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Resonant tunneling in an interacting one-dimensional electron gas

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We study the resonant tunneling of a single-channel interacting electron gas through a double-barrier structure. In striking contrast to the noninteracting electron gas, which exhibits resonances with a temperature-independent Lorentzian line shape at low T, we find that with repulsive interactions present the resonances have a width which vanishes as $T\rightarrow 0$. Moreover, at low T the resonance line shapes are determined by a universal scaling function, with power law but non-Lorentzian tails. These predictions should be accessible to experiments in single-channel wires in gated GaAs.

The recent observation[1-3] of oscillations in the conductance through a quantum dot as a function of gate voltage has been attributed to the Coulomb blockade.[4,5] The Coulomb barrier which impedes the electron from passing through the dot vanishes when the chemical potential (i.e., gate voltage) on the dot is tuned to match precisely the energy cost to add an extra electron to the dot. In this situation, there is a state on the dot at the Fermi energy through which the electrons may resonantly tunnel. Recent theories of this phenomenon[6] have focused on the importance of the electron interactions on the quantum dot which give rise to the Coulomb barrier. A Landauer[7] type of theory was developed, in which the electrons in the leads are treated as noninteracting. In a recent paper,[8] it was shown that when the leads are one dimensional (1D), the electron interactions in the *leads* have a profound effect on the transport across a single barrier.

In this paper we study the more interesting case of a single-channel 1D wire with a *double*-barrier constriction. In this case, a model of noninteracting electrons (or a Fermi liquid) predicts resonances in the transmission with a Lorentzian line shape as a function of incident energy. As we show below, though, in the presence of electron interactions, which destabilizes the Fermi liquid in one dimension leading to a Luttinger liquid, the resonance line shapes and temperature dependence are modified *qualitatively*. Specifically, we find resonances with *non*-Lorentzian line shapes which have a width which vanishes as $T\rightarrow 0$. Differing qualitatively from the noninteracting case, such resonances, if detected in gated GaAs wires, would provide the first experimental evidence for non-Fermi-liquid behavior in the 1D interacting electron gas. In practice, backscattering in the 1D wire away from the double barrier might cause complications. However, this "spurious" backscattering can be minimized by applying a strong magnetic field, which spatially separates right- and left-moving electrons. Since such a field will also spin polarize the electrons, we focus below on the case of spinless electrons.

A spinless, single-channel electron gas can be described as a Luttinger liquid,[8,9] characterized by a dimensionless two-terminal conductance g. The noninteracting electron gas corresponds to $g=1$, whereas $g>1$ for attractive interactions. For repulsive electron interactions, g is given roughly by the expression,[8] $g^2\simeq(1+U/2E_F)^{-1}$, where U is the (screened) Coulomb interaction between neighboring electrons and E_F is the Fermi energy. The ratio U/E_F is proportional to r_s, the electron spacing divided by the Bohr radius, so that as the electron density is decreased, g also decreases. In Ref. 8 it was argued that for repulsive interactions ($g<1$) a Luttinger liquid at $T=0$ is completely reflected by a single barrier,[10,11] i.e., the conductance across the barrier is zero. Despite this, we argue below that such a Luttinger liquid incident upon a symmetric double-barrier structure can exhibit perfect resonant transmission, provided the electron density is not too low. Our central results for this case are summarized in Fig. 1. For very low electron densities, to the left of the shaded region in the figure, the resonant tunneling is suppressed at $T=0$, however, as in the case of a single barrier there will be power-law corrections at $T\neq 0$. For higher electron densities, in the shaded region in the figure, we find resonances with perfect transmission, so that the conductance on resonance, denoted G^*, is $G^*=ge^2/h$. In this case, the resonances are predicted to be *infinitely sharp* at $T=0$, in striking contrast to the res-

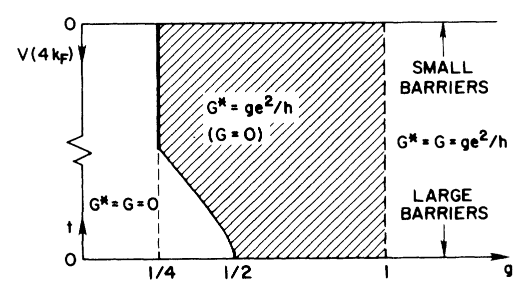

FIG. 1. Phase diagram for resonances of a spinless 1D interacting electron gas incident on a double-barrier structure. Here G^* and G refer to the conductance at $T=0$ on resonance and off resonance, respectively. Resonances occur in the shaded region for repulsive interactions $g<1$, and on the thick solid line at $g=\frac{1}{4}$ for small barriers, where G^* varies continuously.

