Potential and current distribution in an ideal Hall bar

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The local potential and current distributions in a confined ideal two-dimensional electron gas with perpendicular magnetic field are calculated. The nondissipative current-carrying state of the system is described by a generalized equilibrium density operator appropriate for the boundary conditions. The Coulomb interaction of the electrons is taken into account in Hartree approximation. The results obtained by numerical computation allow the interpretation of seemingly contradictory experimental data.

I. INTRODUCTION

It is well known that the *global* Hall conductivity of a confined system of noninteracting electrons can be expressed in terms of edge states. In contrast to this the *local* potential and current distributions are determined by both the edge states and the bulk states. Obviously, we are facing here a problem of self-consistency: on the one side the distributions are determined by all quantum-mechanical states which, on the other side, depend on the distributions (e.g., via the confining potential).

In the past, several attempts have been made to calculate the local quantities in ideal Hall bars. Some of these are restricted to fully occupied Landau levels[1–4] and others omit the self-consistent treatment of the screening as a function of the filling factor,[5,6] the importance of which was pointed out in Refs. 7 and 8.

The basic assumption of the present work is a generalized equilibrium density operator which takes into account the conservation of the total momentum in the direction of the Hall current maintained by periodic boundary conditions (Sec. II). The corresponding single electron density operator is significantly different from the velocity-shifted Fermi operator postulated by Heinonen and Taylor[9,10] which is nonstationary, since, in contrast to the momentum, the velocity is not a conserved quantity. At zero temperature the distribution operator used in this work reduces to the one introduced by Li and Thouless.[11] Using the generalized equilibrium density operator, we calculate the effective Coulomb potential (Sec. III). By identifying the difference of the calculated potentials at any two sample sites with the local voltage drop we can compare our results with experiments[12–14] and interpret apparently contradictory data (Sec. IV). The calculations show that the current density consists of two parts: a large diamagnetic current which is concentrated at the edges of the sample and does not contribute to the total current, and a current which is distributed over the whole sample (Sec. V). Whether the *net* current is dominated by edge or bulk contributions depends on the value of the filling factor.

II. THE MOMENTUM ENSEMBLE

We consider an ideal interacting two-dimensional electron gas in a perpendicular magnetic field $\mathbf{B}$. The system is assumed to be confined in the x direction by a potential $V_{\mathrm{conf}}(x)$ and to be translational invariant in the y direction. To allow for a net current along the y axis we impose for the wave functions $\Psi(x,y)$ the periodic boundary conditions $\Psi(x,y+L_y)=\Psi(x,y)$. The system length L_y is assumed to be large compared to the effective system size L_x brought about by the confinement, i.e., $L_x \ll L_y$.

Since the system is ideal, no dissipation occurs and, therefore, it can be described by an equilibrium density operator. In the construction of this operator all additive conserved quantities have to be taken into account. In our case these quantities are the total energy (Hamiltonian) $\hat{\mathcal{H}}$, the number of electrons $\hat{\mathcal{N}}$, and the total momentum in the y direction $\hat{\mathcal{P}}_y$ (script letters denote additive many-electron quantities, e.g., $\hat{\mathcal{H}}=\sum_i \hat{H}_i$). Fixing their average values, the stationary density operator corresponding to maximal entropy is

$$\hat{\rho}^v = \frac{1}{\mathcal{Z}^v}\; e^{-\beta(\hat{\mathcal{H}}+v\hat{\mathcal{P}}_y-\mu\hat{\mathcal{N}})}, \qquad \mathcal{Z}^v = \mathrm{Tr}\; e^{-\beta(\hat{\mathcal{H}}+v\hat{\mathcal{P}}_y-\mu\hat{\mathcal{N}})}. \tag{1}$$

The value of the parameter (Lagrange multiplier) v is fixed by the total current in the y direction,

$$I = -\frac{e}{L_y}\mathrm{Tr}\,(\hat{\mathcal{V}}_y\hat{\rho}^v) = e\,v\,\frac{N}{L_y}, \tag{2}$$

where $N=\mathrm{Tr}(\hat{\mathcal{N}}\hat{\rho}^v)$ is the average number of electrons in the system and $\hat{\mathcal{V}}_y$ is the y component of the velocity operator. For $v=0$ the density operator $\hat{\rho}^v$, which characterizes a momentum ensemble, reduces to the usual grand canonical operator $\hat{\rho}^0$. Formally, the momentum ensemble can be generated from the grand canonical ensemble by a Galilei transformation to a system of reference (laboratory system) moving with velocity $\mathbf{v}=(0,\,-v)$ relative to the rest frame,

$$\hat{\mathcal{H}} \to \hat{\mathcal{H}}^v = \hat{\mathcal{H}} + v\hat{\mathcal{P}}_y. \tag{3}$$

