Effects of the phase-dependent dissipative term on the supercurrent decay of Josephson junctions

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We study the decay of the zero-voltage state of an underdamped Josephson junction, including in the equivalent circuit model the interference $\cos\phi$ term. From a theoretical point of view, we provide an expression for the lifetime of the metastable state, which takes into account the dependence on the voltage of both the quasiparticles tunneling and the interference-term conductances. We also present a comparison of the theory with our experimental results, which clearly indicates the importance of these intrinsic mechanisms of dissipation on the supercurrent decay. We show that the interference term must be included in the theory if agreement between theory and data is to be achieved within the experimental uncertainty.

The problem of the effects of dissipation in the supercurrent decay of Josephson junctions has recently received much attention. It is, in fact, an important aspect in the thermal limit and it plays a crucial role in some secondary quantum effects, such as macroscopic quantum tunneling[1] or Bloch oscillations.[2]

It is clear that the resistively shunted junction (RSJ) model,[3] till now widely used to fit the experimental data, is becoming inadequate to describe the always more sophisticated experiments on the subject. Within this model, the dissipation in the junction is accounted for in the simplest way one can imagine, namely, through an Ohmic resistor R. The current-carrying states of the junction are then described in terms of a simple current biased lumped circuit model, in which the distributed junction capacitance is considered as a lumped element in parallel with the resistor R and a nonlinear Josephson element $I_j \sin\phi$. Here ϕ is the quantum phase difference between the two superconductors forming the junction. This model presents evident limits to describe the dynamics of Josephson tunnel junctions where the dissipation is due to the combination of highly nonlinear mechanisms,[3] but it is difficult to avoid completely resorting to the equivalent-circuit model because of the complexity of the problem in its full generality. It is possible to remove some of the most evident approximations by taking into account, within the general framework of an equivalent RSJ circuit, some effects neglected in the original schematization.

In a recent paper[4] we have approached a "modified" RSJ model in which the dissipative element was assumed to be voltage dependent. This allowed us to consider the effect of the voltage-dependent conductance due to the quasiparticle tunneling.

In any real experiment the external circuit biasing the junction can also influence the junction damping, and in some experimental configuration the system damping may be dominated by any external shunt or load line.[5] However, in view of many applications, it is desirable to have the system damping dominated by intrinsic mechanisms, which sets the lower limit for the junction dissipation.[6] In this case the voltage and phase dependence of the dissipation is particularly relevant.

As far as intrinsic mechanisms are concerned, we can obtain, within the microscopic theory, the total tunneling current flowing through the junction. Its general expression is rather complicated but in the case of time-independent voltage V across the junction, it can be cast in the form[3]

$$I(V,T)=I_j(V,T)\sin\phi + [\sigma_1(V,T)\cos\phi+\sigma_0(V,T)]V \ . \quad (1)$$

The first term $I_j(V,T)\sin\phi$ describes processes in which phase-coherent tunneling of Cooper pairs occurs, and for $V=0$ represents the dc Josephson current. The dissipative term $I_{\rm qp}=\sigma_0(V,T)\ V$ represents the quasiparticle tunneling. The phase-dependent dissipative term $\sigma_1(V,T)\cos(\phi)V$ could be interpreted as describing a quasiparticle tunneling process which involves a concomitant destruction and creation of pairs on the two superconductors forming the junction, therefore involving phase-coherence effects. It is interesting to observe that this term is ignored in the RSJ model widely used to describe the junction dynamics in the presence of noise. In Ref. 7, an extension of the theory that accounts for the effect of the $\cos\phi$ term was given in the overdamped case; in the same reference, the effect of noise on externally shunted Josephson junctions was also experimentally investigated and the data agreed very well with theory. The work[7] studied the rounding on the current-voltage (I-V) characteristics that occurs in overdamped structures, but the noise was mainly due to the external shunt rather than to the intrinsic mechanisms of dissipation in tunnel junctions.

