Flux-flow resistivity in model high-temperature superconductors

K. H. Lee and D. Stroud
Department of Physics, The Ohio State University, Columbus, Ohio 43210
(Received 6 April 1992)

We calculate the resistivity of a model "high-temperature superconductor" consisting of a simple-cubic arrangement of superconducting "grains" coupled together by resistively shunted Josephson junctions. The effects of temperature are simulated by Langevin noise in each junction. We find a strong magnetoresistance for magnetic fields both parallel and perpendicular to the applied current, in agreement with the results of Kwok *et al.* [Phys. Rev. Lett. **64**, 966 (1990)]. When the magnetic field **B** and the current density **J** make an angle ϕ, the resistivity at sufficiently high temperatures roughly obeys the law $\rho(B,T,\phi)=\rho_0(B,T)+\Delta\rho\sin^2(\phi)$ in agreement with experiment. The resistivity is strongly dependent on current density. At zero magnetic field it is found to satisfy the scaling relation $E=\xi^{-1-z}F_{\pm}(J\xi^{d-1}\Phi_0/ck_BT)$, where E is the electric field, c is the speed of light, J is the current density, d is the dimensionality, and $F_{\pm}$ are scaling functions which apply above and below T_c. The dynamical critical exponent is estimated for this model as 1.5 ± 0.5.

I. INTRODUCTION

Kwok and collaborators[1] have recently studied the resistivity ρ_{ab} of single-crystal $YBa_2Cu_3O_{7-\delta}$ with both current density **J** and magnetic field **B** applied parallel to the *ab* plane. In untwinned single crystals, for temperatures $T<T_{c0}$, where T_{c0} is the temperature at which superconducting fluctuations first become substantial, they found a roughly $\sin^2\phi$ angular dependence on the angle ϕ between **B** and **J**:

$$\rho_{ab}(B,T,\phi)\approx\rho_{ab}(B,T,0)+\Delta\rho(B,T)\sin^2\phi \tag{1}$$

with $\Delta\rho>0$. It is not surprising that the resistivity is greatest for perpendicular field and current, since this configuration maximizes the Lorentz force acting on the flux lines.[2] However, even for fields parallel to the currents, Kwok *et al.* found a substantial magnetoresistance $\Delta\rho_{ab}(B,T,0)\equiv\rho_{ab}(B,T,0)-\rho(0,T,0)$, the origin of which is unknown. Their results are reproduced in the inset of Fig. 1.

In this paper, we present a simulation of flux flow resistivity in a very simplified model of a "high-temperature superconductor." We qaulitatively reproduce some of the features observed by Kwok *et al.*,[1] including the substantial magnetoresistance with **B** parallel to **J**. Furthermore, in zero magnetic field, we find that the current-voltage characteristics satisfy a scaling relation proposed by Fisher, Fisher, and Huse,[3] with dynamical critical exponent $z\approx1.5\pm0.5$, and correlation length exponent $\nu\approx0.7\pm0.2$. The latter value agrees with expectations for the classical three-dimensional ($d=3$) XY Hamiltonian, which is the static limit of our dynamical model.

The remainder of this paper is organized as follows. In Sec. II we summarize our model. Section III describes our results. A brief discussion follows in Sec. IV.

II. MODEL

Our model consists of a simple-cubic network of $N=(L/a)^3$ superconducting grains coupled together by resistively shunted Josephson junctions. Current I is uniformly injected into each node on the $X=0$ face of the network, parallel to one of the planes of the network and extracted from the opposite face (at $X=L$). Temperature is simulated by a Langevin-noise current of the appropriate strength, added in parallel to each junction. A magnetic field **B** is added in the XY plane at an angle ϕ to the X axis.

The equations describing this model system have been given previously.[4] They are

$$I_{ij}(t)=\frac{V_{ij}}{R_{ij}}+I_{c;ij}\sin(\theta_i-\theta_j-A_{ij})+I_{L;ij}(t)\ , \tag{2}$$

$$V_{ij}=\frac{\hbar}{2e}(\dot{\theta}_i-\dot{\theta}_j)\ , \tag{3}$$

$$\sum_j I_{ij}=I_{i,\mathrm{ext}}\ , \tag{4}$$

$$A_{ij}=\frac{2\pi}{\Phi_0}\int_{\mathbf{x}_i}^{\mathbf{x}_j}\mathbf{A}\cdot d\mathbf{x}\ . \tag{5}$$