onances for noninteracting electrons. At nonzero temperatures the resonances are broadened with a width which varies as a nontrivial power of temperature: T^{1-g}. Moreover, the resonance line shapes at finite temperature have *non-Lorentzian* tails, falling off with a power $(2/g)$ *larger* than 2. For g between $\frac{1}{4}$ and $\frac{1}{2}$, as the strength of the double barrier is increased, the resonance disappears at a sharp $T=0$ transition, which is in the Kosterlitz-Thouless[12] universality class.

Employing a bosonized representation for the Luttinger liquid, we analyze the transport through a double-barrier structure perturbatively in the limits of very weak scattering and very strong barriers. We assume the separation between the two barriers, d, is small and focus on temperatures low compared to $\hbar v_F/d$, where v_F is the Fermi velocity. At larger temperatures, backscattering from the two barriers will not add coherently, so they will behave essentially as two single barriers in series. In the small barrier limit, we consider scattering from a weak potential $V(x)$. For noninteracting electrons, the Born approximation can be used in this limit, and the condition for perfect resonant transmission is simply that the Fourier transform of the potential at $2k_F$ vanishes: $V(2k_F)=0$. For a symmetric potential $[V(x)=V(-x)]$, $V(2k_F)$ is real, so that the resonance condition may be reached by tuning one parameter, e.g., the wave vector k_F via a gate voltage V_G. For an asymmetric potential, two parameters must be tuned to achieve a "true" resonance, that is, a resonance which has *perfect* transmission. In the presence of interactions, a similar analysis may be performed, however, it is necessary also to account for scattering at $4k_F$, $6k_F$, etc. An effective Lagrangian $\mathcal{L}=\mathcal{L}_0+\sum_{n=1}^{\infty}\mathcal{L}_{2n}$ may be derived along the lines of Ref. 8, perturbative in $V(x)$, with

$$\mathcal{L}_0=(1/2g)\int_x(\partial_\mu\theta)^2\,, \tag{1a}$$

$$\mathcal{L}_{2n}\sim V(2nk_F)\cos[2n\sqrt{\pi}\theta(x=0,\tau)]\,. \tag{1b}$$

The pure Luttinger liquid Lagrangian in (1a) describes a fixed point under a renormalization-group (RG) transformation. As found in Ref. 8, $2k_F$ scattering grows under the RG for all $g<1$ and is thus a relevant perturbation. However, on resonance, $V(2k_F)=0$. The behavior will then be determined by the next most relevant parameter, namely, $V(4k_F)$. The leading order renormalization-group flow equation for $v\equiv V(4k_F)$ is $dv/dl=(1-4g)v$. Thus, for $g<\frac{1}{4}$, $4k_F$ scattering is relevant and grows stronger at low energies. For $g>\frac{1}{4}$, weak $4k_F$ scattering is irrelevant, and the conductance may be calculated perturbatively. To leading order we find a two-terminal[8] conductance

$$G=ge^2/h-c[V(4k_F)]^2T^{2(4g-1)}\,, \tag{2}$$

so that for $g>\frac{1}{4}$ and at zero temperature *perfect* transmission is predicted on resonance, as shown in the figure. For $g=\frac{1}{4}$, there is a fixed line, and the conductance on resonance, G^*, may take on any value between zero and one. This is reminiscent of the behavior of the single-barrier problem for the noninteracting case, $g=1$.