In this paper, we wish to study the effect of the intrinsic dissipative phase-dependent $\cos\phi$ term on the supercurrent decay of underdamped tunnel junctions, restricting ourselves to the thermal limit. This study can be performed in connection with the general problem of Brownian motion with a friction coefficient depending both on the velocity and position of the Brownian particle. In fact, adding to Eq. (1) the contribution due to the junction capacitance C, and equating the final expression for the total current flowing through the junction to the bias current I, we obtain a Langevin-like equation for the phase ϕ, which in dimensionless units assumes the typical form of the RSJ model[4]

$$\ddot{\phi}+\varepsilon\dot{\phi}=-dU/d\phi+\sqrt{\varepsilon}\xi(t) , \tag{2a}$$

where $U(\phi)=-(\alpha\phi+\cos\phi)+\text{const}$. Here α is the bias current normalized to the critical one I_c, $\alpha=I/I_c$, and Φ_0 is the magnetic flux quantum. The dots indicate the derivation with respect to the normalized time $t=\omega_j t$, being $\omega_j=\sqrt{2\pi I_c/\Phi_0 C}$ the plasma frequency. In the second member of Eq. (2a) appears the stochastic term $\xi(t)$ which represents the noise due to the dissipative term in the equation. We assume that the statistical properties of the random term $\xi(t)$ are

$$\langle \xi(t)\rangle=0, \quad \langle \xi(t)\xi(t')\rangle=(4/\gamma)\delta(t-t') , \tag{2b}$$

where $\gamma=\Phi_0 I_c/\pi kT$. The friction coefficient is now given by

$$\varepsilon=\frac{\sigma_1(V,T)\cos(\phi)+\sigma_0(V,T)}{C\omega_j} . \tag{3}$$

Before solving the Langevin equation for our model, we outline its limit of applicability. While it is clear that it is possible to write the general Langevin equation (2a), the statistical properties of the stochastic term $\xi(t)$ (which define the actual form of the Langevin equation in our problem) are described by Eq. (2b) only approximately in the presence of a voltage- and phase-dependent friction coefficient. The mean value of $\xi(t)$ must still be zero; indeed the average behavior of the fluctuations must coincide with the macroscopic behavior described by the deterministic equations. The δ-function correlation means that the values of $\xi(t)$ are completely uncorrelated. Strictly speaking, this implies that the fluctuation power spectral density is constant, namely, it is flat like white light (white noise; this well-known result is a consequence of the Wiener-Khintchine theorem).[3,8] This condition holds exactly only within the RSJ model approximation, which assumes that the junction conductance is frequency independent and the fluctuations are due to the Johnson noise associated with the resistor R.[3,8] The contributions to the current noise spectral density that arise from the quasiparticle tunneling as well as from the quasiparticle pair interference term have been studied in Ref. 8 (see also Ref. 3). As a consequence of the Callen-Welton fluctuation-dissipation theorem,[8] we may expect that, in case of a voltage- and phase-dependent junction conductance, the power spectrum is proportional to $I(\hbar\omega/e)/(\hbar\omega/e)$, and therefore no longer constant.[3] Note, however, that for all the frequencies of physical interest in Josephson junctions, namely, up to the plasma frequency, the voltage $V=\hbar\omega/e$ is typically a small value ($\sim\mu V$). In this low-voltage region the junction I-V curves differ only slightly from the linear behavior. Therefore the spectral density is almost constant over all the frequencies of physical interest, and we expect that the time interval over which correlation extends between values of the process $\xi(t)$ (i.e., the correlation time)[9] is very short, in fact smaller than all the other relevant time constants of the system. Under these conditions we can approximate the actual process by a δ-correlated process,[9] in order to obtain concrete results by using the mathematical methods of Markov process theory. Equation (2b) is a consistent way of replacing the real process by a δ-correlated process.[10] We wish to stress, however, that Eqs. (2a) and (2b) implicitly assume that only small voltage values are relevant so that the differential junction resistance is a smooth function of the voltage. This condition is typically very well satisfied, as will be clear in the following. The final expression for the lifetime of the $V=0$ state obtained by our approach will be in fact very similar to the RSJ result, but with a very important implication: The effective resistance to describe the junction dissipation, which was an arbitrary parameter in the RSJ model, can be now obtained in terms of measurable junction parameters and bias conditions. It can be related to the junction resistance measured on the dc current-voltage characteristics at a certain temperature-dependent voltage. This allows a direct comparison of data with theory.

