Generalized one-dimensional t-J models with longer-range spin exchange

J. Bonča and P. Prelovšek
Jožef Stefan Institute, University of Ljubljana, 61111 Ljubljana, Slovenia
(Received 16 January 1992)

The behavior of holes in the one-dimensional (1D) t-J model differs substantially from the one in planar correlated systems. We show that within the 1D model one can recover some essential properties of 2D systems by including longer-range spin exchange. Be means of analytical calculations for the anisotropic exchange and the exact diagonalization of small systems, it is established that this generalization leads to the large mass enhancement for a single hole, to the hole binding at strongly reduced J/t, and to substantial changes in the phase diagram at electron densities $n \lesssim 1$. It is shown that the longer spin exchange changes qualitatively the phase-separation line $J_c(n)$, which obtains an n dependence analogous to the 2D systems, i.e., opposite to the one in the 1D t-J model. The correlation exponent K_ρ calculated in the Luttinger-liquid regime $J < J_c(n)$ indicates also that the region of dominant pairing correlations can move even to $J/t < 1$.

I. INTRODUCTION

Unusual electronic properties and the unknown mechanism of superconductivity in copper oxides have stimulated numerous studies of low-dimensional models for strongly correlated systems. Of particular interest is the ground state of the prototype t-J (Ref. 1) and Hubbard models on a planar lattice. So far much better understood are 1D systems. Their typical feature is a separation of charge and spin excitations[2,3] in the low-energy regime. Several models show such phenomenon in the whole energy range: the Luttinger model,[2] the Hubbard model in the limit $U/t \gg 1$,[4] or equivalently the t-J model at $J/t \ll 1$.

Properties of 2D systems appear to be quite different.[5-9] There are several indications for a stronger coupling between spin and charge degrees of freedom, at least beyond the low-excitation regime. Hole masses are found to be strongly enhanced for $J/t \ll 1$.[5,6,8,9] Two holes added to the undoped antiferromagnet (AFM) seem to bind in a singlet in a broad range of J/t.[7,9] There is also enough evidence for essential differences in the phase diagram. The phase diagram of the planar t-J model has been analyzed so far by the means of the exact diagonalization[10] and by the high-temperature expansion.[11] The shape of the phase separation line is still to some extent questionable, in particular near the half filling $n \lesssim 1$. Nevertheless all results show, that on increasing J the phase separation takes place first at $n \lesssim 1$, while $J_c(n)$ monotonically increases toward lower densities $n \to 0$. This is just opposite to the behavior found in the 1D model.[12]

In this paper we show that several 2D phenomena can be simulated within a generalized 1D t-J model, where the nearest-neighbor exchange J is supplemented with longer-range exchange terms J_l, $l > 1$. Such a model can be advocated in terms of the 2D t-J model, which can be reduced to the generalized 1D model after a mean-field-type decoupling between chains. This procedure and the model are described in Sec. I. Section II is devoted to the analysis of single hole and hole pair properties. The difference to the usual t-J model can be best understood and approached analytically within the simpler model with an anisotropic (Ising-like) spin exchange, while the isotropic case is studied by the exact diagonalization of small systems. Of particular interest are the strong renormalization of the coherent mass and the appearance of the hole binding at modest $J/t < 1$. These effects are quite established in the 2D t-J model, where they appear mainly as a consequence of the local AFM order across the hole. In 1D similar effects may be achieved by a longer-range spin exchange. Results for other electron densities n and for the phase diagram are presented and discussed in Sec. III. One of the most interesting observations here is that in the generalized model the phase-separation line behaves as in 2D. Before the onset of the phase separation we find as well a regime dominated by pairing correlations, which can now appear even at $J/t < 1$.

