# Electrons in the $t$-$J$ model as bound states of spinons and holons

Y. Chen, D. Förster, and A. Larkin*
*Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany*
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We identify the quasielectron band in the $t$-$J$ model at small hole doping using a gauge-theoretic approach. We find that quasielectrons are formed as bound states of spinons and holons. At zero doping, the residue of the bound state vanishes at the Fermi momentum of the spinons. At this point, the inelastic spectrum develops a fractional-power singularity that is reminiscent of an x-ray singularity. The spectral weight of the Green's function of the bound state adds up to the spinon density as required by the Hubbard algebra. We also derive an effective field theory for many quasielectrons and their interactions.

## I. INTRODUCTION

The $t$-$J$ model[1] is the simplest of all models of strongly correlated electrons. It is also believed[2] to describe the motion of singlet holes in the Emery model[3] of cuprate superconductors. The normal-state properties of this model and its implications for tunneling spectra have been considered previously,[4] and extensive numerical analysis has been performed.[5] A distinctive feature that has emerged from these studies is the strong inelastic contribution to the spectral function that implies strong interactions between spin and charge degrees of freedom in this model (see Fig. 1).

In spite of considerable efforts invested into analytical and numerical investigations of this model, the nature of the quasiparticles both in this model and in the cuprate superconductors themselves remains unclear. An important question is whether the hole-doped cuprate superconductors may be treated within the standard-Fermi-liquid framework or whether a more radical approach such as marginal-Fermi-liquid theory is required.[6,7]

In a previous paper we demonstrated the existence of attractive forces among the spin and charge degrees of freedom of strongly correlated electrons that are described by the $t$-$J$ model. In the present paper, we show that these forces bind spinons and holons together at small doping to form physical quasielectrons as bound states.

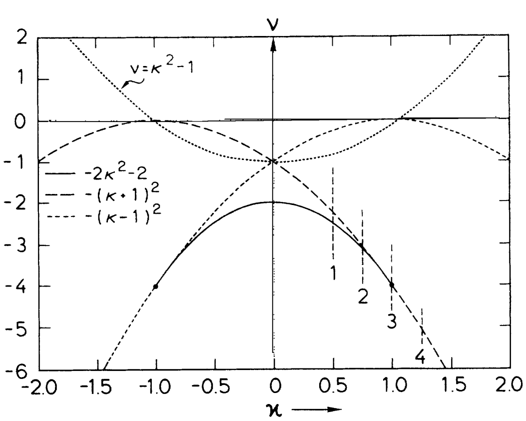


FIG. 1. Spectral densities for various values of $\kappa$ with the pole contribution.

The considerations of this paper serve to clarify the nature of the quasiparticles in the $t$-$J$ model. They also set the stage for a detailed analysis of superconducting and other instabilities in this model.

This work is organized as follows. Section II reviews our earlier results on the interactions among spinons and holons. Section III describes the structure of the bound state in the spinon antiholon channel. Section IV describes an effective-field theory for many bound states and their interactions, and Sec. V gives our conclusions. Technical matters have been relegated to the Appendices.

## II. ATTRACTIVE FORCES BETWEEN HOLONS AND ANTISPINONS

We begin by recalling our previous arguments[8] on the interaction between spin and charge degrees of freedom in the $t$-$J$ model. We use a continuum formulation at $J=2t$ to simplify our analysis as much as possible. At $J=2t$, the $t$-$J$ Hamiltonian reads as follows:

$$H=-\frac{J}{2}\sum_{\langle i,j\rangle a,b=0,1,2}(A_i^{a\dagger}A_j^a)(A_j^{b\dagger}A_i^b)\ . \tag{1}$$

where the slave operators $A_i^a$ annihilate fermionic spinons for $a=1,2$ and bosonic holes for $a=0$. The continuum Lagrangian is obtained by a gradient expansion and reads

$$\mathcal{L}=\sum_a \bar{y}^a\partial_\tau y^a+\frac{Jb^2}{2}\sum_a\overline{D_\mu y^a}D_\mu y^a\ , \tag{2}$$

where $b$ is the lattice size and where $D_\mu$ is a covariant derivative defined as

$$D_\mu y^a=(\partial_\mu+iA_\mu)y^a\ ,\quad A_\mu=-i\bar{y}^a\partial_\mu y^a\ . \tag{3}$$

The absence of direct contact interactions between the slaves in the continuum limit is due to an extra symmetry of the model at $J=2t$.[9] It is analogous to the lack of interactions among magnons of small momenta, and it greatly simplifies our subsequent analysis by suppressing charge density or superconducting instabilities. The gauge field $A_\mu$ that arises in the formal continuum limit of the model reflects the local U(1) gauge invariance of the slave operator description. The constraint $\sum_{a=0,1,2}\bar{y}_i^a y_i^a=1$ is due to the single-occupancy constraint of the slaves and is treated within a $1/N$ approximation. The gauge field picks up dynamics via quantum fluctuations at order $1/N$; see Ref. 8 for more details.