The behavior predicted by Eqs. (2a) and (2b) is easily understood in terms of the mechanical analog, i.e., that of a Brownian particle of unit mass performing its motion in the washboard potential $U(\phi)$, where ϕ is the position of the particle. The voltage across the junction is related to the particle velocity,[3] $V=(\Phi_0\omega_j/2\pi)\dot{\phi}$, and the friction coefficient Eq. (3) is velocity and position dependent accordingly.

The Josephson superconducting state can be visualized as the particle trapped in a well of the potential energy $U(\phi)$, performing oscillations in the potential minimum. We restrict ourselves to the extremely underdamped case,[11] $\varepsilon \ll 1$, so that we are dealing with a quasiconservative motion. This limit is the most interesting to study in connection with applications to hysteretical Josephson junctions. The dissipation plays in fact a significant role only in extremely underdamped systems. In the Kramers moderate underdamped limit,[11] for instance, the RSJ model gives an expression for the lifetime of the $V=0$ state essentially independent of the damping. Moreover, the intrinsic dissipation produces typically a low damping level.[6] Because of the small value of ε, Eq. (2a) describes a particle performing nonlinear oscillations under a weak fluctuating force in the presence of a small damping. In such a situation the energy $E=\frac{1}{2}\dot{\phi}^2+U(\phi)$ is conserved over a large number of oscillations and it is possible to describe the system in terms of its energy rather than velocity and position. The mathematical formulation of this physical aspect is that one can associate with Eqs. (2a) and (2b) a one-dimensional Fokker-Planck (FP) equation along the energy axis for the probability density $P(E,t)$ of finding the system at time t with an energy value E. The resulting FP equation assumes the form[13]

$$\frac{\partial P}{\partial t}=-\frac{\partial}{\partial E}[K_1(E)P(E)]+\frac{1}{2}\frac{\partial^2}{\partial E^2}[K_2(E)P(E)] , \tag{4}$$

where

$$K_1(E)=\frac{1}{\gamma\varphi'(E)}\left[2\frac{d\varphi(E)}{dE}-2\lambda(E)-\varphi(E)+1\right],$$

$$K_2(E)=\frac{4}{\gamma}\frac{\varphi(E)}{\varphi'(E)}.$$

The functions $\varphi(E)$, $\lambda(E)$, and $\varphi'(E)$ are defined as

$$\varphi(E)=\int_{R(E)}\varepsilon\sqrt{E-U(\phi)}\,d\phi\ ,\tag{5a}$$

$$\lambda(E)=\int_{R(E)}\frac{\partial\varepsilon}{\partial E}\sqrt{E-U(\phi)}\,d\phi\ ,\tag{5b}$$

$$\varphi'(E)=\frac{1}{2}\int_{R(E)}\frac{1}{\sqrt{E-U(\phi)}}\,d\phi\ .\tag{5c}$$

Here the integration is performed over the region $R(E)$ of ϕ values where $U(\phi)<E$. Let us confine our attention to the solution of this Eq. (4) valid inside a potential well of $U(\phi)$. We are interested to the lifetime of the $V=0$ state or, in the mechanical analog, to the mean time spent by the Brownian particle in the well before its escape over the potential barrier U_0. In statistical terms, we can exclude from consideration any realization of the random process as soon as it overcomes the potential barrier. This is equivalent to state that the statistical population of particles in the metastable state is decreasing in time due to the escape process. The mathematical apparatus of Markov process theory enables us to solve this kind of classical problem, called the first-passage-time problem.[9,12] The technique is a very powerful one when we deal with a one-dimensional FP equation, as in the case of extremely underdamped systems. In this case the mean first-passage time can be always expressed in terms of quadratures by solving a simple second-order differential equation.[9,12] The first-passage-time equation for the lifetime of the $V=0$ state associated with our FP Eq. (4) assumes the form[9,12]

$$-1=K_1(E_0)\frac{d\tau_{E_0}}{dE_0}+\frac{1}{2}K_2(E_0)\frac{d^2\tau_{E_0}}{dE_0^2}\ .\tag{6}$$