II. MODEL

In this paper we consider the following model, which is a generalization of the 1D t-J model,

$$H = -t \sum_{i,s} (c^\dagger_{i,s} c_{i+1,s} + \text{H.c.}) + \sum_l (-1)^{l-1} J_l \sum_i (\mathbf{S}_i \cdot \mathbf{S}_{i+l} - \tfrac{1}{4} n_i n_{i+l}) , \quad (1)$$

where J_l, $l > 1$ denote the spin exchange beyond nearest neighbors. The signs are already chosen so that $J_l > 0$ enhance the AFM ordering. In Eq. (1) the density-density correlations are subtracted in all exchange terms to keep the analogy with the usual t-J model.

The underlying idea is straightforward. The exchange $J_l, l > 1$ can establish an AFM (Néel) ordering across the hole. This is the case in 2D where it is achieved by an indirect exchange around the hole. It is evident however that on a chain the long-range AFM order, which exists in a planar system at small (vanishing) doping, can be ob-

tained only within an infinite range $L \to \infty$ model. On the other hand, discussing the finite doping more realistic are finite range models $L \sim \bar{a} < \infty$, where $\bar{a}$ denotes the average hole-hole separation.

In a more formal way we can establish the relation with the planar t-J model on the level of the mean-field-type approximation (MFA). Let us assume a system of 1D correlated chains coupled only via the spin exchange $\tilde{J}$, with an additional external inhomogeneous field h_i acting on the central 0th chain,

$$H = \sum_n H_n^0 + \tilde{J} \sum_i \mathbf{S}_{ni} \cdot \mathbf{S}_{n+1,i} - \sum_i S_{0i}^z h_i \ , \tag{2}$$

where H_n^0 represents the t-J Hamiltonian for the nth chain. Performing the MFA-type decoupling of the interchain spin exchange term and calculating the local spin averages within the linear response to the effective mean fields on each chain, we obtain a set of equations in the wave vector q representation

$$\langle S_{nq}^z \rangle = \chi_q^0 h_q \delta_n - \tilde{J} \chi_q^0 (\langle S_{n+1,q}^z \rangle + \langle S_{n-1,q}^z \rangle) \ , \tag{3}$$

where χ_q^0 is the susceptibility of the 1D t-J model. If there is no spontaneous ordering for $h_q = 0$, i.e., if $2\tilde{J}\chi_q^0 < 1$, the set of Eqs. (3) yields the solution

$$\langle S_{nq}^z \rangle = \frac{\chi_q^0 h_q}{\sqrt{1 - 4\tilde{J}^2 \chi_q^{0^2}}} \ . \tag{4}$$

Treating in an analogous MFA also the longer range $l > 1$ spin exchange in the generalized 1D model (1), with the same additional external field as in Eq. (2), we get as a result the correspondence

$$J_l = \frac{1}{N} \sum_q e^{-i(q+\pi)l} (\chi_q^{0^{-1}} - \sqrt{\chi_q^{0^{-2}} - 4\tilde{J}^2}) \ , \quad l > 1 \ , \tag{5}$$

which reduces for small $\tilde{J}$ to

$$J_l = \frac{1}{N} \sum_q e^{-i(q+\pi)l} 2\tilde{J}^2 \chi_q^0 \ , \quad l > 1 \ . \tag{6}$$

Since at low doping χ_q^0 has a maximum (singularity) at $q \sim \pi$ we reproduce the desired form with $J_l > 0$ only slowly decreasing with l.

It is evident that the previous analysis where the correspondence between models has been established on the level of the linear response is appropriate only for qualitative arguments within the paramagnetic phase, characterized by a short-range AFM order. On the other hand at $T = 0$ the singularity of χ_q^0 at $q = 2k_F$ leads in Eq. (3) to the instability of the assumed paramagnetic state toward a spin-ordered phase. This tendency to ordering in 2D is however overestimated due to the MFA-type approach and in particular due to the neglected interchain hopping. Nevertheless we expect that hole properties are mostly governed by short-range AFM order, so that the proper description of the longer-range order is not so crucial.

