad (4.2)$$

Here,

$$Z\equiv\sum_{s\in S}\exp[-E(\mathbf{s})] , \quad (4.3)$$

and $E(\mathbf{s})$ represents the effective Hamiltonian. After using the familiar transfer-matrix method, for example, one may finally arrive to expressions for the quantities of interest as a function of K_e, h_e, K, and μ. In particular, one gets

$$U=-JN\{1+2(x-1)^{-1}[1-c_hy^{-1/2}]\}-\mu Ns_hy^{-1/2} , \quad (4.4)$$

$$C_v=-JN(\partial K_e/\partial T)x(x-1)^{-1}\{8(x-1)^{-1}[c_hy^{-1/2}-1]-4c_hy^{-3/2}x^{-2}\} -\mu N[\partial(\beta h_e)/\partial T]c_hy^{-1/2}[1-s_h^2y^{-1}]-2N[J\partial(\beta h_e)/\partial T+\mu\partial K_e/\partial T]x^{-1}s_hy^{-3/2} , \quad (4.5)$$

$$\sigma_U^2=J^2Nx(x-1)^{-1}\{8(x-1)^{-1}[c_hy^{-1/2}-1]-4c_hy^{-3/2}x^{-2}\}+\mu^2Nc_hy^{-1/2}[1-s_h^2y^{-1}] +2\mu JNx^{-1}s_hy^{-3/2}+\sigma_h^2Ny^{-1}[Ns_h^2+c_h(y^{1/2}-s_h^2y^{-1/2})] , \quad (4.6)$$

$$M=Ns_hy^{-1/2} , \quad (4.7)$$

$$\sigma_M^2=Nc_hy^{-1/2}[1-s_h^2y^{-1}] , \quad (4.8)$$

and

$$\chi_T=\sigma_M^2\partial(\beta h_e)/\partial\mu+2Nx^{-1}s_hy^{-3/2}\partial K_e/\partial\mu . \quad (4.9)$$

Here, $c_h\equiv\cosh(\beta h_e)$, $s_h\equiv\sinh(\beta h_e)$, $x\equiv\exp(4K_e)$, and $y\equiv s_h^2+x^{-1}$. Notice as a general remark that *soft kinetics* is such that $K_e=K$ and $\partial K_e/\partial\mu=0$, and that *hard kinetics* requires even distributions $p(h)$ implying $\mu=0$ and $h_e=0$; these facts notably simplify the above formulas when referring to specific cases.

The possible existence of a critical point in an Ising-like system for some values of the temperature T and applied field h may be determined by analyzing the behavior of the correlation length, say

$$\xi=\xi(h,T) .$$

That is, the spin-spin correlation function is usually expected to behave at long distances as

$$g(r) \sim \exp(-r/\xi) \quad \text{as } r \to \infty \ ,$$

and ξ will then eventually diverge for the values of T and h locating the critical point. For example, using standard notation,

$$\xi(0,\epsilon) \sim \epsilon^{-\nu} \quad \text{as } \epsilon \to 0^+$$

and

$$\xi(0,\epsilon) \sim (-\epsilon)^{-\nu'} \quad \text{as } \epsilon \to 0^-$$

for the familiar Ising model. More precisely, the correlations for the NN one-dimensional Ising model under a uniform field h, which is defined via the Hamiltonian

$$H(s) = -\sum_r s_r(Js_{r+1}+h) \ , \tag{4.10}$$

are such that

$$g(r) = \sin^2[2\tau(\Phi_-/\Phi_+)^r] \tag{4.11}$$

and

$$\xi = [\ln(\Phi_-/\Phi_+)]^{-1} \ . \tag{4.12}$$

Here,

$$\begin{aligned} &\Phi_\pm = e^K\cosh(\beta h) \pm [e^{2K}\sinh^2(\beta h) + e^{-2K}]^{1/2} \ , \\ &\cot(\tau) = e^{2K}\sinh(\beta h) \ . \end{aligned} \tag{4.13}$$

This implies the existence of a critical point. That is, the correlation length diverges as

$$\xi(0,\epsilon) \approx \epsilon^{-\nu} \quad \text{with } \nu = 1 \ ,$$

when one approaches $T=0$ with $h=0$. One may also notice[19] that, in the presence of a zero-temperature critical point, the relevant temperature parameter for investigating thermal critical exponents is $\epsilon \equiv \exp(-2K)$, and it is then convenient to consider the scaling form $m = \Theta|\Theta|^{1/\delta-1}\Psi(\epsilon|\theta|^{-1/\beta\delta})$, where m represents the magnetization for small values of both ϵ and $\Theta \equiv \beta h$, and Ψ is some undetermined *scaling function*. When $|\Theta| \ll 1$, that scaling behavior is precisely confirmed for the one-dimensional Ising model with $\Psi(x) = (1+x^2)^{-1/2}$ and with $\beta\delta = 1$ and $\delta = \infty$ in such a way that $\beta = 0$ is implied. Also, the scaling law $2-\alpha = \gamma = \nu$, together with the fact that $\nu = 1$, leads to $\alpha = 1$ for the one-dimensional Ising model.

Concerning our one-dimensional models, we know (cf. Sec. III) that, when the GDB property holds, they have an effective Hamiltonian with the structure of (4.10). Consequently, they will reveal critical behavior, as far as (3.4) is satisfied, when $h_e = 0$, and $T \to 0^+$. Those conditions, however, are not as simple as they may appear at first glance. That is, our models involve a distribution $p(h)$ of fields whose specific form strongly affects both conditions, (3.4) and $h_e = 0$, as already stated, and the fact that the effective parameters h_e and K_e depend on temperature necessitates a careful study of the limit $T \to 0^+$. In fact, it follows from (4.12) that the critical length in our system diverges when $\Phi_-\Phi_+^{-1} \to 1$. This occurs in practice for

$$\exp(-4K_e)[1-\tanh^2(\beta h_e)] + \tanh^2(\beta h_e) \to 0$$

and, given that

$$\tanh^2(\beta h_e) \geq 0$$

and that

$$\exp(-4K_e)[1-\tanh^2(\beta h_e)] \geq 0 \ ,$$

it is required both that

$$\tanh^2(\beta h_e) \to 0 \ ,$$

which implies $\beta h_e \to 0$, and that

$$\exp(-4K_e) \to 0 \ ,$$

which implies $K_e \to \infty$. A simple situation will then occur when $h_e \to 0$ as the mean μ of $p(h)$ goes to zero, assuming this limit causes no extra problems in K_e, which corresponds to the existence of a proper critical point at $T=0$ in our system. One may expect more complex situations in general, however, as becomes clear below.

V. MACROSCOPIC BEHAVIOR FOR SPECIFIC FIELD DISTRIBUTIONS

In this section, we consider the thermodynamic formulas derived in the preceding section for specific rates and field distributions. In particular, we investigate in detail the system behavior in the zero temperature limit, and, in addition to the nonequilibrium version of the usual random-field case, we also refer to an *impure* system in which the distribution $p(h)$ involves strong fields "freezing" the spin direction with a given probability.

A. Even distributions

Consider first any generic, even field distribution, so that the mean is zero, $\mu = 0$. When the system evolves according to *soft kinetics,* one simply has that $K_e = K$ and $h_e = 0$. This seems an evidence that the system reduces, in practice, to the canonical Ising system. It also follows in that case, however, that

$$\begin{aligned} &U = -2NJ\sinh^2(K)[\sinh(2K)]^{-1} \ , \\ &C_v = (4NJ^2/k_BT^2)\sinh^2(K)[\sinh(2K)]^{-2} \ , \\ &M = 0, \quad \sigma_M^2 = N\exp(2K), \end{aligned}$$

and

$$\sigma_U^2 = k_BT^2C_v + \sigma_e^2 = k_BT^2C_v + N\sigma_h^2\exp(2K) \ .$$

Consequently, a fluctuation-dissipation relation does not hold, in the sense that σ_U^2 differs from $k_BT^2C_v$, even in the present case, which happens to bear very simple nonequilibrium features. In fact, this system has an *excess* of energy fluctuations, which is given by

$$\sigma_e^2 = N\sigma_h^2\exp(2K) = \sigma_h^2\sigma_M^2 \ ,$$

due to the existence of a field distribution having nonzero variance. Otherwise, the behavior seems, indeed, the same as for the Ising model in the absence of a field, including the fact that

$$\sigma_U^2 U^{-2} \to 0$$

in the thermodynamic limit $N \to \infty$. The critical behavior is the same as in equilibrium, i.e., there is a zero-temperature critical point, and one finds

$$\nu = \eta = 1 \quad \text{as } T \to 0^+ .$$

Some some specific even field distributions for *hard kinetics,* with results more intriguing than the *soft* case, are considered below.

