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Forming of wave packets by one-dimensional tunneling structures having a time-dependent potential

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We study the influence of a relatively small (in a classical sense) arbitrary time-dependent electrical pulse on the transmission amplitude of one-dimensional tunneling and resonant-tunneling structures. The transmission amplitude is found with use of semiclassical perturbation theory. First, the simplest tunneling structure—a potential barrier—is considered. It is shown that the form of the outgoing wave packet is sensitive to the presence of poles of the potential in the complex plane of time. For instance, a potential pulse with a single-peak time dependence can generate an outgoing wave packet with a number of peaks related to complex poles. Next, an expression for the transmission amplitude of arbitrary one-dimensional structure is obtained under the assumption that the time-dependent part of the potential is independent of the coordinate inside the structure. Then a resonant-tunneling double-barrier structure is considered. The expression for the transmission amplitude is simplified in this case, making use of the Breit-Wigner approximation. As examples we consider the switching-on of a potential, constant in time, and a potential with a linear time dependence. It is shown, in the nonadiabatic case, that at the moment of switching the outgoing flux begins to oscillate, with the amplitude of oscillation vanishing with time. We consider also the charge-accumulation process during one-dimensional resonant tunneling of monoenergetic electrons and study the conditions of intrinsic stability of a double-barrier structure.

I. INTRODUCTION

The classical problem of tunneling through a potential barrier and resonant tunneling has recently been generalized to the case of a potential having an increment with a harmonic dependence on time for both simple tunneling[1–4] and resonant tunneling.[5–14] These investigations were stimulated mostly by the progress in the preparation of nanometer tunneling and resonant-tunneling devices,[15–17] including devices subjected to an alternating field. The recent advances in manipulating atoms with a scanning tunneling microscope allows one to make artificial structures consisting of a finite number of atoms with atomic-size accuracy.[18,19] Therefore one can believe that in the not-too-distant future artificial tunneling and resonant-tunneling devices working on the levels of a finite system of atoms deposited at preselected positions will be created.

Investigating the properties of such devices under the influence of radiation, one can speak also about the nanometer detector of radiation.[9,12–14] An alternating electrical field can also be generated inside the structure, e.g., owing to the charge-accumulation process in the resonance-tunneling device.[20–25] Thus the time dependence of an applied field can in general have an arbitrary form (from the simplest harmonic to pulselike) generated by a laser or charged wave packets passing by at small distance from or just inside the structure investigated.

One of the versions of a nanoelectronic device is a structure having good ballistic qualities of electrons. It can be modeled by a potential profile containing quantum wells and barriers and a number of electrodes.

In Fig. 1 we show the simplest structures we consider. Figures 1(a) and 1(b) demonstrate a tunnel junction and a resonant-tunnel junction subjected to a radiation. In Fig.

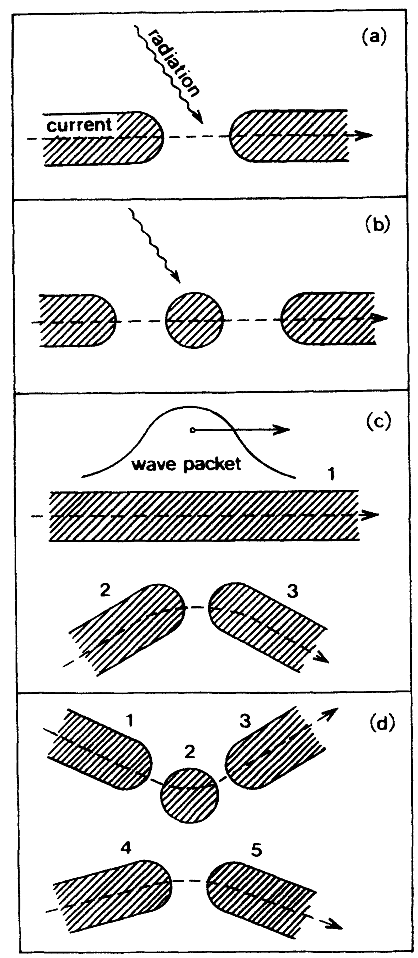

FIG. 1. Simplest nanometer tunneling and resonant-tunneling structures.

1(c) we denote the situation when changing the potential barrier between electrodes 2 and 3 is caused by a charged wave packet passing by inside wire 1. In Fig. 1(d) we show a device where the nonstationary charge-accumulation process in its upper, resonant-tunneling, part (see below) may induce barrier oscillations and hence the related alternating current in the lower tunnel junction. The sequence of structures shown in Fig. 1 can be easily continued.

The principal aim of the present work is the investigation of both the pulse transformation by the tunneling structure and the pulse transformation and generation by the resonant-tunneling structure. These problems might be regarded as a basis for developing a general theory of more complicated nanometer structures where the time-dependent potential is induced both due to the external radiation and intrinsic charging. By this, we mean, e.g., the the generalization of the stationary theory of complicated resonant-tunneling devices.[26]

Below we consider one-dimensional structures having a time-dependent potential. In Secs. II–IV we assume that the time-dependent form of the device potential is known. In Sec. V the time dependence of potential appears as a result of the charge accumulation in quantum wells, considered using the self-consistent-field approximation.

To calculate the outgoing electron wave function, we use semiclassical perturbation theory. It allows us to obtain the time-dependent increment of the action staying in the exponent of the expression for the wave function. The action for tunneling processes has an imaginary part, and such an increment can change the wave function by an order of magnitude. In Sec. II we formulate the scattering problem under consideration and given an account of semiclassical pertubation theory. In Sec. III we derive a semiclassical expression for the outgoing electron flux for the simplest tunneling structure: a time-dependent potential barrier. As examples, we find the analytical expressions for the outgoing flux for the potentials having poles in the complex plane of time. We show that, under definite conditions, a pole can generate a peak in the flux dependence of time and coordinates. For example, a potential pulse with the time dependence $(\gamma^4 t^4+1)^{-1}$, considered in Sec. III C, has only one maximum, but it can generate a wave packet with two maxima related to the real parts of complex poles of $(\gamma^4 t^4+1)^{-1}$. Another example considered in Sec. III is a potential with the Lorentz dependence of time (Sec. III B) and potential subjected to the periodic sequence of Lorentz pulses (Sec. III D). In Sec. IV A we obtain the expression for the outgoing amplitude for an arbitrary one-dimensional potential structure submitted to a potential pulse independent of the coordinates in the region of the structure. In Sec. IV C the double-barrier resonant-tunneling structure is considered. The formula for the transmission amplitude obtained in Sec. IV A is simplified using the Breit-Wigner approximation. As examples, in Sec. IV D we assume that the structure is subjected to a potential, constant in time, and a potential having a linear dependence of time, switched on at a definite moment. We show that in this moment, in the nonadiabatic case, the outgoing flux starts to oscillate. The amplitude of oscillations vanishes with time. In Sec. V we study the charge-accumulation process during one-dimensional resonant tunneling of monoenergetic electrons using the self-consistent-field approximation. The equation for the transmission amplitude is derived. Using this equation, we study the intrinsic stability of a one-dimensional resonant-tunneling device.