III. SINGLE HOLE AND TWO HOLE STATES

The main novel feature of the longer-range spin exchange in the model Eq. (1) is the spin string effect connected with the hole motion, typical for 2D systems,[5,7] but not present in the usual 1D t-J model.[13] This is best seen in *the Ising-type model* with an extreme anisotropic spin exchange, where we replace

$$\mathbf{S}_i \cdot \mathbf{S}_{i+l} \to S_i^z S_{i+l}^z \ . \tag{7}$$

Without holes the ground state is in this case an ordered Néel spin configuration. The advantage of the Ising case is that the problem of one and two additional holes, i.e., $N_h = 1, 2$, can be reduced to a potential problem and treated analytically.

Let us for illustration first discuss a single hole $N_h = 1$ case for a specific exchange range $L = 3$. Starting with a localized hole at the origin $r = 0$ in a Néel spin background, as shown in Fig. 1(a), one can generate different configurations by hole hopping, as presented in Figs. 1(b) and 1(c) (for distances $r = 1, 2$). The corresponding potential energies, expressed relative to the $N_h = 0$ system, are

$$\begin{aligned} \epsilon_0 &= J_1 + J_2 + J_3 \ , \\ \epsilon_1 &= \epsilon_0 + \tfrac{1}{2}J_1 + J_2 + J_3 \ , \\ \epsilon_2 &= \epsilon_0 + \tfrac{1}{2}J_1 + \tfrac{3}{2}J_2 + 2J_3 \ , \\ \epsilon_{r \geq 3} &= \epsilon_0 + \tfrac{1}{2}J_1 + \tfrac{3}{2}J_2 + \tfrac{5}{2}J_3 \ , \end{aligned} \tag{8}$$

while $\epsilon_{-r} = \epsilon_r$. It is easy to see that ϵ_r increases with distance until $|r| = L$, if we choose all $J_l > 0$. This is a typical string effect, which has the correspondence with the analogous phenomenon found in 2D with the long-range AFM order.[5] In the latter case however the energy increases linearly to arbitrary distances (if one neglects loops). Therefore, to simulate the ordered 2D state we would need in 1D an infinite range $L \to \infty$.

It is now easy to calculate the ground-state energy E_0 ($N_h = 1$) by solving the potential problem with local energies Eqs. (8) and hopping matrix elements t between neighboring localized states. The solution is always a bound state of the hole (holon) and the AFM domain wall (spinon).[14]

There is however another topologically different configuration for open boundary conditions, as shown in Fig. 1(d). Here, the position of the localized hole coincides with the AFM domain wall, the configuration

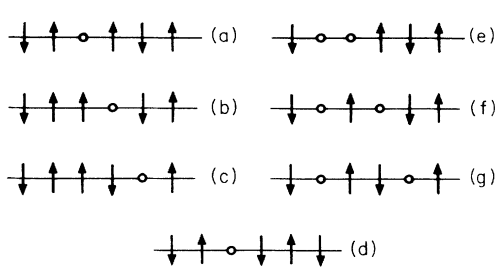

FIG. 1. Several spin configurations as they evolve by motion of a single hole and a hole pair, respectively.

representing thus a system with a single holon but without spinons. Such a configuration is freely propagating, since the potential energy is not the function of the distance r. Hence the solution for the energy is

$$\tilde{E}_0(N_h=1)=-2t+\epsilon_0+\tfrac{1}{2}J_2+J_3 \ . \tag{9}$$

Although $\tilde{E}_0$ is higher in the localized case ($t=0$), the solution becomes the stable one for smaller J_l/t.

