Phenomenology of the superconductive state of a marginal Fermi liquid

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We consider an extension of the marginal-Fermi-liquid model to the s-wave superconducting state by phenomenologically incorporating the superconducting gap into the scattering spectrum. The linear-in-temperature scattering rate due to the high density of low-energy electronic excitations naturally leads to a large pair-breaking rate, which suppresses T_c. Below T_c the low-energy excitations are self-consistently suppressed due to the opening of a superconducting gap. This leads to a vanishing of both the inelastic-scattering rate and pair breaking below T_c. There are a number of consequences not found in traditional BCS electron-phonon–induced s-wave superconductors. For energies below 3Δ, the quasiparticles become well defined in the superconducting state, while they are marginal (scattering rate proportional to the energy) in the normal state. This produces a two-peaked structure in the one-particle spectra—a sharp feature between Δ and 3Δ (depending on momentum) and a broad hump with an onset at 3Δ (independent of momentum). The transport properties do not obey the usual BCS rules. Transport properties in the $q \rightarrow 0$ limit and for low frequencies $\omega \ll \Delta$ show peaks below T_c. These are observable in microwave conductivity and in electronic thermal conductivity. Local or momentum-averaged response properties such as the nuclear relaxation rate show no peak, but a sharp drop below T_c. The superconductive gap opens very rapidly below T_c, and the value of $2\Delta/T_c$ can cover a wide range, depending on parameters. The physical origin of these results is discussed, and comparison to experiment is made.

I. INTRODUCTION

The normal state of the high-temperature cuprate superconductors exhibits a number of anomalous properties, of which the linear T dependence of the resistivity $\rho(T)$ is the most familiar.[1] Others include a linear bias dependence of the tunneling conductance $g(V) = dI/dV = g_0 + g_1|V|$;[2,3] an unusual line shape from angle-resolved photoemission data, suggesting a quasiparticle damping $1/\tau \propto |\omega - \mu|$ growing linearly in energy away from the Fermi energy;[4] excess absorption in the mid-ir over that expected from a Drude theory;[5–12] a flat electronic background in inelastic light scattering down to energies $\sim T$;[13] and anomalous T dependence of both the NMR T_1^{-1} (Refs. 14 and 15) and dynamic structure factor $S(q,\omega)$ seen in neutron scattering.[16–21] These properties are not consistent with a simple Fermi-liquid description[22–25] and may be crucial to the understanding of the high-T_c superconductors.

These unusual properties were found to be consistent with an anomalous form for the quasiparticle lifetime $1/\tau \propto \mathrm{Im}\Sigma \propto \max(\omega, T)$, which becomes long at low temperatures or energies; however, the spectral weight $Z(\omega) = |\ln(\omega/\omega_c)|^{-1}$ vanishes logarithmically. These can be subsumed by a quasiparticle self-energy of the form

$$\Sigma(\omega) = \lambda[\omega \ln(x/\omega_0) + i\pi x] , \tag{1}$$

where $x = \max(\omega, T)$ and ω_0 is a high-energy cutoff. Such behavior can arise from strong inelastic scattering of quasiparticles from a bosonic spectrum which is flat over a large frequency range $T < \omega < \omega_c$. All of the response functions (spin fluctuations, Raman scattering, optical conductivity) have a polarizability of the approximate form

$$\mathrm{Im}[P(q,\omega,T)] = \begin{cases} \omega/T, & \omega \ll T \\ \text{const}, & T \ll \omega \ll \omega_0 , \end{cases} \tag{2}$$

with a possible multiplicative factor $F(\mathbf{q})$, although some modifications are needed for conserved quantities for which $\mathrm{Im}P \propto q^2$ for $qv_F < \omega$. This state of affairs was termed a "marginal" Fermi liquid (MFL).[24,25]