The following results were found for the effective gauge Lagrangian by integrating out spinons and holons from the partition function:

$$S_{\rm eff}=\tfrac{1}{2}\int A_\mu \Pi_{\mu\nu} A_\nu \ , \tag{4}$$

$$\Pi_{\mu\nu}=\Pi_t\left[\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]+\Pi_l\frac{k_\mu k_\nu}{k^2} \ , \tag{5}$$

where

$$\Pi_t=\Pi_t^B+\Pi_t^F=\frac{\rho_B}{m}+\chi k^2-i\frac{\rho_F}{m}y \ , \tag{6}$$

$$\Pi_l=\Pi_l^B+\Pi_l^F=-\frac{\rho_F}{2m}(2y^2+iy^3)-\frac{\rho_B}{m}\frac{\omega^2}{E^2(\mathbf{k})} \ , \tag{7}$$

$y=\omega/v_F k$, $E(\mathbf{k})=k^2/2m$, $\chi$ is the Landau diamagnetic susceptibility, and $\rho_B,\rho_F$ are, respectively, the holon density and the spinon density of one polarization.

With this effective Lagrangian, we found the following interaction term in the electric field energy of two static slave charges:

$$U(\mathbf{x})=-Q^2\int\frac{d^2k}{(2\pi)^2}\frac{\cos(\mathbf{k}\cdot\mathbf{x})}{k^2\epsilon(0,k)} \ . \tag{8}$$

Here $\mathbf{x}$ is the position vector connecting the static slaves and $\epsilon(\omega,\mathbf{k})$ is related to the longitudinal polarization by

$$\epsilon(\omega,\mathbf{k})=\frac{\Pi_l(\omega,\mathbf{k})}{\omega^2} \ . \tag{9}$$

As a check of Eq. (9), we note that with $\epsilon=1$ it produces the correct Coulomb attraction $(1/2\pi)\ln x$ between opposite slave charges. Furthermore, when a dynamical photon is present initially, we may use the polarization $\Pi_l$ of Eq. (7) to find Debye screening in $U(\mathbf{x})$. Having convinced ourselves that Eq. (8) is reasonable, we shall now use it when there is no dynamical photon present initially.

At vanishing hole doping $\delta$ we see from Eq. (8) that $U(\mathbf{x})$ becomes a delta function

$$U(\mathbf{x})=-\frac{2\pi}{m}(1-\delta)\delta(x)=-V\delta(x) \ . \tag{10}$$

It is important that there is attraction between opposite slave charges. Attraction between like charges would imply pairing in the holon-holon channel and lead to an unconventional mechanism for superconductivity.

## III. A BOUND STATE IN THE SPINON-ANTIHOLON CHANNEL

One anticipates attraction among opposite charges to lead to the formation of electrons as bound states of spinons and holons. For the instantaneous and pointlike interaction of Eq. (10), the usual Bethe-Salpeter equation for the spinon-holon bound state reduces to a much simpler algebraic equation that is analogous to the Bardeen-Cooper-Schrieffer (BCS) equation in the theory of superconductivity.[10,11]

Summing up the bubble diagrams for $\langle \bar{y}^\alpha y^0(1)\bar{y}^0 y^\alpha(2)\rangle$ we obtain

$$G=G_0[1+VG_0+(VG_0)^2+\cdots$$
$$=\frac{G_0}{1-VG_0}=\frac{1}{G_0^{-1}-V} \ . \tag{11}$$

$G_0$ is the convolution of spinon and holon Green's functions[4]

$$G_0(\omega,\mathbf{k})=-\int\frac{d^2p}{(2\pi)^2}\frac{n_B(\mathbf{p}-\mathbf{k})+n_F(\mathbf{p})}{\omega+i0^+-E_F(\mathbf{p})+E_B(\mathbf{p}-\mathbf{k})} \ , \tag{12}$$

where $E_B(\mathbf{k})=k^2/2m-\mu_B$, $E_F(\mathbf{k})=k^2/2m-E_F$, and $\mu_B$ ($<0$) and $E_F$ ($>0$) are, respectively, the chemical potentials of the holon and spinon.