B. Symmetric distributions with nonzero mean

When the field distribution is

$$p(\mu + h) = p(\mu - h) \quad \text{with } \mu \neq 0$$

for any h, the system with *hard kinetics* satisfies no GDB condition except for $J=0$ in (2.4), which is a trivial case. Concerning *soft kinetics,* it is convenient to consider two subcases separately. For rates (3.6a), we have demonstrated before that $K_e = K$ and $h_e = \mu$. Consequently, it follows the same behavior as for the Ising model under a field $h = \mu$, except for the magnitude of some fluctuations. Namely, there is now an *excess* of energy fluctuations, which is given by

$$\sigma_e^2 = \sigma_h^2 (\sigma_M^2 + M^2) .$$

This is quite consistent with the situation described in Sec. V A for the same rate and $p(h) = p(-h)$, provided that $M=0$ there. It also follows that

$$\sigma_U^2 U^{-2} \sim \sigma_h^2 (M/U)^2 \quad \text{as } N \to \infty ,$$

implying that

$$\sigma_U^2 U^{-2} \sim \sigma_h^2 (J+\mu)^{-2} \quad \text{as } T \to 0$$

in the thermodynamic limit. The latter is a distinguishing property of the MSURF, which does not hold for the NERFM, as indicated in Sec. II. Concerning critical behavior, the system exhibits a critical point as $\mu \to 0$ and $T \to 0^+$, which is characterized by equilibrium critical exponents, namely,

$$\nu = \eta = 1, \ \delta = \infty , \ \text{and } \beta = 0$$

in such a way that

$$\beta\delta = 1 .$$

The system behavior for the other subcase of *soft kinetics,* (3.6b) and (3.6c), is described in the next section for a more specific field distribution.

C. The case $p(h) = \frac{1}{2} q \delta(h - [\mu + \kappa]) + \frac{1}{2} q \delta(h - [\mu - \kappa]) + (1-q)\delta(h - \mu)$

Concerning *soft kinetics,* contrary to case (3.6a) described in the preceding section, there is no general result for any symmetric distribution of nonzero mean when the elementary rates are either (3.6b) or (3.6c). Consequently, we have studied the case

$$p(h) = \tfrac{1}{2} q \delta(h - [\mu + \kappa]) + \tfrac{1}{2} q \delta(h - [\mu - \kappa]) + (1-q)\delta(h - \mu) ,$$

which produces $K_e = K$ and expression (3.14) for h_e. The resulting formulas for the macroscopic quantities of interest, which are rather involved, do not directly reveal any interesting general fact; thus, instead of writing them explicitly, we shall only refer here to the system behavior in the zero-temperature limit.

Three different asymptotic behaviors follow depending on the relation between the parameters μ and κ, which characterize the distribution $p(h)$.

Case a occurs for $\mu > \kappa$. It then follows within the limit $T \to 0$ that

$$\tanh(\beta h_e) \to 1 , \quad \partial(\beta h_e)/\partial\mu \approx \beta ,$$

$$\partial(\beta h_e)/\partial T \approx k_B \beta^2 (\kappa - \mu) ,$$

and

$$h_e \approx \mu - \kappa + (2\beta)^{-1} \ln(2q^{-1}) .$$

Case b occurs for $\mu = \kappa$. This is characterized as $T \to 0$ by

$$\tanh(\beta h_e) \to 1 - \tfrac{1}{2} q , \quad \partial(\beta h_e)/\partial\mu \to 0 ,$$

$$\partial(\beta h_e)/\partial T \to 0 ,$$

and

$$h_e \approx (2\beta)^{-1} \ln[(4-q)q^{-1}] .$$

In both cases, *a* and *b*, the system tends to the following asymptotic behavior as $T \to 0$:

$$U \approx -N(J+\mu) , \quad C_v \to 0 , \quad M \approx N , \quad \chi_T \to 0 ,$$

$$\sigma_U^2 \approx N \sigma_h^2 , \quad \text{and } \sigma_M^2 \to 0 .$$

The only relevant distinction between the two cases concerns some details of the asymptotic regime, which may influence critical behavior, as is discussed below.

Case c occurs for $\mu < \kappa$. This is characterized by

$$\tanh(\beta h_e) \to 1 - q , \ \partial(\beta h_e)/\partial\mu \to 0 , \ \partial(\beta h_e)/\partial T \to 0 ,$$

$$\text{and } h_e \approx (2\beta)^{-1} \ln[(2-q)q^{-1}] .$$

The latter is perhaps the most fascinating situation for *soft kinetics.* That is, while $q \neq 1$ essentially reproduces the above cases (in particular, case *b*, as shown later on), a field distribution

$$p(h) = \tfrac{1}{2} \delta(h - [\mu + \kappa]) + \tfrac{1}{2} \delta(h - [\mu - \kappa])$$

gives rise to an extremely rich behavior as one varies the relation between μ, κ, and J (maintaining $\mu < \kappa$, however).

It then turns out to be convenient to distinguishing *cases* c_1, c_2, *and* c_3 associated, respectively, with $\kappa - \mu < J$, $\kappa - \mu = J$, and $\kappa - \mu > J$.

In *case* c_1, the strength J of the spin interactions is strong enough to compensate the action of the fields

$$h_1=\mu+\kappa \text{ and } h_2=\mu-\kappa \text{ with } h_2<0 ,$$

and it still follows a relatively regular situation. That is, U, C_v, and M are given as in cases a and b above,

$$\chi_T \approx 2N\beta \exp[4\beta(\kappa-\mu-J)] \to 0 ,$$

$$\sigma_M^2 \approx N \exp\{\beta[6(\kappa-\mu)-4J]\} ,$$

and

$$\sigma_U^2=\sigma_h^2(\sigma_M^2+N^2) ,$$

the latter two going to infinity, N, or zero according to whether $3(\kappa-\mu)$ is greater than, equal to, or smaller than $2J$.

Case c_2 is a changeover situation where it is noticeable that

$$U \approx -N(J+2^{-1/2}\mu) , \quad \sigma_U^2 \to \infty , \quad M \approx 2^{-1/2}N ,$$

$$\chi_T \approx 2^{-1/2}\beta N \to \infty , \text{ and } \sigma_M^2 \approx (22^{1/2})^{-1}Ne^{2K} \to \infty .$$

Finally, the interactions cannot compensate the action of the fields in *case* c_3, and one gets

$$U \approx -NJ , \quad C_v \to 0 , \quad \sigma_U^2 \approx N(\mu^2+\sigma_h^2)e^{2K} \to \infty ,$$

$$M \approx 0 , \quad \chi_T \approx 2N\beta \exp[-2\beta(\kappa-\mu-J)] \to 0 ,$$

and

$$\sigma_M^2 \approx Ne^{2K} \to \infty .$$

Those differences imply a rich critical behavior. In particular, one may distinguish two classes of critical points when the rates are (3.6b). The first one is for $\mu \to 0$ and $T \to 0^+$. When $q \neq 1$, everything follows as in the equilibrium system, except that the magnetization m now scales with $\Theta=\beta\mu(1-q)$. When $q=1$, however, one finds equilibrium critical exponents and also a novel behavior, namely, that $\beta\delta=1-\kappa/J$ and that spontaneous magnetization exists for $\kappa<J$, which scales with $\Theta'=4\beta\mu$, while it does not when $\kappa>J$. The second class of critical points occurs for $q=1$ and $\mu \neq 0$ as $T \to 0^+$. As far as $\mu<\kappa$, a line of critical points then exists, where $\xi \to \infty$, which are characterized by $\nu=\min[1,(\kappa-\mu)J^{-1}]$. Notice that this essentially differs from the case $q=1$ in the first class, where one obtains $\nu=1$ by taking $T \to 0^+$ after $\mu \to 0$. In fact, it follows now that

$$\nu=\min[1,\kappa J^{-1}]$$

if one makes $\mu \to 0$ so that ν may differ from 1, i.e., the two limits do not commute in general. As far as $\mu>\kappa$, on the contrary, ξ shows no divergence.