We have introduced the notion of scattering from the boson above—which is presumably a composite of quasiparticle-hole pairs—as a convenient approximation to treat what is undoubtedly a more complicated phenomenon. Such a picture carries an unwritten implication of low-order perturbation theory, which might be (and has been[26]) questioned given the singular behavior derived at low energies. However, the three-point vertex function for a coupling of a boson of form (2) to quasiparticles (see Fig. 1) is by use of the Migdal theorem, a constant with corrections $O(\omega_0/E_F \ln[\omega_0/T])$. Since $\omega_0/E_F \approx 10^{-1}$ and $\ln(\omega_0/T_c) \approx 2$, the concept is at least self-consistent for calculational purposes. The presence of a logarithmic factor in the vertex correction should raise questions about whether a correct microscopic picture will allow this electron-boson picture in the normal state. In the superconducting state, the low-energy cutoff in Eq. (2) is $\max(2\Delta(T), T)$ (see the discussion in the next section), and so there would seem to be less reason to worry.

Any response function can be separated in principle into the response of noninteracting quasiparticles and corrections due to the four-point vertex Γ. Note that at long wavelengths ($qv_F \ll \omega$), the noninteracting bubble gives already the form of Eq. (2) when self-energy corrections are included. Hence long-wavelength response functions for nonconserved quantities (conductivity, Raman scattering) can be calculated (to within an overall magnitude) from pure, noninteracting, quasiparticle response functions.[25] At large momenta this is not true, and we shall discuss this more carefully later. For the moment we remark that the strongly momentum-dependent part of the result is again to be found in the noninteracting bubble, because this is sensitive to the band structure, which is assumed to be unrenormalized in the MFL picture. Thus peaks in the spin-fluctuation spectrum arising from nesting features in the band structure will be well represented by the noninteracting diagram. If these features dominate the overall (i.e., local) response, this will give a good account of, e.g., the NMR relaxation rate T_1^{-1}.[27–30]

All of this, of course, should be justified by a microscopic theory. The anomalous normal-state properties have been claimed to result, at the microscopic level, from resonating valence-bond theories,[22,23,31,32] from local charge-transfer fluctuations,[33] and from near nesting of the Fermi surface.[34] However, we can make progress at a phenomenological level without much microscopic input, and we shall follow that approach here. We note that the phenomenology of Anderson's Luttinger liquid[23] is close to that of a MFL, with the principal important differences being the role of two-dimensional confinement of the spinons and the finite mass of the fermions (in the MFL we have $m^* \propto 1/Z$). Comparison with the properties of the superconducting state, such as those calculated here, can in principle allow a distinction between Luttinger liquid[35] and MFL ideas.

The phenomenological marginal Fermi-liquid spectrum of particle-hole excitations, when treated as bosons in an Eliashberg-like approximation, leads to a superconductive state at low temperatures. In the normal state, the spectrum has no low-energy scale other than the temperature itself. Below T_c, however, there exists the natural scale of the superconducting gap, $2\Delta(T)$.[36] We incorporate this idea in calculations of observable properties of an s-wave superconductive state, which is the symmetry favored by the bulk of experiments. A primary effect is the rapid increase of quasiparticle lifetime below T_c, and some remarkable deviations of observable properties from those in customary electron-phonon superconductors are predicted. These are the disrespect of the BCS coherence rules in transport properties—a peak is predicted in electromagnetic absorption, but not in the nuclear relaxation rate; a two-peaked quasiparticle spectrum—with features at Δ and 3Δ—observable in angle-resolved photoemission and tunneling; a possible optical gap of 4Δ in the clean limit. Absent a microscopic theory of the MFL spectrum, most of these conclusions have dubious quantitative validity, but there are certain general aspects which, it can be argued, are likely to survive in a more appropriate theoretical scheme. A similar approach has been used by Nicol, Carbotte, and Timusk to calculate the optical response at high[37] and low frequencies,[38] the quasiparticle damping,[39] and other physical quantities.[40] We have presented some preliminary results elsewhere,[41] and the low-frequency dynamics was discussed by Nuss *et al.*[42]

II. SUPERCONDUCTIVE STATE

As already discussed,[24,36] the MFL spectrum treated as a boson scattering off quasiparticles leads to an s-wave superconducting ground state, provided that the phenomenological coupling constant λ_ρ in the charge channel is larger than λ_σ, that in the spin channel. Since the vertex coupling to such bosons is frequency independent in the normal state, such phenomenological constants may safely be used for a linear instability analysis of the normal state, i.e., to determine T_c and the properties close to T_c.