$G_0$ is evaluated by the proper-time method in Appendix A. At zero hole doping we find

$$G_0=\frac{m}{2\pi}\frac{k_F}{k}[x-\sqrt{x^2-1}] \ , \tag{13}$$

where

$$x=\frac{-\omega-i0^+-E(\mathbf{k})-(E_F-\mu_B)}{v_F k} \ , \tag{14}$$

and $v_F$ is the spinon Fermi velocity. We absorb $\mu_B$ in the definition of $\omega$ and introduce dimensionless variables $v=\omega/E_F$, $\kappa=k/k_F$. In these variables, the relation between $\omega$ and $x$ reads

$$-v=\kappa^2+2\kappa x+1 \ . \tag{15}$$

The propagator of the physical electron in Eq. (11) is now given by

$$G=\frac{m}{2\pi\kappa}\frac{1}{x+\sqrt{x^2-1}-1/\kappa} \ . \tag{16}$$

We can see from the expression for $G$ that, in general, there is both a pole contribution and a cut. The cut is located at $-1<x<1$ and, by the relation between $v$ and $x$, implies inelastic processes for $v$ lying between the curves $v_\pm=-(\kappa\pm1)^2$.

### A. Spectral sum rules

To understand the properties of $G$, it is best to make the singularities of $G$ uniform as a function of $x$ first by extending $x$ to the complex variable $z$ and then by mapping the complex $z$ plane onto the region $|y|>1$ in the $y$ plane via

$$z=\frac{1}{2}\left[y+\frac{1}{y}\right] \ , \quad dz=\frac{1}{2}\left[1-\frac{1}{y^2}\right]dy \ . \tag{17}$$

This map neatly disentangles the square-root singularities as it sends the first and second Riemann sheets to $|y|>1$ and $|y|<1$, respectively. Under this conformal transformation, the upper and lower branch of the cut $(-1,+1)$ in the $z$ plane map onto the upper and lower part of the unit circle in the $y$ plane. In terms of the new variable $y$, the propagator $G$ takes on a particularly simple form

$$G=\frac{m}{2\pi\kappa}\frac{1}{y-y_0}\ ,\quad y_0=1/\kappa\ . \tag{18}$$

We recognize a pole in the propagator at $y=y_0$. By relating $y$ to $x$ and $x$ to $v$ this generates the following dispersion of the pole:

$$v=-2(\kappa^2+1)\ . \tag{19}$$

On the upper half of the unit circle at $e^{i\phi}$ in $y$, $G$ has a spectral density $\rho(\omega,\mathbf{k})$ given by

$$\rho=-\frac{1}{\pi}\,\mathrm{Im}G=\frac{m\pi}{2\pi^2}\frac{\sin\phi}{\kappa^2-2\kappa\cos\phi+1}\ . \tag{20}$$

Here $\cos\phi$ is related to $v$ via Eqs. (15) and (17), i.e., $-v=\kappa^2+2\kappa\cos\phi+1$. As the unit circle maps onto the cut in the $z$ plane, this spectral density has support between the curves $\omega_\pm(k)$ found earlier (see Fig. 1). We may expect the angle $\phi$ $(-\pi<\phi<\pi)$ to parametrize the scattering kinematics of spinons and holons.

In the $y$ plane, it is easy to identify a sum rule for the total spectral density at each value of $\kappa$. The total spectral weight at each $\kappa$ is given by the contour integral

$$\mathcal{A}(\mathbf{k})=\frac{1}{2\pi i}\int_C d\omega\, G(\omega,\mathbf{k})\ , \tag{21}$$

where $C$ wraps the real axis in a counterclockwise direction. In the complex $y$ plane, $C$ maps onto a curve $(C^*)$ that wraps the unit circle and the half lines $(-1,\infty)$ and $(\infty,1)$. Using $d\omega=(kk_F/m)dx$, we find

$$\mathcal{A}(\mathbf{k})=\rho_F\frac{1}{2\pi i}\int_{C^*}dy\frac{1-1/y^2}{(y-y_0)}\ , \tag{22}$$

Here we have used the fact that the spinon density of one polarization is given by $\rho_F=k_F^2/4\pi$. An easy calculation

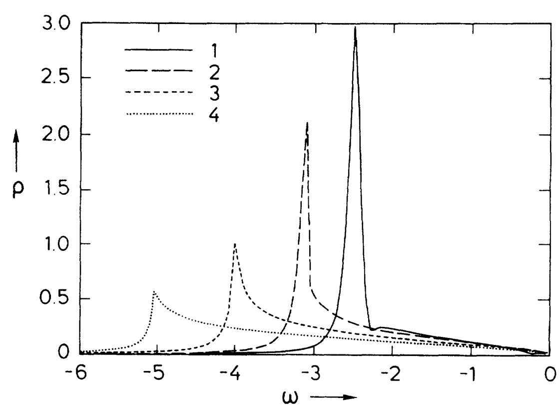


FIG. 2. Dispersion of the bound-state pole and support of inelastic spectrum.