Here $I_{ij}(t)$ is the current from grain i to grain j at time t; $V_{ij}\equiv V_i-V_j$ is the potential difference between grains i and j; $I_{c;ij}$ is the critical current, R_{ij} is the shunt resistance, and $I_{L;ij}(t)$ is the Langevin-noise current of the (ij)th junction; θ_i is the phase of the order parameter on the ith grain; $\mathbf{x}_i$ is the position of the ith grain; $\Phi_0=hc/2e$ is the flux quantum; and **A** is the vector potential. Equation (1) describes the total current through junction (ij) as the sum of a normal current through the shunt, a supercurrent, and a Langevin-noise current. Equation (2) represents the Josephson relation between current and phase. Equation (3) is Kirchhoff's Law, which enforces current conservation at each "grain." Finally, Eq. (4) describes the magnetic phase factor, which must be added to the phase difference between grains i and j to ensure gauge invariance in the presence of a vector potential.

We assume that the noise currents in a given junction have δ-function correlations in time[5] and that noise currents associated with different junctions are uncorrelated:

$$\langle I_{L;ij}(t)I_{L;kl}(t')\rangle_e=\frac{2k_BT}{R_{ij}}\delta(t-t')\delta_{ij;kl}\ , \tag{6}$$

where $\delta_{ij;kl}$ is a Kronecker δ function and $\langle\cdots\rangle_e$ denotes an average with respect to a canonical ensemble.[6]

Equations (2)–(4) can be combined into N equations in the N unknown phases, which can be solved by straightforward iteration in time, as described in several previous papers.[7] Periodic boundary conditions are imposed in the two transverse (Y and Z) directions. A magnetic field $\mathbf{B}$ is applied in the XY plane, making an angle ϕ with respect to the direction of current flow along the X axis, using the gauge $\mathbf{A}=B\cos\phi Y\hat{Z}+B\sin\phi Z\hat{X}$.[8] We assume that all critical currents and shunt resistances are identical and equal to I_c and R_0, respectively. Resistivities are obtained by computing the voltage differences across the cubic sample, averaged over the sample faces and averaged over a time interval of order $(500-1000)\tau_0$, where $\tau_0=\hbar/(2eR_0I_c)$ is the natural unit of time. The equations of motion are iterated in time steps of typically $\Delta T=0.05\tau_0$, starting from random initial phase configurations for each temperature, current, and magnetic field. The Langevin-noise current in each time step is drawn from a uniform distribution between $I_m/\sqrt{\Delta t}$ and $-I_m/\sqrt{\Delta t}$, where I_m is a cutoff current chosen to satisfy Eq. (6).[9]

III. RESULTS

Figure 1 shows the resistivity $\rho\equiv\langle V\rangle/I$ of the network, calculated as a function of temperature for zero magnetic field and for a magnetic field $f\equiv Ba^2/\Phi_0=\frac{1}{8}$ (that is, at a field corresponding to $\frac{1}{8}$ of a flux quantum per plaquette) at a current level per junction $I=0.1I_c$. The results shown come from averages over two independent runs, each with a different Langevin random number seed, as least-squares-fitted to a polynomial of the form $\sum_{n=1}^{7}a_nT^n$ in the range $0.1<k_BT/E_J<2.0$.

At zero magnetic field, ρ drops sharply near $k_BT=2E_J$, where $E_J=\hbar I_c/(2e)$ is the Josephson coupling energy for a single junction. To understand this drop, we note that in the limit of zero current our model corresponds to the three-dimensional ($d=3$) classical XY model with Hamiltonian $H=-E_J\sum_{\langle ij\rangle}\cos(\theta_i-\theta_j)$ (the sum running over all distinct nearest-neighbor pairs of grains). This model is known to have a phase transition into a phase-ordered state at a temperature $k_BT_c(B=0)=2.21E_J$.[10] In our calculation, the sharp drop in resistivity occurs slightly below this temperature because of the finite current density.

Figure 1 also shows the magnetoresistivities $\rho(B,T)$ with fields parallel and perpendicular to the current at $f=\frac{1}{8}$, fitted to a polynomial as at zero field. Clearly, the network resistivity at any temperature is increased by a magnetic field, whatever the orientation of the field relative to the current. In our model, such behavior occurs because the field substantially lowers the superconducting transition temperature $T_c(\mathbf{B})$ of the cubic network. This lowering is due to the "frustration" of the Josephson coupling in the plane perpendicular to the magnetic field. With a finite vector potential, the coupling energy between nearest neighbors is $-E_J\cos(\theta_i-\theta_j-A_{ij})$. In the absence of a magnetic field, $A_{ij}=0$, and the coupling energy of all the junctions can be minimized simultaneously, by a parallel arrangement of phases. Correspondingly, the superconducting transition temperature of the *network* is maximized for zero field. When a magnetic field is present, it is no longer possible simultaneously to maximize all the Jospehson-coupling energies for all the junctions. This "frustration" causes the ground-state energy to be less negative in a field at zero field. The transition temperature is correspondingly reduced.

