Thermoelectric power and transport entropy of dirty type-II superconductors

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The relation $\int_0^{T_{c2}} S(B,T)dT \approx B\hbar/(2mc)$ presented recently for the thermoelectric power S of a dirty high-T_c superconductor [V. V. Gridin *et al.*, Phys. Rev. B **40**, 8814 (1989)] is considered. It is shown that measurements which have been reported for the thermomagnetic coefficients of nearly reversible conventional type-II superconductors provide additional support for this relation. The result obtained from time-dependent microscopic theory for vortex motion (time-dependent Ginzburg-Landau theory and linear-response theory) is also discussed.

Recently Gridin *et al.*[1] reported measurements of the thermoelectric power S of a high-T_c superconductor (Bi-Sr-Ca-Cu-O) as a function of the average magnetic induction B and the absolute temperature T. They find that, for

$$B \ll H_{c2}(0) , \tag{1}$$

the measured values of $S(B,T)$ satisfy the simple relation[1]

$$\int_0^{T_{c2}} S(B,T)dT = gBe\hbar/2qmc \tag{2}$$

with $q = e$ and

$$g \approx 1,13 . \tag{3}$$

Here $H_{c2}(0)$ is the upper critical field at $T=0$, T_{c2} is the transition temperature in the presence of a field B, m is the electronic mass, e is the magnitude of the electronic charge, and q is the charge of the charge carriers. These authors also gave a derivation which yielded the formula (2) with $g \approx 1$; they emphasized the approximate nature of their analysis and suggested that a more rigorous treatment would be of interest.

The purpose of this paper is twofold. Firstly, we show that measurements of transport properties (thermomagnetic coefficients) that have been reported for nearly reversible conventional type-II superconductors also provide support for (2) and (3). Secondly, we compare the result obtained from time-dependent microscopic theory for vortex motion [time-dependent Ginzburg-Landau (TDGL) theory and linear-response theory] with (2) and (3).

For our purposes it is useful to consider the transport entropy of a moving vortex $\sigma_d(B,T)$, which is the amount of deliverable entropy carried per vortex per unit length of vortex.[2] The thermoelectric power is given in terms of σ_d by

$$S(B,T) = \frac{B}{Nq\phi_0}\sigma_d(B,T) , \tag{4}$$

where $\phi_0 = hc/2e$ is the flux quantum and N is the total electron density.[3] We start with a discussion of the results of experimental work on the transport entropy.

There have been detailed experimental studies of thermomagnetic effects in superconductors.[2,4–8] In these studies the transport entropy has been obtained from measurements of both the Ettingshausen and Nernst effects; the values of σ_d obtained from these two effects in nearly reversible alloys are in good agreement.[4,6] A typical dependence of the measured $\sigma_d(B,T)$ on B for fixed T is shown in Fig. 1 for a thin slab of superconductor in perpendicular magnetic field (i.e., demagnetizing factor ≈ 1). The transport entropy is zero at B_{c2} and it increases linearly as B decreases below B_{c2}; in dirty type-II superconductors this linear dependence persists down to fields $B \approx (0.3-0.5)B_{c2}$.[4–6] At lower fields the curve becomes concave upwards, and $\sigma_d(0,T)$ is larger than the value $\sigma_d'(T)$ obtained by extrapolating the linear portion to $B=0$. For fixed B the measured values of $\sigma_d(B,T)$

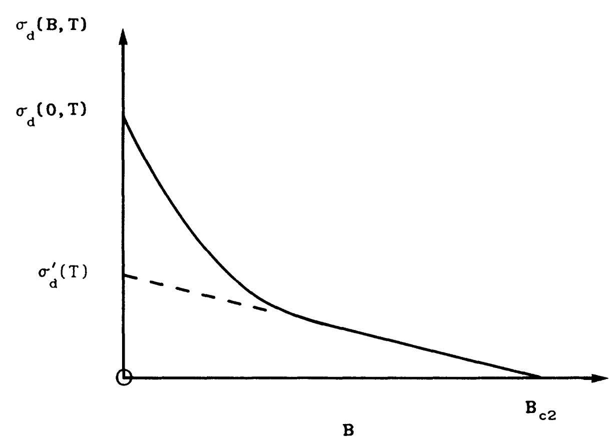

FIG. 1. Schematic illustration the the magnetic field dependence of the transport entropy $\sigma_d(B,T)$ at fixed T for dirty type-II superconductors (see, for example, Fig. 7 of Ref. 2 and Fig. 3 of Ref. 6).

tend to zero as $T \to 0$ (as required by the third law of thermodynamics[9]) and as $T \to T_{c2}$. At fixed low fields ($B \ll B_{c2}$), the plot of σ_d vs T has a maximum at $T \approx 0.5 T_c$ (see Refs, 2, 4–8, and Fig. 2). The accuracy of measurements of $\sigma_d(B,T)$ at low fields depends on the reversibility of the sample. The two most reversible alloys on which measurements have been performed are $Nb_{85}Mo_{15}$ and $Nb_{80}Mo_{20}$,[4,6] and in the following we consider these two alloys.