The bosons have an upper frequency cutoff ω_0, which is determined from fits to experiments to be of $O(0.1E_F)$. Therefore the Migdal approximation and Eliashberg theory of superconductivity may be used to determine T_c. Before we come to the crucial question of the spectrum and calculations below T_c, we briefly summarize the Eliashberg formalism.

We use the Nambu notation, with a matrix Green's function

$$G(k,i\omega_n)=[i\omega_n Z_n-\varepsilon_k\tau_3-\phi_n\tau_1]^{-1} , \tag{3}$$

where $\omega_n=(2n+1)\pi T$ is a fermion Matsubara frequency, Z_n the mass renormalization, and $\Delta_n=\phi_n/Z_n$ the gap function. Assuming a flat density of states for the electronic eigenvalues ε_k, the Eliashberg equations become[43,44]

$$\omega_n(1-Z_n)=-\pi T\sum_m \lambda^+_{n-m}\frac{\omega_m Z_m}{[(Z_m\omega_m)^2+\phi_m^2]^{1/2}} , \tag{4}$$

$$\phi_n=\pi T\sum_m \lambda^-_{n-m}\frac{\phi_m}{[(Z_m\omega_m)^2+\phi_m^2]^{1/2}} . \tag{5}$$

The coupling constants $\lambda_n^\pm$ are given by

$$\lambda_n^\pm=\lambda^\pm\int_{-\infty}^{\infty}d\omega\frac{B(\omega)}{i\omega_n-\omega} , \tag{6}$$

B is the boson spectrum, and $\lambda^\pm=\lambda_\rho\pm\lambda_\sigma$, with $\lambda_{\rho(\sigma)}$ the coupling in the charge (spin) channel.

The MFL spectrum for the normal state may be parametrized as

$$B_0(\omega)=\tanh(\omega/2T)/[1+(\omega/\omega_0)^2] . \tag{7}$$

The Eliashberg equations are expected to be valid for the prediction of the superconducting T_c when $T_c \ll \omega_0 \ll E_F$; elsewhere, they can only be approximately relied upon. However, when the mediating boson changes its structure below T_c, even in the weak-coupling limit the theory must be supplemented to understand physical properties below T_c. It is worth remembering that even in the electron-phonon case, there are three Eliashberg equations, with the third being the renormalization of the phonon propagator. The Eliashberg equations can be derived from a variational minimization of the free energy with regard to the diagonal and off-diagonal electron self-energies [leading to Eqs. (4) and (5)] as well as the phonon self-energy. In the weak-coupling limit of the electron-phonon problem, the renormalization of the phonons is not strongly affected by the superconducting transition, and $\alpha^2F(\omega)$ is temperature independent. The correct way to proceed in the superconducting state would be to replace Eqs. (4) and (5) by the self-energy graph shown in Fig. 1 (including the full vertex Γ), where *all* the intermediate propagators are given by Eq. (3) self-consistently. These equations need to be supplemented by an equation for Γ in the superconductive state, i.e., the equivalent of the phonon self-energy in conventional theory. Without a microscopic theory even in the normal state, that is where the difficulty lies; we do not have a proper variational theory.