D. The case $p(h)=\frac{1}{2}q\delta(h-\kappa)+\frac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h)$

Following the study of even distributions initiated in Sec. V A, we report now on some singular situations arising as well for *hard kinetics* when

$$p(h)=\tfrac{1}{2}q\delta(h-\kappa)+\tfrac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h) \text{ with } q \neq 0 .$$

We first note that such a distribution has zero mean and, consequently, χ_T cannot be defined. Thermodynamics follow now from

$$U=-2NJx ,$$

$$C_v=-4NJx[\sinh(2K_e)]^{-1}(\partial K_e/\partial T) ,$$

$$M=0 ,$$

$$\sigma_U^2=4NJ^2x[\sinh(2K_e)]^{-1}+\sigma_h^2 N \exp(2K_e) ,$$

$$\sigma_h^2=q\kappa^2 , \text{ and } \sigma_M^2=N \exp(2K_e) ;$$

here $x \equiv \sinh^2(K_e)[\sinh(2K_e)]^{-1}$.

For rates (3.6d), one obtains three situations as $T \to 0^+$ corresponding to the three different behaviors in (3.23). There is always a critical point, which may present two quite distinct kinds of behavior. When $2J>\kappa$ and $T \to 0^+$, one obtains

$$U \to -NJ , \quad C_v \to 0 , \quad M=0 ,$$

$$\sigma_M^2 \approx N(2/q)^{1/2}\exp[\beta(2J-\kappa)] \to \infty ,$$

$$\sigma_U^2 \approx \sigma_h^2 N(2/q)^{1/2}\exp[\beta(2J-\kappa)] \to \infty ,$$

$$K_e \approx K(1-\tfrac{1}{2}\kappa/J)+\tfrac{1}{4}\ln(2/q) ,$$

and

$$\xi \approx (\tfrac{1}{8}q)^{1/2}\epsilon^{-\nu} \text{ with } \nu=1-\tfrac{1}{2}\kappa J^{-1} .$$

When $2J=\kappa$, on the other hand, it follows that

$$U \approx NJ[(1-y)(1+y)^{-1}] , \quad C_v \to 0 ,$$

$$\sigma_U^2 \approx 2NJ^2[1-4/(2+qy)]+N\sigma_h^2 y ,$$

and

$$\sigma_M^2 \approx Ny ,$$

and one has the result $K_e \approx \frac{1}{2}\ln y$ implying $\xi \approx \frac{1}{2}y$; here $y \equiv [(4-q)q^{-1}]^{1/2}$. This is, the disorder makes the system *hot* enough at $T=0$ to avoid the usual (thermal) critical point, while there is still some critical behavior as $q \to 0$, which is characterized by $\nu_q=\frac{1}{2}$. This is also the critical behavior characterizing the system when $2J<\kappa$, a case for which one gets

$$U \approx -NJ[1-qz][2(1-q)]^{-1} , \quad C_v \to 0 ,$$

$$\sigma_U^2 \approx 2NJ^2[1-2/(1+qz)]+N\sigma_h^2 z ,$$

and

$$\sigma_M^2 \approx Nz \text{ as } T \to 0^+ ;$$

here $z \equiv [(2-q)q^{-1}]^{1/2}$. Moreover, a very interesting situation corresponds to the latter case (i.e., when the interactions cannot balance out the peaks of the field distribution) for $q=1$; it then follows that

$$K_e \to 0 , \quad U \to 0 , \quad C_v \to 0 ,$$

$$\sigma_U^2 \to N\sigma_h^2 , \text{ and } \sigma_M^2 \to N .$$

Figure 1 illustrates the situation for rates (3.6d).

For rates (3.6f), one may again distinguish three main

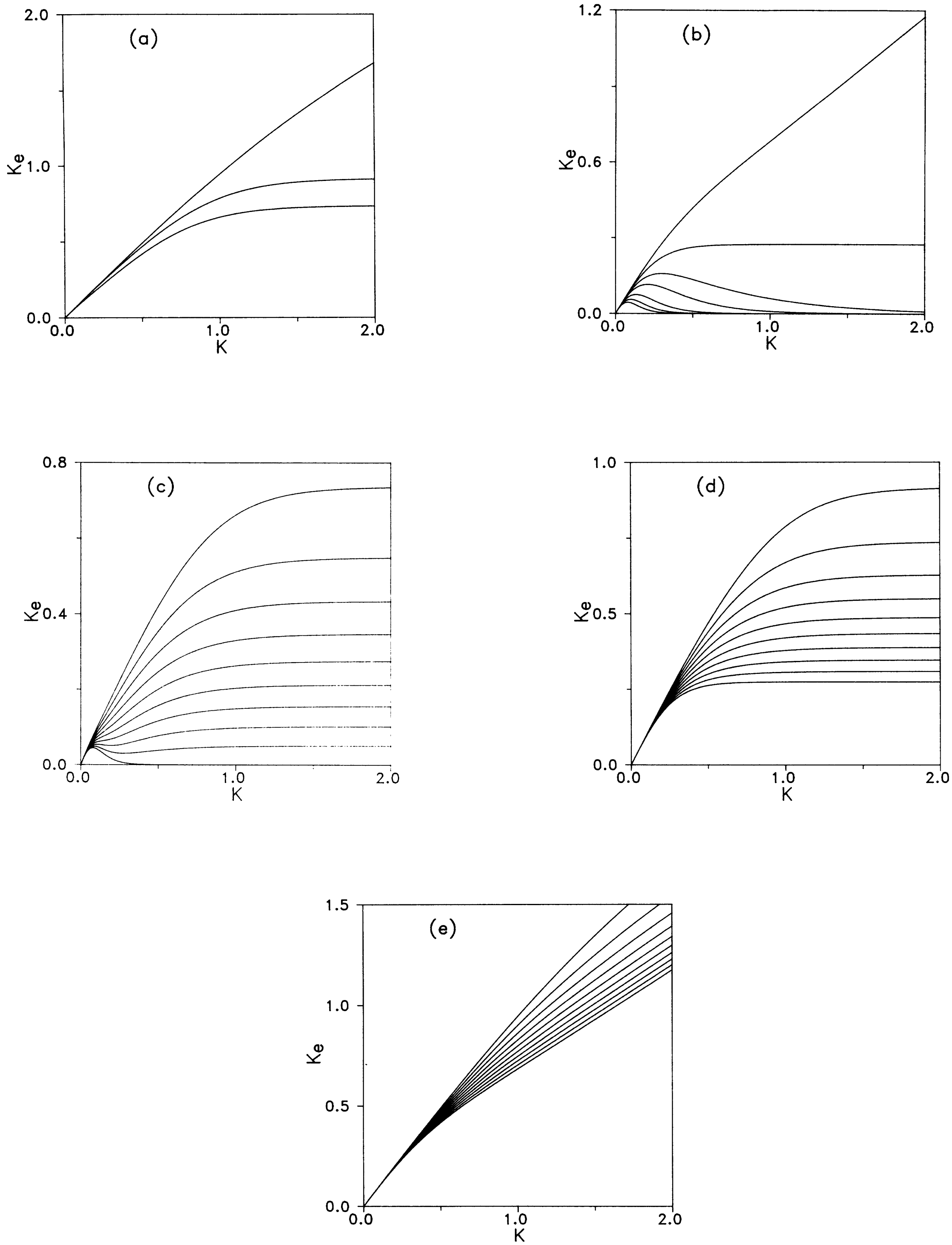

FIG. 1. The dependence with $K \equiv J/k_B T$ of the effective parameter K_e, cf. Eqs. (3.11) and (3.12), for the one-dimensional system driven by Kawasaki rates (3.6d) (hard kinetics) and acted on by the even field distribution $p(h)=\frac{1}{2}q\delta(h-\kappa)+\frac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h)$, as described in Sec. V D, for different values of q and $\lambda \equiv \kappa/2J$. (a) $q=0.1$ and $\lambda=0.5$, 1, and 5 from top to bottom. (b) The same as (a) but for $q=1$ and $\lambda=0.5,1,1.5,\ldots,5$ from top to bottom. The changes occur continuously between (a) and (b). (c) $\lambda=5$ and $q=0.1,0.2,\ldots,1$ from top to bottom. (d) The same as (c) but for the critical value $\lambda=1$. (e) The same as (c) but for $\lambda=0.5$.

cases, and the corresponding critical behavior is rather involved and interesting.

When $2J>\kappa$, one has

$$K_e \rightarrow K(1-\kappa/2J)$$

implying that

$$U\approx -NJ\ ,\quad C_v\rightarrow 0\ ,\quad M=0\ ,$$

$$\sigma_M^2 \approx N\exp[\beta(2J-\kappa)]\rightarrow\infty\ ,$$

$$\sigma_U^2\approx\sigma_h^2 N\exp[\beta(2J-\kappa)]\rightarrow\infty\ ,$$

and

$$\xi\approx\tfrac{1}{2}\epsilon^{-\nu}\ \text{ with } \nu=1-\tfrac{1}{2}\kappa J^{-1}\ ,$$

i.e., the critical behavior is similar to the one for rates (3.6d) and $2J>\kappa$. The case limit of that occurs for $2J=\kappa$; then,

$$K_e\rightarrow 0 \ \text{ and } U\rightarrow 0\ ,$$

$$C_v\rightarrow 0\ ,\quad M=0\ ,\quad \sigma_M^2\rightarrow N\ ,$$

and

$$\sigma_U^2\rightarrow\sigma_h^2 N\ .$$

That is, the system is now *extremely hot* at $T=0$, and it presents quantitative differences with the case of rates (3.6d).