In the above equation the lifetime τ_{E_0} is considered as a function of the energy E_0 of the particle at the time $t=0$. The solution of Eq. (6) with the suitable boundary conditions[4] can be cast in the form[4,13]

$$\tau_{E_0}=\int_{E_0}^{U_0}\frac{dE}{\varphi(E)}\exp\left[\frac{\gamma}{2}E+\beta(E)\right]\times\int_0^E\frac{\gamma}{2}\varphi'(E')\exp\left[-\frac{\gamma}{2}E'-\beta(E')\right]dE'\ ,\tag{7}$$

where $\beta(E)$ is given by

$$\beta(E)=\int\frac{\lambda(E)}{2\varphi(E)}dE\ .$$

In Josephson junctions, the barrier height is a function of the normalized bias current

$$U_0=-\alpha\pi+2[\alpha\sin^{-1}\alpha+\sqrt{1-\alpha^2}]\ .$$

The expression (7) for the lifetime depends on the actual form of the dependence of the friction parameter ε on E and φ, which appears in the definitions (5a) and (5b), and therefore on the voltage and phase dependence of the dissipation in the junction.

Note that the integrations along the energy axis in Eq. (7) are performed up to the maximum value U_0, corresponding to a maximum velocity at the bottom of the well equal to $\sqrt{2U_0}$. In terms of the junction, this corresponds to the fact that the voltage region to be considered for computing the lifetime is ranging between $-V_k$ and V_k, where $V_k=(\Phi_0/2\pi)[\omega_j\sqrt{2U_0}]$. Typically V_k is a very low voltage value ($\sim\mu V$). In this low-voltage subgap region, the conductances σ_0 and σ_1 can be calculated within the microscopic theory.[3] For symmetrical junctions (i.e., formed by the same superconductor on both sides of the barrier) we have[3,4]

$$\sigma_0=\frac{\ln(\mu/V)}{R^*}\ ,\tag{8}$$

where, for our convenience, we defined $\mu\equiv\min(kT/e,\Delta)$ and the temperature-dependent Ohmic resistance:

$$R^*=\frac{4R_NkT}{e\Delta}\cosh^2\left[\frac{e\Delta}{2kT}\right],\tag{9a}$$

Here R_N is the junction normal resistance, $e\Delta$ and kT are the energy gap and the thermal energy, respectively. This resistance R^* will be useful for the comparison with the experimental data. Note that R^* can be interpreted as the resistance measured on the quasiparticle branch of the I-V characteristics at the temperature-dependent voltage $V^*=\mu/e$ ($V^*\ll\Delta$).

At low temperature and low voltage, σ_1 should tend to σ_0.[3,14] In computing the lifetime from Eq. (7) we will then assume the following voltage and phase dependence for the friction parameter

$$\varepsilon=\eta^*(1+\cos\phi)\ln(\mu/V)\ ,\tag{9b}$$

where we defined a constant damping coefficient related to the resistance R^*; $\eta^*=(\omega_jR^*C)^{-1}$. In terms of the dimensionless energy E and of the phase ϕ, we have

$$\varepsilon=\eta^*(1+\cos\phi)\ln\left[\frac{2e\mu}{\hbar\omega_j}\frac{1}{\sqrt{2[E-U(\phi)]}}\right],\tag{10a}$$

$$\frac{\partial\varepsilon}{\partial E}=-\frac{\eta^*(1+\cos\phi)}{2[E-U(\phi)]}\ ,\tag{10b}$$

and for the relevant functions

$$\varphi(E)=\eta^*\int_{R(E)}\ln\left[\frac{b}{\sqrt{E-U(\phi)}}\right]\times(1+\cos\phi)\sqrt{E-U(\phi)}\,d\phi\ ,\tag{10c}$$

$$\lambda(E)=-\eta^*\int_{R(E)}\frac{1+\cos\phi}{\sqrt{E-U(\phi)}}\,d\phi\ ,\tag{10d}$$

where $b=\sqrt{2}e\mu/\hbar\omega_j$ depends on the junctions' parameters.

The form (10a) of the dependence of ε on E and ϕ does not allow us to perform analytically the quadratures appearing in Eq. (7) for computing the lifetime. However, an analytical expression for τ can be obtained within some approximations for the potential $U(\phi)$ and the $\cos\phi$ term: We consider the parabolic expansion of $U(\phi)$ around the minimum $\phi_1=\sin^{-1}(\alpha)$ of the potential well, namely, assuming $U(\phi)\cong\frac{1}{2}\sqrt{1-\alpha^2}(\phi-\phi_1)^2$; for $\cos\phi$ we will take only the first-order approximation, assuming it as a constant: $\cos\phi\cong\cos\phi_1=\sqrt{1-\alpha^2}$.