II. ASSUMPTIONS AND THE DESCRIPTION OF THE CALCULATION METHODS

A tunneling structure will be described below by the potential $V(x)$ (one-dimensional model) and additional (applied) alternating potential $W(x,t)$. The electron wave function of the problem satisfies the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+[V(x)+W(x,t)]\psi\ . \tag{2.1}$$

Let $V(x)$ and $W(x,t)$ be constants to the far left and to the far right of the structure and, in particular, $\lim_{x\to-\infty}W(x,t)=0$. Suppose that an electron is given to the far left by the stationary incident wave with energy E:[27]

$$\psi_{\rm in}(x,t)=v^{-1/2}\exp\left[\frac{i}{\hbar}\left[-Et+m\int^x v\,dx\right]\right],$$
$$v(x)=\left[\frac{2}{m}(E-V(x)\right]^{1/2}. \tag{2.2}$$

The corresponding outgoing wave to the right of the structure is convenient to present in the form

$$\psi_{\rm out}(x,t)=T(x,t)v^{-1/2}\exp\left[\frac{i}{\hbar}\left[-Et+m\int^x v\,dx\right]\right]. \tag{2.3}$$

To find the transmission amplitude $T(x,t)$, we use the following methods.

A. Semiclassical pertubation theory

We suppose $V(x)$ and $W(x,t)$ to be rather slow (semiclassical) functions everywhere except in a finite number of points (interfaces) where they or their derivatives may have discontinuities. We assume also that $W(x,t)$ is small compared with the kinetic energy of electron $mv^2/2$, as usually it takes place in practice. To solve Eq. (2.1) in semiclassical regions, we consider the corresponding Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t}+\frac{1}{2m}\left[\frac{\partial S}{\partial x}\right]^2+V(x)+W(x,t)=0 \tag{2.4}$$

and expand the action S in powers of $W(x,t)$ up to the first order:

$$S=S_0+S_1+\cdots ,$$

$$\frac{\partial S_0}{\partial t}+\frac{1}{2m}\left[\frac{\partial S_0}{\partial x}\right]^2+V(x)=0 , \tag{2.5}$$

$$\frac{\partial S_1}{\partial t}+\frac{1}{m}\frac{\partial S_0}{\partial x}\frac{\partial S_1}{\partial x}+W(x,t)=0 .$$

Solving (2.5), we obtain the general semiclassical solution of Eq. (2.1) in the form

$$\psi=v^{-1/2}\exp\left[\frac{i}{\hbar}(S_0+S_1)\right] ,$$

$$S_0(x)=-Et+m\int^x v\,dx, \tag{2.6}$$

$$S_1(x,t)=\int^x W\left[x',t+\int_x^{x'}\frac{dx''}{v(x'')}\right]\frac{dx'}{v(x')}+\eta\left[\int^x\frac{dx'}{v(x')}-t\right] ,$$

where $\eta(\tau)$ is an arbitrary function that is small compared with S_0 [the derivatives of $\eta(\tau)$ must be small compared with the derivatives of S_0 also]. Particular cases of the approximation (2.6) were used earlier in the calculations.[1–3,5,7,8]

B. Current and charge densities of the outgoing electrons

In the approximation considered, the outgoing wave (2.3) can be presented as a linear combination of solutions (2.6). Because of the inequality $S_{1x} \ll S_{0x}$, we have $\psi_x=iv\psi$, and according to (2.3), the flux of the wave function to the right of the structure,

$$\mathcal{J}(x,t)=\frac{i\hbar}{2m}\left[\psi_{\rm out}\frac{\partial\psi^*_{\rm out}}{\partial x}-\psi^*_{\rm out}\frac{\partial\psi_{\rm out}}{\partial x}\right] , \tag{2.7}$$

is

$$\mathcal{J}(x,t)=|T(x,t)|^2 . \tag{2.8}$$

The corresponding probability density of $\psi_{\rm out}$ is

$$\mathcal{D}(x,t)=[v(x)]^{-1}|T(x,t)|^2 . \tag{2.9}$$

According to (2.6), the important property of the function $\mathcal{J}(x,t)$ is that it only depends on the argument of the function η in (2.6):

$$\mathcal{J}(x,t)=\mathcal{J}\left[t-\int^x\frac{dx}{v(x)}\right] . \tag{2.10}$$

It means also that for $v(x)=$const (i.e., in a homogeneous medium) the wave packet $|\psi_{\rm out}(x,t)|^2$ propagates along the x axis without changing its form, having velocity v. To calculate $\mathcal{J}(x,t)$ it is obviously enough to find this function for one value of x. Below we will put x equal to the right boundary of the structure x_2.

The values (2.8) and (2.9), considered for one electron, only have a probability meaning. However, in a real situation, there may be many electrons having initial energies close to E. Then the value $\mathcal{J}(x,t)$ is proportional to the electron-current density, and the value $\mathcal{D}(x,t)$ is proportional to the electron-charge density.

III. SINGLE BARRIER

Let us consider the case of simple barrier shown in Fig. 2(a). We assume first that $W(x,t)$ is an analytical function having no poles in the complex plane of time. Then the expression for the outgoing flux $\mathcal{J}(t)$ can be found by straightforward matching the solutions (2.6) in points x_1 and x_2 and also with the incident wave (2.2). As a result, we have[28]

$$\mathcal{J}(t)=\mathcal{J}_0\exp[I_0(t)] ,$$

$$\mathcal{J}_0=\exp\left[\frac{2}{\hbar}\left[-m\int_{x_1}^{x_2}|v|dx\right]\right] ,$$

$$I_0(t)=\frac{2}{\hbar}{\rm Im}\left[\int_{-\infty}^{x_1}\frac{dx}{v(x)}W\left[x,t+i\tau_0+\int_{x_1}^{x}\frac{dx'}{v(x')}\right]+\int_{x_1}^{x_2}\frac{dx}{v(x)}W\left[x,t+\int_{x_2}^{x}\frac{dx'}{v(x')}\right]\right] , \tag{3.1}$$

$$\tau_0=\int_{x_1}^{x_2}\frac{dx}{|v|} ,$$

where $x=x_2$, v means $i|v|$ for v image, and τ_0 is the tunneling time.[1] One can see that two integrals of W in (3.1) can be rewritten as one integral over time on the circuit C_0 in the complex plane of time with cuts along the segments $({\rm Im}\tau=0,\ -\infty<{\rm Re}\tau<0)$ and $({\rm Im}\tau=-i\tau_0,\ 0<{\rm Re}\tau<\infty)$ [see Fig. 2(b)], so that

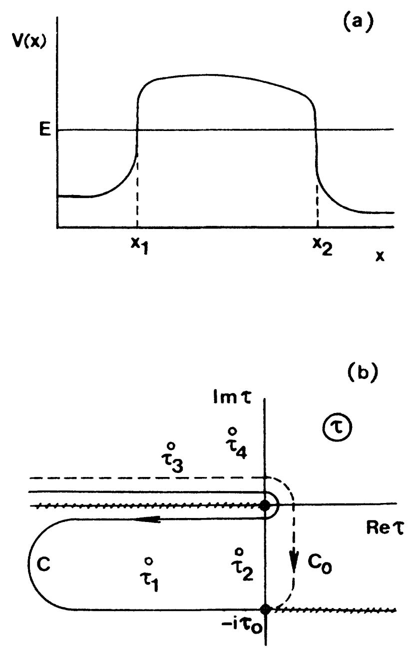

FIG. 2. (a) Potential barrier and (b) the circuits of integration in the complex plane of time: C_0 staying in Eq. (3.2) and C staying in Eq. (3.12).

$$I_0(t)=\frac{2}{\hbar}\mathrm{Im}\int_{C_0}d\tau\, W(x(\tau),t+i\tau_0+\tau)\ , \qquad (3.2)$$

$$\tau=\int_{x_1}^{x}\frac{dx'}{v(x')}\ , \qquad (3.3)$$

where Eq. (3.3) defines the function $x(t)$ standing in Eq. (3.2).

