Problem of Josephson-vortex-lattice melting in layered superconductors

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A layered superconductor is studied in the presence of a magnetic field along the layers. With the help of a transformation to a Coulomb-gas representation it is shown that even in the limit of high field the phase transition into the phase with effectively decoupled layers is impossible. Contrary to the suggestion by Efetov [Sov. Phys. JETP **49**, 905 (1979)] the coherence between the layers will be lost simultaneously with the total destruction of superconductivity.

I. INTRODUCTION

More than ten years ago Efetov put forward a conjecture[1] that a strong magnetic field applied to a layered superconductor in parallel to the layers can induce the loss of phase coherence between the layers, whereas each of the layers will remain superconducting. In terms of the vortices this phase transition can be described as a melting of the vortex lattice formed by the Josephson vortices lying between the layers. Discovery of high-T_c superconductors with well-developed layered structure has led to revival of interest in this problem. A more detailed investigation of properties of the quasi-two-dimensional phase in which the vortex lattice is assumed to be melted has been undertaken recently.[2,3] It seems worth mentioning that according to Kes *et al.*[4] even in the absence of the external field many of the properties of the high-T_c superconductors can be satisfactorily explained on the assumption that there is no effective coupling between the layers.

At the same time some serious doubts appeared on the possibility of the existence of such a phase. The question of whether a transition into a quasi-two-dimensional phase can really take place at temperatures lower than that required for the destruction of superconductivity was addressed both in terms of the vortex lattice melting[5,6] and in the fermionic representation.[7] It was shown both for strongly interacting[5] and for noninteracting[6] vortices (neglecting the possibility of vortex hopping between valleys and considering only the in-plane fluctuations of vortices) that the vortex lattice will remain unmelted in the whole domain of parameters where such an approximation is applicable. This makes the existence of the intermediate phase with effectively decoupled layers impossible. The conclusion obtained by Horovitz,[7] with the help of the transformation to the fermionic representation, is that the transition into the quasi-two-dimensional phase may be possible only in the limit of large core energy and only in the narrow region of fields corresponding to penetration of flux lines between every nine or ten layers.

However the problem cannot be considered as being completely solved. The vortex representation[5,6] is restricted to the case of not very strong magnetic field for which the average distance between vortices is large in comparison with the size of the core. On the other hand the fermionic representation[7] is formulated in terms of very strongly interacting fermions (the density of which is proportional to the field), so one cannot be sure whether the results obtained in the framework of the renormalization approach can be trusted even for small fields.

Taking also into account that the original suggestion by Efetov[1] related the destruction of the coherence between the layers with the increase in the magnetic field, one has to conclude that the high-field limit corresponding to overlapping of vortex cores deserves special consideration; this is the subject of the present paper. Our conclusion is that for such fields the existence of the intermediate phase with effectively decoupled but still superconducting layers is also impossible.

II. THE MODEL

Let us start with the semiphenomenological Hamiltonian describing the fluctuations of the phase of the order parameter in a layered superconductor:[1]

$$H = \sum_n \int\int dx\,dy \left[\frac{J_\parallel}{2}\left(\nabla_\parallel \varphi_n - \frac{2e}{c}\mathbf{A}_n\right)^2 - J_z \cos(\varphi_{n+1} - \varphi_n - hx)\right] + \frac{1}{8\pi}\int\int\int dx\,dy\,dz (\mathrm{rot}\mathbf{A})^2, \quad (1)$$

where φ_n stands for the phase of the order parameter on the nth superconducting layer,

$$J_\parallel = \frac{\phi_0^2 d}{16\pi^3 \lambda_{ab}^2}$$

is the stiffness constant characterizing the energy of phase fluctuations in a single layer,

$$J_z = \frac{\phi_0^2}{16\pi^3 \lambda_c^2 d}$$

is the Josephson coupling between the layers, $\lambda_{ab}(T)$ and $\lambda_c(T) \gg \lambda_{ab}(T)$ are the London penetration depths for different directions, d is the interlayer distance, $\mathbf{A} \equiv \mathbf{A}(x, y, z)$ is the fluctuating part of the vector potential, and

$$\mathbf{A}_n(x, y) = \mathbf{A}(x, y, nd)$$

is the value of $\mathbf{A}$ at the n-th superconducting layer. It is convenient to choose a gauge in which the fluctuating part of the vector potential $\mathbf{A}$ has only in-plane (x and y) components and the nonfluctuating part describing the constant field along the y axis has only a z component:

$$\frac{2e}{c}A_z = hx, \quad h = \frac{2\pi Bd}{\phi_0}, \quad \phi_0 \equiv 2\pi\frac{c}{2e}.$$

