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# Vortex motion under the influence of a temperature gradient

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We show that fulfillment of the boundary conditions at the core boundary causes, in the presence of a temperature gradient, the Magnus force acting on the vortices together with the well-known thermal force $\mathbf{F}_{\rm th} = -S_\Phi \nabla T$ ($S_\Phi$ is a transport entropy per vortex unit length). By measuring the thermoelectric power $S$ and the resistivity $\rho$ of the single-crystalline $Bi_2Sr_2CaCu_2O_x$ and $YBa_2Cu_3O_{7-\delta}$ in $H \perp ab$ and $H \parallel ab$ we observe qualitatively different responses of Abrikosov vortices and Josephson vortices to the temperature gradient and attribute this difference to the absence of the normal cores of Josephson vortices.

The flux motion parallel to the temperature gradient leads to the electric field in the $\mathbf{H} \times \nabla T$ direction: $\mathbf{E}_\perp = \nu \mathbf{H} \times \nabla T$, where $\nu$ is the Nernst coefficient. The flux motion perpendicular both to $\nabla T$ and to the magnetic field leads to the electric field in the direction of the temperature gradient: $\mathbf{E}_\parallel = S \nabla T$, where $S$ is the thermoelectric power (the Seebeck coefficient). Thermomagnetic effects in the mixed state of type-II superconductors are well known to exist (for review, see Ref. 1). In the high-$T_c$ superconductors such phenomena attracted a lot of interest during the last two years. In high magnetic fields and at high temperatures the vortices can move freely under the influence of a driving force of a temperature gradient. Pronounced Ettingshausen and Nernst effects have been observed on $YBa_2Cu_3O_{7-\delta}$ (Y 1:2:3) single crystals[2] and epitaxial films,[3,4] $Bi_2Sr_2CaCu_2O_x$ (Bi 2:2:1:2) single crystals,[5] and $Tl_2Ba_2CaCu_2O_8$ $c$-oriented films.[6] The main conclusion from experiments on the thermoelectric power[5,7,8] of the high-$T_c$ superconductors in the mixed state is that the $S(T)$-transition curves are similar to the resistivity transition curves, and that the thermoelectric power is much larger than the conventional flux flow model[1] value $\nu H \tan\theta$, where $\theta$ is the Hall angle.

A variety of models have been proposed to explain the pronounced thermoelectric effect observed in the mixed state of the high-$T_c$ superconductors. It has been argued[7] that granularity may cause the large Seebeck effect for ceramic samples. For epitaxial films and single crystals this effect should be reduced compared with ceramic samples, which does not take place. Therefore, it is natural to assume that such a mechanism is not essential.

Huebener and co-workers have proposed a model[9] based on an idea of Ginzburg[10] that in the presence of a temperature gradient in zero magnetic field there is a normal excitation current $\mathbf{j}_n \propto \nabla T$ locally cancelled by a supercurrent $\mathbf{j}_s$, so that at any point of a superconductor $\mathbf{j}_s = -\mathbf{j}_n$, and both $\mathbf{j}_s$ and $\mathbf{j}_n$ are spatially homogeneous. They suppose that only $\mathbf{j}_s$ interacts with the vortices causing their motion.

We will argue that although this local cancellation takes place far enough from the vortex line centers, there is another cause for the flux motion in the $\mathbf{H} \times \nabla T$ direction and, consequently, the Seebeck voltages in the mixed state. The role of the boundary conditions at the core boundary is underlined in our consideration. It has been observed that the magnetothermoelectric power is very small for the highly anisotropic Bi 2:2:1:2 in $H \parallel ab$ (when the vortices are believed to be coreless), supporting our point of view.

