Spin-flip relaxation time of quantum-well electrons in a strong magnetic field

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We present theoretical estimates of the spin-flip relaxation time of conduction electrons in a quantum well subjected to a strong magnetic field B. We find that the spin-flip relaxation time increases as $B^{1/2}$ if the scatterers are pointlike defects.

Several experimental and theoretical studies[1–11] have recently been devoted to the spin relaxation in semiconductor quantum wells. The experimental evidence, obtained by studying the relaxation of photogenerated spin-polarized electrons and holes, ranges over a wide variety of situations. In particular, it is not clear which species (electrons or holes) relaxes its spin more effectively in undoped materials. The problem is further complicated by strong excitonic effects, which may considerably distort the single-particle picture. High-field magnetoluminescence and magnetoluminescence excitation experiments have been interpreted in terms of an inhibition of the spin-flip scattering in a strong magnetic field.[3,4] The Landau-level spectrum of ideal quasi-two-dimensional electrons is discrete. However, it is macroscopically degenerate (in the sense that the δ-function-like singularities of the unperturbed Landau spectrum have a height which is proportional to the sample area, as a result of the degeneracy of each Landau level with respect to the center of the cyclotron orbit). When imperfections are taken into account the δ-function singularities are broadened. We shall be interested in a situation of weak disorder where there still exist peaks, of finite height and width, in the density of states of the imperfect material. These broadened peaks are still separated by energy gaps of the order of $\hbar\omega_c - \Gamma$, where $\hbar\omega_c$ is the cyclotron energy and Γ is a typical Landau-level broadening. Inside each Landau band one thus deals with a quasicontinuum. The finite separation between Landau bands clearly implies a slowing down of any kind of relaxation (spin conserving or spin flipping) which involves a change of the Landau-level index or the spin index when compared to the zero-field situation. The purpose of the present paper is to present a simplified model of the spin-relaxation time for quantum-well electrons subjected to a quantizing magnetic field and elastic scatterers. Thus, because of our weak disorder approximation, we shall only take into account collisions which conserve that Landau subband index. However, we shall allow changes of the spin sublevel, since the spin splitting is smaller or much smaller than the Landau splitting $\hbar\omega_c$ (unresolved spin subbands).

There are two kinds of difficulty in our problem. First, it is well known that a parabolic approximation of the hosts' dispersion relations leads in many quantum-well structures (say GaAs-like) to conduction-band states which are eigenstates of the spin operator σ. Since it is fair to assume that there are no localized magnetic moments in these materials (or at least a negligible amount) no relaxation mechanism can lead to spin flipping. Second, since the Landau-level spectrum is discrete, there is no way to evaluate an kind of relaxation time by using the regular Born approximation (since the density of final unperturbed states is either zero or infinite). Broadening has to be introduced and it has been known for some time that this is not an easy task.[12]

To introduce spin mixing in the conduction-band states of a quantum well (grown along the z direction) we use the lowest-order correction to the parabolic dispersion law, which is the anisotropic cubic term:[13,14]

$$H_{\rm SF} = \alpha \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}/2 \; , \tag{1}$$

where

$$\Omega_x = p_x(p_y^2 - p_z^2) \; , \quad \Omega_y = p_y(p_z^2 - p_x^2) \; , \quad \Omega_z = p_z(p_x^2 - p_y^x) \; , \tag{2}$$

$$\alpha = \alpha^*[m_c^{3/2}(2E_g)^{1/2}]^{-1} \; . \tag{3}$$

α^* is a dimensionless constant [0.07 in GaAs (Ref. 9)], $\boldsymbol{\sigma}$ the dimensionless Pauli spin matrices, and E_g is the band gap of the well material (GaAs in the case of GaAs-$Ga_{1-x}Al_xAs$ quantum wells). In a situation of pronounced two-dimensionality it is possible to simplify $H_{\rm SF}$ by replacing p_x and p_z^2 by their averages over the ground conduction bound state of the quantum well for the z motion. These averages are equal to 0 and $2m_cE_1$, respectively, where E_1 is the confinement energy of the ground state. Then $H_{\rm SF}$ becomes equal to[9]

$$H_{\rm SF} = \alpha^* E_1 (2m_cE_g)^{-1/2}(-p_x\sigma_x + p_y\sigma_y) \; . \tag{4}$$