When $2J<\kappa$, one may distinguish up to seven different subcases within the limit $T\rightarrow 0^+$, and the system then presents the richest critical behavior. That is, one obtains,

$$K_e\approx K(1-\tfrac{1}{2}\kappa J^{-1})\ \text{ for } \tfrac{8}{3}J>\kappa>2J\ ,$$

$$K_e\approx -(1/3)K+\tfrac{1}{4}\ln[(2-q)/q]\ \text{ for } \kappa=\tfrac{8}{3}J\ ,$$

and

$$K_e\approx -K(1-\kappa/4J)+\tfrac{1}{4}\ln[2(1-q)q^{-1}]\quad \text{for } 4J>\kappa>\tfrac{8}{3}J\ .$$

These three cases are such that $K_e\rightarrow-\infty$ when $T\rightarrow 0^+$, revealing a kind of effective antiferromagnetic situation at low temperatures. It is also interesting to notice that, analyzing the behavior of K_e with T, there follows a change of sign implying the existence of a temperature, say T^*, such that $K_e(T^*)=0$. On the other hand,

$$K_e\approx\tfrac{1}{4}\ln[2(1-q)q^{-1}]\ \text{ for } \kappa=4J\ ,$$

$$K_e\approx -K(1-\tfrac{1}{4}\kappa J^{-1})+\tfrac{1}{4}\ln[2(1-q)q^{-1}]\rightarrow\infty\ ,\quad \text{for } 8J>\kappa>4J$$

and

$$K_e\approx\tfrac{1}{4}\ln[2(1-q)q^{-1}],\ \text{ for } \kappa=8J\ .$$

The critical behavior of the correlation length in these three cases may be represented by

$$\xi\approx(\tfrac{1}{2})^{1/2}\epsilon^{-\nu}q^{-\nu_q}\ \text{ with } \nu=\tfrac{1}{4}\kappa J^{-1}-1$$

and

$$\nu_q=\tfrac{1}{2}\ \text{ when } 8J>\kappa>4J\ ,$$

and with

$$\nu=0\ \text{ and } \nu_q=\tfrac{1}{2}\ \text{ when } \kappa=4J\ \text{ or } 8J\ .$$

That is, the disorder may preclude, as before, the existence of a *thermal critical point* when $T\rightarrow 0^+$, but there is still critical behavior in any case as $q\rightarrow 0$. Finally, when $\kappa>8J$, one obtains

$$K_e\approx K\rightarrow\infty$$

and it follows the existence of a standard critical behavior with $\nu=1$. Figure 2 illustrates the situation for rates (3.6f).

E. Random freezing

It also seems interesting, in principle, to consider distributions involving strong fields, which are able to *freeze* the direction of the spins. As an example, we shall study the case

$$p(h)=q\delta(h-\mu_1)+(1-q)\delta(h-\mu_2)$$

in the limit $\mu_2\rightarrow\infty$. As shown before, such uneven distribution only tolerates GDB when the system evolves according to *soft kinetics,* and one then may distinguish two different cases.

The first case occurs for rates (3.6a), when $\Phi_-\Phi_+^{-1}\rightarrow 0$ as $\mu_2\rightarrow\infty$. This is characterized by

$$h_e\approx\tfrac{1}{2}(\mu_1+\mu_2)+\tfrac{1}{2}\beta^{-1}\ln[(1-q)q^{-1}]\rightarrow\infty\ ,$$

$$\tanh(\beta h_e)\approx\tanh(\beta\mu_2)\rightarrow 1\ ,$$

and

$$\partial(\beta h_e)/\partial T\approx -\tfrac{1}{2}k\beta^2(\mu_1+\mu_2)\rightarrow-\infty\ ,$$

the indicated limiting values occurring for $\mu_2\rightarrow\infty$, and by

$$\partial(\beta h_e)/\partial\mu\approx\tfrac{1}{2}\beta[q^{-1}+(1-q)^{-1}]\ .$$

This implies the following thermodynamics as $\mu_2\rightarrow\infty$:

$$U\approx -N[J+q\mu_1+(1-q)\mu_2]\rightarrow-\infty\ ,\quad C_v\rightarrow 0\ ,$$

$$\sigma_U^2\approx q(1-q)(\mu_1-\mu_2)^2N^2\rightarrow\infty\ ,$$

$$\sigma_U^2U^{-2}\approx q(1-q)^{-1}\ ,\quad M\rightarrow N\ ,\quad \chi_T\rightarrow 0\ ,\ \text{ and } \sigma_M^2\rightarrow 0\ .$$

It also seems interesting to consider the double limit $\mu_2\rightarrow\infty$ and $q\rightarrow 1$ with $\mu_2(1-q)=\kappa$ remaining finite. For the same rates, (3.6a), it follows the same qualitative behavior as in the above case for finite q, except that U remains bounded,

$$U\approx -N(J+\mu_1+\kappa)\ ,$$

and $\sigma_U^2U^{-2}$ diverges, according to our previous comments on the amplitude of energy fluctuations.