Our resistivity curves are broadened over a much wider temperature range than are the experimental resistivities. We attribute this difference to the absence of temperature-dependent coupling between grains in our model. We believe that the experimental measurements correspond to a strongly temperature-dependent coupling, which leads to a much narrower temperature width of the transition than in our model. This point is discussed again below.

The *static* properties of this frustrated $d=3$ XY model have been previously calculated by Shih *et al.*[11] and by Li and Teitel,[11] using Monte Carlo methods. For small values of the frustration f, both groups found that $T_c(B)$

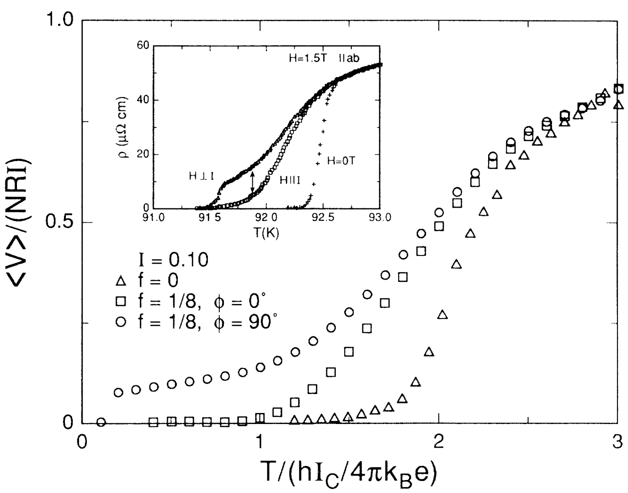

FIG. 1. Resistivity $\rho(B,I,T,\phi)$ of an $8\times8\times8$ sample of coupled Josephson junctions at a current level of $0.10I_c$, vs temperature. Three curves: zero magnetic field; $f=\frac{1}{8}$, $\phi=0$ ($\mathbf{B}$ parallel to the current); and $f=\frac{1}{8}$, $\phi=90°$ ($\mathbf{B}$ perpendicular to the current). ρ is defined as the voltage drop per junction, divided by the applied current per grain. Temperature is given in units of E_J/k_B, where E_J is the coupling energy of the junction, $E_J=\hbar I_c/(2e)$. f is the flux per plaquette, in units of the flux quantum. The points shown are least-squares polynomial fits to an average over two independent runs, each with a different Langevin random-number seed. Inset: measured resistivity of $YBa_2Cu_3O_{7-\delta}$ at an applied field $H=0$, and at $H=1.5$ T, with currents parallel and perpendicular to the field, as quoted in Ref. 1.

indeed falls off with B, just as suggested by the above argument.

Since T_c is reduced in the presence of a field, the resistivity is expected to be nonzero *no matter what the direction of the current relative to the field.* This reduction is shown in Fig. 2, where $\rho(B,T)$ is plotted as a function of angle for several temperatures at $f=\frac{1}{8}$ and $I=0.1I_c$. In order to reduce the noise associated with the Langevin simulation, we have plotted a least-squares fit of the quantity

$$\tfrac{1}{4}[\rho(B,T,\phi)+\rho(B,T,\pi-\phi)+\rho(B,T,\pi+\phi)+\rho(B,T,2\pi-\phi)]$$

to a polynomial of the form $\sum_{n=1}^{7}a_n\phi^n$ in the range $0<\phi<\pi/2$. This form takes advantage of the symmetry of ρ since ρ must be symmetric about $\phi=\frac{1}{2}\pi$ and $\phi=\pi$. For a given B and T, Fig. 2 shows that ρ is largest when B and I are perpendicular because of the extra resistivity produced by flux flow arising from the $\mathbf{J}\times\mathbf{B}$ (Lorentz) force acting on the flux lines. But even when $\mathbf{B}$ and $\mathbf{J}$ are parallel, the magnetoresistance is nonzero. This finite "phase-slip" resistivity[12] might still arise, however, from a kind of fluctuating Lorentz-force-driven flux flow. In this picture, above the freezing temperature of the flux lattice, the vortex lines are mobile and floppy. Hence, although oriented parallel to the applied field *on average,* the vortex lines also have components of their length at an angle to the applied field. They could therefore experience a Lorentz force even when current and field are nominally parallel.