shows that the contributions to the contour integral from the poles at $y=0$ and $y_0$ add up to one, irrespective of whether the pole at $y_0$ is inside or outside the unit circle. The extra factor $\rho_F$ reflects that the formation of a bound state requires the existence of spinons. In other words, the spectral weights of pole and cut always add up to $\rho_F$. More explicitly, the residue of the pole in the $\omega$ plane is given by $Z_k=\rho_F\theta(1-\kappa)(1-\kappa^2)$ and vanishes when the pole joins the cut at $\kappa=1$.

The above spectral sum rule also follows directly from the original Hubbard algebra at equal time and zero doping:

$$\langle\Omega|[X_i^{\alpha 0},X_j^{0\beta}]_+|\Omega\rangle=\delta_{ij}\langle\Omega|X_i^{\alpha\beta}|\Omega\rangle\ . \tag{23}$$

It is well known[12] that such a relation implies a total spectral weight given by $\langle\Omega|X^{\alpha\alpha}|\Omega\rangle$, i.e., the density of spinons. The fact that this essential relation has survived the $1/N$ expansion and taking the continuum limit encourages us to believe that our approximations capture the essential physics of the $t$-$J$ model.

### B. Threshold singularities of the spectral function

The above discussion on the elastic and inelastic contribution to $G$ are summarized in Figs. 1 and 2, where the dispersion laws and spectral densities are displayed. The inelastic contribution shows threshold singularities that can be understood by expanding $\rho(\omega)$ as given in Eq. (20) at the boundaries $\omega_+(\kappa),\omega_-(\kappa)$ of its support:

$$\rho(\omega,\kappa)\sim\sqrt{\pm(\omega_\pm-\omega)}\ . \tag{24}$$

At $\kappa=1$ the inelastic spectral density coalesces with a pole of vanishing spectral weight. One deduces from Eq. (20) that at this point, $\rho(\omega)$ has a one-sided fractional-power singularity:

$$\rho(\omega,\kappa=1)\sim\frac{\theta(\omega+4)}{\sqrt{\omega+4}} \tag{25}$$

for $\omega>-4$.

We would like to point out now that the above conclusions are only weakly affected by the bosonic contribution to $G_0$ in Eq. (12). The full expression for $G$ that includes the bosonic contribution can be shown to have the following form:

$$G=\frac{P_\delta(y)}{Q_\delta(y)}\ , \tag{26}$$

for $P_\delta$ and $Q_\delta$ are second- and third-order polynomials in $y$, respectively, with coefficients that depend on the doping $\delta$. At zero doping these polynomials simplify to

$$G\sim\frac{y^2+1+2\kappa}{(y^2+1+2\kappa y)(y-1/\kappa)}=\frac{1}{y-y_0}\ . \tag{27}$$

This implies that at finite doping the two extra conjugate zeros of $Q_\delta$ are nearly canceled by the zeros of $P_\delta$. This explains the small spectral weight of the extra branch at $y^2+2\kappa y+1=0$ or $v=-(\kappa^2+1)$ that is seen numerically at small doping. (See Fig. 2.)

### C. Fermi energy of the bound state

We now have to discuss the chemical potential of the bound states, i.e., up to what energy are the bound states of the electron occupied? We may hope to clarify this by getting a better understanding of $G_0$. We shall recompute $G_0$ in a conventional way:

$$G_0^h = \langle \Omega | T\Psi^{\alpha\dagger}(t,k)\Psi^{\alpha}(0,k)|\Omega\rangle \ . \tag{28}$$

Here $h$ stands for the hole channel. On using $\Psi^\alpha = y^{0\dagger} y^\alpha$ and the lack of holons in the undoped state $|\Omega\rangle$, we recognize that $G(t,k)$ only propagates forward in time. Its Fourier transform is given by

$$G^h(\omega,k) = \int_0^\infty dt \int \frac{d^2p}{(2\pi)^2} n_F(\mathbf{p}) \times \exp\{it[\omega + i0^+ + E_F(\mathbf{p}) - E_B(\mathbf{p}+\mathbf{k})]\} \ , \tag{29}$$

which is exactly the starting point of the explicit calculation in Appendix A. The above simple consideration shows that $G_0$ is unbounded above and below (at finite doping) simply due to the convolution of particle and hole dispersion laws that are unbounded in a continuum approximation. On a lattice, the dispersion laws would be bounded, and we must introduce an explicit ultraviolet cutoff in our continuum theory with $k_{\text{cutoff}} > k_F$. Apparently, it takes a finite mount of hole doping before we reach the momentum $k_F$, where the pole forms.