Expressions (3.1) and (3.2) assume the continuation of $W(x,t)$ into the complex t plane and can be used for investigation of different particular cases. The case of harmonic pertubation $W(x,t)=\Omega(x)\cos(\omega t)$ was studied in detail in Refs. 1–3. It was shown in particular that the $\Omega(x)\cos(\omega t)$ contribution into $P(t)$ has a double-exponent dependence on the adiabatic parameter $\omega\tau_0$ and can be large for very small $\Omega(x)$, but large enough $\omega\tau_0$.

A. Potential $W(x,t)$ having poles in the complex plane of time

In general, the function $W(x,t)$, being a slowly varying function for real t, can nevertheless have poles for complex t, which may significantly change $\mathcal{J}(t)$ under definite conditions.

Let $t_n(x)$ be a pole of $W(x,t)$, so that in a small vicinity of $t_n(x)$ the potential $W(x,t)$ has the form

$$W(x,t)\approx\frac{a(x)}{t-t_n(x)}\ , \qquad (3.4)$$

with complex residue $a(x)$. Then the pole of integrand in (3.2) will be

$$\tau_n(x,t)=t_n(x)-i\tau_0-t\ . \qquad (3.5)$$

The time t being changed, the point τ_n is moving in the complex plane of time from the right to the left in parallel to the real axis. Suppose that in the moment $t=t_n^*$ the point τ_n is intersecting the imaginary part of the circuit C_0 in the point τ_n^*. The values t_n^* and τ_n^* are the roots of the equation $\tau_n(x(\tau),t)=\tau$. The latter can be rewritten in the form

$$t_n^*=\mathrm{Re}t_n\ ,$$
$$\tau_n^*=i(\mathrm{Im}t_n-\tau_0)\ . \qquad (3.6)$$

The criterion of intersection is the inequality

$$0<\mathrm{Im}t_n<\tau_0\ . \qquad (3.7)$$

In the near vicinity of τ_n^* and t_n^*, we can write

$$W(x(\tau),t+i\tau_0+\tau)\approx\frac{a(x_n^*)}{(t-t_n^*)+(\tau-\tau_n^*)[1-(\partial t_n/\partial x)(x_n^*)'v(x_n^*)]}\ , \qquad (3.8)$$

where $x_n^*=x(t_n^*)$. Let $a(x(\tau))$ and $t_n(x(\tau))$ be smooth function in a vicinity of τ_n^*. Then, substituting Eq. (3.8) into Eq. (3.2), we find the jump of $I_0(t)$ in the moment t_n^*:

$$\Delta I(t_n^*)=2\pi\,\mathrm{sgn}(c)[c\ \mathrm{Re}a(x_n^*)-d\ \mathrm{Im}a(x_n^*)]\ , \qquad (3.9)$$

where the real values c and d are defined by the equality

$$c+id=\left[1-\frac{\partial t_n}{\partial x}(x_n^*)'v(x_n^*)\right]^{-1}\ . \qquad (3.10)$$

Thus the function $I_0(t)$ is discontinuous in the point $t=t_n^*$.

To find the correct solution of the problem, we must match semiclassical solutions of Eq. (2.1) in every point t_n^*, demanding the continuity of the wave function or, simply, the continuity of $\mathcal{J}(t)$ in these points. It means that the function $I_0(t)$ in expressions (3.1) and (3.2) must be changed by the continuous function

$$I(t)=I_0(t)-\sum_{\substack{\mathrm{Re}t_n<t,\\ 0<\mathrm{Im}t_n<\tau_0}}\Delta I(t_n^*)\ , \qquad (3.11)$$

where the sum is taken over all poles t_n that have intersected the circuit C_0 before the moment t. Suppose that the circuit C passes around the points $\tau_n=t_n-i\tau_0-t$, as shown in Fig. 2(b). Then Eq. (3.11) can be rewritten in the form similar to Eq. (3.2):

$$I(t)=\frac{2}{\hbar}\mathrm{Im}\int_{C}d\tau\, W(x(\tau),t+i\tau_0+\tau)\ , \qquad (3.12)$$

where the circuit of integration C stands instead of C_0.

The potentials $V(x)$ and $W(x,t)$ may be represented by different analytical functions in different regions of the axis x. Each of these analytical functions has its own analytical continuation into its own complex plane of time. The only common points of these planes are the points $\tau^{(k)}$ corresponding to the points $x_k=x(\tau^{(k)})$, where $V(x)$ or $W(x,t)$ lose their analyticity. In this case the circuit C must be fixed in the points $\tau^{(k)}$.

Suppose that $a(x(\tau))$ or $\partial t_n/\partial x(x(\tau))$ has a discontinuity in $\tau=\tau^{(k)}$ and that the value τ_n^* coincides with $\tau^{(k)}$. It can be shown then, using (3.8), that the integral (3.12) diverges for $t\rightarrow t_n^*$ as $\ln(|t-t_n^*|)$. However, the value of $I(t)$ is restricted by the validity of semiclassical perturbation theory used [$\hbar I(t)$ must be small compared with the characteristic value of the action S_0]. Nevertheless, if $I(t)>0$ near such point, then, being the exponent of $I(t)$, the outgoing flux $\mathcal{J}(t)$ can reach anomalously large values here. We will consider in Secs. IIIB–IIID the examples of $W(x,t)$ with simple poles and discontinuity that can cause such a phenomenon.

In the case more generalized than that considered above, the potential $W(x,t)$ can have a pole of Nth order. Then the discontinuity in the Nth derivative of $W(x,t)$ with respect to x may also lead to an infinite peculiarity in $I(t)$. We will not dwell here on the investigation of such cases.

B. Pulse with the Lorentz dependence of time

Consider the example when

$$V(x)=\begin{cases}0, & x<x_1,\ x>x_2\\ V_0, & x_1<x<x_2\ ,\end{cases}$$

$$W(x,t)=\frac{\Omega(x)}{\gamma^2t^2+1}\ , \tag{3.13}$$

$$\Omega(x)=\begin{cases}0, & x<x_0\\ W_0\dfrac{x-x_0}{x_1-x_0}, & x_0<x<x_1\\ W_0, & x>x_1\ .\end{cases}$$

The barrier in this case is submitted to the total movement and remains rectangular, and the interaction with alternating field occurs only in region $x_0<x<x_1$ in front of the barrier.

Calculations by formulas obtained above give[29]

$$\mathcal{J}(t)=\mathcal{J}_0\exp[I(t)]\ ,$$
$$\mathcal{J}_0=\exp\left[-\frac{2}{\hbar}|2m(E-V_0)|^{1/2}(x_2-x_1)\right]\ , \tag{3.14}$$

$$I(t)=-\frac{W_0}{\gamma^2\tau_1\hbar}\text{Im}[f^+(t)+f^-(t)-f^+(t-\tau_1)-f^-(t-\tau_1)]\ ,$$
$$f^{\pm}(t)=[1\pm\gamma(\tau_0-it)]\ln[1\pm\gamma(\tau_0-it)]\ , \tag{3.15}$$

$$\tau_0=\frac{m^{1/2}(x_2-x_1)}{[2(V_0-E)]^{1/2}},\quad \tau_1=\frac{m^{1/2}(x_1-x_0)}{(2E)^{1/2}}\ . \tag{3.16}$$

Here τ_0 is the tunneling time and τ_1 is the time for an electron to cross the region $x_0<x<x_1$ of interaction with alternating field. Suppose that this time is small, so that $|1\pm\gamma(\tau_0-it)|\gg\gamma\tau_1$. Then

$$I(t)=\frac{W_0}{2\gamma\hbar}\ln\left[\frac{(1-\gamma\tau_0)^2+\gamma^2t^2}{(1+\gamma\tau_0)^2+\gamma^2t^2}\right]\ . \tag{3.17}$$

This formula is exact for $\tau_1=0$, i.e., for the case when $\Omega(x)$ is zero to the left of the barrier and is constant W_0 in the barrier region [such a steplike x dependence was considered in Ref. 1, but for the $\cos(\omega t)$ time dependence]. In this situation $I(t)$, and hence $\mathcal{J}(t)$ for $W_0<0$] is infinite for $t=0$ and $\gamma\tau_0=1$. This is just the case discussed at the end of the preceding section. Actually, in this case, the zero of the complex τ plane coincides, for $t_1=i\gamma^{-1}$, with both the point of discontinuity of $W(x,t)$ and the pole of the integrand in Eq. (3.12).

