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Two-mode electrodynamics of superconductors in the mixed state

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The surface impedance of type-II superconductors in the mixed state has been studied, taking into account the vortex elasticity and nonlocality (long-range interaction of vortices). An electromagnetic wave penetrates into a superconductor as a superposition of two exponentially decaying modes with different penetration lengths, in contrast with one mode in a normal conductor or in a superconductor in the Meissner state. The second mode is an elastic mode of the vortex array; it is crucial for incorporation of vortex elasticity and surface pinning into electrodynamics.

The electrodynamics of the type-II superconductor in the mixed state is governed by the dynamics of vortices. Elastic properties of vortex lines are of great interest since they are fingerprints of different phases of the vortex array (lattice, fluid, glass, and others) and are now intensively discussed for high-T_c superconductors.[1-4] The effect of vortex elasticity on the surface impedance was studied by Gorkov and Kopnin[5] for a superconductor in the resistive state, when the bulk pinning is not important. They showed that at low frequencies the ac response and the penetration depth are the same as those of the normal conductor possessing resistance equal to the flux-flow resistance ρ_f. In order to determine the differential magnetic permeability $\mu = dB/dH < 1$ of the mixed state, which takes into account the mixed-state diamagnetism due to circular currents in the vortex-array cell, Gorkov and Kopnin[5] used the Labusch elastic moduli[6] of the vortex lattice. However, at high frequencies, when the penetration depth of the wave (the skin-layer width) is smaller than the penetration depth λ for the static magnetic field, this approach becomes invalid. In this case one should take into account long-range interaction between vortices with λ being its cutoff radius. This nonlocal effect manifests itself in spatial dispersion of the elastic moduli which was discussed by Brandt.[7] Nonlocality cancels the proportionality relation between the average magnetic field and the vortex density in a nonuniformly deformed vortex structure.[8] The wave penetration depth becomes close to the static penetration depth λ and the ac response is governed by the vortex motion in this layer. The theory of the ac response for this case was developed by Gittleman and Rosenblum[9] (see also Ref. 10) for the Abrikosov vortices and by Sonin and Tagantsev[11] for vortices in the Josephson-junction network (the Josephson medium). Recently a general analysis of the electromagnetic wave penetration into a superconductor has been undertaken incorporating the effects of image vortices, bulk pinning, and vortex creep.[12,13] All theories of the ac response presented in the above-mentioned papers assume that the incident electromagnetic wave generates inside the superconductor only one exponentially decaying mode. However, the theory that considers simultaneously the effect of the vortex elasticity and that of nonlocal interaction, requires a model of the ac response that invokes two modes of the collective field-vortex-lattice motion inside the superconductor.[14] The second mode is connected with the additional degree of freedom related to the elasticity of vortices.

We have already demonstrated[14] that the effect of elasticity influences the frequency dependence of the surface impedance at high frequencies. In the present paper we present the two-mode electrodynamics approach to the ac response in the low-frequency limit. We show that the additional elastic mode is essential for the derivation of the correct electrodynamical relations that take into account diamagnetism of the mixed state and for the analysis of the surface pinning which is able to strongly suppress the absorption of the electromagnetic wave in the bulk superconductor. The surface impedance has been analyzed, but the developed approach is relevant also for other experiments related to electrodynamics, such as ultrasound propagation or vibrating reeds.

Let us consider a type-II superconductor placed in a dc external magnetic field applied along the z axis normal to the superconductor surface. Assume that the intervortex distance a satisfies the condition $\lambda \gg a \gg r_c$, where λ stands for the London penetration length and r_c for the vortex core radius. We neglect the normal current $\mathbf{j}_n$ of quasiparticles as being small in respect to the supercurrent $\mathbf{j}_s$, i.e., $\mathbf{j} \approx \mathbf{j}_s$. Then the macroscopic electrodynamics of the superconductor may be described by the London equation together with the equation of vortex motion and the Maxwell equation. All these equations are averaged over the vortex-array cell:

$$\mathbf{h} - \lambda^2 \frac{\partial^2 \mathbf{h}}{\partial z^2} = B_0 \frac{\partial \mathbf{u}}{\partial z} , \tag{1}$$