Figure 2 shows that at relatively high temperatures $(k_BT\approx1.4E_J)$ the resistivity has a *roughly* $\sin^2\phi$ angular dependence for this applied current. This dependence is similar to that found by Kwok *et al.*[1] but is noisier, probably because of random fluctuations in our Langevin calculation averaged over a finite time.

At lower temperatures $(k_BT=0.6E_J)$ at this current density, ρ is very small except for $\phi\approx\pi/2$. This temperature probably lies below $T_c(B)$ for this field, but the flux lines are only weakly pinned by the periodic lattice. Hence, they can be set in motion by the relatively small current of $0.1I_c$, *provided* that the current is perpendicular to the magnetic field lines to maximize the Lorentz force.

Our remaining results (Figs. 3 and 4) are consistent with this picture. The results in Fig. 3 are least-squares-fitted to polynomials as in Fig. 1, except that an average is taken over *five* different Langevin random-number seeds. Doubling the field to $f=\frac{1}{4}$ is expected to lower the zero-current transition temperature slightly[11] and hence further to broaden the resistivity curve at any finite current. Indeed, Shih *et al.*[11] estimate the zero-current transition temperature at $f=\frac{1}{4}$ to be $(1.05\pm0.05)E_J/k_B$,

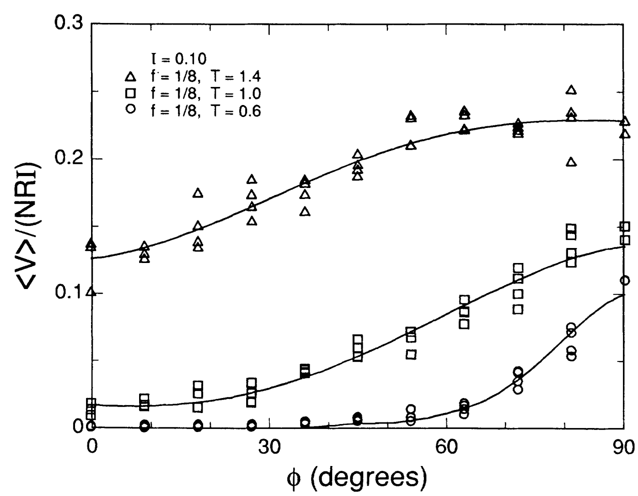

FIG. 2. $\rho(B,I,T,\phi)$, plotted at several temperatures as a function of angle ϕ for $I=0.1I_c$, $f\equiv B/\Phi_0=\frac{1}{8}$. Temperature is in units of E_J/k_B. The different points at a given angle and temperature represent the resistivity at ϕ, $\pi-\phi$, $\pi+\phi$, and $2\pi-\phi$, while the solid curves represent least-squares polynomial fits to the data.

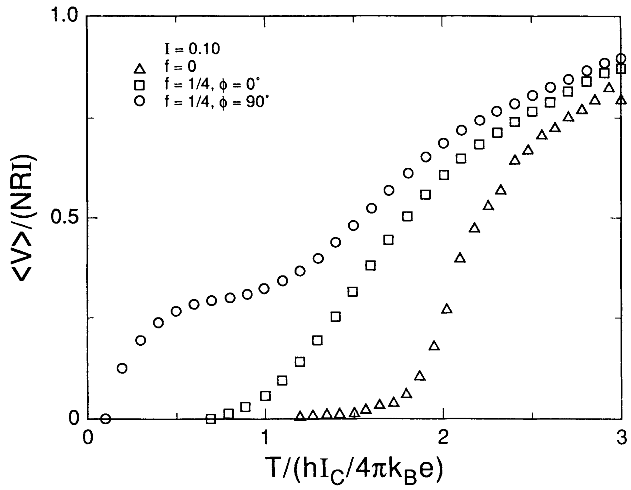

FIG. 3. Same as Fig. 1, but for $f=\frac{1}{4}$. The polynomial fits are averaged over five independent runs.