As we have pointed out in Ref. 5, fulfillment of the boundary conditions at the core boundary requires the normal currents inside the core. In the vortex frame of reference the electric field $\mathbf{E}'$ is zero because in this frame the supercurrent distribution is stationary, and from the London equation[1] we obtain

$$\mathbf{E}' = m \frac{\partial \mathbf{v}_s}{\partial t} = -m \mathbf{v}'_L \nabla \cdot \mathbf{v}_s = 0, \tag{1}$$

where $m$ is the electron mass, $\mathbf{v}'_L = 0$ is a vortex line velocity in this frame of reference, and $\mathbf{v}_s$ is the superfluid velocity locally connected to the supercurrent density: $\mathbf{j}_s = n_s e \mathbf{v}_s$, where $n_s$ is the superfluid electron density. The tangential component of the electric field must be continuous. It has been assumed in the Bardeen-Stephen model[11] that there is a contact potential at the core boundary due to the normal component of the electric field whereas the superfluid velocity is taken to be continuous. This point of view has been criticized in Ref. 12. It has been suggested[12] that for a superconductor with the Ginzburg-Landau parameter $\kappa \gg 1$ the superfluid velocity changes discontinuously, and there is no contact potential. So, the normal current inside the core $\mathbf{j}_n^{(\rm in)}$ should arise to compensate the electric field due to the normal thermoelectric power $S_n$:

$$\rho_n \mathbf{j}_n^{(\rm in)} + S_n \nabla T = 0, \tag{2}$$

where $\rho_n$ is the normal-state resistivity. We introduce the superscript (in) to distinguish $\mathbf{j}_n^{(\rm in)}$ flowing in the cores from the normal current due to the Ginzburg mechanism. The charge conservation requires a counterflow of the supercurrent $\mathbf{j}_s^{(\text{out})} \sim -\mathbf{j}_n^{(\text{in})}$, where the superscript (out) reflects the fact that the supercurrents flow outside the core. The supercurrent causes the Magnus force on the vortices[12,13] $\mathbf{F}_M = \Phi_0 n_s e(\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{z}$, which can be written in our case as

$$\mathbf{F}_M = \Phi_0[(S_n/\rho_n)\nabla T - n_s e \mathbf{v}_L] \times \mathbf{z}, \tag{3}$$

where $\mathbf{z}$ is a unit vector in the direction of magnetic field, $\Phi_0$ is the flux quantum, and $\mathbf{v}_L$ is a vortex line velocity in the laboratory frame of reference. This force should be compensated by the thermal force $\mathbf{F}_{\text{th}} = -S_\Phi \nabla T$ ($S_\Phi$ is a transport entropy per vortex unit length) and the drag force[11,12] $\mathbf{f} = -\eta \mathbf{v}_L$, where $\eta$ is the viscous drag coefficient. We neglect for simplicity the term $-\eta' \mathbf{z} \times \mathbf{v}_L$ introduced[14] to explain the sign change of the Hall voltage in the mixed state. This simplification will not change the final result [Eqs. (10) and (11)]. The vortex line velocity is found from the equation $\mathbf{F}_M + \mathbf{F}_{\text{th}} + \mathbf{f} = 0$. One can express $\mathbf{v}_L$ through $\nabla T$:

$$\mathbf{v}_L = \gamma \nabla T \times \mathbf{z} + \alpha \nabla T, \tag{4}$$

where

$$\gamma = \Phi_0 \frac{\eta S_n/\rho_n + S_\Phi n_s e}{(\Phi_0 n_s e)^2 + \eta^2}, \tag{5}$$

$$\alpha = \frac{\Phi_0^2 n_s e S_n/\rho_n - S_\Phi \eta}{(\Phi_0 n_s e)^2 + \eta^2}. \tag{6}$$

Using the Josephson relation[15] $\mathbf{E} = -\mathbf{v}_L \times \mathbf{H}$, where $\mathbf{E}$ is the electric field in the laboratory frame of reference, we obtain

$$\mathbf{E} = H(\alpha \mathbf{z} \times \nabla T + \gamma \nabla T). \tag{7}$$

Taking into account that $\mathbf{E} = \mathbf{E}_\perp + \mathbf{E}_\parallel$ and that[12]

$$\rho = \Phi_0 H \frac{\eta}{(\Phi_0 n_s e)^2 + \eta^2}, \tag{8}$$

$$\rho_H = \Phi_0 H \frac{\Phi_0 n_s e}{(\Phi_0 n_s e)^2 + \eta^2}, \tag{9}$$

where $\rho$ and $\rho_H$ are the longitudinal and Hall resistivities, respectively, we obtain for the Nernst effect:

$$-\nu H/\rho = S_\Phi/\Phi_0 - (S_n/\rho_n)\tan\theta. \tag{10}$$

Note that for the vortices moving from the hot side of a sample to the cold one the Nernst coefficient is negative. Similarly, one obtains for the thermoelectric power