The harmonic approximation for the potential is often assumed[4,11,12] since one expects that in the small damping limit the lifetime depends essentially on barrier height rather than on the particular potential shape.[15] The approximation of $\cos\phi$ as a constant seems, at first glance, difficult to justify. We expect, however, that it is acceptable for high-γ values, as typically occurs in the experiments. In this case the measured values of τ refer to current values close to the critical one ($\alpha\sim 1$) and for low potential barrier height U_0[$(\gamma/2)U_0$ typically ranges between 3 and 15 in any experimental situation; for $\gamma\sim 1000$, U_0 is very small].[5,6] The region $R(U_0)$ of ϕ values inside the well is therefore very narrow, and only small fluctuations of $\cos\phi$ around $\cos\phi_1$ can occur. This point will be discussed in more detail in the following. It will be also useful to compare the analytical expression for τ obtained within these approximations with the exact values for the lifetime obtained by numerical integration of Eq. (7). The comparison will justify the use of the approximate result in a wide range of junction parameters and noise conditions. Moreover, an empirical correction to the approximate lifetime expression can be given in an analytical form, independent of the junction parameters. This will finally provide an analytical expression for τ deviating from the exact one by only a few percent in all the typical experimental situations.

Within the mentioned approximations for $U(\phi)$ and $\cos\phi$, the quadratures appearing in the definitions (5a) and (5c) can be performed analytically[16] and Eq. (7) for the lifetime reduces to[4]

$$\tau_{E_0}=\int_{E_0}^{U_0}dE\frac{\exp[(\gamma/2)E]}{E}\int_0^E\frac{\gamma}{2\eta(E')}\exp\left[-\frac{\gamma}{2}E'\right]dE' , \tag{11}$$

where we defined the energy-dependent damping coefficient

$$\eta(E)=\eta^*(1+\sqrt{1-\alpha^2})\left[\ln\left[\frac{2e\mu}{\hbar\omega_j}\frac{1}{\sqrt{2E}}\right]+\delta\right] ,$$

$$\delta=-\frac{2}{\pi}\int_{-1}^{1}\sqrt{1-x^2}\ln(1-x^2)dx\cong 0.19 .$$

In the experiments the noise parameter γ is typically a large number ($\gamma\sim 1000$; $\gamma/2U_0\gg 1$), therefore the main contribution to the integral in E' is restricted within a narrow region around the energy value E_1 where the function $\exp[-(\gamma/2)E']/\eta(E')$ has its maximum. The value E_1 is implicitly defined by the relationship $\ln(b/\sqrt{E_1})+\delta=1/\gamma E_1$; ($b=\sqrt{2}e\mu/\hbar\omega_j$).

Due to the slight dependence of η on E, for large γ, the expression for the lifetime Eq. (11) can be then cast in the form[4]

$$\begin{aligned}\tau_{E_0}\cong\tau_A&=\frac{\gamma}{2\eta_1}\int_{E_0}^{U_0}dE\frac{\exp[(\gamma/2)E]}{E}\\&\quad\times\int_0^E\exp\left[-\frac{\gamma}{2}E'\right]dE'\\&=\frac{F(U_0)-F(E_0)}{\eta_1}\\&=\frac{\mathrm{Ei}[(\gamma/2)U_0]-\mathrm{Ei}[(\gamma/2)E_0]-\ln(U_0/E_0)}{\eta_1} ,\end{aligned} \tag{12}$$

where $F(E)=\sum_{n=1}^{\infty}\{[(\gamma/2)E]^n/n!n\}$ and $\mathrm{Ei}(\cdots)$ is the exponential integral. Here $\eta_1=\eta(E_1)$. Note that expression (12) reproduces the RSJ results (with a constant resistance),[12] provided that one assumes an effective dissipation coefficient η_1, which can be now obtained in terms of the measurable junction parameters and bias conditions. Within these approximations the effect of the $\cos\phi$ term is accounted for by a multiplicative factor, $1+\sqrt{1-\alpha^2}$, in the effective dissipation.[17] We wish to define also an effective resistance R_{eff} as the Ohmic resistance in the RSJ model, which reproduces the effective dissipation η_1. It is interesting to note that R_{eff} is smaller than R^* by a factor

$$R^*/R_{\mathrm{eff}}=(1+\sqrt{1-\alpha^2})[\ln(b/\sqrt{E_1})+\delta] ,$$

which in typical experimental situations assumes values between 10 and 20. This result will be used for the comparison with experimental data.