For rates (3.6a) and a field distribution

$$p(h)=q\delta(h-\mu_1)+(1-q)\delta(h-\mu_2)\ ,$$

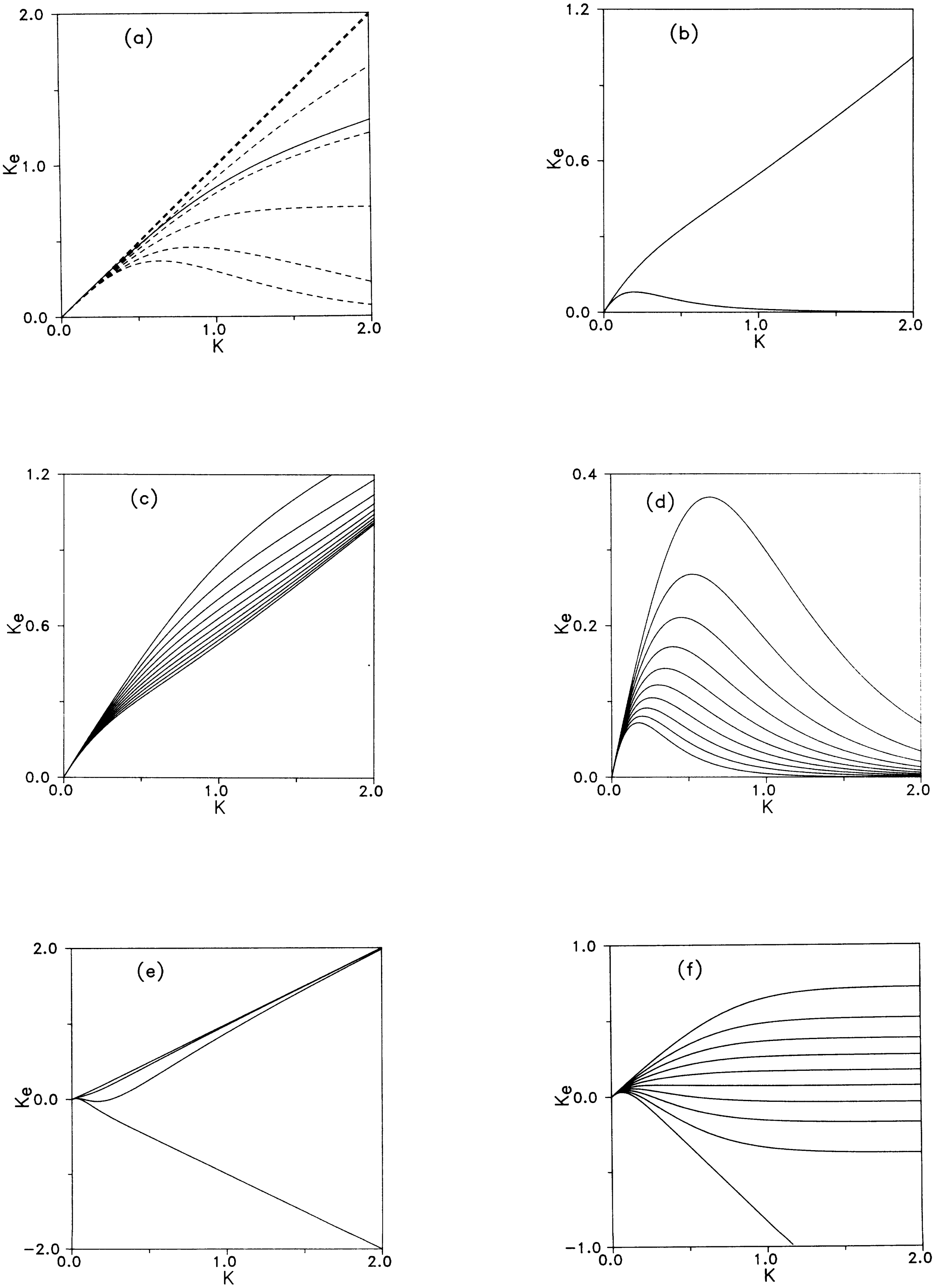

FIG. 2. The same as Fig. 1 but for the modified Metropolis rates (3.6f) (hard kinetics), as described in Sec. V D. (a) $q=0.1$ and $\lambda=0.5$ (solid line) and, from bottom to top, $\lambda=1,1.5,2,\ldots,5$. (b) $q=0.9$ and $\lambda=0.5$ (upper curve) and 1. (c) $\lambda=0.5$ and $q=0.1,0.2,\ldots,1$, from top to bottom. (d) The same as (c) but for $\lambda=1$. (e) The same as (c) but for $\lambda=2$. (f) $\lambda=5$ and, from top to bottom, $q=0.1, 0.5, 0.9$, and 1.

the system has no critical point except for $\mu_1=-\mu_2$. In that case, the critical point occurs at $T\to 0^+$ where

$$\xi \approx -\tfrac{1}{2}|1-2q|^{-1}(1-\tfrac{1}{3}|1-2q|^2-\tfrac{4}{45}|1-2q|^4+\cdots) ,$$

i.e., ξ diverges as $q\to\frac{1}{2}$ with $\nu=1$.

The rates (3.6b) and (3.6c) produce, instead, the following macroscopic behavior as $\mu_2\to\infty$:

$$U\approx -N(1-q)\mu_2 y^{-1}\to\infty ,$$

$$C_v\approx\{2J+\tfrac{1}{2}q\mu_1[1-\tanh^2(\beta\mu_1)][(1-x^2)x]^{-1}\}$$
$$\times Nk\beta^2(1-q)(y^2-1)y^{-3}\mu_2\to\infty ,$$

$$\sigma_U^2\approx\mu_2^2\{N[(1-q)^2+(1-q)qx^2]$$
$$\times x^{-2}(y^2-1)y^{-3}+N^2(1-q)qy^{-2}\}$$
$$+\mu_2\{2N(1-q)q(1-x)x^{-1}\mu_1$$
$$+4NJ(1-q)-2N^2(1-q)qy^{-2}\}\to\infty ,$$

$$M\approx Ny^{-1} ,$$

which remains finite,

$$\chi_T\approx\tfrac{1}{2}\beta[1-\tanh^2(\beta\mu_1)](1-x^2)^{-1}\sigma_M^2$$

and

$$\sigma_M^2\approx N(y^2-1)(xy^3)^{-1} ;$$

here

$$x\equiv(1-q)+q\tanh(\beta\mu_1)$$

and

$$y\equiv[1+e^{-4\beta J}(1-x^2)x^{-2}]^{1/2} .$$

When $\mu_2\to\infty$ and $q\to 1$ in such a way that $\mu_2(1-q)=\kappa$ remains finite, we get $h_e=\mu_1$, and it follows that

$$U=U^*-N\kappa y^{*-1} ,$$

$$C_v=C_v^*+Nk\beta^2\kappa(y^{*2}-1)y^{*-3}[2J+\mu_1/\tanh(\beta\mu_1)]$$

and

$$\sigma_U^2=\sigma_U^{*2}+N\kappa(y^{*2}-1)y^{*-3}[\kappa/\tanh(\beta\mu_1)$$
$$+2/\tanh(\beta\mu_1)+4J]$$
$$+\sigma_h^2 N[(y^{*2}-1)/y^{*3}\tanh(\beta\mu_1)+Ny^{*-2}] ,$$

where

$$\sigma_h^2\to\infty ;$$

here,

$$y^*\equiv[1+e^{-4\beta J}\{1-\tanh^2(\beta\mu_1)\}\tanh^{-2}(\beta\mu_1)]^{1/2}$$

and the other quantities with an asterisk represent the corresponding quantity for an equilibrium system under a field μ_1. That is, when the rates are (3.6b) or (3.6c) one obtains the equilibrium behavior expect for some additional terms having a simple interpretation.

Finally, concerning critical behavior, the system with rates (3.6b) has no critical point, except for $\mu_2>0>\mu_1$ when one essentially obtains cases considered before.

VI. COMPARISONS WITH RELATED SYSTEMS

The system we have studied here essentially differs from two related models of disorder studied before, namely, from the more familiar quenched and annealed versions of the random-field system.[20] This is illustrated in this section by comparing some one-dimensional versions of our system with some existing exact results for the two above-mentioned equilibrium cases. The (few) similarities one may draw in this way are only occasional, but their consideration is interesting enough. In fact, the comparison confirms the great influence dynamics may have on the steady-state properties of frustrated systems, and provides further indication of the possible relevance of the NERFM to the better understanding of disordered systems. It also seems worthwhile to mention that the main intrinsic difference between the quenched, nonequilibrium and annealed random-field models concerns the behavior of the random-field distribution. This distribution is, respectively, fixed in time, changing in time at random, i.e., as if the local fields were driven by some sort of completely random or infinite-temperature process, and evolving in such a way that it remains in equilibrium at temperature T with the other degrees of freedom. Only the latter, annealed case, where disorder seeks the most convenient (correlated) distribution, which minimizes its influence, e.g., by moving towards the interface and thus lowering the system free energy, seems physically rather unimportant for understanding the behavior of most real systems. We also describe, in this section, a nonequilibrium version of the diluted antiferromagnetic system under a uniform field, which, in light of an equilibrium result, one might expect to be equivalent in a sense[21] to the NERFM.

A. The quenched random-field Ising model

The familiar, quenched random-field model[14,20] is an equilibrium ferromagnetic Ising system whose spins at different sites experience random local magnetic fields h, which are independent and *spatially* assigned according to some distribution $p(h)$. Although this has been actively investigated, both theoretically and experimentally, exact solutions are partial at present.[22–25] Nevertheless, the original solution for a one-dimensional Ising model with random exchange energy (or a spin-glass model, as it is customarily known nowadays) under a uniform field[26] may easily be adapted to write an *implicit* solution for the one-dimensional random-field Ising model. This has been analyzed, among other authors, by Grinstein and Mukamel.[27] The latter have considered the particular case (to be denoted GMM in the following) in which the field is (spatially) distributed according to a distribution having mean μ and variance $\sigma_h^2=q\kappa^2$, namely,

$$p(h)=\tfrac{1}{2}q\delta(h-\mu_+)+\tfrac{1}{2}q\delta(h-\mu_-)+(1-q)\delta(h-\mu)\ ,\quad \mu_\pm=\mu\pm\kappa\ , \tag{6.1}$$

in the limit $\kappa\to\infty$. We quote this case because it is the only one we found comparable to one of our model cases here, particularly to our case in Sec. V C. In fact, those authors recognized that a main motivation for their study arose from the noted relevance of the parameter κ/J, a fact that has also been pointed out by us in Sec. V C.

In addition to the latter fact, we may compare the corresponding results for the spin-spin correlation function defined

$$g(r)\equiv[\langle\langle s_R s_{R+r}\rangle\rangle]_{\rm av}\ ,$$

where $[\langle\langle\cdots\rangle\rangle]_{\rm av}$ represents the double average in (2.6), i.e., the usual ensemble average plus the disorder average with respect to $p(h)$. Grinstein and Mukamel found that, for $z\equiv\tanh(\beta J)\neq 1$ $(T\neq 0)$ and $rz^r\ll 1$ (thus involving $T\to\infty$, in particular), one has

$$g(r)\sim(qr+1)(1-q)^r z^r$$

neglecting terms of order $r(1-q)^r z^{3r}$ or smaller. Our model is characterized by

$$g(r)=[\tanh(K_e)]^r\ .$$

In order to reduce this to a comparable result, we need to consider the rates (3.6d) (hard kinetics), and the distribution (6.1) with zero mean, $\mu=0$, and large enough values of κ (actually, any $\kappa>2J$). It then follows in the same limit as before that

$$g(r)\sim(1-q)^r z^r$$

neglecting terms of order $r(1-q)^r z^{r+2}$ or smaller. That is, the high-temperature nonequilibrium correlations for $d=1$ show a purely exponential behavior, unlike the one reported for the GMM. The correlation length, however, behaves the same way in both cases, namely,

$$\xi^{-1}\approx-\ln z-\ln(1-q)\ .$$

When $T=0$, on the other hand,

$$g(r)\sim\text{const}\times(1-q)^r$$

in the GMM and here

$$g(r)\sim[\tfrac{1}{2}(1-q)]^r\ .$$

Note that those similarities when $d=1$ between the random-field Ising model (or the GMM) and our nonequilibrium model cannot be extrapolated beyond the conditions stated above. In fact, the two models essentially differ, for instance, when the (nonequilibrium) kinetics is driven by rates other than (3.6d). For example, distribution (6.1) with $\mu=0$ and rates (3.6f) leads to $K_e\to K$ as $\kappa\to\infty$ (actually for $\kappa>8J$), which cannot be compared with the GMM case.