An analogous analysis can be performed for a hole pair $N_h=2$. Corresponding configurations are presented in Figs. 1(e)–1(g) ($r=1,2,3$), whereby the potential energies $\tilde{\epsilon}_r$ for holes at the distance r are

$$\begin{aligned}
\tilde{\epsilon}_1&=\tfrac{3}{2}J_1+2J_2+2J_3 \ , \\
\tilde{\epsilon}_2&=\tilde{\epsilon}_1+\tfrac{1}{2}J_1+\tfrac{1}{2}J_2+J_3 \ , \\
\tilde{\epsilon}_3&=\tilde{\epsilon}_1+\tfrac{1}{2}J_1+J_2+\tfrac{3}{2}J_3 \ , \\
\epsilon_{r\geq 4}&=\tilde{\epsilon}_1+\tfrac{1}{2}J_1+J_2+2J_3 \ .
\end{aligned} \tag{10}$$

It is evident that $\tilde{\epsilon}_r$ also increases with r until it reaches the plateau at $r>L$. Taking into account the hopping between states with different $r>0$, we calculate the ground-state energy E_0 ($N_h=2$). Due to the hard-core repulsion between holes requiring $r>0$ a hole pair forms a bound state only above certain threshold $J_l/t>J_{lc}/t$.

This threshold is strongly dependent on the spin exchange range L. Let us consider further in this section a model with equal $J_l=J$, $l\leq L$. In this case we get the following threshold values for the pair binding: $J_c/t=4.00$ for $L=1$, $J_c/t=0.93$ for $L=2$, and $J_c/t=0.31$ for $L=3$. The rapid decrease of J_c with L can be attributed partly to larger cumulative spin exchange due to the increasing range as well as to the broader potential well with the width $d\sim 2L$. Underlying prerequisite is a longer-range Néel-type spin coordination across holes, which is absent for $L=1$. It should be noted that the binding of the hole pair is enabled by the efficient correlated pair hopping allowing for a large (negative) kinetic energy (or small effective pair mass), as found in 2D systems as well.[5,7]

The isotropic case, Eq. (1), cannot be analyzed analytically. Hence we evaluate the quantities of interest via the exact diagonalization of small 1D systems. It is now well established that the ground state in short chains is in general at a nontrivial wave vector q_0.[13] This can be obtained by studying the dependence $E_0(\theta)$ on the phase θ, entering kinetic term of Eq. (1) as $t\rightarrow t\exp(\pm i\theta)$.[13]

Let us investigate further the question of the hole binding in the isotropic case. It is known that for the usual t-J model the binding of two holes appears at $J/t\sim 3.5$ (Ref. 12), which is the value very close to the result within the Ising limit $J/t=4.00$. In Fig. 2 (open circles) we present numerical results for the binding energy

$$\epsilon_b=E_0(N_h=2)-2E_0(N_h=1)+E_0(N_h=0) \ , \tag{11}$$

as obtained for $N_h=14$ sites and $L=2,3$. It should be noted that here the energy minimum for $N_h=1$ is at nontrivial θ_0, while for $N_h=0,2$ it is at $\theta_0=0$. For comparison also the results in the Ising limit (dashed curves) are

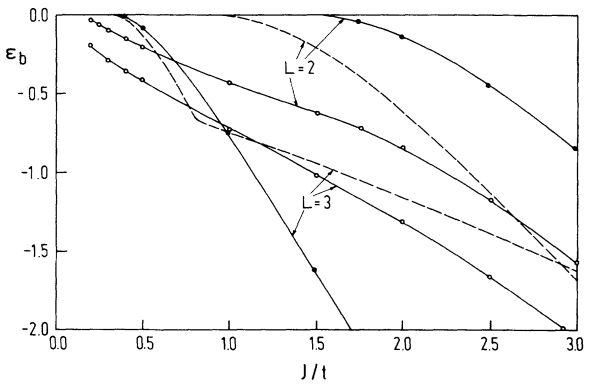

FIG. 2. Binding energies ϵ_b in units of t vs J/t for $L=2,3$. With open and solid circles we present numerical results, as obtained from Eqs. (11) and (12), respectively. Dashed curves represent exact results in the Ising limit. Solid lines are guides to the eye only.

presented. The cusp in the latter curves (for $L=2$ it appears at $J>3t$) at $J=J^*$ comes from the change of the character of the stable single-hole state, as described above and represented by Figs. 1(a)–1(c) for $J>J^*$ and by Fig. 1(d) for $J<J^*$, respectively. Such discontinuity naturally does not exist in the isotropic case.