In the adiabatic limit when $\gamma\tau_0$ and $\gamma\tau_1$ are small, the result (3.17) gives

$$I(t)=-\frac{2W_0\tau_0}{\gamma\hbar(\gamma^2t^2+1)}\ . \tag{3.18}$$

This is just the same as the result obtained for time-independent $W(x,t)$ with t assumed as a parameter.

It is not difficult to find from Eq. (3.15) that $I(t)$ for $W_0<0$ has one maximum $t=\tau_1/2$ only (for $W_0>0$ it will be minimum) and that the maximum of $|I(\tau_1/2)|$ is reached for $\gamma\tau_0=1$. In Fig. 3(a) the function $\tilde{I}(t)=(\gamma\hbar/W_0)I(t)$ is shown for $\gamma\tau_0=1$ and different $\gamma\tau_1$. For $\gamma\tau_1=0$, as follows from Eq. (3.17), we have $\tilde{I}(t)\sim\ln|t|$ for $t\to0$. In Fig. 3(b) we show $\tilde{I}(t)$ for different $\gamma\tau_0$ and fixed $\gamma\tau_1=2$.

C. Pulse with the time dependence $(\gamma^4t^4+1)^{-1}$

Consider the pulse

$$W(x,t)=\frac{\Omega(x)}{(\gamma t)^4+1}\ , \tag{3.19}$$

where the function $\Omega(x)$ and potential barrier $V(x)$ are taken to be the same as in the preceding section. The potential (3.19) has four poles

$$t_n=\gamma^{-1}\exp\left[\frac{i\pi}{4}(5-2n)\right],\quad n=1,2,3,4\ . \tag{3.20}$$

In this case the function $I(t)$ has the form

$$I(t)=\frac{W_0}{2\gamma^2\tau_1\hbar}\text{Im}\sum_{n=1}^{4}[f^{(n)}(t)-f^{(n)}(t-\tau_1)]\ ,$$
$$f^{(n)}(t)=\gamma t_n z_n\ln(iz_n)\ , \tag{3.21}$$
$$z_n=\gamma(t+i\tau_0-t_n)\ ,$$

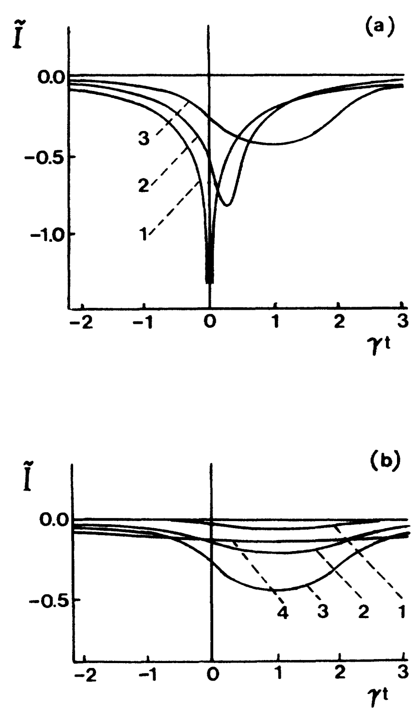

FIG. 3. Form of the normalized exponent $\tilde{I}(t)=(\gamma\hbar/W_0)I(t)$ in the expression for the outgoing flux in point $x=x_2$ for the Lorentz pulse. (a) $\gamma\tau_0=1$: (1) $\gamma\tau_1=0$, (2) $\gamma\tau_1=0.5$, and (3) $\gamma\tau_1=2$; (b) $\gamma\tau_1=2$: (1) $\gamma\tau_0=0.1$, (2) $\gamma\tau_0=0.5$, (3) $\gamma\tau_0=1$, and (4) $\gamma\tau_0=3$.

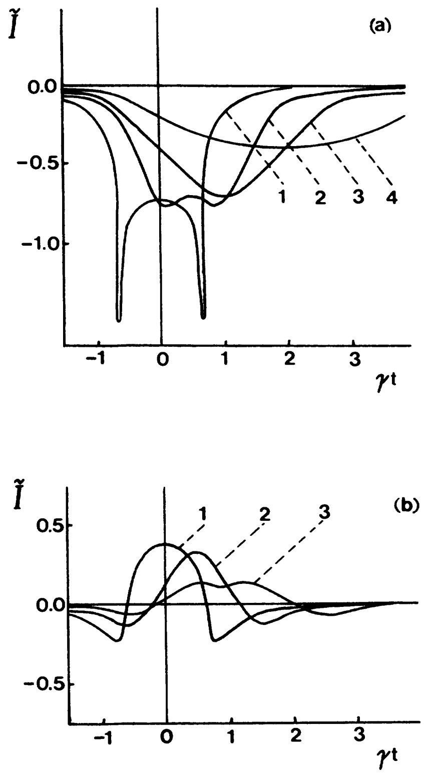

FIG. 4. Form of the normalized exponent $\tilde{I}(t)=(\gamma\hbar/W_0)I(t)$ in the expression for the outgoing flux in point $x=x_2$ for the pulse (3.19). (a) $\gamma\tau_0=0.707$: (1) $\gamma\tau_1=0$, (2) $\gamma\tau_1=1$, and (3) $\gamma\tau_1=2$; and (4) $\gamma\tau_1=4$; (b) $\gamma\tau_0=0.8$: (1) $\gamma\tau_1=0$, (2) $\gamma\tau_1=1$, (3) $\gamma\tau_1=2$.

where τ_0 and τ_1 are defined by formula (3.16).

If $\gamma\tau_1\to 0$ and $\tau_0=\mathrm{Im}\,t_{1,2}=2^{-1/2}\gamma^{-1}$ [i.e., if $W(x,t)$ has a discontinuity in $x=x_1$ and if $\tau_n=t_n-i\tau_0-t$ can cross the zero of complex t plane], then $I(t)$ has a logarithmic peculiarity for $t\to\mathrm{Re}\,t_n$. In Fig. 4(a) the cases are shown when τ_0 is slightly smaller than $\mathrm{Im}\,t_{1,2}$. Two maxima in these dependences correspond to the moments when t passes $\mathrm{Re}\,t_1$ and $\mathrm{Re}\,t_2$. For the dependences shown in Fig. 4(b), the value τ_0 is slightly larger than $\mathrm{Im}\,t_n$. The interesting feature of these dependences is that $\tilde{I}(t)$ can change its sign in spite of the constant sign of $W(x,t)$ considered.