B. The annealed random-field system

We reach the same main conclusion when the nonequilibrium one-dimensional system is compared with a random-field Ising model (with $d=1$) whose impurities are not quenched but annealed, i.e., they have reached equilibrium with the other degrees of freedom instead of remaining frozen in.[20,28] This equilibrium situation may be defined via the partition function.

$$Z_N=\langle\langle Z_N(T,h)\rangle\rangle \equiv\int\prod_{j=1}^{N}dh_j p(h_j)\sum_{\mathbf{s}}\exp\{\beta\Sigma_i s_i[Js_{i+1}+h_i]\}\ . \tag{6.2}$$

It simply follows that Z_n may be written as

$$(C_h^2-S_h^2)^{(1/2)N}\equiv\zeta\ ,$$

where

$$C_h\equiv\langle\langle\cosh(\beta h)\rangle\rangle\ ,\quad S_h\equiv\langle\langle\sinh(\beta h)\rangle\rangle\ ,$$

and the average $\langle\langle\cdots\rangle\rangle$ is defined in (2.2), times the partition function of the familiar NN one-dimensional Ising model under a field Λ, with the latter satisfying

$$\tanh(\beta\Lambda)=S_h/C_h\ .$$

Then, given that ζ is independent of $\mathbf{s}$, it follows that the stationary solution $P^{\rm st}(\mathbf{s})$ for the annealed system equals the one for the Ising model under a field Λ, and also the one for the nonequilibrium system with rates (3.6a) (soft kinetics). That is, annealed configurational quantities such as the magnetization, its fluctuations, and spin-spin correlation functions are identical to the ones for those two mentioned systems, while thermal quantities such as U and C_v will, in general, differ essentially showing a strong dependence on $p(h)$ in addition to the one involved by Λ. For instance, when one defines ΔU as the difference between the energies for the nonequilibrium [with rates (3.6a)] and annealed systems, it follows for (6.1) that

$$\Delta U=\kappa\tanh(\beta\kappa)\quad\text{when } q=1\ ,$$

$$\Delta U\to 0\quad\text{as } T\to\infty\ ,$$

and

$$\Delta U\to\kappa\quad\text{as } T\to 0\ .$$

In any case, our comment at the end of the last section also applies here.

C. A nonequilibrium diluted antiferromagnetic system

It also seems valuable to recall that some of the interest on the random-field Ising model followed after the recognition[21] that it describes the practical case[29] of a diluted antiferromagnetic, whose spins are present at each lattice site with a probability that is independent of other spins, in a uniform field. Consequently, we ask ourselves whether such a relation also holds for nonequilibrium systems.

A nonequilibrium diluted antiferromagnet under a constant field may be modeled by following the philosophy in Sec. II. That is, we shall now consider a competing kinetics involving (only) different exchange energies, namely, having a distribution

$$g(J)=p\delta(J+J_0)+q\delta(J)\ , \tag{6.3}$$

with $p+q=1$ and $J_0>0$. This is essentially similar to a system considered before,[5] except that the existence of an external uniform magnetic field, h, is assumed here. That is, the original competing Hamiltonians have the structure (2.4), where J is a random variable with distribution (6.3) and $h=$const. For $d=1$ and $h\neq 0$, the condition (3.4) produces an effective Hamiltonian, which has the structure (3.11), only when the system is driven by the *soft* rates (3.6a). The corresponding parameters are

$$K_e=\tfrac{1}{4}\ln\left[\frac{p\exp(-2K_0)+q}{p\exp(2K_0)+q}\right]\ ,\quad h_e=h\ , \tag{6.4}$$

with $K_0\equiv\beta J_0$. This is precisely the effective Hamiltonian of our nonequilibrium random-field system in Sec. V B, i.e., under any field distribution

$$p(\mu+h)=p(\mu-h)\quad\text{with } \mu\neq 0\ ,$$

when the latter evolves with rates (3.6a) and has a coupling constant given by $J=-\frac{1}{2}J_0$. It may be proved that no other simple equivalence exists between those two nonequilibrium models. That is, the only relation occurs for rates (3.6a), which, as discussed before, devise a case that is almost identical to the (equilibrium) one considered in Ref. 21, a trivial case from the point of view of the present work.

VII. SOME EXACT RESULTS FOR ARBITRARY DIMENSION

The formalism outlined in Sec. III gives no significant explicit information concerning the model versions with $d>1$, except the following general theorem:[7] When an effective Hamiltonian exists and the transition rates are local having certain symmetry properties, which is the case, in particular, of the transition rates enumerated in Sec. III, the effective Hamiltonian necessarily has the NN Ising structure of the original one, e.g., (2.4). The problem is that, at the present stage, the practical computation of such an effective Hamiltonian involves the GDB condition, and one may prove this is not satisfied by any version of our model system when $d>1$. This has two main consequences. On the one hand, two- and three-dimensional versions of our model system cannot have the quasi-canonical behavior that we have discussed in Sec. III; actually, this is also the case for some one-dimensional versions, as indicated before. This seems to guarantee that those cases, where (3.4) does not hold, will be characterized by a full nonequilibrium behavior, which is even more interesting than the one in Sec. V. On the other hand, it follows that one needs to study the case $d>1$ by different methods.

Some interesting exact results concerning systems of arbitrary dimension may still be derived, for example, from the following theorem:[9] The rates in (2.1) may be written as

$$c(\mathbf{s}^r|\mathbf{s})=\tfrac{1}{2}c(\mathbf{r})\left[1-s_{\mathbf{r}}\sum_{\alpha}P_\alpha(\mathbf{r})s_\alpha\right]\ ,$$

where $c(\mathbf{r})>0$, the sum is over all possible different sets of spins α, one defines

$$s_\alpha=\prod_{r'\in\alpha}s_{r'}\ ,$$

and $P_\alpha(\mathbf{r})$ are real functions. Consequently, once $c(\mathbf{s}^r|\mathbf{s})$ is known, one gets

$$c(\mathbf{r})=2^{1-N}\sum_{\mathbf{s}}c(\mathbf{s}^r|\mathbf{s})$$

and

$$P_\alpha(\mathbf{r})=-\sum_{\mathbf{s}}s_\alpha s_r c(\mathbf{s}^r|\mathbf{s})\left[\sum_{\mathbf{s}}c(\mathbf{s}^r|\mathbf{s})\right]^{-1}\ .$$

Then, it may be shown that, when a given lower bound to the minimum possible value of $c(\mathbf{s}^r|\mathbf{s})$ is positive, namely, when

$$\delta\equiv\inf_r\left\{c(\mathbf{r})\left[1-\sum_\alpha|P_\alpha(\mathbf{r})|\right]\right\}>0\ , \tag{6.5}$$

the process is exponentially ergodic. This means that, for almost any probability measure $\mu\in\Omega$, it is $|\langle s_\alpha\rangle_{t,\mu}-\int dv\,s_\alpha|\le 2e^{-\delta t}$ indicating that the system will relax exponentially fast in time towards the invariant measure v.