From Fig. 2 (open circles) the resemblance to the 2D result[7] with $\epsilon_b\sim -J$ is evident. We notice however that the binding calculated via Eq. (11) is overestimated due to the odd-even effect in the particle number. Namely, a single hole in an even chain introduces in a system an additional spinon, not present for $N_h=2$. This artificially enhances the binding. Related is the observation that the onset of the binding ϵ_b does not coincide with the evidence from the hole density-density correlations which appears only at somewhat higher J/t.

To remedy the even-odd effect one can choose in the evaluation of the binding energy only systems with an even number of electrons, but with a different number of sites $N_0=N_2$, $N_1=N_0\pm 1$. The expression (11) should be then modified

$$\begin{aligned}
\tilde{\epsilon}_b=&E_0(N_h=2)-2E_0(N_h=1) \\
&+\left[\frac{2N_1}{N_0}-1\right]E_0(N_h=0) \ ,
\end{aligned} \tag{12}$$

and results (solid circles) are also shown in Fig. 2. Although the onset value for binding J_c/t is substantially larger, it is still in the regime $J_c/t<1$ for $L=3$. We expect that the analysis via Eq. (12) gives more reliable threshold values J_c/t, being compatible with the evidence from the density-density correlations. In Fig. 2 we also notice, that in the region $J_c<J<J^*$ $\tilde{\epsilon}_b$ from Eq. (12) follows closely the analytical curve in the Ising case. On the other hand, above the (Ising-case) cusp $J>J^*$, ϵ_b from Eq. (11) gives a better agreement, while Eq. (12) yields unphysically large binding energies. These observations confirm that in a finite 1D system the numerical analysis should be adapted to the investigated regime, e.g., to the holon-spinon decoupling at $J_c<J<J^*$.

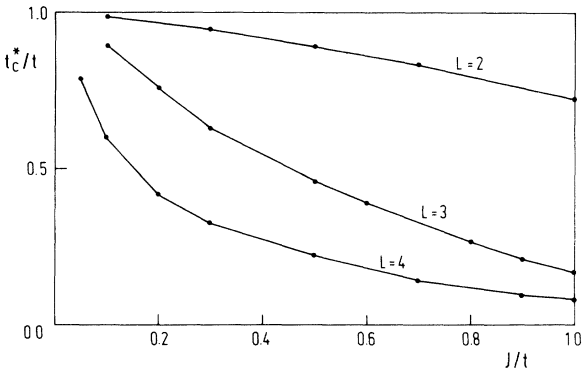

FIG. 3. Coherent hopping integral t_c^*/t vs J/t for different L as obtained by the exact diagonalization of the system with $N=16$, $N_h=1$.

Let us further consider the coherent motion of a single hole, $N_h=1$. The coherent mass m_c^* and the related hopping integral $t_c^*=(2m_c^*)^{-1}$ can be calculated via the curvature of $E_0(\theta)$ around the minimum,[13]

$$2t_c^*=\left.\frac{\partial^2 E_0(\theta)}{\partial\theta^2}\right|_{\theta=\theta_0} . \tag{13}$$

In this way it has been shown that in the t-J model ($L=1$) the hopping remains only weakly renormalized,[13] i.e., $t_c^* \lesssim t$ even for large $J/t \gg 1$. In particular the value $t_c^*=0.938t$ at $J/t=2$ is known via the Bethe ansatz exact solution.[13] On contrary results change dramatically on increasing L, as shown in Fig. 3. It is obvious that t_c^*/t is decreasing strongly with L. At the same time the kinetic energy and the related incoherent hopping t_i are only weakly reduced in the investigated regime $J/t<1$ for all considered L.