D. Periodic sequence of pulses

It is interesting to examine the case of a periodically driven potential with $W(x,t)$, as shown in Fig. 5. The pulses in this case can be obtained as a superposition of harmonics. Now let us calculate the tunneling probability averaged over the period T of $W(x,t)$:

$$\langle \mathcal{J}(t)\rangle=\frac{1}{T}\int_0^T \mathcal{J}(t)dt \; . \tag{3.22}$$

Let the peaks of $W(x,t)$ be negative and large enough so that the main contribution into $\mathcal{J}(t)$ is made by their vicinities small compared with the period T. Now we will not deal with a new T-periodic model of $W(x,t)$.[30] But assume that in these vicinities $W(x,t)$ has the form (3.13) with $W_0<0$ and large $|W_0|/(\hbar\gamma)$. In this case the contribution of $W(x,t)$ is large and the integral (3.22) can be calculated by the saddle-point method. Putting the

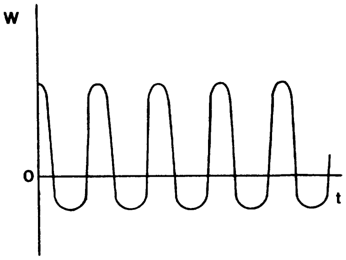

FIG. 5. Periodic sequence of pulses.

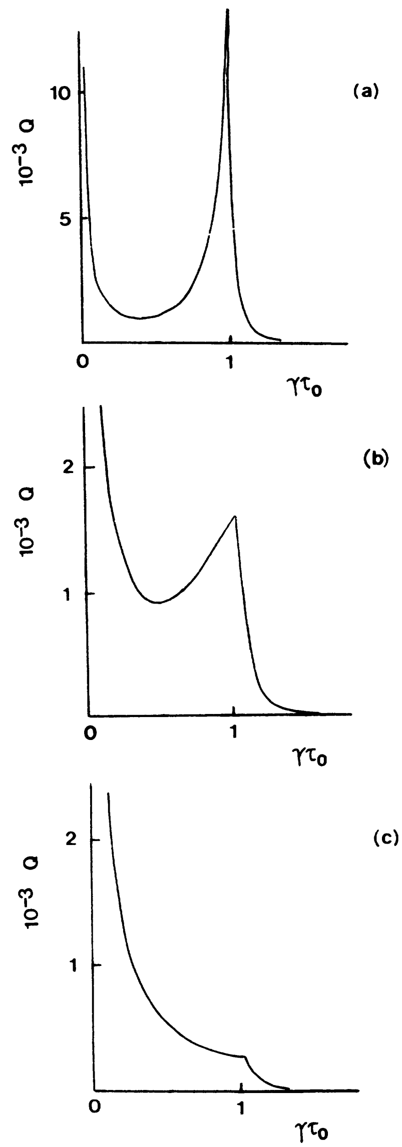

FIG. 6. The normalized average tunneling probability $Q(t)=(\langle\mathcal{J}(t)\rangle/\mathcal{J}_0)(T/\tau_0)$ as a function of $\gamma\tau_0$ for the periodic sequence of Lorentz pulses, $W_0\tau_0/\hbar=-3$. (a) $\tau_1/\tau_0=0.5$, (b) $\tau_1/\tau_0=1$, and (c) $\tau_1/\tau_0=2$.

derivative $I_t(t)$ equal to zero, we find the saddle point $t=\tau_1/2$ and

$$\langle \mathcal{J}(t)\rangle = \frac{\mathcal{J}_0}{T}\left[\frac{2\pi}{-I_{tt}(\tau_1/2)}\right]^{1/2}\exp[I(\tau_1/2)] ,$$
$$I_{tt}(\tau_1/2)=\frac{-4W_0\gamma^2\tau_0}{\hbar\{[1+\gamma^2(\frac{1}{4}\tau_1^2-\tau_0^2)]^2+\gamma^4\tau_0^2\tau_1^2\}} , \qquad (3.23)$$

with $I(t)$ defined by Eq. (3.15). The main contribution to $\langle \mathcal{J}(t)\rangle$ is made by the vicinity

$$|t-(\tau_1/2)| \lesssim [-I_{tt}(\tau_1/2)]^{-1/2} ,$$

which is smaller than τ_0 for $\gamma\tau_0 \sim \gamma\tau_1 \sim 1$. In Fig. 6 the normalized averaged tunneling probability $Q = (\langle \mathcal{J}(t)\rangle/\mathcal{J}_0)(T/\tau_0)$ is shown as a function of dimensionless inverse pulse width $\gamma\tau_0$ for $\tau_1/\tau_0=0.5,1,2$ and $W_0\tau_0/\hbar=-3$. For $\gamma\tau_0=1$ the function $Q(\gamma\tau_0)$ has a discontinuity of the derivative reserved from Eq. (3.15). Note that large values of Q and sharp peaks appear for the parameters $\gamma\tau_0$ and $\gamma\tau_1$ having an order of unity.

IV. CONDUCTANCE OF COMPLICATED DEVICE: DOUBLE-BARRIER RESONANT-TUNNELING STRUCTURE

In the examples of Secs. III C and III D, the potential $W(x,t)$ was considered to be independent of x in the barrier region. In other words, it was proposed that the interaction of electrons with alternating fields takes place outside the barrier only. Such a situation might be realized by the special design of the device. In general, such an approximation is valid when the characteristic dimensions of $W(x,t)$ are large compared with the dimensions of nanometer tunneling devices.[31] For instance, this approximation is good when $W(x,t)$ describes the interaction with infrared radiation with relatively large wavelength. Under definite conditions it is also good for $W(x,t)$ describing the long-range interaction with a charged wave packet changing with time.

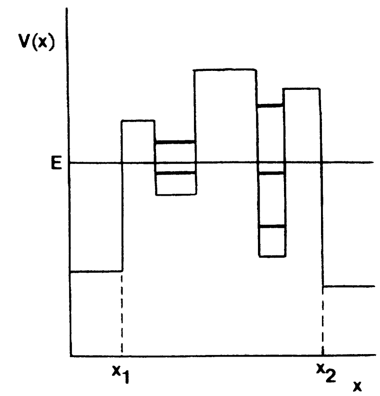

FIG. 7. Potential $V(x)$ for an arbitrary one-dimensional structure.

A. Outgoing flux for the potential $W(x,t)$ independent of x inside the device

Owing to the reasons indicated above and because of the computational difficulties in solving the general problem for complicated structures, we assume below that $W(x,t)$ is independent of x inside the device.

In front of the device, near x_1 (Fig. 7), where $W(x,t)=W_0(t)$ is already independent of x, the incident wave (2.2) is transformed into the solution

$$\psi(x,t)=v^{-1/2}\exp\left[\frac{i}{\hbar}\left\{-Et+m\int_{x_1}^{x}v\,dx-\int_{-\infty}^{0}d\tau\left[W\left[x(\tau),\tau+t-\int_{x_1}^{x}\frac{dx}{v}\right]-W_0\left[\tau+t-\int_{x_1}^{x}\frac{dx}{v}\right]\right]-\int_{-\infty}^{t}d\tau\,W_0(\tau)\right\}\right] . \qquad (4.1)$$

In the region near x_1, where $W(x,t)=W_0(t)$ is independent of x, this function can be expanded into a Fourier integral of elementary solutions:

$$\psi(x,t)=v^{-1/2}\int_{-\infty}^{\infty}d\lambda\,C(\lambda)\exp\left\{\left[-Et+m\int_{x_1}^{x}v\,dx+\lambda\left[\int_{x_1}^{x}\frac{dx}{v}-t\right]-\int_{-\infty}^{t}d\tau\,W_0(\tau)\right]\right\} . \qquad (4.2)$$

In the approximation considered, we suppose that the values of λ, contributing to integral (4.2), are small compared with the electron kinetic energy $mv^2/2$.