In the opposite limit, of very strong barriers, we consider two semi-infinite leads which are very weakly connected to an island (or "dot"). We model the island by supposing that the Coulomb energy on the island is very large, so that away from resonance, the charge on the island is fixed, and transmission is only possible by virtual tunneling through the Coulomb barrier. As the chemical potential is tuned through resonance, there will be two charge states on the island, differing by one electron, which become degenerate. The island is thus modeled as a two-level system in which hopping on and off the island corresponds to switching back and forth between the two levels. The partition function may be expanded perturbatively in powers of a weak hopping matrix element, t, which connects the island to the leads. The bosonic fields describing the leads [i.e., $\theta(x,\tau)$ in Eq. (1)] may be integrated out, and we are left with a one-dimensional statistical-mechanics problem of interacting charges, which represent hopping events across one of the two barriers. The hopping matrix element t plays the role of the fugacity of these "charges," and the Luttinger liquid leads mediate a logarithmic interaction between them. If $q_i=\pm1$ denotes the charge transferred to the right in a hopping event and $r_i=\pm1$ denotes the change in the charge on the island, the partition function in this Coulomb gas representation can be written as

$$Z=\sum_n\sum_{\{q_i,r_i\}}t^{2n}\int_0^\beta d\tau_{2n}\cdots\int_0^{\tau_2}d\tau_1\exp\left[\frac{1}{2g}\sum_{i<j}(Kr_ir_j+q_iq_j)\ln(\tau_i-\tau_j)/\tau_c\right]\,, \tag{3}$$

where $\tau_c\simeq E_F^{-1}$ is a short-time cutoff. The parameter K is initially equal to 1, however, its value is renormalized. We analyze the above model as a perturbative RG in t, following closely the treatment of the Kondo problem by Anderson, Yuval, and Hamann.[13] The leading order flow equations are

$$dK/dl=-8Kt^2\,, \tag{4a}$$

$$dt/dl=t[1-(1+K)/4g]\,. \tag{4b}$$

The consequences of these flow equations are depicted in the lower part of the figure. For $g>\frac{1}{2}$, (4b) shows that t is a relevant perturbation. This is consistent with our analysis in the small $V(x)$ regime, and it is highly plausible that the flows join together, giving us perfect resonant transmission for $g>\frac{1}{2}$ (shaded region in the figure). For $g<\frac{1}{4}$, on the other hand, t is irrelevant, and flows to zero, implying perfect reflection at zero temperature (see Fig. 1). For $\frac{1}{4}<g<\frac{1}{2}$, as t is increased, a separatrix at t^* is crossed which separates flows to $t=0$ from flows to large t. The separatrix flows into a Kosterlitz-Thouless fixed point[12] on the $t=0$ line, with a critical value $K_c=4g-1$. Thus in this range of g's, as t is decreased the resonance will disappear at a sharp Kosterlitz-Thouless transition.[12] Right at this transition, when $t=t^*$, the conductance is zero. For the special case

$g=\frac{1}{4}$, $K_c=0$, and the flows for $t>t^*$ terminate on a $K=0$ fixed line along which the conductance will vary continuously. Thus for $g=\frac{1}{4}$, the conductance on resonance, G^*, will increase continuously from zero for $t>t^*$. (The precise value of t^* for $g=\frac{1}{4}$ is not perturbatively accessible.) The fixed line for $g=\frac{1}{4}$, which is entirely consistent with the fixed line that we found in the small $V(x)$ limit, is shown as a thick line in the figure.

The above analysis can be generalized to allow for asymmetric barriers, with $t_1 \neq t_2$. We find that this asymmetry is a relevant perturbation, which destroys the resonance for all $g<1$. Thus the only resonances which survive repulsive electron interactions are the "true" resonances which have perfect transmission in the noninteracting case.

The above discussion has focused on the conductance through a double barrier when precisely on resonance and at $T=0$. A more crucial issue experimentally is the width of the resonances and the line shapes of the resonance peaks. Consider then the shaded region in the figure, where resonances with perfect transmission are present. Precisely on resonance, the RG flows are toward the fixed point described by Eq. (1) in which $V(2nk_F)=0$ for all positive integers n. Since all scattering except the $2k_F$ scattering is irrelevant, the resonance condition is that the *renormalized* value of $V(2k_F)=0$. As the chemical potential of the dot (or the gate voltage, V_G) is moved slightly off resonance, the conductance will be determined by the behavior of this single relevant parameter as it flows away from that unstable fixed point. Near resonance, the initial value of this parameter will be proportional to the distance from resonance, $V(2k_F, l=0) \sim \delta \equiv V_G - V_G^*$. Under renormalization the $2k_F$ scattering grows as $dV(2k_F)/dl=(1-g)V(2k_F)$. Associated with this relevant direction there is a single critical time scale which diverges as $\delta \to 0$ as $\delta^{-1/(1-g)}$. From this we deduce a characteristic frequency scale, denoted Ω, which vanishes as $\Omega \sim \delta^{1/(1-g)}$. Near the resonance for small δ the conductance at finite temperatures should depend only on the ratio T/Ω. More specifically, one expects the conductance for small T and δ to be described by a *universal scaling function:*