$$\eta \frac{\partial \mathbf{u}}{\partial t} = \frac{\phi_0}{c} [\mathbf{j} \times \hat{\mathbf{z}}] + \frac{\phi_0}{B_0} C_{44}^* \frac{\partial^2 \mathbf{u}}{\partial z^2} , \tag{2}$$

$$\mathbf{j} = \frac{c}{4\pi} \operatorname{curl} \mathbf{h} . \tag{3}$$

Here B_0 is the magnetic induction inside the superconductor, $\mathbf{u}$ is the vortex displacement, $\mathbf{h}$ is the ac magnetic field in the x-y plane, η is the friction coefficient for the vortex motion, ϕ_0 is the magnetic flux quantum, and

$$C_{44}^* = C_{44} - \frac{B_0^2}{4\pi} = \frac{B_0(H_0 - B_0)}{4\pi} = B_0 |M_0| \tag{4}$$

is the renormalized tilt modulus. Observe, that C_{44}^* is different from the Labusch[6] tilt modulus $C_{44}=B_0H_0/4\pi$, $H_0=B_0-4\pi M_0$. Using the known dependence of B_0 on H_0, the explicit expression for the renormalized tilt modulus is

$$C_{44}^{*}=\frac{B_0H_{c1}}{4\pi}\frac{\ln(a/r_c)}{\ln(\lambda/r_c)}, \quad (5)$$

where H_{c1} is the lower critical field.

The eigenmodes of the set of Eqs. (1)-(3) may be found in the form $u\propto h\propto j\propto\exp(ikz-i\omega t)$ where the wave number k and the frequency ω fulfill the following dispersion equation:

$$\frac{i\omega}{\omega_B}=k^2\lambda^2\left[\frac{1}{1+k^2\lambda^2}+\frac{\omega_c}{\omega_B}\right], \quad (6)$$

where

$$\omega_c=\frac{\phi_0C_{44}^{*}}{B_0\lambda^2\eta}=\frac{\phi_0H_{c1}}{4\pi\lambda^2\eta}\frac{\ln(a/r_c)}{\ln(\lambda/r_c)}, \quad \omega_B=\frac{\phi_0B_0}{4\pi\lambda^2\eta}. \quad (7)$$

Neglecting the nonlocal effect described by the term $\propto\lambda^2$ in the left-hand side of Eq. (1), one obtains the one-mode theory of Gorkov and Kopnin.[5] Within this theory the ac magnetic field $\mathbf{h}$ is equal to the right-hand side of Eq. (1) representing the variation of the magnetic-flux density due to the tilt $d\mathbf{u}/dz$ of the vortex lines. Whereas the theories that take into account the nonlocal effect, but neglect the vortex elasticity in the final expressions for the surface impedance,[9-13] correspond to $C_{44}^{*}=0$ in Eq. (2) and to $\omega_c=0$ in Eq. (7). In both cases the dispersion equation (7) is linear with respect to k^2. In general the dispersion equation is quadratic with respect to k^2 and has the following two solutions:

$$k_{1,2}^2=\frac{1}{\lambda^2}\left\{-\frac{1}{2}\left(1+\frac{\omega_B}{\omega_c}-\frac{i\omega}{\omega_c}\right)\pm\left[\frac{1}{4}\left(1+\frac{\omega_B}{\omega_c}-\frac{i\omega}{\omega_c}\right)^2+\frac{i\omega}{\omega_c}\right]^{1/2}\right\}. \quad (8)$$