The drift motion with constant velocity $\mathbf{v}$ gives rise to a constant electric field $\mathbf{E} = (E, 0)$,

$$E = -\frac{v}{c} B \tag{4}$$

in the rest frame. Note that the momentum ensemble and the canonical ensemble in the presence of the electric field $\mathbf{E}$ characterized by the density operator

$$\hat{\rho}^E = \frac{1}{Z^E} e^{-\beta(\hat{\mathcal{H}}+eE\hat{\mathcal{X}}-\mu^E\hat{\mathcal{N}})} \tag{5}$$

are substantially different. Whereas the averages of the particle density with respect to the two ensembles coincide,

$$n^v(x) = n^E(x) \equiv n(x), \tag{6}$$

the averages of the current densities differ by a drift term,

$$j_y^v(x) = j_y^E(x) + evn(x). \tag{7}$$

Due to the confinement, the total current corresponding to the current density $j_y^E(x)$ vanishes.[15,7] The chemical potentials are related by $\mu = \mu^E + mv^2/2$.

III. THE ELECTROSTATIC POTENTIAL

In the Hartree approximation the self-consistent equations are

$$n(\mathbf{r}) = \mathrm{Tr}[\hat{\rho}^v(\hat{\mathcal{H}})\delta(\mathbf{r} - \hat{r})], \tag{8}$$

$$\hat{\mathcal{H}} = \sum_i \hat{H}_0(\mathbf{r}_i, \mathbf{p}_i) + \hat{V}(\mathbf{r}_i) + \hat{V}_{\text{back}}(\mathbf{r}_i), \tag{9}$$

$$V(\mathbf{r}_i) = e^2 \int \frac{n(\mathbf{r})}{|\mathbf{r}_i - \mathbf{r}|} d^3r. \tag{10}$$

Here $\hat{H}_0$ is the Hamiltonian of a free electron moving in the (x, y) plane in a perpendicular magnetic field and $\hat{V}_{\text{back}}(\mathbf{r}_i)$ is the potential induced by a constant positive background charge density guaranteeing charge neutrality. In our case the particle density is given by $n(\mathbf{r}) = n(x)\delta(z)$. In the numerical solution of Eqs. (8)–(10) the single-electron wave functions are assumed to vanish at $x = \pm L_x/2$ (Dirichlet boundary conditions) and the length scales are choosen to $L_x/L_y = 0.1$, $L_x/\epsilon a_0^* = 20$, $l/L_x = 0.1$, and $v/l\omega_c = 0.1$.

Here ϵ is the dielectric constant, a_0^* the effective Bohr radius of the host material, $l = \sqrt{\hbar c/eB}$ the magnetic length, and $\omega_c = eB/mc$ the cyclotron frequency. For GaAs we have $\epsilon a_0^* = 97.9$ Å and the parameters given above characterize a Hall bar of length $L_y = 2$ μm and width $L_x = 200$ nm subjected to a magnetic field of 1.6 T. The drift velocity is approximately 8×10^3 m/s.

The calculated effective electrostatic potential $V^v(x) \equiv V(x) = V_c(x) + V_{\text{back}}(x)$ as a function of the filling factor $\nu = 2\pi l^2(N/L_xL_y)$ is shown in Fig. 1. While $V(x)$ changes slowly with ν *near the edges* ($x \simeq L_x/2$), it drops rapidly *at the center* of the bar ($x = 0$) when a new Landau level begins to be populated. The reason for this is the drastically different polarizability of nearly completely filled (or nearly empty) and half-filled Landau levels.[8] This becomes more apparent in Fig. 2 where the effective electrostatic potential is given for different values of the filling factor. In the "bulk," three different types of behavior can be distinguished: for filling factors far from integer values ($\nu = 0.19$ and $\nu = 0.49$) $V(x)$ increases linearly from the left side of the sample to the right. When the Landau level becomes nearly filled ($\nu = 0.88$), all states located in the right half of the sample are occupied and increasing the number of electrons leads to a decreasing dipole moment. Thus, in the middle of the sample the potential saturates. With increasing filling factor the region of constant potential grows until the first states in the next Landau band become available. While the electrons in the filled lower Landau band build up nearly symmetric density and potential distributions, the dipole moment required to screen the Lorentz electric field has to be generated by the few electrons occupying