$$S/\rho = S_n/\rho_n + (S_\Phi/\Phi_0)\tan\theta. \tag{11}$$

It follows from Eq. (10) that to calculate the transport entropy from the Nernst effect the contribution from the normal thermoelectric power should be taken into account. Using the experimental data from Ref. 5 for Bi 2:2:1:2 single crystals, $-\Phi_0 \nu H/\rho = 1.5\times10^{-15}$ J K$^{-1}$ m$^{-1}$, $S_n$=3.2 $\mu$V/K, $\rho_n$=2 $\mu\Omega$ m, $\tan\theta$=0.07 at $T$=65 K, $H$=4 T, we conclude that the second term in Eq. (10) may reach 15% of the first one whereas in conventional type-II superconductors it is negligible because $S_n$ is small.

Two contributions to the thermoelectric power in the mixed state are found to exist. The first one is $10^2$–$10^3$ times larger in the high-$T_c$ superconductors than the second one.[4,6] In conventional superconductors these contributions are comparable.[16] We show that our model leads to Eq. (11), in agreement with the Maki microscopic theory.[17] The same result for the thermoelectric power has been obtained in Ref. 9. However, the reason of the flux motion in the $\mathbf{H} \times \nabla T$ direction differs from that proposed by us.

In our opinion, the model[9] has a point of controversy. Indeed, Ginzburg developed his theory of thermoelectricity of superconductors[10] for $H$=0. We think that at low vortex line density the local cancellation of $\mathbf{j}_n$ and $\mathbf{j}_s$ does take place far enough from the vortex line centers (at the distances larger than the penetration depth). However, as follows from the arguments presented above, the supercurrent and the normal current are strongly redistributed in space in the vicinity of the vortex cores. Furthermore, experiments on the thermoelectric power in the high-$T_c$ superconductors in the mixed state are carried out in a magnetic field of a few tesla. For instance, for $H$=4 T the average intervortex distance is $d \simeq \sqrt{\Phi_0/H} \simeq$25 nm. If we consider a vortex line as a cylinder of normal region with diameter $2\xi_{ab} \simeq$6 nm, where $\xi_{ab}$ is the in-plane coherence length, then it becomes clear that indeed the normal currents and the supercurrents are nowhere equal in magnitude and compensate each other if averaged over a square of the order of $d^2$. In a dense vortex lattice (or vortex fluid) with $\nabla T \neq 0$ the normal currents flow presumably inside the vortex cores, and the supercurrents flow between the cores.

In view of the present consideration one may expect that the Magnus force (3) will not arise in the case of coreless Josephson vortices. It is well known[18] that a magnetic field parallel to the layers penetrates into layered superconductors in the form of the Josephson vortices if $\xi_c < s/\sqrt{2}$, where $\xi_c$ is the coherence length in the direction perpendicular to the layers, $s$ is the interlayer distance, and the supercurrent distribution around the vortex center differs significantly from that for the Abrikosov vortex line (see Ref. 19). For Bi 2:2:1:2 with $\xi_{ab}(T=0) \simeq$3.2 nm,[20] the anisotropy parameter $\Gamma = (\xi_{ab}/\xi_c)^2 \simeq 3000$ (Ref. 21) and $s$ =1.2 nm the condition $\xi_c < s/\sqrt{2}$ is satisfied at temperatures $T_c - T$ >0.4 K and, consequently, at these temperatures the vortices are of Josephson type. The Josephson coupling between the layers in Bi 2:2:1:2 single crystals has been demonstrated recently by Kleiner *et al.*[22] On the contrary, for less anisotropic Y 1:2:3 with $\xi_{ab}(T=0) \simeq$2 nm,[20] $\Gamma \simeq 26$ (Ref. 21) and $s$ =8.3 nm for $T_c - T$ <10 K the vortices are of Abrikosov type.