Here B is the magnetic field penetrating between the layers of the superconductor which only slightly differs from the external field.

Hamiltonian (1) is Gaussian in the fluctuations of the vector potential, therefore they can be integrated out of the partition function corresponding to it. In the following we will be mostly interested in low temperatures for which the possibility of the formation of two-dimensional vortices or vortex pairs in this or that layer (i.e., the possibility of flux-line penetration across the layer) can be neglected. That means that the order-parameter phase $\varphi_n(x, y)$ can be assumed to be continuous on every layer. In that case the terms in the Hamiltonian containing a nonpotential part of $\mathbf{A}$ decouple from the other terms and can be omitted.

Integration over the potential part of $\mathbf{A}$ leaves us then with the relatively simple nonlocal Hamiltonian:

$$H = \int\int dx\, dy \left(\frac{J_\parallel}{2} \sum_{n,n'} g(n-n')(\nabla_\parallel \varphi_n) \cdot (\nabla_\parallel \varphi_{n'}) - J_z \sum_n \cos(\varphi_{n+1} - \varphi_n - hx) \right), \quad (2)$$

where

$$g(n) = \int_{-\pi}^{\pi} \frac{dq}{2\pi} e^{iqn} g(q), \qquad g(q) = \frac{2(1-\cos q)}{(d/\lambda_{ab})^2 + 2(1-\cos q)}, \quad (3)$$

and q is the dimensionless z component of the wave vector. In real layered superconductors the inequality $\lambda_{ab} \gg d$ is always fulfilled. In that case $g(q)$ for almost all q is close to 1 and $|g(n)|$ for $n \neq 0$ is much less than 1.

For high enough field for which the vortices penetrate between all the layers, the distribution of the order parameter phase in the ground state can be described with the help of just one function $\Phi(x)$. For example, φ can be chosen in the form

$$\varphi_n(x, y) = (-1)^n[\pi/4 - \Phi(x)], \quad (4)$$

where $\Phi(x)$ is the solution of the equation:

$$g(\pi)J_\parallel \frac{\partial^2 \Phi}{\partial x^2} + 2J_z \cos 2\Phi \cos hx = 0$$

and for

$$\Phi_0 = \frac{2J_z}{h^2 g(\pi) J_\parallel} \approx \frac{2J_z}{h^2 J_\parallel} \ll 1 \quad (5)$$

has the form[8]

$$\Phi(x) \approx \Phi_0 \cos hx. \quad (6)$$

In the ground state given by Eqs. (4)–(6) the individuality of each vortex is lost but the periodicity of this state can still be associated with the periodicity of the vortex lattice. This ground state differs only slightly from the ground state in the absence of interlayer coupling for which φ_n does not depend on x. Therefore it would not be very surprising if even at relatively low temperatures thermal fluctuations destroy the periodic ordering, mediating thus a transition to the intermediate phase with effectively decoupled layers.

Inequality (5) can be rewritten as a lower bound for the magnitude of the magnetic field:

$$B \gg B_* = \frac{\phi_0}{2\pi d}\left(\frac{J_z}{J_\parallel}\right)^{1/2} \equiv \frac{\phi_0}{2\pi d^2}\frac{\lambda_{ab}}{\lambda_c}$$

which determines the domain of applicability of our approach. From the side of the very high fields such analysis is restricted only by the fields that destroy superconductivity in each individual layer.