A convenient unit in which to express the transport entropy at low fields is

$$\bar{\sigma}_d = \frac{\phi_0 H_{c2}(0)}{4\pi T_c \kappa^2} , \tag{5}$$

where κ is the Ginzburg-Landau parameter. In Table I we list experimental values of the parameters appearing in (5), and the corresponding values of $\bar{\sigma}_d$ for $Nb_{85}Mo_{15}$ and $Nb_{80}Mo_{20}$. The data in Table I were obtained from Refs. 4, 6, 10, and 11. [The values for $H_{c2}(0)$ were obtained by fitting the data for $H_{c2}(T)$ to Eq. (16) below.] In Fig. 2 we show measured values of the transport entropy at low fields, taken from Refs. 4 and 6. The data points in Fig. 2 have been expressed in terms of the values $\bar{\sigma}_d$ given in Table I. The solid curve in Fig. 2 has been drawn to agree, to within the experimental error of $\sim 15\%$, with the data points for $0.3T_c \lesssim T \lesssim T_c$. Below $0.3T_c$, we have extrapolated the transport entropy to zero as required by the third law of thermodynamics.[9] We note that a bell-shaped curve for $\sigma_d(0,T)$ vs T is characteristic of all superconductors on which measurements have been performed, and that the extrapolation procedure is supported by experiments on Nb and $In_{60}Pb_{40}$, where results are available at a lower, reduced temperature.[2,7,8] An evaluation of the area under the curve in Fig. 2 yields

$$(T_c \bar{\sigma}_d)^{-1} \int_0^{T_c} \sigma_d(0,T) dT \approx 0.5 \pm 25\,\% , \tag{6}$$

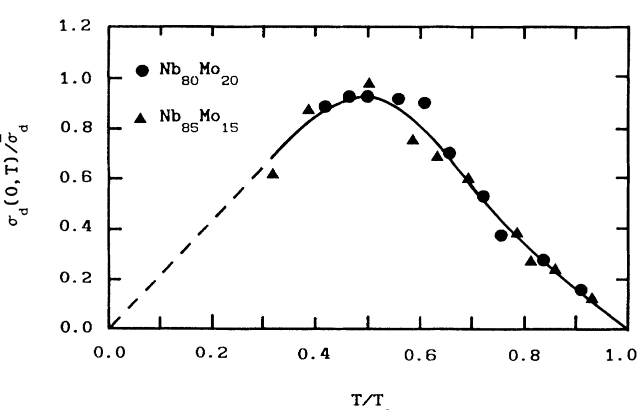

FIG. 2. Experimental values of the transport entropy at low fields in $Nb_{85}Mo_{15}$ and $Nb_{80}Mo_{20}$. The data, which are from Refs. 4 and 6, have been expressed in terms of $\bar{\sigma}_d$ defined by (5). The numerical values of $\bar{\sigma}_d$ are given in Table I. The solid curve and the dashed curve have been constructed as described in the text.

TABLE I. Experimental parameters for $Nb_{85}Mo_{15}$ and $Nb_{80}Mo_{20}$. See text.

	T_c (K)	κ	$H_{c2}(0)$ (kOe)	$\bar{\sigma}_d$ (erg cm^{-1} K^{-1})
$Nb_{85}Mo_{15}$	5.16	3.6	7.2	18×10^{-7}
$Nb_{80}Mo_{20}$	4.15	4.1	5.4	13×10^{-7}

where the indicated error is our estimate of the combined uncertainties in the measurement of $\sigma_d(0,T)$ and the extrapolation in Fig. 2.

From Eqs. (4)–(6) we obtain (2) with

$$g = (0.5 \pm 25\,\%) \frac{mcH_{c2}(0)}{Neh\kappa^2} . \tag{7}$$

For a dirty type-II superconductor we have the relations[12]

$$H_{c2}(0) = \sqrt{2}\kappa_1(0) H_c(0) , \tag{8}$$

$$H_c(0) = Nhe\kappa_3(0)/(\sqrt{2}mc) , \tag{9}$$

where $\kappa_1(0) = 1.20\kappa$ and $\kappa_3(0) = 1.54\kappa$. Equations (7)–(9) yield

$$g = 0.9 \pm 25\,\% . \tag{10}$$

We conclude that measurements of the transport entropy in these alloys provide additional support for Eqs. (2) and (3).