## IV. EFFECTIVE-FIELD THEORY OF THE BOUND STATE

So far, we have idealized the $t$-$J$ model system at small doping in terms of nonrelativistic bosons and fermions with an attractive nonretarded pointlike interaction. We now wish to write down an effective-field theory for the bound state in this system that will also clarify the relation between our method and the well-known x-ray edge problem. In the first step, we convert the pointlike interaction into a coupling with an extra auxiliary fermionic field as follows:

$$\mathcal{L} = \bar{y}^\alpha \left[\partial_\tau - \frac{\Delta}{2m} - \mu_F\right] y^\alpha + \bar{y}^0 \left[\partial_\tau - \frac{\Delta}{2m} - \mu_B\right] y^0 + \frac{1}{V}\bar{\chi}_\alpha\chi_\alpha + i\bar{\chi}_\alpha y^\alpha \bar{y}^0 - i\bar{y}^\alpha y^0 \chi_\alpha \ . \tag{30}$$

It is easy to check that we obtain the pointlike interaction $V\bar{y}^\alpha y^\alpha \bar{y}^0 y^0$ upon tracing out the auxiliary field. In the second step, we now integrate over the holon and spinon degrees of freedom, which is possible because the Lagrangian is, by construction, only quadratic in the spinon and holon fields. In fact, it is a "graded" quadratic form of mixed statistics in $y^\alpha, y^0$, and we perform the integrating by use of the general (and elementary) relation

$$\int d\bar{z}\, dz\, e^{-\bar{z}Mz} = \frac{1}{\text{sdet} M} \ , \tag{31}$$

where the superdeterminant of a graded matrix $M$,

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \ , \tag{32}$$

is given by[13]

$$\text{sdet} M = \frac{\det A}{\det(D - CA^{-1}B)} \ . \tag{33}$$

Hence, the resulting effective Lagrangian of the $\bar{\chi},\chi$ field is

$$S[\bar{\chi},\chi] = -\ln \text{sdet} \left[\begin{bmatrix} \partial\tau - \dfrac{\Delta}{2m} - \mu_B & i\chi \\ -i\bar{\chi} & \partial_\tau - \dfrac{\Delta}{2m} - \mu_F \end{bmatrix}\right] . \tag{34}$$

In order to construct the quantum field $(\bar{\Psi}_\alpha, \Psi_\alpha)$ that creates and destroys physical electrons, we shall recast the partition function $\Xi$ of our effective-field theory described by Eq. (30) in an alternative form,

$$\Xi = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi e^{-S_{\text{eff}}[\bar{\Psi},\Psi]} \ , \tag{35}$$

where

$$S_{\text{eff}}[\bar{\Psi},\Psi] = -V\int \bar{\Psi}\Psi + W[\bar{\Psi},\Psi] \ , \tag{36}$$

and

$$e^W = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \exp\left[i\int \chi\bar{\Psi} - i\int \Psi\bar{\chi}\right](\text{sdet}M[\bar{\chi},\chi])^{-1} \tag{37}$$

is expressed in terms of a functional Fourier-Legendre transform.[14]

The bubble diagrams, which give the physical-electron Green's function, are reproduced here by expanding $\ln \text{sdet} M$ to second order in $\bar{\chi},\chi$:

$$\ln \text{sdet} M = -\int d1 \int d2 \bar{\chi}(1) \mathcal{G}_F(1-2)\mathcal{G}_B(2-1)\chi(2) \ , \tag{38}$$

where

$$\left[\partial_{\tau_1} - \frac{\Delta_{x_1}}{2m} - \mu_F\right] \mathcal{G}_F(x_1,\tau_1;x_2,\tau_2) = -\delta(x_1 - x_2)\delta(\tau_1-\tau_2) \ , \tag{39}$$

$$\left[\partial_{\tau_1} - \frac{\Delta_{x_1}}{2m} - \mu_B\right] \mathcal{G}_B(x_1,\tau_1;x_2,\tau_2) = -\delta(x_1 - x_2)\delta(\tau_1-\tau_2) \ . \tag{40}$$