III. EFFECTIVE HAMILTONIAN FOR SLOW VARIABLES

At finite temperatures the fluctuations on the background of the ground state should be considered. Low-energy fluctuations can be described by introducing slowly changing variables $u_n \equiv u_n(x, y)$, $v_n \equiv v_n(x, y)$, and $s_n \equiv s_n(x, y)$ so that Eq. (4) is substituted by

$$\varphi_n(x, y) = (-1)^n(\pi/4 - u_n \cos hx - v_n \sin hx) + s_n. \quad (7)$$

Here u_n and v_n can be interpreted as the variables describing the displacement and distortion of the vortex lattice whereas s_n describes the slow changes of the phase proper.

Assuming that $u_n(x, y)$, $v_n(x, y)$, and $s_n(x, y)$ are changing with x much more slowly than $\cos hx$ and $\sin hx$ we can carry out in Eq. (1) integration over x at short scales. It reduces to substitution:

$$\int dx \sin hx \cdots \Rightarrow 0,$$

$$\int dx \cos hx \cdots \Rightarrow 0,$$

$$\int dx \sin^2 hx \cdots \Rightarrow \tfrac{1}{2}\int dx \cdots ,$$

$$\int dx \sin hx \cos hx \cdots \Rightarrow 0,$$

$$\int dx \cos^2 hx \cdots \Rightarrow \tfrac{1}{2}\int dx \cdots ,$$

and for $|u_n|$ and $|v_n|$ much smaller than 1 yields the effective Hamiltonian:

$$H = \int\int dx\,dy \left[\frac{J_\parallel}{2}\int\frac{dq}{2\pi}\left(\frac{g(\pi+q)}{2}[|hu(q)-\nabla_x v(q)|^2 + |hv(q)+\nabla_x u(q)|^2 + |\nabla_y u(q)|^2 + |\nabla_y v(q)|^2] + g(q)|\boldsymbol{\nabla}_\parallel s(q)|^2\right) - \frac{J_z}{2}\sum_n\{u_n[\cos(s_{n+1}-s_n)+\cos(s_n-s_{n-1})] + v_n[\sin(s_{n+1}-s_n)+\sin(s_n-s_{n-1})]\}\right]. \quad (8)$$

It is possible to obtain Eq. (8) only if u_n, v_n, and s_n are changing with x much slower than $\cos hx$ and $\sin hx$. That means that a more complete form of the effective Hamiltonian should incorporate also terms of higher order in gradients that introduce an effective cutoff when the absolute value of the in-plane momentum $\mathbf{k}$ is of the order of $k_{\max} \sim h$.

Hamiltonian (8) is Gaussian in fluctuations of u_n and v_n therefore they can be integrated out giving thus the effective Hamiltonian which depends only on s_n. If only a leading contribution

$$\int\int dx\,dy \frac{J_\parallel g(\pi)h^2}{4}\sum_n (u_n^2 + v_n^2)$$

is retained among the terms which are quadratic in u_n and v_n this effective Hamiltonian will have a simple sine-Gordon structure:

$$H = \int\int dx\,dy \left(\frac{J_\parallel}{2}\sum_{n,n'} g(n-n')(\boldsymbol{\nabla}_\parallel s_n)(\boldsymbol{\nabla}_\parallel s_{n'}) - Y\sum_n \cos(s_{n+1}-2s_n+s_{n-1})\right) \quad (9)$$

with

$$Y = \frac{J_z^2}{2J_\parallel g(\pi)h^2} = \frac{J_z\Phi_0}{4}.$$

For the properties of the Coulomb gas which is introduced in the next section the approximation neglecting the gradient terms in u and v is not very important and is used here only to obtain a Hamiltonian of compact form.

IV. ANALYSIS OF THE PHASE TRANSITION IN THE COULOMB-GAS REPRESENTATION

The partition function corresponding to Hamiltonian (9):

$$Z = \int D\{s\}\exp\left(-\frac{H\{s\}}{T}\right) \quad (10)$$

can be reduced to that of a quasi-two-dimensional (layered) Coulomb gas. If Eq. (10) is expanded in powers of the second (nonquadratic) term in the Hamiltonian and then in each term the Gaussian integration in s is carried out the expansion will have a form of the partition function of the Coulomb gas:

$$Z = \sum_{l=0}^{\infty}\left[\prod_{i=0}^{2l}\left(\int d\mathbf{r}_i \sum_{n_i}\right)\right]\left(\frac{Y}{2T}\right)^{2l}\exp\left(-\frac{1}{2}\sum_{i,j}^{2l} m_i G(\mathbf{r}_i-\mathbf{r}_j, n_i-n_j)m_j\right), \quad (11)$$

where half of the charges m_i are positive and the other half are negative, for example:

$$m_i = \begin{cases} +1 \text{ for } i = 1,\ldots,l \\ -1 \text{ for } i = l+1,\ldots,2l. \end{cases}$$