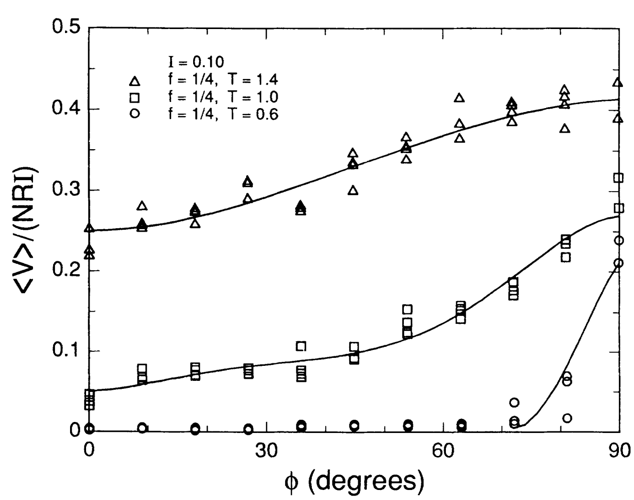

FIG. 4. Same as Fig. 2, but for $f=\frac{1}{4}$.

in good agreement with our findings at finite current density. Similarly, we have found that doubling the current level to $0.2I_c$ lowers the apparent zero-field transition (as measured by the sharp drop in resistivity) and broadens the low-temperature tails of ρ at finite fields (not shown in the figures). In general, the dissipation is quite non-Ohmic: the restivities $\rho \equiv \langle V \rangle / I$ are generally larger at $I = 0.2I_c$ than at $I = 0.1I_c$. This is consistent with the picture that larger currents more easily depin the vortex lines than do smaller currents, and hence produce greater dissipation per unit current. The striking plateau in the resistivity at $f = \frac{1}{4}$ with $\mathbf{J} \perp \mathbf{B}$ is also consistent with this picture: the zero-current transition occurs near $k_B T = 1.05 E_J$; the plateau below this temperature is simply flux-flow resistivity produced by depinnig the vortex lattice that forms at lower temperatures. The second decrease in resistivity near $k_B T = 0.1 E_J$ occurs when the depinning current rises above the applied current density of $0.1 I_c$ per junction.

To further investigate the non-Ohmic character of the I-V characteristics, we plot in Fig. 5(a) the resistivity ρ at zero field and at several current levels. As expected, the application of a higher current lowers the apparent superconducting transition temperature, as measured by the position of the sharp drop in the resistivity: the larger the current, the lower the apparent transition temperature. Several authors[3,13] have predicted that the I-V characteristics of a *homogeneous superconductor* at zero magnetic field can be scaled onto a single universal curve described by the relation

$$E = \xi^{-1-z} F_{\pm} \left[\frac{J \xi^{d-1} \Phi_0}{c k_B T} \right], \tag{7}$$

where E is the electric field, J is the current density, c is the speed of light, d is the dimensionality, $F_+(x)$ and $F_-(x)$ are scaling functions which apply above and below T_c, and ξ is the correlation length characterizing the transition. It is expected that, near the transition temperature, ξ will vary according to a power law, $\xi \propto (|T - T_c| / T_c)^{-\nu}$. In the present problem, since T_c is the transition temperature of the $d = 3$ XY model, we expect $\nu \approx \frac{2}{3}$.[10,14]

In Fig. 5(b) we show a scaling plot of the variable $\rho(J,T) \equiv E/J$. According to Eq. (7), ρ should satisfy

$$\rho(J,T) \propto \xi^{d-2-z} F_{\pm} \left[\frac{J \xi^{d-1} \Phi_0}{c k_B T} \right]. \tag{8}$$

Figure 5(b) shows that Eq. (8) is well obeyed over the limited current range considered, with $T_c \approx 2.21 E_J / k_B$, $z \approx 1.5 \pm 0.5$, and $\nu \approx 0.7 \pm 0.2$. The error bars are simply subjective estimates of the amount by which our exponents could be changed while still allowing satisfactory collapse of the data onto two scaling curves. The value of ν is in the range of the expected value $\nu = 0.67$.[10,14] For z, the value of 2.0 has been proposed.[3] We cannot explain why our apparent value of 1.5 deviates from this prediction. Possibly our model is in a different dynamical universality class from that considered in Refs. 3 and 13.