We now wish to analyze the effects of the approximations that allowed us to obtain the analytical expression Eq. (12) for the lifetime. We then numerically calculated the values of the lifetime τ from Eq. (7) and compare them with the analytical expression τ_A, Eq. (12). The analysis has been performed for values of γ ranging between 20 and 8000 ($\gamma=20$, 50, 100, 500, 1000, 2000, 4000, and 8000) and values of b ranging between 2 and 40 ($b=2$, 5, 10, 20, and 40). The results are summarized in Fig. 1, where the ratio τ/τ_A is plotted for the various values of the considered parameters as a function of $(\gamma/2)U_0$, in the experimentally significant range 2–20. First we wish to stress that τ/τ_A is essentially independent of the value of b (the maximum fluctuations are contained within 1–2 %). Note that for $\gamma>50$ the deviation of the exact values of τ from the analytical approximation τ_A is always less than 20%. We also observe that for $\gamma>500$, the deviation presents a well-defined behavior as a function of $(\gamma/2)U_0$, and just a slight dependence on the γ value. In this region of γ the behavior is well fitted by the following analytical expression:

$$\begin{aligned}&\tau/\tau_A=f((\gamma/2)U_0) ,\\&f\left[\frac{\gamma}{2}U_0\right]=A+\frac{B}{c^2+[(\gamma/2)U_0-d]^2} ,\end{aligned} \tag{13}$$

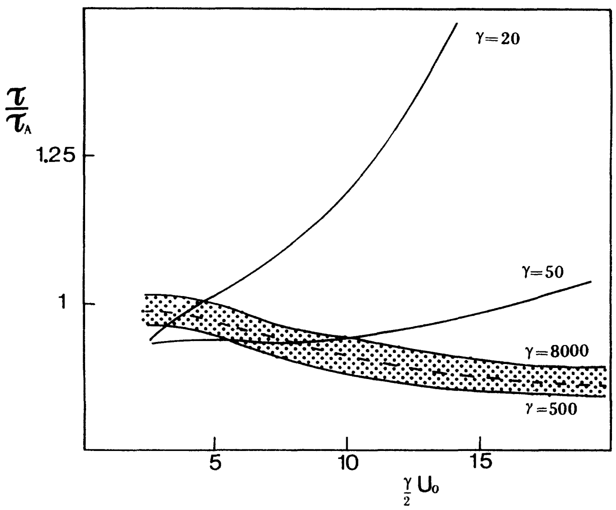

FIG. 1. Exact values τ of the lifetime numerically calculated from Eq. (7) normalized to the analytical approximate expression τ_A, Eq. (12). The ratio τ/τ_A is plotted vs $(\gamma/2)U_0$ at various values of γ. The marked area contains all the curves with the values of γ ranging between 500 and 8000. The dashed line is an example of the behavior predicted by the empirical correction to τ_A, $f((\gamma/2)U_0)$ [Eq. (13)]: It corresponds to $\gamma=2000$. The curves are independent of the value b considered ($b=2$–40).