Thus $H_{\rm SF}$ can be viewed as a Zeeman term due to a magnetic field $\mathbf{b}_{\rm SF}$ which lies in the layer plane and whose modulus squared $b_{\rm SF}^2$ is proportional to the electron in-plane kinetic energy. When an external magnetic field $\mathbf{B}$ is applied parallel to the z axis, p_x is left unchanged while p_y becomes $p_y + eBx/c$ in Eq. (4) if ones uses the Landau gauge [$\mathbf{A} = (0, Bx, 0)$]. It happens that the total in-plane Hamiltonian H_0,

$$H_0 = [p_x^2 + (p_y + eBx/c)^2]/2m_c + g^*\mu_B \boldsymbol{\sigma} \cdot \mathbf{B} + H_{\rm SF} \; , \tag{5}$$

can be exactly diagonalized on the $|n,\uparrow\rangle$, $|n,\downarrow\rangle$ basis of the H_0-H_{SF} eigenstates. This is because H_{SF} couples only $|n+1,\uparrow\rangle$ with $|n,\downarrow\rangle$. Specifically, on this 2×2 basis we find

$$H_0=\begin{matrix}\langle n,\downarrow| \\ \langle n+1,\uparrow|\end{matrix}\begin{bmatrix}\varepsilon_{n\downarrow} & i\beta(n+1)^{1/2} \\ -i\beta(n+1)^{1/2} & \varepsilon_{n+1\uparrow}\end{bmatrix}, \tag{6}$$

where

$$\beta=\alpha^* E_1(\hbar\omega_c/4E_g)^{1/2}, \tag{7}$$

$$\varepsilon_{n\downarrow}=(n+1/2)\hbar\omega_c-1/2g^*\mu_B B, \tag{8}$$

$$\varepsilon_{n\uparrow}=(n+1/2)\hbar\omega_c+1/2g^*\mu_B B. \tag{9}$$

In Eqs. (6)–(9) g^* is the Landé g factor of the conduction electron at the zeroth order in H_{SF} ($g^*=-0.44$ in GaAs). The eigenenergies of H_0 are therefore

$$\varepsilon_{n\pm}=(\varepsilon_{n+1\uparrow}+\varepsilon_{n\downarrow})/2\pm\{[(\varepsilon_{n+1\uparrow}-\varepsilon_{n\downarrow})/2]^2+(n+1)\beta^2\}^{1/2}, \tag{10}$$

while the normalized eigenstates will be written

$$\Psi_{n\pm}=a_\pm(n)|n,\downarrow\rangle+b_\pm(n)|(n+1),\uparrow\rangle. \tag{11}$$

β is always small in GaAs ($\beta/E_1\le5\times10^{-3}$). Thus the spin mixing is weak in this material. To a very good approximation ε_{n+} equals $\varepsilon_{n+1\uparrow}$ and ε_{n-} equals $\varepsilon_{n\downarrow}$. It can also be checked that the average of σ_z over the $\Psi_{n\pm}$ states is very close to $\pm\frac{1}{2}$ (the departure from the $\pm\frac{1}{2}$ value being at most a few percent). It is also worth noting that the $|0,\uparrow\rangle$ level is unadmixed with any other level. Thus the spin relaxation from the Ψ_{0-} level to the $|0,\uparrow\rangle$ level exists only to the extent that Ψ_{0-} contains a small component on $|1,\uparrow\rangle$. For the other spin deexcitations, i.e., the transitions from Ψ_{n-} to Ψ_{n-1+}, the relaxation is due to the spin mixing in the initial and final states. One may therefore anticipate that the spin relaxation in the lowest-lying pair of levels ($\Psi_{0-},\Psi_{-1+}\equiv|0,\uparrow\rangle$) will be the less effective. Also, it is clear that the spin relaxation in general will be faster for the narrower wells, where E_1, and thus β, are the larger.

