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# Kramers degeneracy and quantum jumps in the persistent current of disordered metal rings

V. E. Kravtsov*
*Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg, and Max-Planck Institut für Kernphysik, 6900 Heidelberg, Germany*

M. R. Zirnbauer
*Institut für Theoretische Physik, Universität zu Köln, 5000 Köln, Germany*
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The ground state of any time-reversal-invariant system of an odd number of electrons moving in a disordered normal-metal ring is doubly degenerate by Kramers' theorem. We argue that the degenerate ground state is current carrying if the electrons experience scattering by a spin-orbit field. The insertion of an Aharonov-Bohm flux $\phi$ lifts the degeneracy, thereby causing the zero-temperature persistent current to vary discontinuously with $\phi$ at $\phi=0$. This conclusion holds true irrespective of the presence of electron-electron interactions and potential scattering by impurities.

When a Hamiltonian system does not have a unique ground state but possesses a continuous family of degenerate ground states instead, a number of interesting phenomena take place: the state of the system is determined by history (or preparation); there exists some observable, called the "order parameter," that depends nonanalytically on the external field which it couples to; and there may exist some spontaneously broken symmetry.[1] The paradigm of such behavior are ferromagnets, where magnetization is a nonanalytic function of the applied magnetic field, and rotational symmetry is spontaneously broken. While spontaneous breakdown of symmetry is found exclusively in systems with an infinite number of degrees of freedom, the other features, namely, a continuous family of degenerate ground states and nonanalytic dependence of the order parameter on the symmetry-breaking field, may occur in finite quantum systems as well. An example thereof is presented in this paper.

One of the exciting recent developments in mesoscopic physics is the demonstration by Levy *et al.*[2] that persistent currents in isolated disordered normal-metal rings, whose existence had been predicted by Büttiker, Imry, and Landauer[3] in 1983, are observable under currently available experimental conditions. Three prerequisites are necessary in order for persistent currents to exist: the multiply connected topology of a ring, phase coherence of the electron wave function over distances of the order of the circumference of the ring, and an Aharonov-Bohm flux threading the ring. Although the phenomenon is thought to be qualitatively understood, it has proved difficult[4-6] to explain the unexpectedly large magnitude of the persistent current observed in the experiment of Ref. 2. According to current opinion, which has received further substance by the very recent experiment of Chandrasekhar *et al.*,[7] a quantitative understanding will come from careful analysis of the screened Coulomb interaction between the electrons, first discussed by Ambegaokar and Eckern[8] in this context.

The present paper is not concerned with the size of the experimentally observed persistent current, but focuses on its qualitative behavior for zero temperature and small values of the Aharonov-Bohm flux $\phi \ll \phi_0 = hc/e$. To establish the context, let us first review what is known[9] about this behavior for *noninteracting* electrons in an *ideal* ring. Assuming the ring to be rotationally symmetric, one has that angular momentum is a good quantum number. At zero flux, (almost) every energy level for a single electron is doubly degenerate, since the energy does not change when the sign of the angular momentum is reversed. The flux couples to the orbital current, which is proportional to angular momentum, and therefore the degenerate levels are split when $\phi$ is changed from zero to some small value. To obtain the ground state of the noninteracting system, one successively fills the single-electron levels up to the Fermi energy $\epsilon_F$. Because of the occurrence and removal of degeneracy, the resulting ground-state energy $E(\phi)$ has a cusp at $\phi=0$, when the total particle number is such that the pair of crossing single-particle levels at $\epsilon_F$ is occupied by exactly one electron. Computing the total current $I(\phi)$ from the relation $I(\phi) = -dE(\phi)/d\phi$, one sees that $I(\phi)$ depends discontinuously on $\phi$ at $\phi=0$. The size of the jump is determined by the single-level current at $\epsilon_F$.

An essential, and unrealistic, ingredient to the above argument is rotational symmetry of the ring. The argument in fact collapses when rotational symmetry is broken by the introduction of an impurity potential. This leads to *avoided* level crossings at zero flux, so that the cusp in the ground-state energy is smoothed and current response to flux becomes linear.[9] So far, the spin degree of freedom of the electron has been neglected. We shall now proceed to show that the inclusion of spin has a significant effect, provided that time-reversal-invariant interactions involving the spin, such as scattering by a spin-orbit field, are present.