Condition (6.5) may easily be checked in our systems. As an illustration, consider first the model for any distribution $p(h)$ driven by the *soft* rates (3.6a). The system is ergodic, so that there is a unique phase (and no phase transition is allowed), when $\beta<\beta_0$ where the latter satisfies

$$[1+\tanh(\beta_0 J)]^{2d}=2\{1+[\langle\langle\sinh(\beta_0 h)\rangle\rangle\langle\langle\cosh(\beta_0 h)\rangle\rangle]\}^{-1}\ .$$

When $p(h)=p(-h)$, the theorem leads to the same bounds as for the Ising model with zero field, i.e., $\beta_0 J=0.44$, 0.19, and 0.123 for $d=1$, 2, and 3, respectively. For $p(\mu+h)=p(\mu-h)$ with $\mu>0$, the bounds are the same as for the Ising model under field μ, i.e., $\beta_0 J=0.266$, 0.149, and 0.104 as d is increased. This confirms the fact that the rates (3.6a) generally induce a rather trivial case in the present problem. When the system with

$$p(h)=\tfrac{1}{2}q\delta(h-\kappa)+\tfrac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h)$$

is driven by the *hard* rates (3.6d), β_0 is defined as follows:

$$|q[T_2^+-T_2^-]+2(1-q)t_2|=2\ ,$$

$$|q[T_4^++2T_2^+-T_4^--2T_2^-]+2(1-q)[t_4+2t_2]|+|q[T_4^+-2T_2^+-T_4^-+2T_2^-]+2(1-q)[t_4-2t_2]|=4\ ,$$

and

$$3|q[T_6^+ +4T_4^+ +3T_2^+ -T_6^- -4T_4^- -3T_2^-] +2(1-q)[t_6+4t_4+3t_2]| +16|q[T_6^+ -3T_2^+ -T_6^- +3T_2^-] +2(1-q)[t_6-3t_2]| +3|q[T_6^+ -4T_4^+ +5T_2^+ -T_6^- +4T_4^- -5T_2^-] +2(1-q)[t_6-4t_4+5t_2]|=32 ,$$

where

$$T_n^{\pm} \equiv \tanh[\beta_0(\kappa \pm nJ)] \text{ and } t_n \equiv \tanh(n\beta_0 J) ,$$

for $d=1$, 2, and 3, respectively.

It also seems worthwhile to mention that the model behavior may be represented at $T=0$ by simple random cellular automata. For example, the latter case of *hard* rates (3.6d) and a distribution

$$p(h)=\tfrac{1}{2}q\delta(h-\kappa)+\tfrac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h)$$

is equivalent at $T=0$ to the cellular automaton

$$c(\mathbf{s}'|\mathbf{s})=1-s_r\sigma_1[1+\tfrac{1}{2}q\theta(\lambda_1)] ,$$

$$1-s_r[x(\sigma_1+\sigma_2)+y(\sigma_1\tau_2+\sigma_2\tau_1)] ,$$

and

$$1-s_r[z\Sigma_l\sigma_l+w(\tfrac{1}{2}\Sigma_{l,m}\sigma_l\tau_m+\sigma_1\sigma_2\sigma_3) +v\Sigma_{l,m,n}\sigma_l\tau_m\tau_n]$$

with $l,m,n=1,2,3$, $l\neq m\neq n$, for $d=1$, 2 and 3, respectively. The following notation has been used here to shorten formulas:

$$\sigma_1\equiv\tfrac{1}{2}(s_{\mathbf{r+i}}+s_{\mathbf{r-i}}) , \quad \sigma_2\equiv\tfrac{1}{2}(s_{\mathbf{r+j}}+s_{\mathbf{r-j}}) ,$$

$$\sigma_3\equiv\tfrac{1}{2}(s_{\mathbf{r+k}}+s_{\mathbf{r-k}}) , \quad \tau_1\equiv s_{\mathbf{r+i}}s_{\mathbf{r-i}} ,$$

$$\tau_2\equiv s_{\mathbf{r+j}}s_{\mathbf{r-j}} , \text{ and } \tau_3\equiv s_{\mathbf{r+k}}s_{\mathbf{r-k}} ,$$

where $\mathbf{r}\pm\mathbf{r}'$ ($\mathbf{r}'=\mathbf{i}$, $\mathbf{j}$, or $\mathbf{k}$) represent NN's of site $\mathbf{r}$,

$$x\equiv\tfrac{1}{8}\{4+q[2+2\theta(\lambda_1)+\theta(\lambda_2)]\} ,$$

$$y\equiv\tfrac{1}{8}[-2-2q\theta(\lambda_1)+q\theta(\lambda_2)] ,$$

$$z\equiv\tfrac{1}{8}\{4+\tfrac{1}{4}q[3\theta(\lambda_1)+4\theta(\lambda_2)+\theta(\lambda_3)]\} ,$$

$$w\equiv\tfrac{1}{8}[-4-3q\theta(\lambda_1)+q\theta(\lambda_3)] ,$$

and

$$v\equiv\tfrac{1}{8}\{1+\tfrac{1}{4}q[5\theta(\lambda_1)-4\theta(\lambda_2)+\theta(\lambda_3)]\} ;$$

$$\theta(X)\equiv\Theta(X)[\Theta(-X)-2] ,$$

where $\Theta(X)=0$ for $X<0$ and $\Theta(X)=1$ for $X\geq 1$, and $\lambda_n\equiv\kappa/2J-n$. By using condition (6.5) here, it follows that, at $T=0$, the system with $\kappa>6J$ is ergodic for any $q>0$ when $d=1$, for any $q>\frac{1}{3}$ when $d=2$, and for any $q>\frac{23}{31}$ when $d=3$.

VIII. CONCLUSIONS

This paper introduces a lattice interacting-spin (or particle) model system whose time evolution is stochastic because of a competing spin-flip (or creation-annihilation) kinetics, which, in addition to the usual heat bath, involves a random external magnetic field (or chemical potential). The competition induces a kind of dynamical frustration that might be present in real disordered systems such as the class of random-field materials. It may also be implemented in the laboratory, e.g., by exposing a magnet to a field that is continuously varying according to $p(h)$ with a period much shorter than the mean time between successive transitions modifying the spin configuration. This will, in general, drive the system asymptotically towards a nonequilibrium steady state, thus producing a situation that crucially differs from those in the annealed and quenched random-field model systems. In fact, while the local field is randomly assigned in space according to a distribution $p(h_r)$, which remains frozen in for the quenched case, and $p(h_r)$ contains essential correlations in the annealed system, where the impurity distribution is in equilibrium with the spin system, our case is similar to the quenched system *at each time* during the stationary regime, but h_r keeps randomly changing with time, also according to $p(h)$, at each site. Consequently, while frustration and randomness turn out to be rather unimportant in the annealed case, they are fundamental for the behavior of the nonequilibrium system in a way, however, which is expected to produce macroscopic differences with the quenched case.

We present exact solutions for some of the model versions when the lattice dimension is $d=1$. They are based on previously derived theorems that state that, when the effective transition rate (2.2) satisfies certain symmetry conditions, including the global detailed balance property (3.4), the system may be represented by a short-ranged effective Hamiltonian. In fact, the theorems assert that the effective Hamiltonian is of the nearest-neighbor Ising type when this is the structure of the original series of "Hamiltonians" (2.4) involved by the elementary transition rates, which are always assumed to be local and canonical in the usual sense. That is, for certain transition rates and distributions $p(h)$, our one-dimensional model turns out to be *quasicanonical,* with the effective Hamiltonian parameters reflecting the action of some imaginary agent that aims to represent the influence of kinetics on the steady-state properties. That constraint is, however, non-Hamiltonian, in general, given that one may prove that the global detailed balance condition (3.4) does not hold for other one-dimensional cases nor when $d>1$. It is true that our theorems do not exclude the existence for those cases of a short-ranged Hamiltonian simply defined via Eqs. (3.1)–(3.3), but the system would then lack the canonical feature (3.4). Instead, we have investigated lattices with arbitrary kinetics and dimension by two different methods, namely, by requiring that a given bound to the transition rates is positive, which leads to a bound region of the phase diagram where the system is necessarily ergodic, and by providing simple representations of the system ground state, which may be useful for further investigations.

The main global conclusion from our analysis is that the system behavior, including critical phenomena, is amazingly rich and certainly different from the one in familiar equilibrium situations, even for $d=1$. This is illustrated, in particular, by Figs. 3 and 4. Moreover, here nonuniversal behavior seems the rule, i.e., model parameters usually exist that are relevant or marginal in the sense of renormalization-group theory, and equilibrium features such as the fluctuation-dissipation theorem do not hold, in general. As a specific example of that, our model, unlike the standard random-field Ising model, may present a zero-temperature critical point under some circumstances when $d=1$.