To calculate the spin-flip relaxation time one still has to cope with the discrete nature of the Landau-level spectrum. Any calculation performed at the regular Born approximation provides meaningless results since the transition rate is either zero or infinite. We shall use an empirical model, reminiscent of the self-consistent Born approximation.[12] We shall write that the broadening Γ ($=\hbar/2\tau$) is given by the usual Fermi golden rule except that the δ function ensuring energy conservation is changed into a Lorentzian with a width parameter given by the total broadening in the final state. By summing up all the contributions to the broadening (spin conserving and spin flipping) we shall end up with self-consistent equations. In principle, these self-consistent equations are integral equations. For uncorrelated short-range scatterers, however, it is well known[12] that the self-consistent equations reduce to algebraic ones which, in the limit of strong magnetic field and weak scatterers ($\Gamma\ll\hbar\omega_c$), gives an analytical result to the self-consistent broadening. As applied to our particular situation, this procedure leads to the following set of equations:

$$\hbar/2\tau_{SF\pm}=\Gamma_{SF\pm}=|\langle\Psi_{n-1+}|V_{def}|\Psi_{n-}\rangle|^2(\Gamma_{SF\pm}+\Gamma_{SC\pm})\{[\varepsilon_-(n)-\varepsilon_+(n-1)]^2+(\Gamma_{SF\pm}+\Gamma_{SC\pm})^2\}^{-1}, \tag{12}$$

$$\hbar/2\tau_{SC+}=\Gamma_{SC+}=|\langle\Psi_{n-1+}|V_{def}|\Psi_{n-1+}\rangle|^2(\Gamma_{SF+}+\Gamma_{SC+})^{-1}, \tag{13}$$

$$\hbar/2\tau_{SC-}=\Gamma_{SC-}=|\langle\Psi_{n-}|V_{def}|\Psi_{n-}\rangle|^2(\Gamma_{SF-}+\Gamma_{SC-})^{-1}, \tag{14}$$

where the matrix elements involve an average over the random locations of the scatterers. In establishing Eqs. (12)–(14) we have assumed that $\Gamma_{SC},\Gamma_{SF}\ll\hbar\omega_c$ in order to truncate the infinite sum (over the Landau-level index of the final states) to a single term ($n=m$). Note that in the absence of spin mixing ($H_{SF}=0$, $\Gamma_{SF}=0$) Eqs. (12)–(14) reduce to the known self-consistent Born results.[12] The spin-flip matrix element is readily evaluated for short-range scatterers. We find

$$|\langle\Psi_{n-1+}|V_{def}|\Psi_{n-}\rangle|^2=[V^2N_{imp}/2\pi\lambda^2]\int dz\,\chi_1^4(z)[|a_-^*(n)a_+(n-1)|^2+|b_-^*(n)b_+(n-1)|^2], \tag{15}$$

where V is the strength of the δ-function scatterers, N_{imp} the volume concentration of defects, and $\chi_1(z)$ the envelope function of the E_1 state. This matrix element can be rewritten in terms of Γ_0, the zero-field broadening evaluated at the Born approximation between unadmixed spin states:

$$2\hbar^2\Gamma_0/m_c=V^2N_{imp}\int\chi_1^4(z)dz. \tag{16}$$

Finally

$$|\langle\Psi_{n-1+}|V_{def}|\Psi_{n-}\rangle|^2=(\Gamma_0\hbar\omega_c/\pi)[|a_-^*(n)a_+(n-1)|^2+|b_-^*(n)b_+(n-1)|^2]. \tag{17}$$

Along the same line we find for the spin-conserving matrix elements:

$$|\langle\Psi_{n-1+}|V_{def}|\Psi_{n-1+}\rangle|^2=(\Gamma_0\hbar\omega_c/\pi)[|a_+(n-1)|^4+|b_+(n-1)|^4], \tag{18}$$

$$|\langle\Psi_{n-}|V_{def}|\Psi_{n-}\rangle|^2=(\Gamma_0\hbar\omega_c/\pi)[|a_-(n)|^4+|b_-(n)|^4]. \tag{19}$$