The quantity $\mathcal{G}_F(1-2)\mathcal{G}_B(2-1)$ is recognized as the Fourier transform of the convolution of the free spinon and holon Green's functions. Up to second order the effective action for the physical electrons reads

$$S_{\text{eff}}[\bar{\Psi},\Psi] = \int \bar{\Psi}(\mathcal{G}_0^{-1} - V)\Psi \ , \tag{41}$$

from which the physical-electron Green's function is seen to be equivalent to the Bethe-Salpeter equation, Eq. (11). We may also adopt the first route by tracing out $\bar{\Psi},\Psi$, thereby arriving at an effective action expressed in terms of the fields $\bar{\chi},\chi$ conjugate to the electron fields,

$$S_{\rm eff}[\bar{\chi},\chi] = \int d1 \int d2\bar{\chi}(1)\Gamma^{(2)}(1-2)\chi(2) + \int d1 \cdots \int d4\Gamma^{(4)}(1,2,3,4)\bar{\chi}(1)\bar{\chi}(2)\chi(3)\chi(4) , \tag{42}$$

where

$$\Gamma^{(2)}=\mathcal{G}_F(1-2)\mathcal{G}_B(2-1)-\frac{1}{V}\delta(1-2) ,$$
$$\Gamma^{(4)}=\mathcal{G}_F(1-2)\mathcal{G}_F(3-4)\mathcal{G}_B(4-1)\mathcal{G}_B(3-2) . \tag{43}$$

We may envisage a calculation scheme whereby the fourth-order term in the effective action is treated by a BCS-type factorization[10] and investigate the possibility of a superconducting instability.

We may question the validity of the bubble approximation by specializing to the following circumstance: Consider a single holon and imagine that it has infinite mass. Note that this would be disallowed in the present model because the supersymmetry is artificially broken. Nevertheless, we may adapt the calculation of the spinon-holon bubble given in Appendix A to the case where the holon is infinitely massive and see that there are no bound states. The effective-field theory now becomes that for the x-ray problem:

$$S_{\text{x-ray}}=\int \bar{y}^{\alpha}\left[\partial_\tau-\frac{\Delta}{2m}-\mu_F\right]y^{\alpha} + \int \bar{y}^0(\partial_\tau-\mu_B)y^0-\int V\bar{y}^0y^0\bar{y}^{\alpha}y^{\alpha} . \tag{44}$$

Observe that in this case the holon has no spatial dynamics and can be integrated out exactly. The amplitude analogous to the physical-electron Green's function becomes the "core-hole–conduction-electron" correlation function,

$$F(\tau_1,\tau_2)=-\langle y^{\alpha}(0,\tau_1)\bar{y}^0(\tau_1)y^0(\tau_2)\bar{y}^{\alpha}(0,\tau_2)\rangle , \tag{45}$$

where

$$\langle \cdots \rangle \equiv \frac{\int e^{-S_{\text{x-ray}}}}{\int e^{-S_{\text{x-ray}}}} .$$

We may exploit the fact the holon degree of freedom is infinitely massive by tracing it out in one step,[15] thereby arriving at the following compact expression:

$$F(\tau_1,\tau_2)=g_B(\tau_2,\tau_1)g_F(0,\tau_1;0,\tau_2) , \tag{46}$$

where $S(\tau;\tau_2,\tau_1)$ is a switch function of $\tau$ of unit strength operating in the interval $(\tau_1,\tau_2)$ and $g_F(x,\tau;x',\tau')$ satisfies the following Schrödinger equation:

$$\left[\partial_\tau-\frac{\Delta_x}{2m}-\mu_F-V\delta(x)S(\tau;\tau_2,\tau_1)\right]g_F(x,\tau;x',\tau')=-\delta(x-x')\delta(\tau-\tau') , \tag{47}$$

and

$$g_B(\tau_2,\tau_1)=\frac{\det(\partial_\tau-\Delta_x/2m-\mu_F-V\delta(x)S(\tau;\tau_2,\tau_1))}{\det(\partial_\tau-\Delta_x/2m-\mu_F)} \tag{48}$$

The reader may consult Ref. [15] for a detailed derivation of the above results. At $T=0$, in the limit of $\mu_F\tau\gg 1$, the determinant part of $F$ in Eq. (47) can be shown to have power-law behavior in $\tau$ up to a shift in the "core level", $\mu_B$

$$g_B(\tau_2,\tau_1)\sim(\mu_F|\tau_2-\tau_1|)^{-\bar{V}^2} , \tag{49}$$

while

$$g_F(0,\tau_1;0,\tau_2)\sim(\mu_F|\tau_1-\tau_2|)^{-(1-2\bar{V}} , \tag{50}$$

where $\bar{V}=\mathcal{N}V$ and $\mathcal{N}$ is the density of states at the Fermi level. For repulsion, we make the replacement; $V\rightarrow -V$.