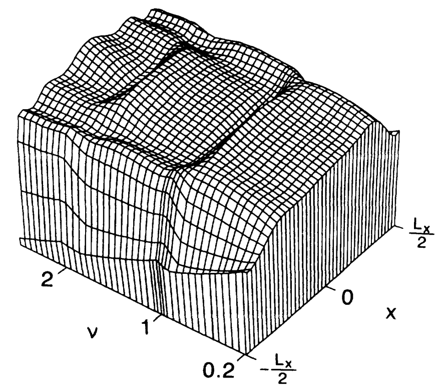

FIG. 1. The electrostatic potential $V(x) = V_c(x) + V_{\text{back}}(x)$ as a function of the filling factor ν.

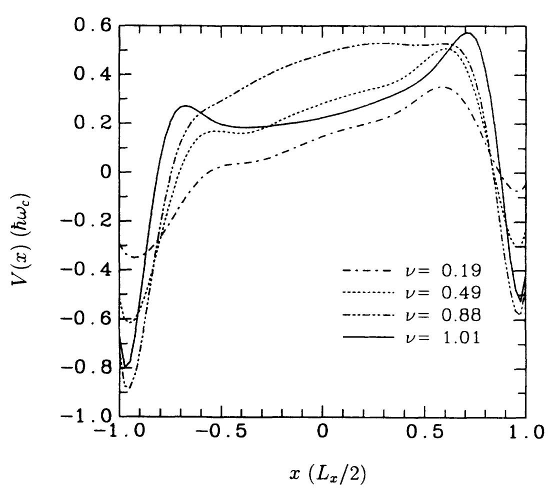

FIG. 2. The electrostatic potential $V(x)$ for different values of the filling factors ν.

the higher level ($\nu = 1.01$). Since, for this purpose, the number of electrons is not sufficient, the main variation of the potential occurs at the right edge of the sample.

A behavior very similar to this has been observed by Ebert, von Klitzing, and Weimann,[12] who measured the voltage drop across ohmic contacts attached to a Hall sample along a line perpendicular to the current direction. By varying the magnetic field, they observed that in a plateau regime of the Hall voltage (i.e., near integer filling) the voltage drop between contacts in the middle and at one edge of the sample changes rather abruptly. At first, the whole Hall voltage drops in the immediate vicinity of one edge, and second, by crossing the magnetic-field value corresponding to integer filling the voltage drop switches from one edge to the other. In contrast to this, in the transition regime between adjacent plateaus the voltage drop was found to be distributed linearly across the sample. The contact measurements by Ebert, von Klitzing, and Weimann probe the electrochemical potential $\mu(x)$. This is related to the effective electrostatic potential occurring in our treatment by $\mu(x) = \mu + V(x)$. Furthermore, in the measurements the total current and the average number of electrons were fixed and the filling was controlled by the magnetic field, whereas in the calculation the drift velocity and the magnetic field were kept constant and the number of electrons was changed. Therefore, in Fig. 3 the calculated voltage divided by the total current is shown as a function of the inverse filling factor $1/\nu \propto B$. The Dirichlet boundary conditions for the wave functions bring about an unphysical rise of the electrostatic potential at the edges of the sample. To eliminate this, the maximum of the potential was chosen as a reference value and its location to define a physical sample width $\tilde{L}_x = 2x_{\max}$, $V(x_{\max}) = V_{\max}$. The measured and calculated potential drops are related by the following expressions:

$$\begin{aligned} eU_{45} &= V(x_{\max}) - V(\tilde{L}_x/4), \\ eU_{46} &= V(x_{\max}) - V(0), \\ eU_{47} &= V(x_{\max}) - V(-\tilde{L}_x/4), \\ eU_{48} &= V(x_{\max}) - V(-x_{\max}). \end{aligned} \tag{11}$$