Since the spin mixing is in fact weak in GaAs, the self-consistent equations (12)–(14) admit the approximate solution ($\Gamma_{SF}\ll\Gamma_{SC}$)

$$\Gamma_{SC+}=[\hbar\omega_c\Gamma_0/\pi]^{1/2}[|a_+(n-1)|^4+|b_+(n-1)|^4]^{1/2}\,, \tag{20}$$

$$\Gamma_{SC-}=[\hbar\omega_c\Gamma_0/\pi]^{1/2}[|a_-(n)|^4+|b_-(n)|^4]^{1/2}\,, \tag{21}$$

$$\Gamma_{SF-}=[\hbar\omega_c\Gamma_0/\pi]^{1/2}[|a_-^*(n)a_+(n-1)|^2+|b_-^*(n)b_+(n-1)|^2][|a_+(n-1)|^4+|b_+(n-1)|^4]^{1/2} \times[|a_+(n-1)|^4+|b_+(n-1)|^4+\pi[\varepsilon_-(n)-\varepsilon_+(n-1)]^2/(\hbar\omega_c\Gamma_0)]^{-1}\,, \tag{22}$$

$$\Gamma_{SF+}=[\hbar\omega_c\Gamma_0/\pi]^{1/2}[|a_-^*(n)a_+(n-1)|^2+|b_-^*(n)b_+(n-1)|^2][|a_-(n)|^4+|b_-(n)|^4]^{1/2} \times[|a_-(n)|^4+|b_-(n)|^4+\pi[\varepsilon_-(n)-\varepsilon_+(n-1)]^2/(\hbar\omega_c\Gamma_0)]^{-1}\,. \tag{23}$$

The numerical solutions of Eqs. (12)–(14) are very close to the approximate ones. Thus we shall discuss the general trend displayed by Eqs. (20)–(23). First, the spin-conserving and spin-flip broadening of the Ψ_{n-} and Ψ_{n-1+} states are almost identical. We also find that the spin-conserving broadenings $\Gamma_{SC\pm}$ are very close to the usual self-consistent Born results (i.e., $[\hbar\omega_c\Gamma_0/\pi]^{1/2}$ (Ref. 12)). The increase as $B^{1/2}$ and the independence of $\Gamma_{SC\pm}$ versus the Landau-level index n are characteristic of short-range scatterers.[12] The spin-flip broadenings display a similar $B^{1/2}$ dependence. However, it is dominated by the B dependence of the mixing term $|a_-^*(n)a_+(n-1)|^2+|b_-^*(n)b_+(n-1)|^2$. [Note that the B dependence of the denominator in Eqs. (22) and (23) is negligible in GaAs quantum wells because the g^* factor is so small. In other materials with larger g^* and if the zero-field broadening is small, the denominator could at large field increase linearly with B and contribute to a further decline of the spin-flip frequency.] The spin mixing term decreases linearly with increasing B. This may readily be understood on the basis that $a_-(n)\cong 1$ while $b_-(n)\cong\beta(n+1)^{1/2}/\hbar\omega_c \ll 1$. In addition, $a_+(n-1)\cong\beta n^{1/2}/\hbar\omega_c$ while $b_+(n)\cong 1$. Thus the mixing term is $\cong(2n+1)(\beta/\hbar\omega_c)^2\cong B^{-1}$ at large field. In summary, one has to expect a $B^{-1/2}$ decrease of the spin-flip broadening with increasing B. This is illustrated in Fig. 1 in the case of a 9-nm-thick GaAs quantum well. Although the precise power-law decline of the spin-flip relaxation frequency depends on the exact shape of the scattering potential, we believe that the ratio between spin-conserving and spin-flip transition rates should be rather shape independent, because it reflects a genuine band-structure effect. Note also that the spin relaxation becomes more effective with increasing Landau-level index. This feature is in qualitative agreement with recent magneto-optical data.[3,4,15] The calculated order of magnitude of the spin-flip broadening with GaAs-like parameters ($m_c=0.07m_0$, $g^*=-0.44$, $L=9$ nm) and the assumption of a good sample ($\Gamma_0=1$ meV) provides a spin-flip relaxation time in the $n=0$ Landau level of 1.4 and 2.3 ns at $B=6$ and 15 T, respectively. This is much longer than the recombination time (a few tenths of ns) and therefore provides a qualitative explanation of the experiments[3,4] which point out the existence of several luminescence lines which involve different spin sublevels in GaAs quantum wells subjected to a strong magnetic field. It is also worth pointing out that the spin-relaxation rate increases with decreasing quantum-well thickness (see Fig. 2). This is due to the linear increase of the coupling constant β with E_1 which leads to a quadratic increase of Γ_{SF} with E_1 (or roughly $\Gamma_{SF}\cong L^{-4}$). This effect is quantitatively important: at $B=6$ T and for $L=3$ nm, $n=0$ and $\Gamma_0=1$ meV, we calculate a spin-flip relaxation time of only 0.11 ns, which is comparable to the recombination time.