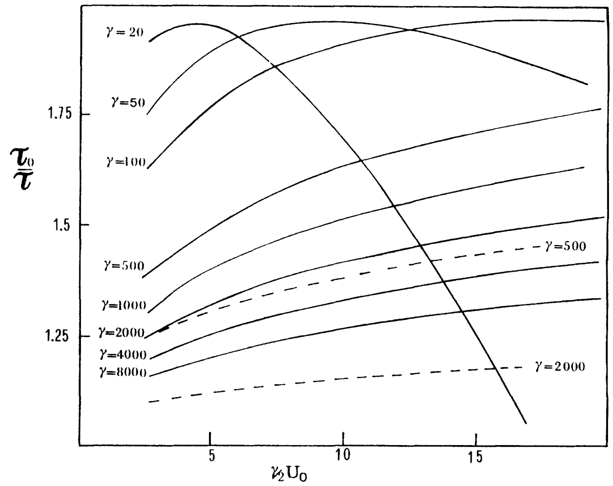

FIG. 2. Lifetime τ_0 obtained from Eq. (7) neglecting the $\cos\phi$ term in the calculations, normalized to the exact value τ. This ratio gives the contribution of the $\cos\phi$ term to the effective dissipation: $\tau_0/\tau \sim \eta_1/\eta_0$, where η_0 and η_1 are the effective dissipations that one obtains by neglecting (η_0) and including (η_1) the $\cos\varphi$ term in the calculations. The ratio τ_0/τ is plotted vs $(\gamma/2)U_0$ at various values of γ. The curves are independent of the value of b. The dashed lines are the plot of the function $1+\sqrt{1-\alpha^2}$ for $\gamma=500$ and $\gamma=2000$.

where the coefficients $B=10$, $c=8$, and $d=2$ are independent of γ and b. The small dependence of τ/τ_A on γ can be accounted for by a logarithmic dependence of A: $A=a+g\log_{10}(\gamma/1000)$ where $a=0.82276$ and $g=0.0434$. The function $f((\gamma/2)U_0)$ represents an empirical correction to the approximate lifetime expression Eq. (12), valid for $\gamma>500$. It is interesting to note that essentially it does not depend on the junction parameters and noise conditions.

As a result, we can conclude that the values of τ given by Eq. (7), in the experimentally important range of γ values ($\gamma>500$), can be obtained within 1–2 % by an analytical expression independent of the value of $b=\sqrt{2}e\mu/\hbar\omega_j$,

$$\tau=f((\gamma/2)U_0)\tau_A \; . \qquad (14)$$

Moreover, the analytical expression τ_A deviates from τ by less than 20% in a very large range of noise conditions, $\gamma>50$, and for any value of junction parameter b. As we expected, at lower γ values, the analytical approximation τ_A starts failing and leads to larger deviations, which increase with increasing $(\gamma/2)U_0$, as shown in Fig. 1 for $\gamma=20$.

We now consider the contribution of the $\cos\phi$ term on the effective dissipation. As we said, within the approximations which lead to τ_A, this contribution is given by the multiplicative factor $1+\sqrt{1-\alpha^2}$. However, this was the weakest part of our approximation. In order to give an insight about this point, we plot in Fig. 2 the ratio τ_0/τ, where τ_0 is the lifetime obtained from Eq. (7) neglecting the $\cos\phi$ term in the calculations. This ratio gives in fact the contribution due to $\cos\phi$ term to the effective dissipation: $\tau_0/\tau \sim \eta_1/\eta_0$, where η_0 is the effective dissipation one obtains neglecting the $\cos\phi$ term. The plot is performed as a function of $(\gamma/2)U_0$, at various values of γ. The results are again independent of the b value. We also report, as dashed lines, the function $1+\sqrt{1-\alpha^2}$ vs $(\gamma/2)U_0$ for $\gamma=500$ and $\gamma=2000$. We observe that the contribution of the $\cos\phi$ term is typically a bit larger than the factor $1+\sqrt{1-\alpha^2}$ that we expect from τ_A, and it is ranging between 20 and 90 % depending on the value of γ.

A comparison of theory with experiments can be done measuring the statistical distributions of the switching current in highly hysteretical Josephson junctions,[6] which are related to the lifetime of the $V=0$ state. The fitting of the data leaving η_1 as a free parameter allows us to get information about the effective dissipation. We have taken data in Nb-NbO_x-Pb Josephson junctions.[18]

In Fig. 3, we report the values of the effective dissipation $\eta_{\rm eff}$ obtained by the fitting of the switching current distributions as a function of the reverse temperature. In order to get a useful quantity for the comparison, we have also measured the subgap resistance $R^*=I^*/V^*$ on the dc current-voltage characteristics at the voltage $V^*=\mu/e$ (this voltage is ranging between 75 and 120 μV). We have then reported in the figure the measured values for the dissipation $\eta^*=(\omega_j R^* C)^{-1}$ (dots), its expected behavior by Eq. (9a) with $\Delta=1.3$ mV (Ref. 19) (the dashed line through the dots), as well as the theoretical extrapolations to the dissipation coefficients η_1 and η_0, obtained within our model including (η_1: solid line) and neglecting (η_0: dashed line) the $\cos\phi$ term.