There are some general features of Fig. 1(a) that can be preserved qualitatively in a simple way in a boson model. The most important is that (at least) three propagators are present in the intermediate state, implying a self-energy whose imaginary part is zero for $|\omega-\mu|<3\Delta$. This can be simulated by a particle scattering off a boson with a low-frequency cutoff 2Δ, given by the diagram in Fig. 1(c). Now Γ is itself made up of graphs with various number of particle-hole lines and is therefore also dependent on Δ—nevertheless, the basic point is that the spectrum in the superconducting state can have only one new parameter, proportional to $\Delta(T)$. In general, the effect of modifications in Γ can be seen in the self-energy to be numerically opposite to that of the propagators, but that can only introduce numerical corrections. So we try using the simplest boson models with a spectrum $B(\omega)=B_0(\omega)f(\omega,T/\Delta(T))$ with some self-consistency condition on f. In practice, we have chosen the following three (others can be imagined, but we find no point at this stage to pursue them):

$$B(\omega)=B_0(\omega)\ , \tag{8a}$$

$$B(\omega)=\omega^{-1}\mathrm{Im}\overline{\Pi}^{\mathrm{I}}(\omega)B_0(\omega)\ , \tag{8b}$$

$$B(\omega)=\omega^{-1}\mathrm{Im}\overline{\Pi}^{\mathrm{II}}(\omega)B_0(\omega)\ . \tag{8c}$$

The polarizabilities $\overline{\Pi}^{\mathrm{I,II}}$ are simply the bare, momentum-averaged polarizabilities with either type-I or -II coherence factors (for a definition of the polarizabilities, see Sec. III). In the normal state, we have $B\equiv B_0$ in all three cases. These choices have been made to incorporate simple physical approximations (i.e., that the excitation spectrum is proportional to the local joint density of states) in a tractable form, without introducing extra complications. The different choices of coherence factors would be appropriate if the coupling were through a densitylike (case B) or a currentlike vertex (case C). The two models (B and C) bracket the qualitative behavior we might expect—comparison between these cases can tell us what the quantitative range of results should be, as well as which of the qualitative features are robust.

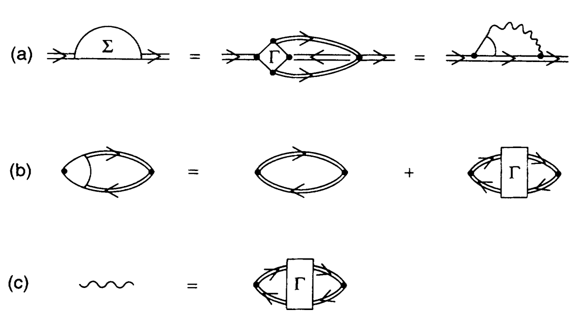

FIG. 1. (a) Self-energy from the four-point vertex and its representation as the interaction with a boson (wiggly line). (b) Separation of response function into noninteracting p-h pair and vertex correction. (c) Approximate form for the boson with the correct qualitative behavior below T_c.

Case A is a "base line" where the spectrum does not develop a gap in the superconducting state, and there are pair-breaking effects at low energy. Cases B and C are similar in that both will have a gap of $2\Delta_0$ at low temperatures, but the onset in case C is smooth, whereas case B is more abrupt. At small temperatures $T<T_c$, both acquire oscillator strength at low energy, which has a different spectral shape for the type-I and -II coherence factors; close to T_c this leads to significant quantitative differences. We have not chosen to normalize the spectral weight in $B(\omega)$ as there is no particular reason that it should be held constant. The shift in oscillator strength away from low frequency means that the effective coupling parameter

$$g^{\pm}=2\lambda^{\pm}\int d\omega\, B(\omega)/\omega \tag{9}$$

is not constant as a function of temperature and falls on entering the superconducting state. This is a negligible effect when $T,\Delta_0\ll\omega_0$, but it can be important at high temperatures or in strong coupling when T_c/ω_0 is not small. In particular, we shall have $2\Delta_0/T_c<3.5$ in some cases, and this number is not a useful guide to the strength of either pairing or pair-breaking effects.