In the expansion (11) $Y/2T$ plays the role of the fugacity of the charges and their interaction is given by

$$G(\mathbf{r},n) = \int\int\frac{d^2\mathrm{k}}{(2\pi)^2}\int\frac{dq}{2\pi}G(\mathbf{k},q)\exp i(\mathbf{r}\cdot\mathbf{k}+nq), \quad (12)$$

$$G(\mathbf{k},q) = \frac{2(1-\cos q)T}{J_\parallel k^2}[2(1-\cos q)+(d/\lambda_{ab})^2].$$

From Eqs. (12) it can be easily seen that the charges of the considered Coulomb gas are interacting logarithmically when they are in the same layer and also logarithmically (but with smaller strength) when they are in neighboring or next-to-neighboring layers. Initially there is no interaction between the charges in more distant layers.

The properties of the similar layered Coulomb gas have been analyzed in Ref. 9 when studying the thermodynamics of the layered superconductor with fluctuating magnetic field in the absence of an external field. For the case of the small dimensionless fugacity $y \equiv Y/2Tk_{\max}^2$ the renormalization-group equations were shown to be of the same structure as those derived by Kosterlitz[10] for the ordinary (two-dimensional) Coulomb gas. That means that the phase transition in such layered Coulomb gas should belong to the Berezinskii-Kosterlitz-Thouless universality class, i.e., should be an infinite-order transition. The same holds true for the Coulomb gas defined by the Hamiltonian (9).

For $Y \to 0$ the phase transition takes place at the temperature at which the prelogarithmic factor in the interaction of charges in the same layer is equal to 4. For the interaction of the form (12) it happens at

$$T_m^0 = \frac{4\pi}{3+(d/\lambda_{ab})^2} J_\parallel . \tag{13}$$

Remarkably the same value of the transition temperature is obtained if the results of the vortex-lattice-melting analysis[5] are extrapolated to the limit of high fields corresponding to penetration of vortices between all the layers. The value of T_m obtained by Horovitz[7] with the help of the fermionic representation for $\lambda_{ab} \gg d$ is lower by a factor of $\frac{3}{4}$.

For finite Y the renormalization of the interaction due to presence of small bound pairs of charges should be taken into account. The renormalization decreases the interaction but the transition will still take place when the renormalized value of the prelogarithmic factor is equal to 4. The temperature of the transition T_m is therefore shifted to higher values:

$$T_m = T_m^0[1 + O(y^2)].$$

In the considered model for $T \sim T_m^0$ the dimensionless fugacity $y \propto \Phi_0^2$ is much smaller than one so the shift of T_m is small and the transition remains continuous. Thus the higher is the magnetic field the more accurate are the results obtained in the small fugacity approximation. In the process of renormalization the interaction of charges on distant layers also appears but it remains decaying exponentially with distance between layers.

In Coulomb-gas representation the main difference between the two phases is that in one of them the free charges are present in the system but in the other all charges are bound in neutral pairs. In terms of the original model the transition between these phases corresponds to the transition between the three-dimensional and quasi-two-dimensional superconductor, that is, to the melting of the Josephson vortex lattice.

V. COMPARISON OF THE TWO TRANSITION TEMPERATURES

Thus we have obtained the lower bound for the temperature of the phase transition to the quasi-two-dimensional phase. But one should bear in mind that all the analysis above was carried out on the assumption that this transition can take place at the temperature low enough for the system of the decoupled layers to be in the superconducting state. After that it should be checked whether this assumption is really self-consistent, that is, the value of T_m should be compared with the temperature at which the superconductivity would be destroyed if the layers were decoupled.