The origin of this renormalization has the analogy to the one in 2D. While for $L=1$ the spin configuration is (nearly) unperturbed by the hole motion,[4,15] the spin coordination over the hole is changing toward more AFM-like (Néel-like) for $L>1$. In particular this holds for larger $J>J^*$. As in 2D the hole coherent motion can happen in the latter regime only due to spin flips (not present in the Ising case) erasing the spin strings. It should be however noted that the dependence of t_c^* on J/t does not follow the one expected in planar systems,[5-8] i.e., $t_c^*/t \propto t/J$ for $J/t \ll 1$. This has the origin in the fact that on a chain at $J/t \ll 1$ very large $L \gg 1$ would be needed to induce a longer-range AFM ordering across the mobile hole.

IV. PHASE DIAGRAM

The 1D t-J model at small J/t falls into a universality class of Luttinger liquids,[3,16] where the asymptotic behavior of correlation functions is determined by a single nontrivial correlation exponent K_ρ. The same should also hold for the generalized t-J model at $J_l/t \ll 1$, since the spin exchange is still of the short range.

The phase diagram of the 1D $t-J$ model has been investigated in Ref. 12. Using the exact diagonalization method the phase separation line $J_c(n)$ between the Luttinger-liquid phase and the phase separated region has been calculated. In the latter region which appears at $J>J_c$ the separation into an undoped Heisenberg chain and an empty chain takes place. The critical values were established at $J_c/t \gtrsim 2.0$ for $n \ll 1$ and at $J_c/t=3.5$ for $n \to 1$. Within the Luttinger-liquid phase the correlation exponent K_ρ was found to vary continuously, from $K_\rho=\frac{1}{2}$ at the boundaries $J=0$ and $n=0,1$, to $K \to \infty$ while approaching the phase separation line $J_c(n)$. This indicates that in the vicinity of the phase separation $J \lesssim J_c$ there is a region of dominant pairing correlations characterized by $K_\rho>1$,[2,12] as confirmed also by Monte Carlo calculations.[17] The slope of the phase separation line $J_c(n)$ in 1D (Ref. 12) is opposite to the one in 2D.[11]

To calculate the ground-state energy of the model equation (1) as a function of the electron density $n=N_e/N$ we use the Lanczos diagonalization method on a chain with $N=14$ sites. To reduce the finite-size oscillations we use periodic boundary conditions for $N_e=4k+2$ electrons and antiperiodic ones for $N_e=4k$. We present results for two different sets of parameters. In the first we choose $L=2$ and $J_2=J_1$, while in the second we put $L=3$ and $J_3=\frac{1}{2}J_2=\frac{1}{2}J_1$. Note that we use here for $L=3$ smaller J_3 to prevent too steep variations in the phase diagram.

We determine the phase separation line by calculating the compressibility κ, which can be expressed in a finite system as[12]

$$\frac{1}{n^2\kappa}=\frac{1}{N}\frac{\partial^2 E_0}{\partial n^2} \sim N\frac{E_0(N_e+2)+E_0(N_e-2)-2E_0(N_e)}{4} . \tag{14}$$

The discrete second derivative is calculated taking into account only states with even N_e to avoid pronounced even-odd finite-size effects. On the other hand we can connect in the Luttinger-liquid-regime κ to the Luttinger-liquid parameters K_ρ, and the charge velocity v_c (Ref. 3)

$$\frac{1}{n^2\kappa}=\frac{\pi}{2}\frac{v_c}{K_\rho} . \tag{15}$$

We can numerically determine also the charge stiffness[3]

$$D=\frac{1}{2N}\left.\frac{\partial^2 E_0(\theta)}{\partial\theta^2}\right|_{\theta=\theta_0}=\frac{v_c K_\rho}{\pi} . \tag{16}$$

Using the relations Eqs. (15) and (16) we can evaluate numerically both K_ρ and v_c.