One of the main problems we see in the above assumption is that the singular behavior of $\Gamma(\omega)$ in the normal state quite likely arises from the interaction between the coherent and incoherent parts of the single-particle spectral functions in a strongly correlated problem.[33] The bulk of the latter may be far from the chemical potential (the residuals of the Hubbard bands of the insulator), but affect the low-energy behavior in two- or more-body propagators. The reasoning above may be justified for simple modifications of the superconducting state if Γ could be made up from quasiparticle interaction processes, but not necessarily if higher-energy transitions are involved.

III. RESPONSE FUNCTIONS

The Eliashberg equations are solved iteratively with the spectrum $B(\omega)$ determined self-consistently in a manner discussed below. Once the solutions are determined, we can evaluate correlation and response functions. Physical response functions are of the general form

$$\Pi(q,i\omega_m;\Gamma)=\sum_{n,k}\mathrm{Tr}[G(k+q,i\omega_{n+m})\times\Gamma(k+q,i\omega_{n+m})\times G(k,i\omega_n)\Gamma(k,i\omega_n)]\ . \tag{10}$$

Here $\Gamma=\gamma(k)_n\tau_3$ for density response (leading to type-I coherence factors) and $\Gamma=\gamma(k)_n I$ for current response (type II). Transport properties such as the conductivity require taking the limit $q\to 0$, whereas local susceptibilities (entering, e.g., NMR relaxation rates) are averaged over momentum q. None of the properties we discuss arises from any sharp momentum dependence of G or Γ, and we shall assume a smooth density of states and average over the momentum.

The local susceptibility becomes [setting $\Gamma=\tau_3$ (type I) and $\Gamma=I$ (type II)]

$$\overline{\Pi}^{\mathrm{I,II}}(i\omega_m)=\sum_q \Pi(q,i\omega_m)\propto \pi T\sum_n C^{\mathrm{I,II}}_{n,n+m}\ , \tag{11}$$

where

$$C^{\mathrm{I,II}}_{n,n+m}=1-(\omega_n\omega_{n+m}\pm\Delta_n\Delta_{n+m})/E_nE_{n+m}\ , \tag{12}$$

with $E_n=(\omega_n^2+\Delta_n^2)^{1/2}$ and upper and lower signs referring to type-I and -II coherence factors, respectively.

At long wavelengths, we have[44]

$$\Pi^{\mathrm{I,II}}(q=0,i\omega_m;\gamma)\propto \pi T\sum_n \gamma_{n+m}\gamma_n C^{\mathrm{I,II}}_{n,n+m}/\tau^{-1}_{n,n+m}\ , \tag{13}$$

where we have defined an effective dynamic scattering rate

$$\tau^{-1}_{n,n+m}=Z_nE_n+Z_{n+m}E_{n+m}+\tau_0^{-1} \tag{14}$$

as well as a Fermi-surface-averaged vertex function $\gamma(i\omega_n)$. Here τ_0^{-1} is the impurity-scattering rate which enters the transport lifetime; thus in the "dirty" limit that $\tau_0^{-1}\gg \mathrm{Im}(\Sigma)$, Eqs. (11) and (13) are proportional.

Physical quantities are obtained by analytic continuation to the real axis, i.e., $i\omega_m\to\omega+i\delta$. We have quoted formulas for quantities evaluated along the imaginary frequency axis, as these are more compact. In earlier work[41] we had used Padé approximants to continue analytically to the real axis.[45,44] These methods are difficult to apply reliably at moderate temperatures, and in the present case, all our calculations were made along the real axis. When written on the real axis, for example, Eq. (8) takes a more familiar form[46]

$$\mathrm{Im}\overline{\Pi}^{\mathrm{I,II}}(\omega)=\int d\omega'\left[\tanh\left[\frac{\omega+\omega'}{2T}\right]-\tanh\left[\frac{\omega'}{2T}\right]\right]\left[\left[\mathrm{Im}\frac{u_1}{(1-u_1^2)^{1/2}}\right]\left[\mathrm{Im}\frac{u_2}{(1-u_2^2)^{1/2}}\right]\mp\left[\mathrm{Im}\frac{1}{(1-u_1^2)^{1/2}}\right]\left[\mathrm{Im}\frac{1}{(1-u_2^2)^{1/2}}\right]\right]\ , \tag{15}$$