In the absence of Josephson coupling between the layers the phase transition in the system of superconducting layers can be also described in terms of a quasi-two-dimensional layered Coulomb gas the charges of which correspond to two-dimensional (pointlike) vortices in this or that layer.[9] A constant magnetic field along the layers have no influence on propeties of such a system. Its partition function will also have the form (11) but with some other value of fugacity and different interaction $\tilde{G}(\mathbf{r}, n)$ corresponding to

$$\tilde{G}(\mathbf{k}, q) = \frac{4\pi^2 J_\parallel}{Tk^2} \frac{2(1-\cos q)}{2(1-\cos q) + (d/\lambda_{ab})^2} . \tag{14}$$

The interaction defined by Eq. (14) depends logarithmically on the in-plane separation of the two-dimensional vortices so at low temperature they can exist only in the form of the small bound pairs. With increase in temperature these pairs will start overlapping and in the high-temperature phase the free vortices will be also present.

As in the case of the layered Coulomb gas discussed in the previous section the phase transition should take place when the prelogarithmical factor in the interaction of the vortices in the same layer is equal to 4. For the interaction of the form (14) in the limit of small fugacities this corresponds to the temperature:

$$T_c^0 = \frac{2\pi}{4 + (d/\lambda_{ab})\sqrt{4+(d/\lambda_{ab})^2} + (d/\lambda_{ab})^2} J_\parallel . \tag{15}$$

But in contrast to the previous case the corrections due to the renormalization of the interaction will make the actual transition temperature T_c not higher but lower than its zero-fugacity limit T_c^0. The difference in the direction in which the renormalization shifts the transition temperature in two cases appears because in one of them the interaction of the Coulomb gas charges is proportional to the temperature and in the other it is inversely proportional.

The existence of the intermediate quasi-two-dimensional phase requires $T_m < T_c$. Comparison of Eq. (13) with Eq. (15) shows that in the zero-fugacity limit the ratio of T_m and T_c for any relation between d and λ_{ab} is not smaller than $\frac{8}{3}$. The finiteness of the fugacity of the two-dimensional vortices can lead only to further increase of this ratio. On the other hand the fugacity of the Coulomb gas charges introduced in Sec. IV is very small so its influence on T_m/T_c can be neglected even if the mutual interaction of vortices and Coulomb gas charges is taken into account. From this we can conclude that the scenario incorporating the existence of the intermediate phase (which requires $T_m/T_c < 1$) is impossible.

VI. CONCLUSION

Thus we have shown that in the presence of a strong magnetic field $B \gg B_* \sim (\phi_0/2\pi d^2)(\lambda_{ab}/\lambda_c)$ the phase transition to the phase in which there is no effective coupling between the layers cannot happen as a separate phase transition preceding the destruction of superconductivity in the system of the decoupled layers. Just as in the case of a smaller field[5,6] the coherence between the layers will be lost simultaneously with the total destruction of superconductivity. The results obtained are applicable not only to layered superconductors but also to the superlattices formed by layers of superconducting and normal metals[11] in which the coupling between the layers can be made rather weak, so that B_* will be much lower than in the bulk superconductor.

The earlier conjecture on the existence of the quasi-two-dimensional phase[1] was based on the calculation of the corrections to the correlation function. It was claimed in Ref. 1 that for $B \gg B_*$ (in our notation) the form of these corrections shows that the behavior of the correlation function for arbitrarily low temperature is qualitatively the same as in the absence of the interlayer coupling. Such calculation cannot be considered as convincing enough since for any value of the field at low enough temperatures fluctuations are small and Gaussian so the three-dimensional ordering cannot be destroyed. Therefore some additional corrections to the correlation function that were not taken into account in Ref. 1 may be also of relevance.

It is interesting to note that according to Glazman and Koshelev[12] in case of a strong magnetic field *perpendicular* to the layers the destruction of the coherence between the weakly coupled layers of a layered superconductor *can* actually happen as a separate phase transition the temperature of which is lower than the temperature of the vortex lattice melting. The disordering in that case is induced by the phase fluctuations related to fluctuations of the vortex lattice.

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