Let $T_0(E)$ be the transmission amplitude of the device for the electron with energy E in the stationary case [i.e., for $W(x,t)\equiv 0$]. Because of potential $W(x,t)$ independence of x in the device region, the integrand in Eq. (4.2) can easily be continued into the region beyond the device, so that the amplitude (2.3) in point x_2 is

$$T(x_2,t)=\int_{-\infty}^{\infty}d\lambda\,C(\lambda)T_0(E+\lambda)\exp\left[\frac{i}{\hbar}(-\lambda t)\right] . \qquad (4.3)$$

Finding $C(\lambda)$ by the inversion of Eq. (4.2) and substituting it into Eq. (4.3), we obtain the result

$$T(x_2,t)=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}d\lambda\, T_0(E+\lambda)\times\int_{-\infty}^{\infty}d\mu\,\Lambda(\mu,t)\times\exp\left[\frac{i}{\hbar}[-\lambda(t+\mu)]\right],$$

$$\Lambda(\mu,t)=\exp\left[\frac{i}{\hbar}\left[-\int_{-\infty}^{0}d\tau[W(x(\tau),\tau-\mu)-W_0(\tau-\mu)]-\int_{-\infty}^{t}d\tau\, W_0(\tau)\right]\right]. \quad (4.4)$$

Here function $x(t)$ is defined by Eq. (3.3).

B. Simple barrier case

For the simple barrier, we have in the approximation considered

$$T_0(E+\lambda)=T_0(E)\exp\left[\frac{\lambda\tau_0}{\hbar}\right]. \quad (4.5)$$

One can make oneself sure that the result (3.12) might be also derived from Eqs. (4.4) and (4.5). To prove this let us formally represent the function $\Lambda(\mu,t)$ in Eq. (4.4) as the Fourier transformation

$$\Lambda(\mu,t)=\int d\omega\exp\left[\frac{i}{\hbar}(\omega\mu)\right]\mathcal{L}(\omega,\mu). \quad (4.6)$$

This representation allows one to continue the function $T(x_2,t)$ into the complex plane of time. Actually, substituting Eqs. (4.5) and (4.6) into Eq. (4.4), we have

$$T(x_2,t)=T_0(E)\int d\omega\exp\left[\frac{i}{\hbar}[-\omega(t+i\tau_0)]\right]\mathcal{L}(\omega,t)=T_0(E)\Lambda(-t-i\tau_0,t). \quad (4.7)$$

The imaginary part of the logarithm of the right part of Eq. (4.7) times 2 gives the result (3.12).

C. Double-barrier structure

Resonant tunneling through a double-barrier structure is a problem that has caused many investigations both in theory and experiment (see Refs. 17 and 16 for a review). In Refs. 5–14 a photon-assisted resonant tunneling was considered, i.e., the case with $W(x,t)=W_0(x)\cos\omega t$. It is interesting to study the influence of the potential $W(x,t)$, having another dependence on t, on the resonant-tunneling process.

Assuming that an incident electron has energy E close to the resonant level E_0 in the quantum well, let us use the Breit-Wigner approximation for the stationary transmission amplitude:

$$T_0(E)=\frac{\Gamma_1\Gamma_2}{(E-E_0)+(i/2)(\Gamma_1+\Gamma_2)}, \quad (4.8)$$

where Γ_1 and Γ_2 are the partial widths of decay of the level E_0 through the left and right barriers. Substituting (4.8) into (4.4), we have

$$T(x_2,t)=-\frac{i}{\hbar}(\Gamma_1\Gamma_2)^{1/2}\int_{-\infty}^{t}d\mu\exp\left\{\frac{i}{\hbar}\left[-\left[(E-E_0)+\frac{i}{2}(\Gamma_1+\Gamma_2)\right](\mu-t)-\int_{-\infty}^{0}d\tau[W(x(\tau),\tau+\mu)-W_0(\tau+\mu)]-\int_{-\infty}^{t}d\tau\, W_0(\tau)\right]\right\}. \quad (4.9)$$

The value $-\infty$ in the last two integrals is not principal here. The integral representation of the amplitude, similar to (4.9), was found earlier in Ref. 5.

D. Examples: switching of the constant increment W and the increment having the linear dependence of time

Assume that the constant potential W_0 is switched on at the moment $t=0$:

$$W(x,t)=W_0\Theta(x-x_1)\Theta(t). \quad (4.10)$$

Then, according to (4.9), we have

$$T(x_2,t)=(\Gamma_1\Gamma_2)^{1/2}\left\{\frac{\exp\left[\frac{i}{\hbar}\{[E-E_0+\frac{i}{2}(\Gamma_1+\Gamma_2)]t\Theta(t)\}\right]}{E-E_0+\frac{i}{2}(\Gamma_1+\Gamma_2)}+\frac{1-\exp\left[\frac{i}{\hbar}\{[E-E_0-W_0+\frac{i}{2}(\Gamma_1+\Gamma_2)]t\Theta(t)\}\right]}{E-E_0-W_0+\frac{i}{2}(\Gamma_1+\Gamma_2)}\right\}. \quad (4.11)$$

For $t<0$ the amplitude $T(x_2,t)$ has the stationary form (4.8). For large $t\gg\hbar/(\Gamma_1+\Gamma_2)$, the amplitude becomes stationary again and has the form (4.8) with $E+W_0$ substituted for E_0. In the intermediate region $t\sim\hbar(\Gamma_1+\Gamma_2)$, the amplitude, and hence the outgoing flux $\mathcal{J}(t)=|T(x_2,t)|^2$, is an oscillatory function of time (see Fig. 8).

Now let $W(x,t)$ be a linear function of t switching on at moment $t=0$:

$$W(t,x)=\Omega_0 t\Theta(x-x_1)\Theta(t) \ . \tag{4.12}$$

Calculation by form (4.9) gives

$$\begin{aligned} T(x_2,t)&=(\Gamma_1\Gamma_2)^{1/2}\exp\left\{\frac{i}{\hbar}\left[\left[E-E_0+\frac{i}{2}(\Gamma_1+\Gamma_2)\right]t\Theta(t)\right]\right\}\left[\frac{1}{E-E_0+(i/2)(\Gamma_1+\Gamma_2)}-D(t)\Theta(t)\right] ,\\ D(t)&=\frac{i}{\hbar}\int_0^t d\tau\exp\left\{\frac{i}{\hbar}\left[-\left[E-E_0+\frac{i}{2}(\Gamma_1+\Gamma_2)\right]\tau+\frac{1}{2}\Omega_0(\tau^2-t^2)\right]\right\} . \end{aligned} \tag{4.13}$$

The integral $D(t)$ can be simply expressed via the error function of complex variable. We will not write here this expression, but only consider some limiting cases.

Assume that $(\hbar\Omega_0)^{1/2}/(\Gamma_1+\Gamma_2)\ll 1$, i.e., that $W(x,t)$ is an adiabatically slow function of time. Then

$$T(x_2,t)=\frac{(\Gamma_1\Gamma_2)^{1/2}}{E-E_0-\Omega_0 t+(i/2)(\Gamma_1+\Gamma_2)} \tag{4.14}$$

and has a usual adiabatic form.

In the opposite case $(\hbar\Omega_0)^{1/2}/(\Gamma_1+\Gamma_2)\gg 1$, we have

$$\begin{aligned} D(t)&=i\left[\frac{\pi}{\Omega_0\hbar}\right]^{1/2}[C(z)+S(z)] ,\\ z&=\left[\frac{\Omega_0}{\pi\hbar}\right]^{1/2}t , \end{aligned} \tag{4.15}$$

where $C(z)$ and $S(z)$ are the Fresnel integrals. In this case the function $D(t)$ is small compared with the previous item in square brackets in Eq. (4.13). According to form (4.13) and (4.15) for $t\sim(\hbar/\Omega_0)^{1/2}$, the flux $\mathcal{J}(t)=|T(x_2,t)|^2$ has small oscillations vanishing in the interval $t\sim(\hbar/\Omega_0)^{1/2}$.