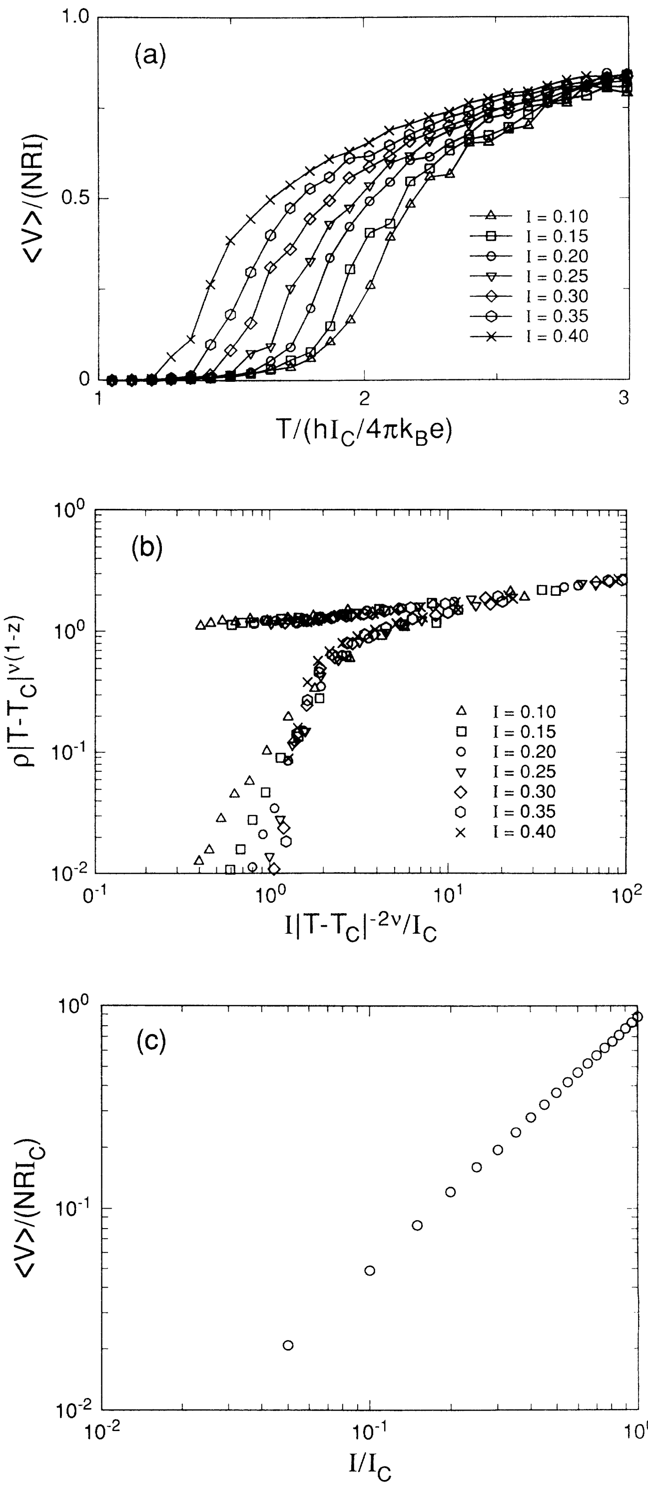

FIG. 5. (a) Resistivity $\rho(B=0,T)$ as a function f of temperature, for an $N \times N \times N$ lattice of Josephson-coupled grains ($N=8$) at several dc current levels I, all given in units of I_c. (b) Plot of the *scaled* variable $\rho|T-T_c|^{\nu(1-z)}$ as a function of the *scaled* current $I|T-T_c|^{-2\nu}/I_c$. A good fit is obtained using the values $z=1.5$, $\nu=0.67$, $T_c = 2.21 E_J/k_B$. (c) Log-log plot of the time-averaged voltage $\langle V \rangle / (NRI_c)$ as a function of the dc current density I/I_c at $T = T_c \equiv 2.21 E_J / k_B$. R is the shunt resistance. The plot can be fitted to a straight line with a slope $x \equiv (1+z)/(d-1) = 1.25$.

More likely, we have simply not reached the asymptotic small-current critical regime in which scaling (with the true z) would be valid.

Further evidence of the scaling form is shown in Fig. 5(c), which shows a plot of $V(I)$ at $T=T_c\equiv 2.21E_J/k_B$. At this temperature, since $\xi\to\infty$, Eq. (7) can be shown to predict a power-law temperature dependence of the form $E\propto J^x$, with $x=(1+z)/(d-1)$. The results of Fig. 5(c) are well fitted by $x=1.25$, corresponding once again to $z=1.5$.