Consider the most general case of disordered system of $N$ interacting electrons moving in the multiply connected geometry of a ring. For such a system, as in fact for any system of $N$ spin-$\frac{1}{2}$ particles, two applications of the time-reversal operator $\mathcal{T}$ result in $\mathcal{T}^2\psi = (-1)^N\psi$ for an arbitrary state $\psi$. Now set $\phi=0$ and assume the Hamiltonian $H$ for zero flux to be invariant under time reversal.

 

Then, if $\psi$ is an eigenstate of $H$ with energy $E$, so is the time-reversed state $\mathcal{T}\psi$. Moreover, for odd $N$ the two states $\psi$ and $\mathcal{T}\psi$ must be linearly independent, since any linear relation $\mathcal{T}\psi=\lambda\psi$ ($\lambda\in\mathbf{C}$) would be inconsistent with $\mathcal{T}^2\psi=-\psi$.[10] The statement we have just proven is known as Kramers' theorem[11] and can be found in standard textbooks on quantum mechanics.[12]

Given a Kramers pair $\psi$ and $\mathcal{T}\psi$ of degenerate energy eigenstates with an odd number of electrons, consider the matrix $j$ of the (orbital) current operator $J$ in the two-dimensional subspace spanned by this pair. Every current in physics is required to be odd under time reversal. Therefore,

$$-(\psi,J\psi)=(\psi,\mathcal{T}^{-1}J\mathcal{T}\psi)=(\mathcal{T}\psi,J\mathcal{T}\psi)^* , \qquad (1)$$

the second equality sign being a consequence of the antiunitarity[13] of $\mathcal{T}$. Equation (1) in conjunction with the Hermitecity of $J$ yields for the matrix elements of the $2\times2$ matrix $j$ the relations $j_{11}=-j_{22}\in\mathbf{R}$ and $j_{12}^*=j_{21}$. These are expressed most succinctly by writing

$$j=\sum_{k=1}^{3} j_k\sigma_k , \qquad (2)$$

where $\sigma_k$ are the Pauli matrices and $j_k$ ($k=1, 2, 3$) are real numbers. There exist no further constraints on $j$ from general symmetry principles beyond those stated and, in particular, there is no cogent reason why $j$ should be zero. (Note, however, that in a *singly connected* isolated system $j=0$ would be in conflict with current conservation.) Therefore, *$j$ does not vanish,* in general.

Now, perturb the system by introducing a small Aharonov-Bohm flux. To calculate the resulting effect on the degenerate energies of a pair of ground states $\psi$ and $\mathcal{T}\psi$, use singular perturbation theory and diagonalize the perturbation in the two-dimensional subspace spanned by $\psi$ and $\mathcal{T}\psi$. The perturbation (equal to current operator times flux) is represented by the $2\times2$ matrix $j\phi$ in this subspace. Diagonalization gives the eigenvalues $\pm\phi[\mathrm{tr}(j^2/2)]^{1/2}$, which are different from zero unless $j$ vanishes identically. We thus see that the Kramers degeneracy of the ground state is lifted by flux, and the situation is qualitatively similar to that for noninteracting electrons in an ideal ring. Hence, by the same reasoning as before, the persistent current for zero temperature depends discontinuously on $\phi$ at $\phi=0$:

$$I(\phi)=[\mathrm{tr}(j^2/2)]^{1/2}\mathrm{sgn}(\phi) . \qquad (3)$$

Let it be clearly stated how all this ties up with our introductory remarks: There exists a two-dimensional manifold (Riemann sphere) of degenerate ground states $\psi^\lambda$ given by $\psi^\lambda=\lambda\psi+\mathcal{T}\psi$ ($\lambda\in\mathbf{C}$); the symmetry-breaking field is the Aharonov-Bohm flux; and nonanalyticity occurs in the order parameter, which is the persistent current.