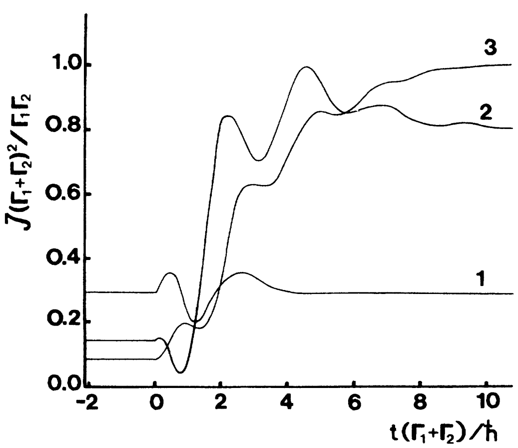

FIG. 8. Outgoing flux as a function of $t(\Gamma_1+\Gamma_2)\hbar$ for a double-quantum-well resonant-tunneling structure and $W(x,t)=W_0\Theta(x-x_1)\Theta(t)$. (1) $E-E_0=0.75(\Gamma_1+\Gamma_2)$, $W_0=1.5(\Gamma_1+\Gamma_2)$; (2) $E-E_0=1.25(\Gamma_1+\Gamma_2)$, $W_0=1.5(\Gamma_1+\Gamma_2)$; and (3) $E-E_0=1.5(\Gamma_1+\Gamma_2)$ $W_0=1.5(\Gamma_1+\Gamma_2)$.

V. CHARGE ACCUMULATION DURING RESONANT TUNNELING

As an application of form (4.9), let us consider the resonant-tunneling process taking into account the charge accumulation in the quantum well.[20–25,17] We will consider below the monoenergetic incident electrons.

Taking the derivative of Eq. (4.9) with respect to t, we have

$$\begin{aligned} i\hbar\frac{dT}{dt}+\left[E-E_0-W_0(t)+\frac{i}{2}(\Gamma_1+\Gamma_2)\right]T\\ =(\Gamma_1\Gamma_2)^{1/2}\exp\left[\frac{i}{\hbar}\left[\int_{-\infty}^{0}d\tau\, W(x(\tau),\tau+t)\right]\right] . \end{aligned} \tag{5.1}$$

To estimate the exponent in (5.1), assume that the characteristic value of W is 0.05 eV and that the characteristic distance in front of the barrier, where W is important (i.e., where $W\sim W_0$), is 100 Å. Then, for a typical electron velocity $v\sim 0.3$ a.u. $\approx 0.6\times 10^8$ cm/s the exponent has an order of unity. It is clear now that under different conditions the exponent can be small or comparable with unity. When it is small (for small W), Eq. (5.1) is simplified:

$$i\hbar\frac{dT}{dt}+\left[E-E_0-W_0(t)+\frac{i}{2}(\Gamma_1+\Gamma_2)\right]T=(\Gamma_1\Gamma_2)^{1/2} . \tag{5.2}$$

According to general estimates,[20] the potential increment $W(x,t)$, caused by charge accumulation in the quantum well, can rise to an order of $1/a$ (in atomic units), where a is the width of the quantum well. For $a\sim 100$ Å, that means $W\lesssim 0.1$ eV.

For the monoenergetic incident electrons, the charge density is simply proportional to the squared absolute value of the wave function, $|\psi(x,t)|^2$, localized in a quantum well. Introducing the total charge $Q(t)\sim\int|\psi(x,t)|^2dx$ in the well, we can find in the approximation considered that $Q(t)\sim|T(x_2,t)|^2$. The latter makes reasonable the following model of $W(x,t)$:

$$\begin{aligned} W(x,t)&=\Omega(x)|T(x_2,t)|^2 ,\\ W_0(t)&=\Omega_0|T(x_2,t)|^2 . \end{aligned} \tag{5.3}$$

Particular cases of this model were used earlier in Refs. 21 and 24. In our case the function $\Omega(x)$ is constant Ω_0 inside the structure. Such an approximation seems to be rather satisfactory because of the long-range character of the Coulomb interaction.

Substituting Eq. (5.3) into Eq. (5.2), we obtain the equation for the transmission amplitude found in Ref. 21. In order to obtain it, the authors[21] used the adiabatic approximation for the wave function and assumed that the potential depends on time inside the quantum well only. Our deduction of Eq. (5.2) shows that it is valid under the broader conditions that were assumed in Ref. 21. Another conclusion from Eq. (5.1) is that the exponent in Eq. (5.1) is not small in general.

Let us derive the conditions of stability of the stationary solution of Eqs. (5.1) and (5.3). For the stationary process, the function $W(x,t)$ is independent of time. Setting $T(t)\equiv T_0=\text{const}$, we find from (4.14) that

$$T_0=\frac{(\Gamma_1\Gamma_2)^{1/2}}{E-E_0-\Omega_0|T_0|^2+(i/2)(\Gamma_1+\Gamma_2)} \times\exp\left[\frac{i}{\hbar}\left[-|T_0|^2\int_{-\infty}^{0}d\tau\,\Omega(x(\tau))\right]\right] \tag{5.4}$$

and

$$\mathcal{J}_0=|T_0|^2=\frac{\Gamma_1\Gamma_2}{(E-E_0-\Omega_0\mathcal{J}_0)^2+\frac{1}{4}(\Gamma_1+\Gamma_2)^2} . \tag{5.5}$$

The value of $\mathcal{J}_0$ can be defined by the cubic equation (5.5), and then the phase of T_0 is simply defined by Eq. (5.4). Note that the stationary flux (5.5) does not depend on the value of potential $W(x,t)$ in front of the structure.

In order to find the conditions of stability of the stationary solution $T(t)\equiv T_0$, we put

$$T(t)=T_0+T_1(t) , \tag{5.6}$$

with small deviation $T_1(t)$. Then the flux

$$\mathcal{J}(t)=|T(t)|^2=\mathcal{J}_0+2\,\mathrm{Re}[T_0T_1(t)] .$$

It is convenient to define the increments

$$\mathcal{J}_1=2\,\mathrm{Re}[T_0T_1(t)],\quad \mathcal{K}_1=2\,\mathrm{Im}[T_0T_1(t)] . \tag{5.7}$$

According to (5.1) and (5.3), these functions satisfy the linear set of equations

$$\begin{aligned}&\hbar\frac{d\mathcal{J}_1}{dt}+\frac{1}{2}(\Gamma_1+\Gamma_2)\mathcal{J}_1+\frac{2}{\hbar}\mathcal{J}_0(E-E_0-\Omega_0\mathcal{J}_0)\int_{-\infty}^{0}d\tau\,\Omega(x(\tau))\mathcal{J}_1(\tau+t)+(E-E_0-\Omega_0\mathcal{J}_0)\mathcal{K}_1=0 ,\\ &\hbar\frac{d\mathcal{K}_1}{dt}-(E-E_0-3\Omega_0\mathcal{J}_0)\mathcal{J}_1+\frac{1}{\hbar}\mathcal{J}_0(\Gamma_1+\Gamma_2)\int_{-\infty}^{0}d\tau\,\Omega(x(\tau))\mathcal{J}_1(\tau+t)+\frac{1}{2}(\Gamma_1+\Gamma_2)\mathcal{K}_1=0 .\end{aligned} \tag{5.8}$$

Seeking $\mathcal{J}_1$ and $\mathcal{K}_1$ in the form $c_{j,k}\exp(\nu t)$, we find from (5.8) the following equation for ν:

$$\hbar^2\nu^2+\hbar\nu[\Gamma_1+\Gamma_2+2\mathcal{J}_0(E-E_0-\Omega_0\mathcal{J}_0)\lambda(\nu)]+\tfrac{1}{4}(\Gamma_1+\Gamma_2)^2+(E-E_0-\Omega_0\mathcal{J}_0)(E-E_0-3\Omega_0\mathcal{J}_0)=0 , \tag{5.9}$$

with

$$\lambda(\nu)=\frac{1}{\hbar}\int_{-\infty}^{0}d\tau\,\Omega(x(\tau))e^{\nu\tau} . \tag{5.10}$$

The roots of Eq. (5.9) define the characteristic inverse times of the system. If $\mathrm{Re}\nu<0$, then the stationary solution T_0 is stable.