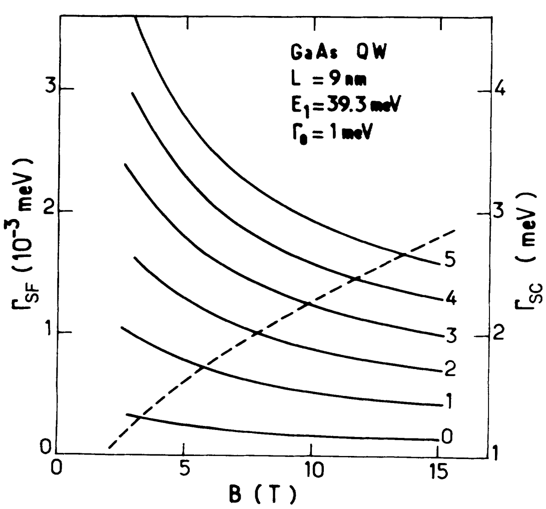

FIG. 1. The calculated spin-flip broadening Γ_{SF} is plotted vs the magnetic-field strength B in a 9-nm-thick GaAs quantum well for several Landau levels n (solid lines, left scale). The broadening due to spin-conserving transitions (Γ_{SC}) is shown for comparison (dashed line, right scale).

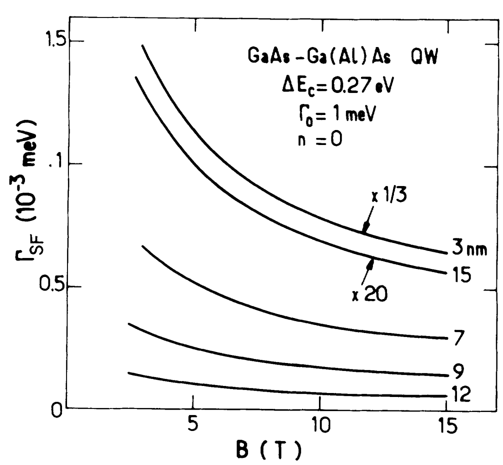

FIG. 2. The calculated spin-flip broadening Γ_{SF} of the $n=0$ Landau level is plotted vs the magnetic-field strength B for several GaAs well thicknesses.

Our calculations have only considered electrons, while optical experiments involve excitons (in undoped wells).[3,4,15] In a strong magnetic field one may think of a very rough exciton model where the lowest-lying exciton states are built out from the ground electron and hole Landau levels. In a narrow quantum well, one may take as the first approximation the hole state as being unadmixed (i.e., corresponding to a spin quantum number $\pm\frac{3}{2}$). Then there are four distinct exciton states. One is optically inactive ($|0_e\uparrow, O_h+\frac{3}{2}\rangle$), two are dipole active ($|0_e\uparrow, O_h-\frac{3}{2}\rangle; |\Psi_{0-}, O_h+\frac{3}{2}\rangle$) in opposite circular polarizations, and one ($|\Psi_{0-}, O_h-\frac{3}{2}\rangle$), in principle inactive, can become weakly active due the spin mixing in the conduction band. There are evidences of relaxation between these states, but it has been suggested that it arises from electron-hole exchange interaction.[3,4] Clearly, with unadmixed hole spin this is impossible. However, the spin mixing in the conduction band allows couplings by a scalar potential between the $|0_e\uparrow, 0_h-\frac{3}{2}\rangle$ and $|\Psi_{0-}, O_h-\frac{3}{2}\rangle$ states on the one hand and between the $|0_e\uparrow, O_h+\frac{3}{2}\rangle$ and $|\Psi_{0-}, O_h+\frac{3}{2}\rangle$ states on the other hand. Of course, numerical evaluations of the relevant transition rates should properly account for the excitonic corrections between the electron and hole wave function. We believe we have shown, however, on qualitative grounds that existing band-structure effects remain good candidates to provide a natural explanation of the observed free carrier and exciton spin relaxation.

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