We wish to stress the generality of our conclusion (3), which was derived on the basis of two conditions only: (i) The Hamiltonian for $\phi=0$ must be *invariant under time reversal* but must possess *no additional symmetries,* and (ii) the total number of electrons must be *odd.* The second condition poses no limitations if instead of a single ring a statistical ensemble of rings is considered whose members contain an odd number of electrons with probability $\frac{1}{2}$. We emphasize, in particular, that the conclusion is affected neither by electron-electron interactions nor by impurity scattering, unless the impurities are of magnetic type.

To illustrate the role of additional symmetries, we shall now consider especially systems *with* disorder but *without* spin interactions. As is well known[9] and was mentioned earlier, the persistent current for such systems vanishes with $\phi$ in the limit $\phi\to0$. It is instructive to see how this follows from the above argument featuring the time-reversal operator $\mathcal{T}$. Neither an Aharonov-Bohm flux nor an impurity potential couples to spin. When the electron spin takes part in none of the other interactions either, there is perfect spin degeneracy. In such a situation we may just as well consider spinless electrons and multiply the result for the persistent current by a factor of 2 at the end. For spinless particles, however, $\mathcal{T}^2=+1$ and $\mathcal{T}$ coincides with complex conjugation when state vectors are represented by functions on position space. In the absence of flux, every eigenstate $\varphi$ of the time-reversal-invariant Hamiltonian $H$ can be turned into a "real" eigenstate of $H$ by the replacement $\varphi\to\varphi_1=\varphi+\mathcal{T}\varphi$. The state $\varphi_1$ so obtained is an eigenstate of $\mathcal{T}$ with eigenvalue $+1$. Let the ground state $\psi$ of $H$ be chosen accordingly, and let there be no additional symmetries, so that $\psi$ is nondegenerate (ignoring spin). Then the relation $\mathcal{T}\psi=\psi$ together with Eq. (1) forces the real number $(\psi,J\psi)$ to be zero, i.e., the ground state carries no current.

Of course, the crucial circumstance making the current for $\phi=0$ vanish by the above argument, is the possibility to choose $\mathcal{T}\psi=\psi$ in the absence of spin. We thus perceive the true reason why the current matrix $j$ is nonvanishing, and the persistent current discontinuous in $\phi$, under the conditions given earlier: It is impossible to arrange for a state with an odd number of spin-$\frac{1}{2}$ particles to be an eigenstate of time reversal.

With the novel phenomenon of quantum jumps in the persistent current of disordered metal rings now firmly established as an effect that exists in principle, various questions related to experimental observability arise. How strong a spin-orbit field is required? What can be said about the typical size and the distribution of current jumps in a statistical ensemble of rings? What are the effects of finite temperature? We will answer these questions in the order in which they are asked. A measure of the strength of the spin-orbit field is the inverse of the spin-orbit scattering time $\tau_{SO}$. We expect the relevant parameter to be $\tau_{SO}\Delta$ with $\Delta$ the single-electron level spacing, since this is known to determine the scale for crossover to the spin-orbit (or "symplectic") universality class for noninteracting systems.[14] According to what has been said, $\mathrm{tr}(j^2)$ vanishes for $\tau_{SO}\Delta\to\infty$, and we expect saturation to occur for $\tau_{SO}\Delta\ll1$.

To gather information about the statistical distribution of $\mathrm{tr}(j^2)$, we pick a cross section $S$ of the ring and compute the current matrix $j$ by integrating the corresponding current density $j(x)$ over $S$. (The choice of cross section is arbitrary by current conservation.) Let us assume the correlation length of $j(x)$ to be much smaller than the thickness of the ring.[15] Under this assumption, the current matrix $j$ [being the integral of $j(x)$ over $S$] is a sum of many independent random variables and each of its components $j_k$ ($k=1$, 2, 3), introduced in Eq. (2), is normally distributed by the action of the central limit theorem. The variance of the distribution is the same in each case because of the isotropic distribution of the spin-orbit field in a thick ring, which leads to isotropy in the two-dimensional manifold of degenerate ground states $\psi^\lambda$. Finally, changing variables to the "length" $I \stackrel{\text{def}}{=} [\mathrm{tr}(j^2/2)]^{1/2} = (j_1^2+j_2^2+j_3^2)^{1/2}$ and "direction" $j/I$ of the vector with components $(j_1, j_2, j_3)$, we obtain for $I$ the (normalized) probability density