For resonant tunneling the time ν^{-1} is usually large compared with the characteristic time τ_c of $\Omega(x(\tau))$. In this case

$$\lambda=\lambda_0=\frac{1}{\hbar}\int_{-\infty}^{0}d\tau\,\Omega(x(\tau)) =\frac{1}{\hbar}\int_{-\infty}^{0}\frac{dx}{v}\Omega(x)\quad(\nu^{-1}\gg\tau_c) \tag{5.11}$$

is independent of ν. The conditions of stability of the stationary amplitude (5.4) then looks as follows:

$$\begin{aligned}&\tfrac{1}{4}(\Gamma_1+\Gamma_2)^2+(E-E_0-\Omega_0\mathcal{J}_0)(E-E_0-3\Omega_0\mathcal{J}_0)>0 ,\\ &\Gamma_1+\Gamma_2+2\lambda_0\mathcal{J}_0(E-E_0-\Omega_0\mathcal{J}_0)>0 .\end{aligned} \tag{5.12}$$

The first of these inequalities, found in Ref. 21, excludes from the $\mathcal{J}_0(E-E_0)$ curve in Fig. 9 the region between the infinities of $d\mathcal{J}_0/dE$ marked by circles. The second inequality holds everywhere if

$$\lambda_0<2/\mathcal{J}_{\max},\quad \mathcal{J}_{\max}=\frac{4\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^2} . \tag{5.13}$$

This inequality is right for $\lambda_0<2$ because $\mathcal{J}_{\max}\le 1$. According to (5.11), for $\nu^{-1}\gg\tau_c$, we can estimate λ_0 by $\Omega_0\tau_c/\hbar$. Then, setting $\hbar\nu\sim\Gamma_1+\Gamma_2$, we obtain that $\lambda_0\ll\Omega_0/(\Gamma_1+\Gamma_2)$. The latter means that for $\Omega_0\sim\Gamma_1+\Gamma_2$ we have $\lambda_0\ll 1$ and the second inequality in (5.12) holds for all E. The value λ_0 can have an order of unity if $\Omega_0\gg\Gamma_1+\Gamma_2$. Then, for $\lambda_0>2/\mathcal{J}_{\max}$, the second inequality in (5.12) defined the region of instability:

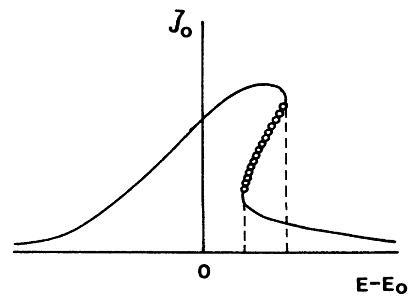

FIG. 9. Typical $\mathcal{J}_0(E-E_0)$ dependence found from (5.5). The region of instability defined by the first inequality in (5.12) is marked by circles.

$$E-E_0-\Omega_0\mathcal{I}_0<0,\quad \mathcal{I}_0^{(1)}<\mathcal{I}_0<\mathcal{I}_0^{(2)},$$
$$\mathcal{I}_0^{(1,2)}=\tfrac{1}{2}\mathcal{I}_{\max}\mp[\tfrac{1}{4}\mathcal{I}_{\max}^2-\lambda_0^{-4}]^{1/2}. \quad (5.14)$$

For λ_0 not too large, this region is covered by the one defined by the first inequality in (5.12). But for very large λ_0, this region covers almost all parts of the $\mathcal{I}_0(E_0-E)$ curve with negative differential conductance.

Suppose now that $\Omega_0(x)$ is extremely long ranged, so that τ_c is large compared with ν.[32] Then (5.11) gives

$$\lambda(\nu)=\Omega_0/\hbar\nu\quad(\nu^{-1}\ll\tau_c). \quad (5.15)$$

Substituting this into (5.9), it is easy to verify that all the $\mathcal{I}_0(E_0-E)$ curve is stable in this case.

The conditions indicated above define the region where the stationary solution for the transmission amplitude exists. If they fail, then the outgoing flux cannot be stationary and oscillates. The particular cases of such oscillations was studied in Refs. 21, 24, and 25.

VI. DISCUSSION

We would like to make some additional comments on the results obtained.

(1) The most important conclusion from the consideration of the simple barrier case in Sec. III is the fact that in the nonadiabatic case the form of the outgoing wave packet may have no similarity to the form of the applied potential pulse. We had shown that the outgoing wave packet may have peaks related to the poles of the pulse in the complex plane of time. Choosing the applied pulse having, e.g., the form

$$\sum_k \frac{\Omega_k(x)}{\gamma_k^4 t^4+1} \quad (6.1)$$

(note that this pulse, as a function of t, has one maximum only), we can obtain the outgoing wave packet with a number of peaks corresponding to complex poles of (6.1).

Note that the anomalous growth of the transmission probability can be obtained only for relatively small time-dependent pertubations of potential. The exponential smallness of the transmission probability remains under such a pertubation, but the latter can change the outgoing flux by an order of its magnitude. Exact calculations made without any pertubation theory will never give, of course, any singularity in the transmission amplitude for a smooth and finite, for t real, potential. Nevertheless, such calculations must confirm the appearance of peaks predicted by the above theory. A similar effect of the double-exponential growth of the transmission probability found analytically in Refs. 2 and 3 was confirmed by computer numerical calculations in Ref. 4.

(2) It is interesting to generalize the results obtained in Sec. IV to the case of complicated multidimensional tunneling devices[26] in alternating fields. It seems to be not difficult to make such a generalization for the case of the time-dependent increment of the potential that is independent of electron coordinates in the region of the device, similar to the treatment of Sec. IV.

If there is more than one quantum well (dot) in the structure, then the mutual influence of the charge accumulating in each quantum well must be taken into account. The latter might be used for modeling a type of resonant-tunneling nanometer device based on the charge-accumulation process.

(3) Above we restricted ourselves to the consideration of one-dimensional problems. There is intrinsic interest in these one-dimensional structures. Another point is that the result obtained can be generalized by taking into account the contribution of electrons having different initial momenta and energies. In this case the model (5.3) for the accumulated charge must also include a sum over the momenta and energies of incident electrons.

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[27]Here and below we omit the normalizing factor of a real electron.
[28]To find the x dependence of the flux, we must substitute $t-\int_{x_2}^{x} dx/v$ for t in (3.1).
[29]Here and in (3.21) we define $\ln(z)$ for complex $z=x+iy$ as $\ln|z|+i\arctan(y/x)$.
[30]For example, $W(x,t)=\Omega(x)[a+b\cos(\omega t)]^{-1}$ or something like it.
[31]For the resonant-tunneling device, this condition must be specified (Ref. 14).
[32]We mean, e.g., the situation that can appear for a three- or two-dimensional medium having one-dimensional conductance. If the resonant states ae closely disposed in a plane perpendicular to the direction of the conductance, then the interaction between the tunneling electrons and charge accumulating in the plane has a long-ranged character with the length defined by the screening properties of the medium.