From these we see that there is indeed no bound state and the effective-field theory reproduces the known results. A simple derivation of Eq. (50) is given in Appendix B.

## V. CONCLUSIONS

Based on the attractive forces among spinons and antiholons that had been found earlier using a $1/N$ and gauge-theory approach to the $t$-$J$ model, we have identified the physical electron band as a found state in this channel. The spectra we found have the peculiar feature that cut and pole coalesce at the spinon Fermi momentum $k_F$, where the weight of the pole vanishes. The validity of a spectral sum rule was proven for the bound state, and this suggests that our approach picks up the essential features of the $t$-$J$ model. At the Fermi momentum of the spinons, our spectra have a cusplike singularity reminiscent of the x-ray singularity. It also bears some resemblance to an interpretation by Anderson and co-workers of the photoemission experiments on hole-doped cuprates.[16]

In addition to the pole and cut contributions to the physical-electron Green's function, which mainly come from the fermionic component of the composite particle, we observe, from Eq. (12) that by treating the bosons as free particles a quasi-particle pole emerges, the weight of which is proportional to δ. With this picture in mind, we could at a rather qualitative level explain the recent photoemission data of Arnold, Mueller, and Swihart,[17] which shows a peak and *associated* with it a broad incoherent contribution at low temperatures. The peak contribution clearly comes from the pure bosonic part of the convolution bubble, while the incoherent part comes from the cut in the spectral function due to the threshold for real production of spinons and holons. However, it may be seen that by integrating out first the spinons and thus providing the gauge field partially with dynamics, followed by integrating out the gauge field, effective interaction between the bosons can be calculated. Once the effective interaction is known, the low-energy mode of the bosonic system can be found. Using this information, we can parametrize the bosonic part of the bubble with the appropriate spectral function, thus providing a more accurate estimate for the quasiparticle pole. The pole contribution is expected to survive the formation of the bound state. Preliminary calculations suggest that this is so. Non-Fermi-liquid behavior will be essentially associated with the production threshold giving the reproducible bump close to the main peak.[17]

At high temperatures, the evaporation of the condensate will suppress the quasiparticle pole, since its weight is proportional to the condensate density. Furthermore, the dissociation of the bound state leaves behind a purely incoherent contribution to the spectral function of the physical electrons (spin-charge separation). This explains the cusplike behavior of the photoemission spectra.[16] Quantitative demonstrations for these statements will be left for a future publication.

Concerning the robustness of the bound state in the $1/N$ approximation, we would like to mention that a higher-order calculation would involve a nonleading gauge field and rather complex combinations to sort out the classification scheme in $1/N$; however, to the order considered in this paper, we may trust the approximation used. The method described above for getting the effective interaction between the holons is equivalent to partially summing a class of diagrams in $1/N$. We have also developed a field theory of interacting quasielectrons, and our calculations set the stage for an analysis of superconducting and other instabilities in the two-dimensional $t$-$J$ model.

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## APPENDIX A: EVALUATION OF THE SPINON-ANTIHOLON CONVOLUTION

Here we sketch the evaluation of the bound state Green's function at zero hole doping. We make use of Schwinger's proper-time parametrization,[18]

$$\frac{1}{x+i0^+}=-i\int_0^\infty dt\, e^{it(x+i0^+)} , \tag{A1}$$

and find

$$\begin{aligned} G^h&=\int \frac{d^2p}{(2\pi)^2}\frac{n_F(\mathbf{p})}{\omega+i0^+ +E_F(\mathbf{p})-E_B(\mathbf{p}+\mathbf{k})} \\ &=-i\int_0^{k_F}\frac{p\,dp}{(2\pi)^2}\int_0^{2\pi}d\theta \\ &\qquad\times\int_0^\infty dt\, e^{\{it[\omega-E(k)-(pk/m)\cos\theta-E_F]\}} . \end{aligned} \tag{A2}$$

Using the identities

$$\int_0^{2\pi}\frac{d\theta}{2\pi}e^{-ia\cos\theta}=J_0(a) , \tag{A3}$$

$$\int_0^a dx\, xJ_0(x)=aJ_1(a) , \tag{A4}$$

and

$$\int_0^\infty \frac{dx}{x}e^{-\alpha x}J_1(\beta x)=\frac{\sqrt{\alpha^2+\beta^2}-\alpha}{\beta} , \quad \mathrm{Re}\alpha>|\mathrm{Im}\beta| \tag{A5}$$

from Ref. 19 we arrive at Eq. (14) of the text.