We have studied the influence of the temperature gradient on the vortices in the high-quality Bi 2:2:1:2 and Y 1:2:3 single crystals. The resistivity transition widths on the 10 – 90% level are 3.5 and 0.35 K, the middle point transition temperatures are 97.0 and 91.3 K for Bi 2:2:1:2 and Y 1:2:3, respectively (see Ref. 23). The magnetic field is aligned in the $ab$ plane with the accuracy 0.5°. We have measured the thermoelectric power and the resistivity simultaneously, at fixed temperatures and temperature gradients, versus magnetic field up to 4 T, and, additionally, the resistivity as a function of temperature in a different magnetic field, with $\nabla T=0$. For all results presented here the electric current and the temperature gradient are parallel to each other and to the *ab* plane and perpenducular to the magnetic field. The accuracy of the thermoelectric power measurements is $0.02S_n$ for Bi 2:2:1:2 and $0.08S_n$ for Y 1:2:3. Most of the resistivity data have been obtained at the dc current density $j \sim 10^5$ A/m$^2$, with reverse of the current direction. When measuring the magnetic field dependences of the resistivity, at each temperature and in selected magnetic field (0, 1, and 4 T), we have checked that the resistivity is current independent over a wide range of the current densities $(3\times10^2$–$3\times10^5)$ A/m$^2$.

In Fig. 1 we plot the normalized resistivity $\rho/\rho_n$ (with $\rho_n=2$ $\mu\Omega$ m) and the normalized thermoelectric power $S/S_n$ (with $S_n=3.2$ $\mu$V/K) of the Bi 2:2:1:2 single crystal in $H \parallel ab$ at different temperatures. In spite of the essential resistivity increase with increase of magnetic field up to 4 T ($0.3\rho_n$ at $T=94.4$ K), $S$ does not depend on magnetic field within the accuracy of the experiment. The only exception is the curve at $T=94.4$ K where the increase of the thermoelectric power is about $0.03S_n$. We adduce the magnetic field dependences of $S$ and $\rho$ in $H \perp ab$ and $H \parallel ab$ for Bi 2:2:1:2 in Fig. 2 and for Y 1:2:3 in Fig. 3. The thermoelectric power and the resistivity depend in the identical manner on $H \perp ab$ for both compounds and on $H \parallel ab$ for Y 1:2:3. For Bi 2:2:1:2 the magnetothermoelectric power $\Delta S \approx 0$ in $H \parallel ab$. This fact has been checked for three Bi 2:2:1:2 single crystals. It is essential to note that the resistivity and the thermoelectric power of Bi 2:2:1:2 change similarly with magnetic field making an angle of about 9° with the *ab* plane.

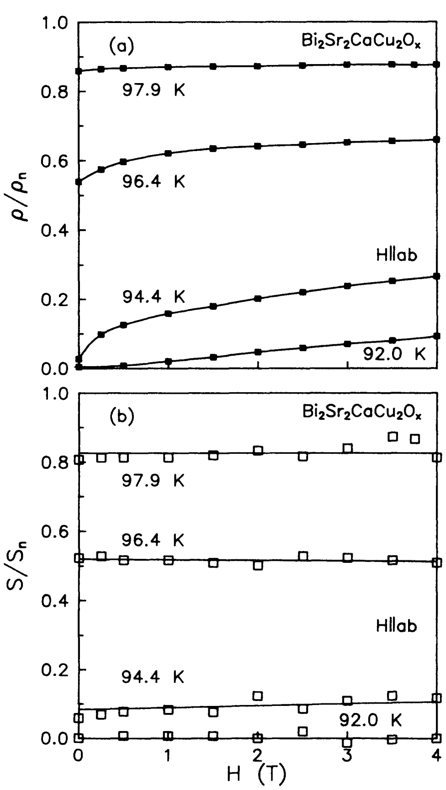


FIG. 1. (a) Magnetic-field dependences of $\rho/\rho_n$ for Bi 2:2:1:2 at $\Delta T$ = 1.5 K, $H \parallel ab$. Temperature is indicated near curves. (b) Magnetic-field dependences of $S/S_n$ for Bi 2:2:1:2 at $\Delta T$ = 1.5 K, $H \parallel ab$. Temperature is indicated near curves.