The surface impedance of any conductor is given by the well-known expression $Z=(E_t/H_t)$ where E_t and H_t are the tangential components of the electric and magnetic field at the surface. In the framework of our two-mode model we take into account the contributions of both modes to the fields. Then using the Maxwell equation $\partial\mathbf{h}/\partial t=-c\,\mathrm{curl}\mathbf{E}$, we write

$$Z=\frac{E_1+E_2}{h_1+h_2}=\frac{\omega}{ck_1}\left(1+\frac{h_2}{h_1}\frac{k_1}{k_2}\right)\left(1+\frac{h_2}{h_1}\right)^{-1}, \quad (9)$$

where the subscripts 1 and 2 correspond to the contributions of two modes to the tangential components of the fields at the surface, the mode 1 referring to the long-wavelength mode. Since there are two components of the ac fields near the superconductor surface, a new boundary condition should be added to the usual electrodynamical conditions:

$$\frac{\partial\mathbf{u}}{\partial z}+b\mathbf{u}=0, \quad (10)$$

where b is the surface-pinning parameter. Without pinning $b=0$, while in the limit of the strong pinning $b\to\infty$. When the surface pinning is absent the boundary condition $\partial\mathbf{u}/\partial z=0$ follows directly from the analysis of the fields generated by vortices and their images near the surface made recently in Refs. 12 and 13.

Using Eq. (1) one finds that the condition Eq. (10) implies the following ratio of the amplitudes of the modes at the surface:

$$\frac{h_2}{h_1}=-\frac{(1+\lambda^2k_1^2)(1+b/ik_1)}{(1+\lambda^2k_2^2)(1+b/ik_2)}. \quad (11)$$

Equations (8), (9), and (11) yield a solution of the problem in the framework of the two-mode electrodynamics, e.g., the results of Ref. 14 can be obtained from these expressions. When the surface pinning is weak, $b\ll|k_1|$, from (8), (9), and (11) we have

$$Z=-\frac{i\omega\lambda}{c}\frac{1-(-\omega_c/i\omega)^{1/2}-\omega_B/i\omega}{\{[1-(-\omega_c/i\omega)^{1/2}]^2-\omega_B/i\omega\}^{1/2}}. \quad (12)$$

In the low-frequency limit, $\omega\ll\omega_c$, the wave numbers of the two modes are

$$k_1^2=\frac{1}{\lambda^2}\frac{i\omega}{\omega_B+\omega_c}, \quad k_2^2=-\frac{1}{\lambda^2}\frac{\omega_B+\omega_c}{\omega_c}. \quad (13)$$

Then the surface impedance and the surface resistance are

$$Z=\left(\frac{-i\omega\mu\rho_f}{4\pi}\right)^{1/2}, \quad \rho_s=\frac{(2\pi\rho_f\mu\omega)^{1/2}}{c}, \quad (14)$$

where

$$\rho_f=\frac{B\phi_0}{c^2\eta}, \quad \mu=\frac{\omega_B}{\omega_B+\omega_c}=\frac{B_0^2}{4\pi C_{44}} \quad (15)$$

stand for the flux-flow resistance and the differential magnetic permeability, respectively. The expressions for Z and ρ_s follow directly from electrodynamics of the continuous medium[15] with the magnetic permeability μ and the resistance ρ_f.

Let us compare the predictions of the two-mode theory to those of the one-mode theory of Gorkov and Kopnin[5] which is expected to be valid in the low-frequency limit. One obtains the one-mode theory neglecting the second-mode amplitude h_2 in Eq. (9). Then the surface impedance in the low-frequency limit is

$$Z=\frac{\omega}{ck_1}=\left(\frac{-i\omega\rho_f}{4\pi\mu}\right)^{1/2}. \quad (16)$$

This expression differs from the one derived within the two-mode theory, as well as from electrodynamics of the continuous media by the factor μ.

The reason why the one-mode theory diverges from electrodynamics of continuous media is following. The one-mode theory takes into account the diamagnetic properties ($\mu<1$) of the bulk, but it neglects the surface currents responsible for the jump of the tangential component of the induction $\mathbf{B}$ (the averaged magnetic field). Indeed, in electrodynamics[15] the tangential component of the thermodynamical magnetic field $\mathbf{H}=\mathbf{B}/\mu$ is continuous, whereas that of $\mathbf{B}$ is not. In the low-frequency limit of the two-mode theory the penetration depth of the additional mode is much smaller than the penetration depth of the basic electrodynamical mode, $k_1 \ll k_2$. Regardless that there is only one mode in the bulk, the short-wavelength elastic mode remains important, since it generates the currents resulting in the jump of the tangential component of **B** at the surface. Indeed, for $k_1 \ll k_2$ one can neglect the term in the first set of parentheses in Eq. (9) and obtain the following expression for the surface impedance:

$$Z = \frac{\omega\mu}{ck_1}, \tag{17}$$

known from electrodynamics, where $\mu = h_1/(h_1+h_2)$ is given by Eq. (15).