The real and imaginary part of Eq. (A2) can be found as a boundary value of Eq. (A5) on the real $\alpha$ axis. We find

$$\begin{aligned} \frac{2\pi}{m\bar{k}}\mathrm{Re}G^h&=x , \quad x\le 1 \\ &=x-\sqrt{x^2-1} , \quad x>1 \end{aligned} \tag{A6}$$

and

$$\begin{aligned} \frac{2\pi}{m\bar{k}}\mathrm{Im}G^h&=-\sqrt{1-x^2} , \quad x\le 1 \\ &=0 , \quad x>1 . \end{aligned} \tag{A7}$$

## APPENDIX B: MORE ON THE RELATION WITH THE X-RAY PROBLEM

$g_F(0,\tau_1;0,\tau_2)$ may be shown to satisfy a singular integral equation of the Cauchy type and was solved by standard procedure.[20] However, to investigate the long-time behavior of $g_F$, a simple approximate calculation will suffice, and we shall present it here. To do this, we may set $\tau_1$ to 0 and study $g_F$ at large $\tau_2$ (renamed $\tau$). The integral equation reads

$$g_F(\tau)=g(\tau)-V\int_0^\tau d\tau_1 g(\tau-\tau_1)g_F(\tau_1) , \tag{B1}$$

where the kernel $g(\tau)$ is given by

$$\begin{aligned} g(\tau)&=\frac{1}{\beta}\sum_\omega e^{-i\omega\tau}\int\frac{d^dk}{(2\pi)^d}\frac{1}{i\omega-[E(k)-\mu_F]} \\ &=-2\mathcal{N}\,\mathrm{Im}\int_0^\infty d\omega\, e^{(i\tau-1/D)\omega} \\ &=-2\mathcal{N}\frac{\tau}{\tau^2+D^{-2}} , \quad T=0 , \end{aligned} \tag{B2}$$

and $\mathcal{N}$ is the density of state at the Fermi level and

$D \sim \mu_F$ is the band-width cutoff. The iterative solution of Eq. (B1) is given as

$$g_F(\tau)=g(\tau)-V\int_0^\tau d\tau_1 g(\tau-\tau_1)g(\tau_1) + V^2\int_0^\tau d\tau_1 g(\tau-\tau_1)\int_0^{\tau_1} d\tau_2 g(\tau_1-\tau_2)g(\tau_2)+\cdots . \tag{B3}$$

Observe that in the limit of $D\tau \gg 1$, we can factor out the free part $g(\tau)$ from every term in the series, thereby arriving at

$$\frac{g_F(\tau)}{g(\tau)} \sim 1-V\int_0^\tau d\tau_1 g(\tau_1) + V^2\int_0^\tau d\tau_1\int_0^{\tau_1} d\tau_2 g(\tau_1-\tau_2)g(\tau_w) + \cdots , \quad D\tau \gg 1 . \tag{B4}$$

We see that all terms in Eq. (B4) exhibit infrared divergence of the type $V^n[\ln(D\tau)]^n$, reflecting the multitude of the low-energy excitations of the Fermi system. Close scrutiny of the series, Eq. (B4), shows that the coefficients of $[\ln(D\tau)]^n$ decrease as $1/n!$.

In order to see this, we may further approximate, for $D\tau \gg 1$,

$$\int_0^\tau d\tau_1\int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{n-1}} d\tau_n g(\tau_1-\tau_2)\cdots g(\tau_n) \sim \int_0^\tau d\tau_1 g(\tau_1)\int_0^{\tau_1} d\tau_2 g(\tau_2)\cdots \int_0^{\tau_{n-1}} g(\tau_n) . \tag{B5}$$

Upon using the explicit form of $g(\tau)$ (valid for all $\tau>0$) we find

$$\frac{g_F(\tau)}{g(\tau)} \sim \sum_{n=0}^{\infty} \frac{\bar{V}^n}{n!}\{\ln[1+(D\tau)^2]\}^n = [1+(D\tau)^2]^{\bar{V}} \sim (D\tau)^{2\bar{V}} , \quad D\tau \gg 1 , \tag{B6}$$

the result reported in the text. Note that despite the approximation used, the exact limiting behavior

$$\frac{g_F(\tau)}{g(\tau)} \to 1 , \quad \tau \to 0$$

is satisfied.

*Permanent address: Landau Institute for Theoretical Physics, Academy of Sciences of the U.S.S.R.

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