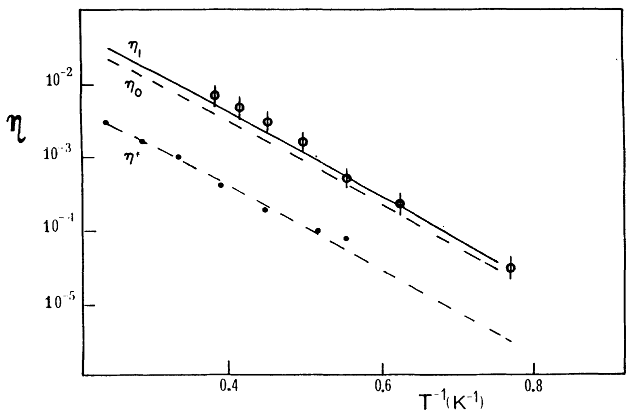

FIG. 3. Some relevant junction dissipation coefficients vs the inverse temperature. The open circles are the effective dissipation values η_{eff} as obtained from data on the switching current distributions (Ref. 18). We also report the η^* values (dots) obtained from the resistances R^* measured on the I-V characteristics at the voltage $V^* = kT/en_e$ and its expected behavior by Eq. (6a) with $\Delta = 1.3$ mV (dashed line fitting the dots). The theoretical extrapolations to the effective dissipation coefficients obtained within our model including (η_1: solid line) and neglecting (η_0: dashed line) the $\cos\phi$ term are also shown.

We wish first to stress that η_{eff} shows a well-defined exponential behavior as due to quasiparticles thermally activated above the superconducting gap. This guarantees that the junction dissipation relevant for the supercurrent decay is in fact dominated by intrinsic mechanisms.[6]

The correlation between the fit parameters did not allow us to get a very precise determination of η_{eff} at each temperature,[18] so that the errors bars are of the same order of expected contribution due to the $\cos\phi$ term. Therefore, the data cannot provide any definitive answer about this effect. We note, however, that the contribution of the $\cos\phi$ term reduces a certain shift observed between data and theory.[6] Moreover, this contribution must be, in fact, included in the model in order to have the remaining deviation within the experimental uncertainty. This gives us confidence in the possibility of our picture to describe the main aspects of the problem. The good agreement obtained here between data and theory gives in fact an answer about the importance of the intrinsic mechanisms of dissipation in this kind of process, as well as a clear indication about a certain relevance of the $\cos\phi$ term. It is clear anyway that our simple model is still a somewhat rough approximation of the junction and it is probably not expected to yield an exact agreement between theory and experiment. A more sophisticated approach to the problem would, however, lead to further complications in the theory.

In conclusion, we have studied the supercurrent decay of underdamped Josephson junctions including in the equivalent-circuit model the $\cos\phi$ term. From a theoretical point of view, we have obtained an expression for the lifetime of the $V=0$ state that takes into account the dependence on the voltage of both the quasiparticles' tunneling conductance σ_0 and of the interference term σ_1. We have then found an analytical approximation for the lifetime, τ_A (Eq. 12), which confirms the results of the RSJ model (with a constant resistance), provided that one assumes an effective dissipation η_1. This dissipation can be seen as given by an effective Ohmic resistance, which comes out to be related, within our model, to a subgap resistance measured on the I-V characteristics, although substantially smaller. The analytical approximation has then been compared with the exact values of the lifetime obtained performing numerically the quadratures appearing in Eq. (7). As a result, we found that the analytical approximation differs by less than 20% from the exact solution over a very large range of junction parameters and noise conditions. Moreover, in the experimentally significant range of γ values ($\gamma > 500$), the difference can be taken into account by a semiempirical analytical expression, Eq. (14). A comparison of the theory with our experimental results has also been discussed. The good agreement observed clearly shows the importance of the intrinsic mechanisms of dissipation on the supercurrent decay. The data also provide an indication about the importance of the $\cos\phi$ term in the model. This term must in fact be included in the theory for agreement between theory and data to be achieved within the experimental uncertainty. About this latter point, however, a greater accuracy in the experiments is necessary to give a definitive answer.

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