We have found an interesting manifestation of the additional mode in the case of strong surface pinning ($b \rightarrow \infty$). In this case from Eqs. (8), (9), and (11) we obtain

$$Z = -\frac{i\omega\lambda}{c}\left[1+\frac{\omega_B}{\omega_c[1+(-i\omega/\omega_c)^{1/2}]^2}\right]^{1/2}. \tag{18}$$

In the low-frequency limit $\omega \ll \omega_c$ the surface resistance is

$$\rho_s = \frac{4\pi}{c}\mathrm{Re}Z = \frac{(2\pi\rho_f\omega^2\mu)^{1/2}}{c\omega_c}. \tag{19}$$

Comparing this result to that obtained for the weak surface-pinning limit [see Eq. (14)], we note that the surface pinning drastically influences the absorption of the incident electromagnetic wave: The frequency dependence of the surface resistance changes from $\propto \omega^{1/2}$ to $\propto \omega^{3/2}$ and the absorption is now ω_c/ω times smaller in respect to the case of weak surface pinning. This behavior can be explained as follows.

In the low-frequency regime the absorption is mostly due to the long-wavelength mode with the penetration depth $|k_1^{-1}|$. If the vortex ends are free to move along the surface, then the energy of the incident wave goes mainly into this mode. The leak of the energy into the second mode is insignificant, therefore its role is of a minor importance. However, the strong surface pinning changes dramatically the situation. Now the wave energy goes mainly to the short-wavelength mode, since $h_1/h_2 \propto \sqrt{\omega}$ in the low-frequency limit. So the share of the long-wavelength mode responsible for the dissipation becomes small. It causes a strong decrease of the energy absorption, i.e., a decrease of the surface resistance of the superconductor. Thus, the surface pinning drastically influences the energy dissipation in the bulk of the superconductor. Let us underline that this phenomenon cannot be described in the terms of the one-mode approach. Note also that in the limit of $\omega \ll \omega_c \ll \omega_B$, Eq. (18) predicts the magnetic-field dependence $\propto B_0^{1/2}$ for the surface impedance or the effective penetration depth, similar to that observed by Hebard *et al.*[16] who explained their result by the bulk pinning. In order to prove whether the bulk or surface pinning is responsible for observed dependence, one may study the frequency dependence of the surface resistance ρ_s: In the low-frequency limit it is expected to be $\propto \omega^2$ for the bulk pinning and $\propto \omega^{3/2}$ for the surface pinning.

Our continuous-medium approach is valid for the penetration depths $2\pi/|k_1|$ and $2\pi/|k_2|$ exceeding the intervortex distance $a \approx (\phi_0/B_0)^{1/2}$. This is equivalent to the condition $\pi\ln(a/r_c) \gg \max(1,\omega_c/\omega_B,\omega/\omega_B)$. It means that the external magnetic field cannot be close to the lower critical field H_{c1} (the condition of the dense lattice $\lambda/a \gg 1$) or to the upper critical field H_{c2} [$\pi\ln(a/r_c) \gg 1$]. When the inequality $2\pi/|k_2| \gg a$ is broken, the two-mode theory fails to describe variation of the magnetic field and the vortex displacements in the boundary layer of the width a where the jump of the tangential component of the magnetic induction **B** occurs. However, even in this case one can use the electrodynamical results in the low-frequency limit $\omega \ll \omega_c$ given by Eqs. (14) and (15).

In conclusion, the two-mode electrodynamics of the mixed state incorporating the nonlocal effects due to long-range intervortex interaction and the effects of the vortex-array elasticity has been developed. It has been shown that in order to obtain the correct macroscopic electrodynamics of superconductors in the mixed state, one has to take into account the additional elastic vortex mode. The new two-mode electrodynamics predicts surface-pinning-induced suppression of the surface resistance.

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