IV. DISCUSSION AND CONCLUSIONS

Many features of our results are qualitatively consistent with experiment. For example, in agreement with Kwok *et al.*, we find (i) a nonzero longitudinal magnetoresistance, (ii) a substantial difference between transverse and longitudinal magnetoresistance, and (iii) an approximately $\sin^2\phi$ angular dependence of the magnetoresistance at certain temperatures. As far as the current dependence of the transition, Blackstead and collaborators[12] have reported a lowering of the transition temperature of $YBa_2Cu_3O_{7-\delta}$ with increasing current, analogous to what we find here. To our knowledge, no detailed experimental verification of the scaling behavior at $B=0$ has been carried out as yet.

In a magnetic field, our transitions are broadened over a far greater relative temperature range than are the experimental transitions of Kwok *et al.* This is undoubtedly due to the simplified nature of our model. In contrast to experiment, our model has an isotropic, temperature-independent coupling between the "grains," and it has a periodic pinning which is an artifact of the simple-cubic grain lattice. Because the real material has a strongly temperature-dependent coupling, the transition occurs over a far narrower temperature range than in our model. As has been discussed by Tinkham and Lobb,[15] for example, the difference between a granular picture and the more traditional fluctuation viewpoint is more linguistic than physical. The "grains," in our language, are to be interpreted as volumes of superconductor of dimensions comparable to the coherence length, and the coupling can be deduced from estimates of the Ginzburg-Landau critical current density of the homogeneous superconductor. The periodic pinning is an artifact of our model, but it could be removed or reduced by introducing disorder in the coupling strengths. Likewise, the absence of anisotropy in our model is probably of only quantitative significance.

An incidental result of our model is that, when $\phi=0$, the zero-temperature critical current is rigorously *independent of* B and equal to I_c per junction. The reason is that, since there is no frustration parallel to the field, one can twist each phase difference between grains up to $\pi/2$ without causing a phase slip. This surprising result agrees with experiment. For example, a recent study[16] shows that the low-temperature critical current density of BiSrCaCuO in the *ab* plane depends only on the component of B in the c direction, not on the *ab* component. The authors of this study conclude that since the parallel component of B is irrelevant, the planes are effectively decoupled at low temperatures. Our results show, however, that this result follows even from an *isotropic* three-dimensional model in which the coupling is the same in all three directions.

To summarize, we have presented in this paper an elementary model, based on a three-dimensional network of coupled resistively shunted Josephson junctions, that describes some observed features of the magnetoresistance of high-temperature superconductors. The results, both numerical and analytical, are in qualitative agreement with a considerable range of experimental data for $YBa_2Cu_3O_{7-\delta}$, and possibly also for other high-temperature superconductors. Thus our model represents a natural starting point for further studies, especially on the kinds of defects which will be most effective in pinning the flux lines and reducing flux-flow dissipation. We will present the results of such studies elsewhere.[17]

ACKNOWLEDGMENTS

We wish to thank H. Blackstead for a number of valuable conversations, and for stimulating our interest in this problem. We also acknowledge useful conversations with J. C. Garland, S. Girvin, and S. E. Hebboul. This work was supported by the Midwest Superconductivity Consortium through U.S. Department of Energy Grant No. DE-FG-02-90ER45427, and by the National Science Foundation, Grant DMR-90-20994. Calculations were carried out on the CRAY Y-MP 8/8-64 of the Ohio Supercomputer Center, with the help of a grant of time.

[1] W. W. Kwok, U. Welp, G. W. Crabtree, K. G. Vandervoort, R. Hulscher, and J. Z. Liu, Phys. Rev. Lett. **64**, 966 (1990).

[2] The conventional explanation for the $\sin^2\phi$ dependence of the flux-flow contribution, as given in Ref. 1, is that the Lorentz force $\mathbf{F}_L$ on a single vortex line varies as $\sin\phi$, where ϕ is the angle between $\mathbf{B}$ and $\mathbf{J}$. The dissipation then varies as $\mathbf{F}_L\cdot\mathbf{v}_d$, where $\mathbf{v}_d$ is the vortex drift velocity resulting from $\mathbf{F}_L$. $\mathbf{v}_d$ is proportional to $\mathbf{F}_L$ and hence to $\sin\phi$. Thus the dissipation, or equivalently the flux-flow resistivity, varies as $\sin^2\phi$. There may also be a flux-flow Hall voltage, as discussed, e.g., by S. J. Hagen, C. J. Lobb, and R. L. Greene, Phys. Rev. B **43**, 6246 (1990).

[3] M. P. A. Fisher, D. S. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).