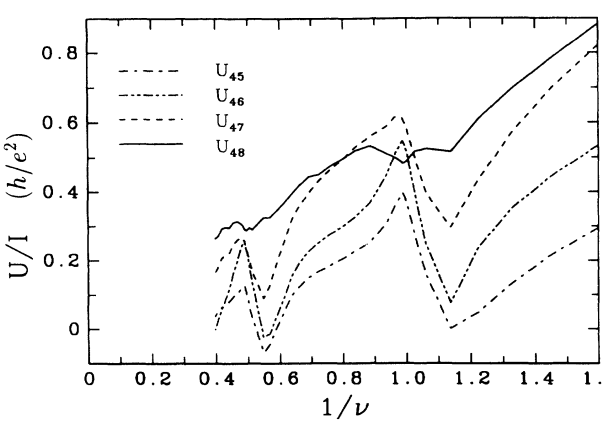

FIG. 3. Calculated voltage drops (see text) as a function of the inverse filling factor $1/\nu$.

The potential drops within the sample are in good qualitative agreement with the measured data. With increasing inverse filling factor, the pronounced maxima at integer filling factors are immediately followed by deep minima in the bulk accompanied by a sudden shift of the voltage drop from one edge to the other. This fact as well as the shoulder in U_{46} at half filling is in good agreement with the experimental data. The calculated Hall voltage U_{48}, however, is too small (in particular at small values of the filling factors). Moreover, the calculations do not show any plateaus, since a random potential giving rise to localization has not been taken into account.

The Ohmic contacts attached to the system give rise to perturbations, which are difficult to estimate. To avoid such perturbations, Fontein *et al.*[13,14] invented a contactless method based on the electro-optical effect for measuring that part of the electrostatic potential which is *induced by the current.* This part of the potential is antisymmetric with respect to the center of the sample, indicating that an equal amount of current induced voltage drop occurs at both edges of the sample. This seems to be in contradiction to the measurements by Ebert, von Klitzing, and Weimann, which show a strongly asymmetric voltage distribution at least for integer filling factors.

In the bulk of the sample the measured induced potential is nearly constant and changes rapidly near the edges for *integer* filling factor (plateau regime). For *half-filled* levels, however, the potential varies linearly in the bulk and its change at the edges is much less pronounced. Thus, in this case, the Hall field is nearly constant within the sample.

The calculations show that the current induced potential is (within an error of about 1%) equal to the antisymmetric part of the self-consistent Coulomb potential, $V_{\rm ind}(x) \equiv V(x) - V_{I=0}(x) \simeq V_a \equiv [V(x) - V(-x)]/2$. [Both $V_{\rm back}(x)$ and $V_{I=0}(x)$ are symmetric with respect to the center of the sample.] In Fig. 4 $V_a(x)$ is depicted as a function of the filling factor. For an infinite classical electron gas the corresponding surface is an inclined plane

$$V_a^{\rm class}(x) = e\frac{v}{c}Bx. \tag{12}$$

For integer filling the quantum-mechanical calculation yields a significant deviation from the classical behavior: in the bulk, the electric field is strongly reduced; the

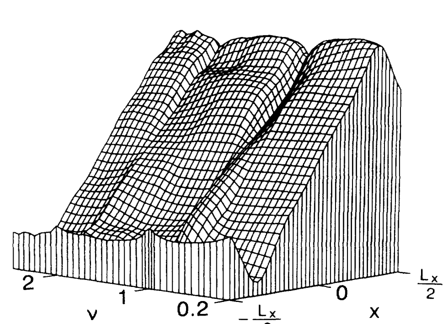

FIG. 4. The antisymmetric part of the electrostatic potential, $V_a(x)$, as a function of the filling factor ν.

potential drop occurs at the edges. For half-filled levels, however, $V_a(x)$ shows a classical behavior (except in the vicinity of the edges). This is in agreement with the results of Gerhardts and Gudmundsson[8] on the screening properties of an ideal two-dimensional (2D) Hall system: For half-filled Landau levels the screening is nearly ideal and $e(dV_a/dx)$ cancels the Lorentz force, whereas for integer filling, screening breaks down resulting in a much weaker Coulomb potential. Note that the canonical equilibrium response of the electric potential to a constant electric field coincides with the one yielded by the momentum ensemble, $V^v(x) = V^E(x)$.