$$G(T,\delta)=\tilde{G}_g(c\delta/T^{1-g}) , \tag{5}$$

where c is a nonuniversal dimensionful constant. For larger δ or T, the irrelevant parameters will provide corrections to this scaling form. For instance, there will also be a dependence in Eq. (5) on $V(4k_F)T^{(4g-1)}$, which, however, vanishes in the zero-temperature limit.

The scaling function $\tilde{G}_g(X)$, which is a symmetric function of its argument, $X=c\delta/T^{1-g}$, can be calculated perturbatively in two limits. An expansion for the conductance to second order in $V(2k_F) \sim \delta$ at finite temperature gives

$$\tilde{G}_g(X)=G^*[1-X^2+O(X^4)] , \tag{6a}$$

where the conductance on resonance is $G^*=ge^2/h$. The behavior of $\tilde{G}_g$ for large X, can be obtained by matching onto the flows into the stable fixed point which describes reflection from a single large barrier. As shown in Ref. 8, off resonance the conductance vanishes as $G \simeq t_{\text{eff}}^2 T^{2(1/g-1)}$. Requiring that the form in (5) matches onto this implies that as $X \to \infty$,

$$\tilde{G}_g(X) \sim X^{-2/g} . \tag{6b}$$

For intermediate values of X, although $\tilde{G}_g(X)$ is not perturbatively accessible, it should be a universal function, depending only on the dimensionless lead conductance g.

The above considerations show that at low temperatures the resonance peaks should have a temperature-dependent width which scales as T^{1-g}. Moreover, rescaled data from different temperatures should collapse onto the same universal curve. The line shapes of the peaks are predicted to be *non-Lorentzian,* with tails which fall off as $\delta^{-2/g}$. Since the resonance peaks are only present for $\frac{1}{4}<g<1$, this exponent will be between 2 and 8. Note that in the limit of noninteracting electrons, $g=1$, the line shape is Lorentzian and temperature independent, as expected. All of these features can, in fact, be confirmed explicitly[14] in an *exact* solution of Eq. (1) which is possible when $g=\frac{1}{2}$, and is also consistent with recent numerical[15] work on the spin-$\frac{1}{2}$ Heisenberg chain with a single impurity.

For a 1D channel with a finite length L which is joined at the ends to metallic (Fermi-liquid) leads, the predicted power-law behavior will be cut off[8] below a temperature $T_L=\hbar v_F/k_B L$. Specifically, the resonance line shapes will cease to sharpen up below this crossover temperature. In addition, at low temperatures a finite applied voltage will serve as a cutoff. Thus, at $T=0$, the I-V curves near resonance (for $V>k_B T_L/e$) should satisfy a scaling form, $I/V=\tilde{G}_g^V(\delta/V^{1-g})$. Although $\tilde{G}_g^V(X)$ should be a *universal* function with the same small and large X dependences as $\tilde{G}_g(X)$ in (5), the two functions will in general be different.

For asymmetric barriers, since two parameters must be tuned to achieve resonance, it is likely that as V_G is varied, $V(2k_F)$ does not go through zero, but exhibits a minimum $\delta_{\min}=\min|V(2k_F)|$, at some value V_G^*. In this situation, δ in Eq. (5) should be replaced by $\bar{\delta}$, proportional to δ for large δ, but approaching a constant, $\delta_{\min}$, as $\delta \to 0$. The "resonance" peak will then ultimately vanish at zero temperature, but at finite temperatures, it will have a temperature dependence determined by $G_{\text{peak}}=\tilde{G}_g(c\delta_{\min}/T^{1-g})$.

Finally we note that in addition to having the resonance condition $V(2k_F)=0$ satisfied, one could in principle simultaneously have $V(4k_F)=0$, though generically this would require careful tuning of four parameters. Nonetheless, this would correspond to a "higher-order resonance," in which perfect transmission would occur for all $g>\frac{1}{9}$. Determining the phase diagram for these higher-order resonances is left for future work.

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