Once features (2.1)–(2.4) are given (note, however, that one could also be interested in a competition between two or more different "Hamiltonian" structures), the model conduct is dominated by details of kinetics such as the form of the functions $p(h)$ for the field distribution and $c(\mathbf{s}^r|\mathbf{s};h)$ for the transition rates. This is reflected in microscopic properties, e.g., (3.4) only holds for certain pairs of functions $p(h)$ and $c(\mathbf{s}^r|\mathbf{s};h)$, and it is also observable. In fact, the study of the cases in which (3.4) holds naturally leads to a classification of the functions that are more familiar in the literature as specific realizations for the elementary rates, namely, one may distinguish *soft* rates, such as (3.6a), (3.6b), and (3.6c), where the function $f_r(\mathbf{s};h)$ defined in (3.5) factorizes with the dependence on the field separated from that on the spin configuration and *hard* rates, such as (3.6d) and (3.6e), where those dependences cannot be separated from each other. As a further indication of the complex behavior our class of systems with competing kinetics may present, note that the recent study of a different type of nonequilibrium impure system,[5] where the competition is between exchange energies and no external field is acting on the system, required a classification of transition rates based on their asymptotic properties. No doubt this and other questions raised in the study of those systems need to receive further investigation from a more general point of view.

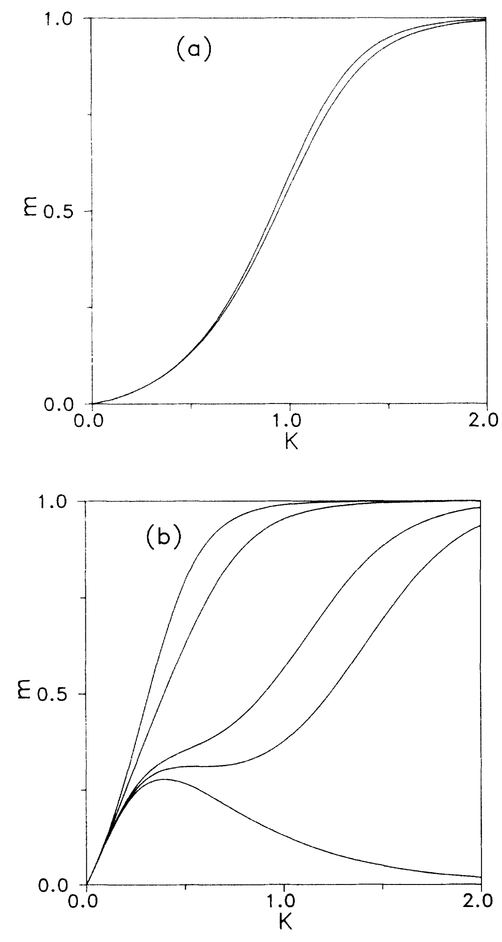

FIG. 3. The magnetization vs K for Glauber rates (3.6b) (soft kinetics), as described in Secs. IV and V. (a) For $p(h)=\frac{1}{2}q\delta(h+0.2J)+\frac{1}{2}q\delta(h-0.4J)+(1-q)\delta(h-0.1J)$, corresponding to the case in Sec. V C when $\mu=0.1J$ and $\kappa=0.3J$, where $q=0.1$ (upper curve) and $q=1$ (lower curve). (b) The same for $\mu=J$, $\kappa=3J$ and, from top to bottom, $q=0.1$, 0.5, 0.9, 0.95, and 1.

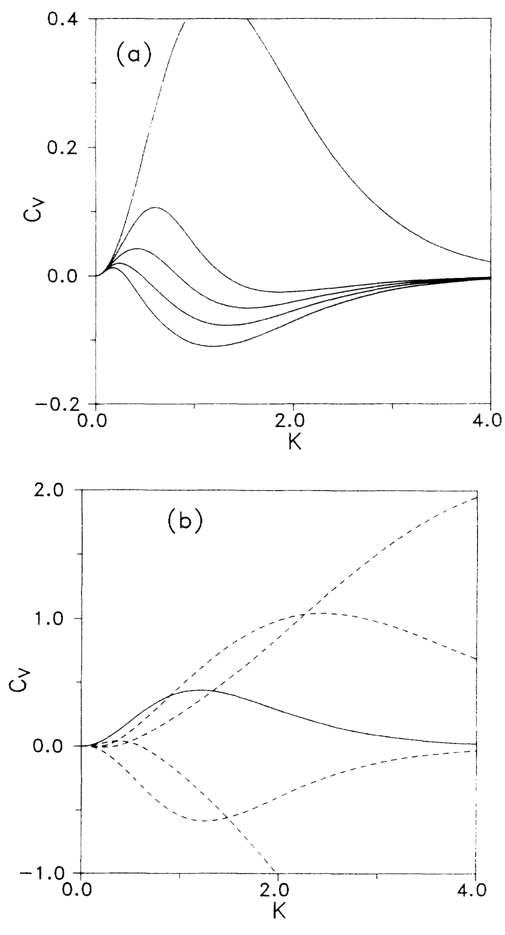

FIG. 4. The specific heat C_v (normalized to Nk_B), as defined in Sec. IV, as a function of K in the case of hard kinetics. (a) For Kawasaki rates (3.6d) when $\lambda=1.5$ and, from top to bottom, $q=0$, 0.25, 0.5, 0.75, and 1. (b) For the modified Metropolis rates (3.6f); the solid line is for $q=0$, while the dashed lines are for $q=0.75$ and $\lambda=3$, $q=0.75$ and $\lambda=2.5$, $q=0.25$ and $\lambda=1.5$, and $q=1$ and $\lambda=2$, respectively, from top to bottom at $K=1$.

In the present case, the classification of rates is motivated by the important consequences the formal distinction mentioned above has both on the global detailed balance condition and on the form for the effective parameters defined in (3.11). That is, (3.4) holds for any $p(h)$ when rates are *soft*, and we then get $K_e=K$ and h_e given, in general, by a complex function of T, $p(h)$, and $c(\mathbf{s}^r|\mathbf{s};h)$. That function is such that $h_e=0$ when $p(h)=p(-h)$, and the system then reduces to an equilibrium system, except for energy fluctuations. In that sense, the simplest situation occurs for the rates (3.6a) used by van Beijeren and Schulman in a different nonequilibrium problem;[15] except for some excess fluctuations, those rates are seen to reduce our system with any distribution

$$p(\mu+h)=p(\mu-h)\ ,$$

even with

$$\mu\neq 0\ ,$$

to the pure Ising model, and we also found for (3.6a) the only similarity (though a loose one) between the nonequilibrium and annealed cases. When the rates are *hard*, the situation is essentially different, beginning with the facts that one has here that (3.4) only holds for even distributions $p(h)$, K_e is a complex function of the model parameters, and $h_e=0$, the latter as a direct consequence of $p(h)=p(-h)$.

The above facts are strongly reflected in critical properties. Given that, on the assumption (2.4) and (3.4), the effective Hamiltonian always has the structure (3.11), the existence of a critical point characterized by condition (4.12), requires that one of the following situations occur: (i) $h_e=0$ (as a consequence of the model features) and $K_e\to\infty$ as $\beta\to\infty$. This is the familiar *path* to the zero-temperature critical point of the pure Ising model; it occurs typically, e.g., for rates (3.6a), when $p(h)=p(-h)$. (ii) $h_e\to 0$ and $K_e\to\infty$, both as $\beta\to\infty$, which has no equilibrium counterpart. This may only occur for *soft* rates, where $h_e\neq 0$, in general. Excluding the "trivial" (cf. the preceding paragraph) case (3.6a), *soft* rates indeed produce an interesting critical behavior of that kind, with two different types of critical phenomena, including the possibility of a critical line, when $p(\mu+h)=p(\mu-h)$. We have revealed that fact in Sec. V C for

$$p(h)=\tfrac{1}{2}\delta(h-[\mu+\kappa])+\tfrac{1}{2}\delta(h-[\mu-\kappa])\ \text{ as } \mu\to 0\ .$$

(iii) It may also occur that K_e has a more complex dependence on temperature, and neither of the two above situations, (i) or (ii), arises. The limit $\beta\to\infty$ then leads to several different situations, i.e., $\beta_e\equiv K_e/J\to\beta^0$, where β^0 is positive (the strong disorder maintains the system *hot* enough at $T=0$ to preclude any critical behavior), negative (the field competition produces an effective antiferromagnetic situation) or infinite (corresponding again to a zero-temperature critical point), depending on the model version. This occurs typically for *hard* rates when the distribution is even, e.g.,

$$p(h)=\tfrac{1}{2}q\delta(h-\kappa)+\tfrac{1}{2}q\delta(h+\kappa)+(1-q)\delta(h)\ ,$$

as described in Sec. V D and in Figs. 1 and 2. It seems remarkable, in particular, the crucial role played by the typical deviation κ, or the way it is related to μ and J, in producing dramatic changes of the observable properties.

We are presently analyzing by other methods those versions of the model in this paper for which condition (3.4) does not hold.

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