with $u(\omega)=\omega/\Delta(\omega)$, $u_1=u(\omega')$, and $u_2=u(\omega+\omega')$. The electrical conductivity[47] is

$$\sigma(\omega)\propto i\Pi^{\mathrm{II}}(q=0,\omega;\gamma=1)/\omega\ , \tag{16}$$

whereas the NMR spin-lattice relaxation rate T_1^{-1} has the same coherence factors but no lifetime effects:

$$T_1^{-1}\propto \lim_{\omega\to 0}\mathrm{Im}[\overline{\Pi}^{\mathrm{II}}(\omega)/\omega]\ . \tag{17}$$

Type-I coherence factors appear in the electronic thermal conductivity

$$K_{\mathrm{el}}\propto \lim_{\omega\to 0}\mathrm{Im}[\Pi^{\mathrm{I}}(q=0,\omega;\gamma=\omega)/\omega]\ , \tag{18}$$

as well as in the attenuation of longitudinal sound waves, but in the latter case without the appearance of the lifetime τ.

IV. RESULTS AND COMPARISON TO EXPERIMENT

The models (A–C) differ slightly from the form we had used earlier[41] and also are different from the prescription of Nicol, Carbotte, and Timusk.[37] They agree in most qualitative respects, but there can be significant quantitative disagreement, particularly near T_c, for those quantities which depend delicately on the quasiparticle scattering rate. In our earlier approach, we used simply a joint density of states for particle-hole pairs, which develops a gap in the spectrum resembling the single-particle density of states. Models B and C have near T_c an extra component to the spectrum at low energies arising from quasiparticles thermally excited across the "gap". This results in a more "smeared-out" crossover from low to high temperatures and removes the weak first-order transition we found in strong coupling with the earlier model.

We shall set energy and temperature scales by letting $\omega_0=1$.

A. General features

In Fig. 2 we show the calculated densities of states for several temperatures for the three different models with the same value of coupling constants $\lambda_+=\lambda_-=0.3$. In all cases these are quite different from BCS theory, reflecting the pair breaking self-energy terms which smear the density of states at moderate temperatures. First, we note that model A, with no gap in the spectrum, actually has the largest low-temperature gap, even though all models have the same T_c (for the same coupling constants). Because models B and C open gaps in the spectrum at low temperatures, both the pairing and pair-breaking interactions are reduced as temperature is lowered, leading to lower values of 2Δ than might have otherwise been expected. Values of $2\Delta/T_c$ less than 3.5 can be obtained within our approach (see Table I). However, if we had chosen to normalize the spectrum to have the same weight above and below T_c different values would have been obtained.

The results of Table I should be viewed as a guide to the range of behavior we can plausibly expect. More important are the qualitative features of common to all three models. There is some region of gaplessness near T_c, more pronounced for models A and C. Model C, because of the type-II coherence factors, has considerable oscillator strength at low frequency and finite temperature; this is much reduced with type-I coherence factors, which show less persistence of the gapless behavior. The qualitative behavior of models B and C is, however, similar, and we shall restrict further discussion to model B. We note that there is a peak in the density of states, that sharpens into a BCS-like singularity as temperature is lowered; the position of this peak is always large and comparable to the zero-temperature gap, even close to T_c, and in contrast with BCS weak-coupling theory.