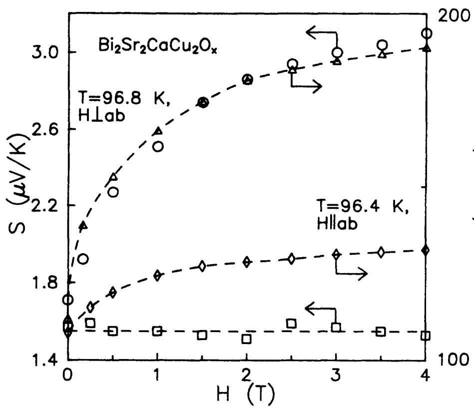


FIG. 2. The resistivity $\rho$ vs magnetic field ($\triangle$, $H \perp ab$; $\diamond$, $H \parallel ab$) and the thermoelectric power $S$ vs magnetic field ($\circ$, $H \perp ab$; $\square$, $H \parallel ab$) for Bi 2:2:1:2.

A possible point of view is that the Magnus force is given by Eq. (3) for both types of vortices, but it is too small to drive the vortices in the direction perpendicular to the layers, and that this is the reason for the smallness of the magnetothermopower in Bi 2:2:1:2 single crystals in $H \parallel ab$. To answer whether this is true, one should compare the magnitude of the supercurrent density $j_s^{(\mathrm{out})} \sim (S_n/\rho_n) \mid \nabla T \mid$ with the transport current densities applied to the samples in the resistivity measurements. The temperature difference $\Delta T=1.5$ K corresponds to the magnitude of temperature gradient $\mid \nabla T \mid \simeq 1.5\times10^3$ K/m. Thus, one obtains $j_s^{(\mathrm{out})} \sim 2.5\times10^3$ A/m$^2$. This value is within the current-

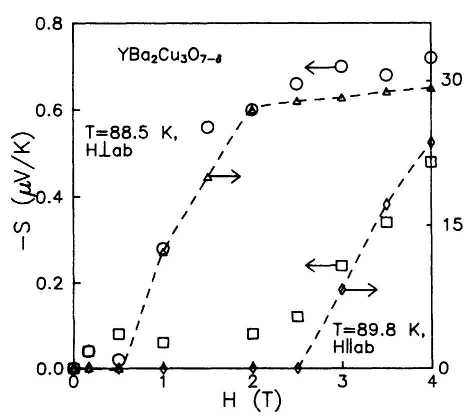


FIG. 3. The resistivity $\rho$ vs magnetic field ($\triangle$, $H \perp ab$; $\diamond$, $H \parallel ab$) and the thermoelectric power $S$ vs magnetic field ($\circ$, $H \perp ab$; $\square$, $H \parallel ab$) for Y 1:2:3.

density range where a linear electric-field response has been observed. Therefore, if the supercurrent of the density $j_s^{(\text{out})} \sim 2.5 \times 10^3$ A/m$^2$ would exist due to the temperature gradient applied in the case of Josephson vortices, the electic field along the temperature gradient induced by the vortex motion in the direction of the $c$ axis would be easily observed.

We conclude that the supercurrent $\mathbf{j}_s^{(\text{out})}$ is not induced by the temperature gradient [or it is much smaller in magnitude than $(S_n/\rho_n) \mid \nabla T \mid$] and that the Magnus force (3) is not active (or small) for the case of the Josephson vortices, since we are not able to detect the magnetothermopower in single-crystalline Bi 2:2:1:2 in $H \parallel ab$.

There is a relatively large amount of literature on the properties of a Josephson $SNS$ junction in a temperature gradient (see Ref. 24 and references therein). It was observed that an electric current and a heat flow had analogous influences upon a Josephson junction. As this differs from our result, one should dwell on differences between our experiment and these works. First, in Ref. 24 a temperature gradient was applied *across* a Josephson junction. (As applied to Bi 2:2:1:2 single crystals with intrinsic Josephson coupling between the layers, it would correspond to a temperature gradient parallel to the $c$ axis.) Second, the magnetic field applied was either zero or of the order of $10^{-7}$–$10^{-6}$ T.[24] It meant that the magnetic field was zero in the bulk of a superconductor and a consideration based on theoretical works[10,25] was appropriate.

To summarize, we propose a model of the thermomagnetic effects in superconductors in the mixed state. The central aspect of our model is the fulfillment of the boundary conditions at the core boundary which leads to the Magnus force (3). The absence of the $N$-$S$ boundaries of the Josephson vortices is, in our opinion, the reason for the smallness of the magnetothermoelectric power in highly anisotropic Bi 2:2:1:2 in $H \parallel ab$.

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