[4] These equations have been studied by a large number of authors in the last several years. Some representative references are K. M. Mon and S. Teitel, Phys. Rev. Lett. **62**, 673 (1989); W. Xia and P. L. Leath, *ibid.* **63**,1428 (1989); K. H. Lee, D. Stroud, and J. S. Chung, *ibid.* **64**, 962 (1990); F. Falo, A. R. Bishop, and P. S. Lomdahl, Phys. Rev. B **41**, 10 983 (1990); H. Eikmans and J. E. van Himbergen, *ibid.* **41**, 8927 (1990); J. U. Free, S. P. Benz, M. S. Rzchowski, M. Tinkham, C. J. Lobb, and M. Octavio, *ibid.* **41**, 7267 (1990); M. Kvale and S. E. Hebboul, *ibid.* **42**, 3720 (1990); L. I. Sohn, M. S. Rzchowski,

J. U. Free, S. P. Benz, M. Tinkham, C. J. Lobb, *ibid.* **44**, 925 (1991); D. Dominguez, J. V. Jose, A. Karma, and C. Wiecko, Phys. Rev. Lett. **67**, 2367 (1991); S. J. Lee and T. C. Halsey (unpublished); J. C. Chung, K. H. Lee, and D. Stroud, Phys. Rev. B **40**, 6570 (1989); S. R. Shenoy, J. Phys. C **18**, 5163 (1985).

[5]V. Ambegaokar and B. I. Halperin, Phys. Rev. Lett. **22**, 1364 (1969).

[6]Note that these equations neglect capacitive terms in the differential equations, which would lead to terms second order in time; this neglect is presumably appropriate for superconducting-normal-superconducting junctions. We also assume that the vector potential **A** is that of the applied field. This is tantamount to assuming that the size of the simulation volume is small in comparison to the penetration depth of the sample.

[7]See especially the papers by K. H. Lee and collaborators in Ref. 4 for a more complete description of our method of calculation.

[8]With our choice of gauge, there is a minor technical problem with the simulations when the field is at an arbitrary angle to the axes of the simple-cubic lattice. Since we use periodic boundary conditions in the transverse directions, the phase factors A_{ij} are also periodic. This implies that, when the field is not parallel to one of the crystal axes, the flux through the "boundary" plaquettes differs from that through the other plaquettes within the body of the sample. This may have some influence on our results when $\phi\neq 0$. The problem can be alleviated by a more sophisticated choice of gauge, as will be discussed elsewhere [K. H. Lee and D. Stroud (unpublished)].

[9]While it is conventional to choose a Gaussian, rather than a uniform, distribution to describe Langevin noise, we found, in initial small-scale simulations, that both distributions gave the same time-averaged voltages to within our numerical accuracy. Since the Gaussian simulation required about 3 times the CRAY time on our CRAY Y-MP 8/8-64, we have used the uniform distribution in the simulations described here.

[10]M. Ferer, M. A. Moore, and M. Wortis, Phys. Rev. B **8**, 5205 (1973); A. Garg, R. Pandit, S. A. Solla, and C. Ebner, **30**, 106 (1984); Y. H. Li and S. Teitel, **40**, 9122 (1989).

[11]W. Y. Shih, C. Ebner, and D. Stroud, Phys. Rev. B **30**, 134 (1984); Y. H. Li and S. Teitel, *ibid.* **45**, 5718 (1992).

[12]The picture of resistivity in high-temperature superconductors as arising from phase slips in networks of Josephson junctions has been discussed by M. Tinkham, Phys. Rev. Lett. **61**, 1658 (1988). A division of magnetoresistivity into phase-slip contributions (ϕ independent) and flux flow (varying as $\sin^2\phi$) has been discussed in detail by H. A. Blackstead [cf., e.g., H. A. Blackstead, D. B. Pulling, P. J. McGinn, and J. Z. Liu, Physica C **174**, 394 (1991).]

[13]A. T. Dorsey, Phys. Rev. B **43**, 7575 (1991).

[14]J. C. Le Guillou and J. Zinn-Justin, Phys. Rev. B **21**, 3976 (1980).

[15]M. Tinkham and C. J. Lobb, in *Solid State Physics,* edited by H. Ehrenreich, F. Seitz, and D. Turnbull (Academic, New York, 1989), Vol. 42, p. 91.

[16]P. Schmitt, P. Kummeth, L. Schultz, and G. Saemann-Ischenko, Phys. Rev. Lett. **67**, 267 (1991).

[17]K. H. Lee and D. Stroud (unpublished).