It is interesting to note that a separate estimate of the value of g can be made as follows. There have also been measurements of the total entropy of vortex, $\sigma_i(B,T)$.[2,4,13] The theoretical value for an isolated vortex,

$$\sigma_i(0,T) = -\frac{\phi_0}{4\pi} \frac{dH_{c1}(T)}{dT} , \tag{11}$$

where $H_{c1}(T)$ is the lower critical field, is in good agreement with experiment in $Nb_{85}Mo_{15}$ and $Nb_{80}Mo_{20}$.[6,14] Although the measured values of σ_d and σ_i in these alloys are, in general, different, the areas under the curves of $\sigma_d(0,T)$ and $\sigma_i(0,T)$ vs T between 0 and T_c are approximately equal[15]

$$\int_0^{T_c} \sigma_d(0,T) dT \approx \int_0^{T_c} \sigma_i(0,T) dT = \frac{\phi_0}{4\pi} H_{c1}(0) . \tag{12}$$

For a dirty superconductor with $\kappa = 3.6$, we have[16]

$$H_{c1}(0) \approx 0.33 H_c(0) . \tag{13}$$

From Eqs. (4), (12), (13), and (9) we obtain the estimate $g \approx 1.3$. Similarly, for $\kappa = 4.1$ we obtain the same estimate of g.

Finally, we consider theoretical values of the transport entropy. There has been considerable interest in calculations of the transport entropy based on microscopic theories of the time-dependent mixed state (TDGL theory or linear-response theory).[17–19] For dirty type-II superconductors, the transport entropy calculated from these theories is given by[17–19]

$$\sigma_d(B,T)=\frac{\phi_0}{4\pi T_c}\frac{B_{c2}(t)-B}{1.16[2\kappa_2^2(t)-1]+1}\frac{L(t)}{t}\ , \qquad (14)$$

where $t=T/T_c$ is the reduced temperature,

$$L(t)=1+\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\ , \qquad (15)$$

and $\rho[\sim B_{c2}(T)]$ is determined by

$$\ln t=\psi(\tfrac{1}{2})-\psi(\tfrac{1}{2}+\rho)\ . \qquad (16)$$

Here ψ, $\psi^{(1)}$, and $\psi^{(2)}$ are, respectively, the digamma function and its higher-order derivatives.[20] The parameter $\kappa_2(t)$ increases slightly from κ at $t=1$ to 1.2κ at $t=0$.[21]. According to (14)–(16), σ_d goes to zero at $T=0$ and $T=T_c$, and σ_d at fixed T increases linearly from zero as B decreases below B_{c2}. Equation (14) is obtained by an expansion to lowest order in the order parameter,[17–19] and this usually requires magnetic fields close to $B_{c2}(t)$. The values given by (14)–(16) are in good agreement with the measured $\sigma_d(B,T)$ for $B\gtrsim(0.3-0.5)B_{c2}$.[5,6]

From (4), (14), (8), and (9), we obtain Eq. (2) with

$$g=0.8\int_0^{T_{c2}/T_c}\frac{\kappa^2}{\kappa_2^2(t)}\frac{B_{c2}(t)}{B_{c2}(0)}\left[1-\frac{B}{B_{c2}(t)}\right]\frac{L(t)}{t}dt\ . \qquad (17)$$

In writing down (17), we have assumed that $\kappa_2\gg 1$. The dimensionless parameter (17) is a decreasing function of B; the maximum value of g occurs at $B=0$ (and hence $T_{c2}=T_c$). This maximum value g_{max} can be determined by a numerical integration: in this calculation we use $B_{c2}(t)/B_{c2}(0)$ given by (16) and the values of $\kappa_2(t)/\kappa$ for a dirty superconductor.[21] The result is

$$g_{max}\approx 0.38\ . \qquad (18)$$

Thus, the above results from the time-dependent microscopic theory yield a value for the parameter g which is smaller than the empirical value (3) by a factor $\gtrsim 3$. The reason for this discrepancy is presumably that (18) corresponds to taking σ_d equal to the extrapolated value σ_d' shown in Fig. 1, whereas the actual value of σ_d at low fields is larger than σ_d'. In fact, the experimental results in Refs. 4–6 show that

$$\sigma_d(0,T)=(3\pm 30\,\%)\sigma_d'\ .$$

We remark that there are considerable theoretical difficulties associated with any attempt to deduce (2) and (3) from microscopic theory at low fields. To our knowledge, the only case in which the TDGL theory has been used to calculate the low-field transport entropy for all $T\le T_c$ is for superconductors containing a sufficiently high concentration of magnetic impurity that they are gapless at low fields.[22] These superconductors are difficult to work with, in practice, because the transition temperature is strongly suppressed by the magnetic impurity; we are unaware of any measurements of thermoelectric power and thermomagnetic coefficients on such materials, and therefore we have not considered them in this paper.

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