Figure 4 shows that our calculations reproduce the qualitative features of the experimental results obtained by Fontein *et al.*[13,14] Moreover, the apparent contradiction between the different experiments can also be clarified: Whereas Ebert, von Klitzing, and Weimann measured the total electrostatic potential which is the sum of the current induced potential, the (nonuniform) equilibrium Coulomb potential, and the background potential, Fontein *et al.* probed the induced part only. The measured quantities behave very differently. In particular, the sudden shift of the voltage drop from one side of the sample to the other at integer filling is entirely due to a rearrangement of the equilibrium ($I = 0$) potential distribution.

IV. THE CURRENT DISTRIBUTION

The calculated current density $j_y^v(x)$ and its symmetric part $j_{ys}^v(x) = [j_y^v(x) + j_y^v(-x)]/2$ which yields the total current $I = eNv/L_y$ are shown in Figs. 5 and 6, respectively. The antisymmetric part of j_y^v (the diamagnetic current) does not contribute to the net charge transfer. For small values of the drift velocity, the current density is dominated by the diamagnetic contribution; especially for filling factors greater than one, the total current distribution is much more complicated than its symmetric part. As already found by Yogeshwar and Brenig,[6] at integer filling the main current path jumps from one edge of the sample to the other.

In Fig. 6 the contribution of edge and bulk states to the current density can be well distinguished. In the bulk (around $x = 0$) a sudden increase of the current density occurs when a Landau level begins to fill. To understand this, note that the contribution of a Landau state of energy ϵ_{nk}, located at $x \simeq X = -l^2 k$, to the total current density is proportional to $\partial\epsilon_{nk}/\partial k$. In the bulk

$$\frac{\partial \epsilon_{nk}}{\partial k} \sim \left.\frac{\partial V_a(x)}{\partial x}\right|_{x=X} . \tag{13}$$

Since here $\frac{\partial V_a(x)}{\partial x}$ is approximately constant, all bulk states carry about the same amount of current which determines the step behavior seen in Fig. 6. At any site x, the current density can only be changed by varying the level occupation. Moreover, from Fig. 6 it becomes evident that for nearly filled Landau levels the net current is carried by the edge states, whereas their contribution is strongly reduced, when a new level begins to be populated. In that case the net current is set up by bulk states.

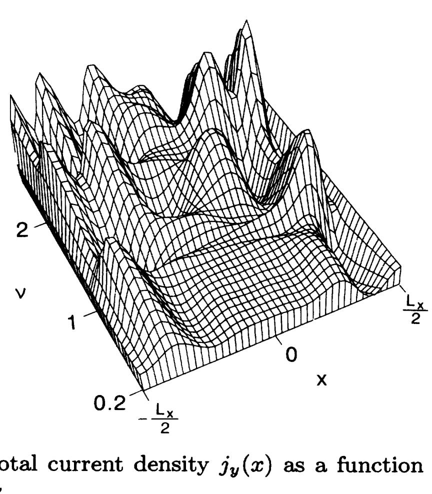

FIG. 5. Total current density $j_y(x)$ as a function of the filling factor ν.

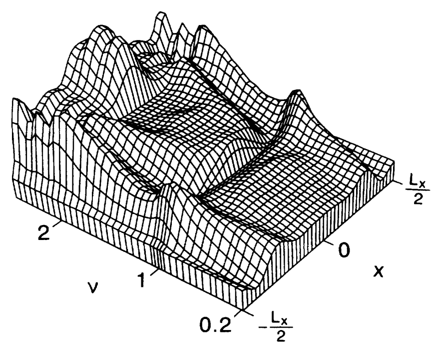

FIG. 6. The symmetric part of the current density, $j_{ys}(x)$, as a function of the filling factor ν.

V. SUMMARY

We calculated the potential and current distribution in an ideal 2D Hall system. The Coulomb interaction of the electrons was taken into account in Hartree approximation. We described the current carrying state by means of an (equilibrium) momentum ensemble. The results for the potential distribution are in qualitative agreement with the data obtained by both Ohmic probing and contactless measurements, and explain the apparent contradiction between the latter. The local current density shows a complicated structure. It was pointed out for which filling factors the net current flow occurs in the bulk or at the edges.

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