Figure 3 shows the superconducting order parameter $\rho_s(T)$, which is proportional to $\lambda^{-2}(T)$, the penetration depth. Here we have compared three different ratios of $\lambda^+/\lambda^- = g$ with the two-fluid result $1-(T/T_c)^4$, which represents the experimental data quite well.[48] Increasing g reduces both T_c and $\Delta(0)$, but the latter by less because the pair breaking is diminished at low temperatures when

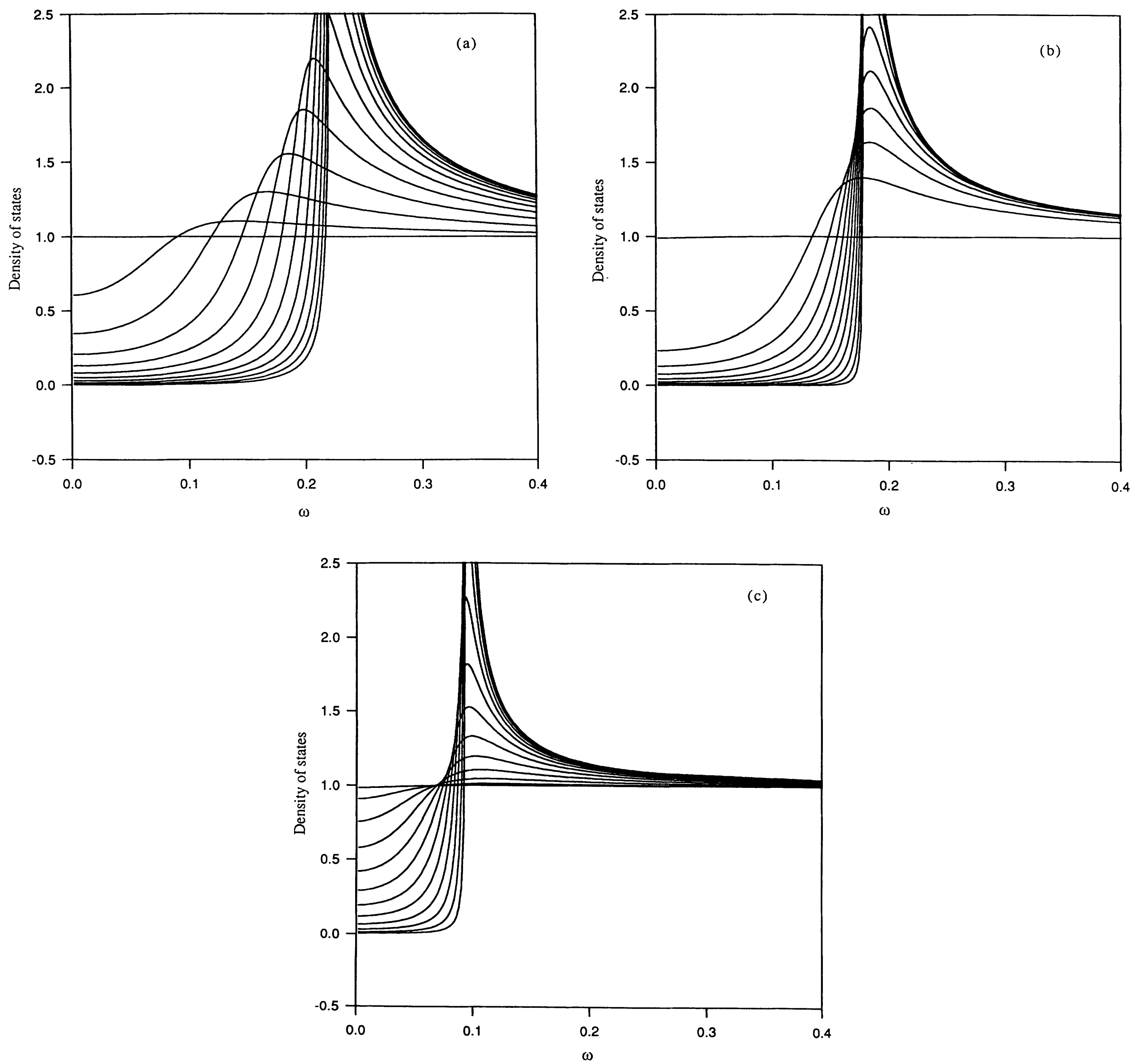

FIG. 2. Density of states calculated for the three models at 12 equally spaced temperatures from T/T_c from 0.35 to 1, with $\lambda^+ = \lambda^- = 0.3$, $\omega_0 = 1$. (Energies are measured in units of ω_0.) (a) Model A; (b) Model B ; (c) Model C.