The phase separation line $J_c(n)$ in the phase diagram is determined by the divergence $\kappa \to \infty$. In Figs. 4(a) and 4(b) we present $J_c(n)$ for the $L=2$ and $L=3$, respectively. Results, obtained numerically for $N=14$, are represented by dots. Besides those we include also exact results for the phase separation in the two limiting cases.

(a) $n \to 0$: The problem of two electrons $N_e=2$ in an infinite chain can be solved exactly. Corresponding critical values are $J_c/t=2.00$ for $L=1$, $J_c/t=3.24$ for $L=2$,

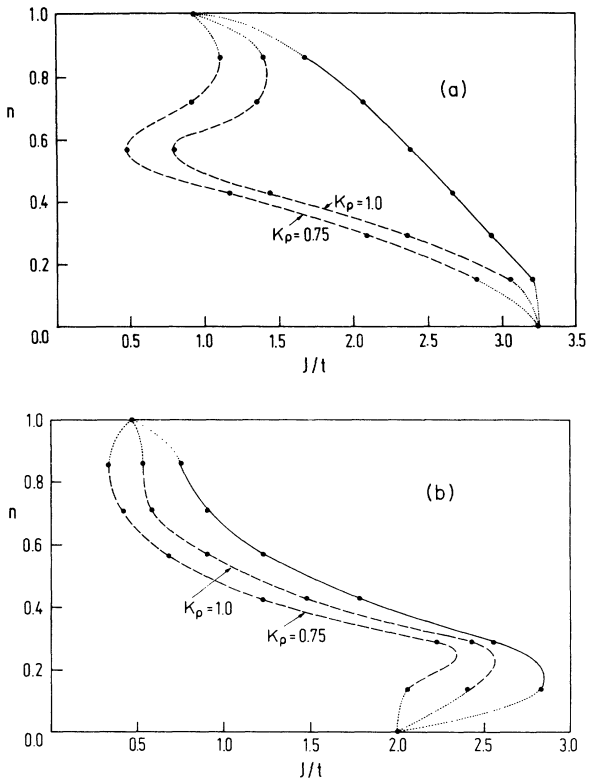

FIG. 4. Phase diagrams for (a) $L=2$, $J_2=J_1$ and (b) $L=3$, $J_3=\frac{1}{2}J_2=\frac{1}{2}J_1$. The phase separation lines $J_c(n)$ are denoted with solid lines. Presented also are points of constant K_ρ, connected with dashed curves. At $n=0$ and $n=1$ exact results for $N_e=2$ and $N_h=2$ (Ising case) are plotted.

and $J_c/t=2.00$ for our particular choice $L=3$.

(b) $n\to 1$: In Sec. II we already presented results for $N_h=2$ in the Ising case and for the corresponding thresholds J_c/t. We note that for the case $L=3$ discussed here we get $J_c/t=0.47$. For the isotropic case exact values for J_c are not available and we can only rely on the exact diagonalization results as discussed in Sec. II.

Comparing Figs. 4(a) and 4(b) with the phase diagram of a standard t-J model in Ref. 12 we find, that slopes of the phase separation line $J_c(n)$ are opposite. In the t-J model the phase separation takes place first by introducing electrons into the empty band.[12] On the contrary in the effective model Eq. (1) for $L\geq 2$ the system becomes unstable first by introducing holes to the half-filled band. The same tendency can be seen already from the exact results for $N_e=2$ and $N_h=2$ (Ising case) discussed above. The main difference between different L appears at $n\lesssim 1$, where the phase separation line moves toward much lower values J_c with increasing L. The effect of longer-range spin exchange is less pronounced at $n\ll 1$. It should be noted that in the latter regime the model is more artificial, since the derivation as presented in Sec. II does not apply here.