$$P(I)dI = \frac{4I^2}{\sqrt{\pi} I_0^3} \exp\left(-\frac{I^2}{I_0^2}\right) dI , \tag{4}$$

where $I_0$ is the typical value of $I$. To deduce the dependence of $I_0$ on the number of "channels" $M \sim$ area ($S$), we observe that $j(x)$ scales with $M$ as $M^{-1}$, by the normalization of wave functions. $I_0^2$, being the variance of $\sqrt{2}I$, is proportional to the number of independent random variables adding up to $j$, and we conclude that $I_0$ scales as $M^{-1/2}$.

At finite temperature $T$, phase-breaking processes lower the coherence of the many-electron wave function, thereby reducing the dependence or $E(\phi)$ on $\phi$ and depressing the persistent current. Here we disregard such processes and consider only the consequences of replacing ground-state energy by the free energy $F = -k_B T \log \mathrm{tr} \exp(-H/k_B T)$. Omitting from the partition sum all terms except those coming from the pair of degenerate (at $\phi = 0$) ground-state energies $E_{\pm}(\phi) \simeq E(0) \pm I\phi$, and differentiating $F$ with respect to flux, we obtain instead of (3) the formula

$$I(\phi) = I \tanh(I\phi/k_B T) , \tag{5}$$

valid for $k_B T \ll \Delta$ and $\phi \ll \Delta/I_0$. Thus, the singular behavior of the persistent current is rounded off by finite temperature, but the typical slope of $I(\phi)$ at $\phi = 0$ diverges as $I_0^2/k_B T$ for $T \to 0$.

All of the above will now be further substantiated and illustrated by direct calculation of $I_0$ for the case where electron-electron interactions are absent. According to our general picture, there exits pairs of crossing energy levels whose splitting at $\phi \neq 0$ ($\phi$ small) is proportional to the current $I$. Clearly, this splitting must be visible in the flux-dependent energy-level correlation functions for a single electron. We are thus led to consider a problem in energy-level statistics.

It is convenient for our purposes to study Dyson's two-level correlation function $R_2(r)$, defined as the probability density for finding two single-electron energy levels at a distance $r\Delta$, regardless of the positions of other levels.[16] In formal terms, $R_2(r) = \Delta^2 \langle \nu(\epsilon_F) \nu(\epsilon_F + r\Delta) \rangle - \delta(r)$, where $\nu$ is the level density, and the subtraction of the $\delta$-function eliminates self-correlations of the levels. Efetov[14] has shown how to calculate $R_2$ for small metallic particles, using Berezin's theory of superintegration and the mapping onto a nonlinear $\sigma$ model. We here consider the limit $\tau_{SO}\Delta \ll 1$, where the system is in the symplectic universality class perturbed by magnetic flux, and we take $\phi$ to be small compared to $\phi_0$, so that the so-called zero-mode approximation[14] may be used. The latter reduces the expression for $R_2$ to an integral over a single supermatrix, which can be computed with the help of a variant of Efetov's method recently introduced by Iida; see Ref. 17. Having completed this computation, we realized that an equivalent problem, which in random-matrix terminology is called the crossover from Gaussian symplectic ensemble to Gaussian unitary ensemble, had already been solved completely by Mehta and Pandey[18] in 1983. After introduction of the proper physical units, our result for $R_2$, reassuringly, coincides with theirs. It is given by

$$R_2(r;\alpha) = 1 - \left(\frac{\sin \pi r}{\pi r}\right)^2 + \int_1^\infty dx\, x \sin(\pi r x) e^{-\alpha^2 x^2} \int_0^1 \frac{dy}{y} \sin(\pi r y) e^{\alpha^2 y^2} . \tag{6}$$

Here $\alpha = (2\pi E_c/\Delta)^{1/2} \phi/\phi_0$, with $E_c = \hbar D (2\pi/L)^2$ the Thouless energy, $D$ the diffusion constant, and $L$ the circumference of the ring.