There is a strong similarity of the phase diagram in Figs. 4(a) and 4(b) to the one of the 2D t-J model obtained in Refs. 10 and 11. On increasing J/t the phase separation in 2D appears first at low hole densities $n\sim 1$ and after that J_c increases toward $n\to 0$ ending at $J_c\sim 3.8$. The lower critical value $J_c(n=1)$ is however far from being settled, claimed to be $J_c\sim 1.2$ from the high-temperature expansion[11] or even $J_c=0$ in Ref. 10.

While constructing $J_c(n)$ in Figs. 4(a) and 4(b) we included known binding thresholds for $N_e=2$ and (approximate) for $N_h=2$. Still there remains a possibility that pair binding (either of holes or of electrons) happens before the phase separation. This question seems to be open even for few electrons in the 1D t-J model. Of particular interest is the behavior at $n\to 1$. Because of finite-size effects the exact diagonalization results for the model Eq. (1) cannot resolve between the pair binding and the phase separation. Therefore we performed an analysis of several 1D few-body models with the interaction being of the potential type as in Eq. (11). Results indicate that the pair formation and the phase separation energy emerge together in these cases, which are however not general enough.

In Figs. 4(a) and 4(b) we also present contours of constant K_ρ.[3,16] Comparing results again with the 1D t-J model[12] we observe, that the region of dominant (superconducting) pairing correlations, which according to the g-ology[2] appear at $K_\rho>1$, moves with increasing L toward lower and physically more relevant values $J/t<1$. Furthermore in the case $L=2$ the area of the regime dominated by pairing correlations in the phase diagram increases in comparison to the 1D t-J model.

V. CONCLUSIONS

In preceding sections we have shown that the 1D generalized t-J model with longer-range spin exchange simulates several properties of the 2D (or higher D) t-J models, in particular those which are connected with the AFM short-range order and with the phenomenon of the spin string formation. Thus we find a clear correspondence of the large mass renormalization, the hole binding, and the phase separation in the generalized 1D model to those in planar models.

The hole pair formation can appear for $L>1$ even at $J/t<1$. This can be attributed to the attractive potential which is a consequence of the perturbed spin background possessing short-range AFM order. The attraction has the range $l\sim L$, at least in the Ising limit. Note that there is no such spin string effect in the standard 1D t-J model.[4,15] The same is the origin of the close relation between 1D and 2D phase diagrams, and in particular of the similarity between the phase separation line in 1D generalized model (for $L\geq 2$) and the one speculated for 2D.[10,11]

We should however mention that certain features of higher-dimensional correlated models cannot be reproduced by Eq. (1). This is the case with the onset of the ferromagnetism and of polarized states at $J/t\ll 1$, $n\lesssim 1$ which arise in $D>1$ due to loop-like motion of holes. It has been shown, that the latter effect can be also simulated in 1D by allowing for the longer-range hopping of electrons, e.g., the next-nearest-neighbor hopping $t'<0$.[18]

Most interesting regime in 2D strongly correlated systems is that of low hole doping $n\lesssim 1$. It is clear that here

our model with fixed range L yields the proper qualitative trend, but cannot be used for quantitative comparisons. To represent the onset of AFM long-range order at $n \to 1$ it would be thus more appropriate to use variable L and renormalized J_l decreasing with L. This can lead to modifications of the phase diagram at $n \lesssim 1$. The threshold J_c/t for pair binding can move even to lower values, as found in 2D t-J model.[7,9] While our analysis of the model (1) seems to indicate on the coincidence of the pair formation and the phase separation, this still remains an open question, in particular for more general (higher-D) models. In the generalized 1D t-J model the pairing correlations become dominant as the precursor of the phase separation. We have shown that this can happen even at $J/t < 1$ for reasonable L. On the basis of this finding it is tempting to speculate that pairing might exist even in the physically relevant regime of more realistic planar models.

ACKNOWLEDGMENTS

Both authors acknowledge the support of the Internationales Büro Jülich and the hospitality at the University of Würzburg, where part of this work has been performed. The authors thank also A. Muramatsu, W. Hanke, M. Ogata, and M. Rice for helpful discussions.

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