The integral over $x$ in Eq. (6) is *nonanalytic* in $\alpha$ at $\alpha = 0$. Asymptotic evaluation for $\alpha \ll 1$ gives

$$R_w(r;\alpha) = R_2^{\mathrm{symp}}(r) + \frac{\pi^{5/2} r^2}{4\alpha^3} \exp\left(-\frac{\pi^2 r^2}{4\alpha^2}\right) , \tag{7}$$

where $R_2^{\mathrm{symp}}$ is Dyson's two-level correlation function for the Gaussian symplectic ensemble,[19] with (nonstandard) normalization $\int_0^\infty [1 - R_2^{\mathrm{symp}}(r)] dr = 1$. The $\alpha$-dependent term in (7) contracts to $\delta(r)/2$ in the limit $\alpha \to 0$, and it is recognized as the peaklike structure that we expect to be present in $R_2$ because of the splitting of degenerate energy levels by the dimensionless flux $\alpha$. From (7) and the relation

$$P(I)dI = 2 \lim_{\phi \to 0} R_2\left(\frac{\phi I}{\Delta}; \alpha(\phi)\right) \frac{\phi}{\Delta} dI , \tag{8}$$

we see that the distribution function $P(I)$ for the single-level current $I$ has the form (4), with the typical single-level current $I_0$ given by

$$I_0 = \frac{2}{\phi_0} \left(\frac{2E_c\Delta}{\pi}\right)^{1/2} . \tag{9}$$

Averaging Eq. (5) over the distribution (4), and using (9), we obtain the following linear response of the persistent current to flux $\phi$:

$$\langle I(\phi) \rangle = \frac{12 E_c \Delta}{\pi k_B T \phi_0^2} \phi . \tag{10}$$

The result (9) for $I_0$ is easy to understand. We argued earlier that $I_0$ scales with the number $M$ of channels as $M^{-1/2}$. In the limit under consideration, i.e., for $\tau_{SO}\Delta \ll 1$ and $\phi \to 0$, there exist no other energy scales beyond $E_c$ and $\Delta$ on which $I_0$ may depend. The only function of these parameters with the physical dimension of a current and the correct dependence on $M$ is $(E_c M)^{1/2}/\phi_0$.

Recall that all of our results apply to systems with an odd number of electrons. For even electron number, where the effect of spin is less dramatic, we expect the analysis of Altland *et al.*[20] to be applicable. These authors consider systems in the orthogonal universality class perturbed by magnetic flux, and they obtain a zero-temperature linear response $\langle I(\phi)\rangle \sim E_c\phi/\phi_0^2$, which differs from (10) by the absence of a factor of $\Delta/k_BT$. Thus, there is a qualitative difference between even and odd particle number, suggesting the striking experimental possibility of discriminating, at temperatures $k_BT<\Delta$, a system with $10^7$ electrons from another one with $10^7+1$.

In summary, we have shown that any time-reversal-invariant *generic* Hamiltonian for an odd number of electrons moving in a disordered metal ring, possesses a two-parameter family of current-carrying degenerate ground states. The lifting of degeneracy by the insertion of an Aharonov-Bohm flux $\phi$ causes the zero-temperature persistent current to jump at $\phi=0$. This singular behavior is destroyed neither by impurity scattering nor by electron-electron interactions, but is rounded off by finite temperature only.

*Note added in proof.* A paper by O. Entin-Wohlman, Y. Gefen, Y. Meir, and Y. Oreg [Phys. Rev. B **45**, 11 890 (1992)] has recently appeared in print which discusses the drastic consequences of spin-orbit scattering and Kramers degeneracy for the paramagnetic susceptibility at zero flux.

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*Permanent address: Institut of Spectroscopy, Russian Academy of Sciences, 142092 Troitsk, Moscow Region, Russia.

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