TABLE I. Values of T_c/ω_0 and $2\Delta(0)/T_c$ for the three models described in the text.

Model	λ_+	λ_-	T_c	$2\Delta/T_c$
A	0.3	0.3	0.084	5.2
A	0.45	0.3	0.046	4.3
A	0.6	0.3	0.022	3.5
B	0.3	0.3	0.084	4.2
B	0.45	0.3	0.046	5.5
B	0.6	0.3	0.022	7.2
C	0.3	0.3	0.084	2.2
C	0.45	0.3	0.046	3.3
C	0.6	0.3	0.022	4.2

there is a gap. Consequently, $2\Delta/kT_c$ increases with coupling. The shape of the curves naturally steepens as g is increased, but the curve for $g=1$ and $\lambda^{\pm}=0.3$ is already anomalous in shape, even though the value of coupling is fairly weak. The explanation for this shape will be clear when we study the optical conductivity later.

It is of interest to follow the detailed behavior of the gap function and self-energy, shown in Fig. 4 for model B at a set of temperatures (the same parameters as in Fig. 2). At low temperatures both $\mathrm{Im}(\Sigma)$ and $\mathrm{Im}(\Delta)$ are nonzero only for frequencies exceeding 3Δ above the chemical potential. This gap arises on account of the 2Δ gap in the scattering spectrum—only quasiparticles excited over 2Δ above the band edge (itself at $\omega=\Delta$) have a decay channel. As the temperature is raised, a small foot develops in the imaginary parts of $\omega=\Delta$ and above; and as $T\rightarrow T_c$, the density of states becomes gapless. We now discuss the consequences for some experiments.

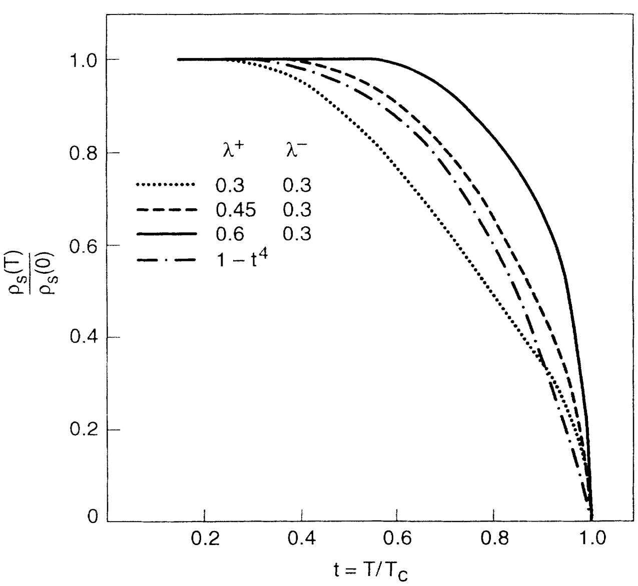

FIG. 3. Temperature dependence of the order parameter $\rho_s \propto \lambda(T)^{-2}$ (model B) for three different values of λ^+/λ^-, compared with the two-fluid result.

B. Single-particle properties: tunneling and photoemission

The two experiments which may be interpreted directly in terms of the quasiparticle spectrum are photoemission (or inverse photoemssion) and tunneling. The angle-resolved photoemission spectrum is proportional to the single-particle spectral function at a fixed momentum $\mathbf{k}$ as a function of energy below μ. It is by now well established that a sharp Fermi edge exists in the cuprate superconductors, at momenta close to those predicted by the band structure, both from photoemission[4,49] and positron annihilation measurements.[50] However, the energy

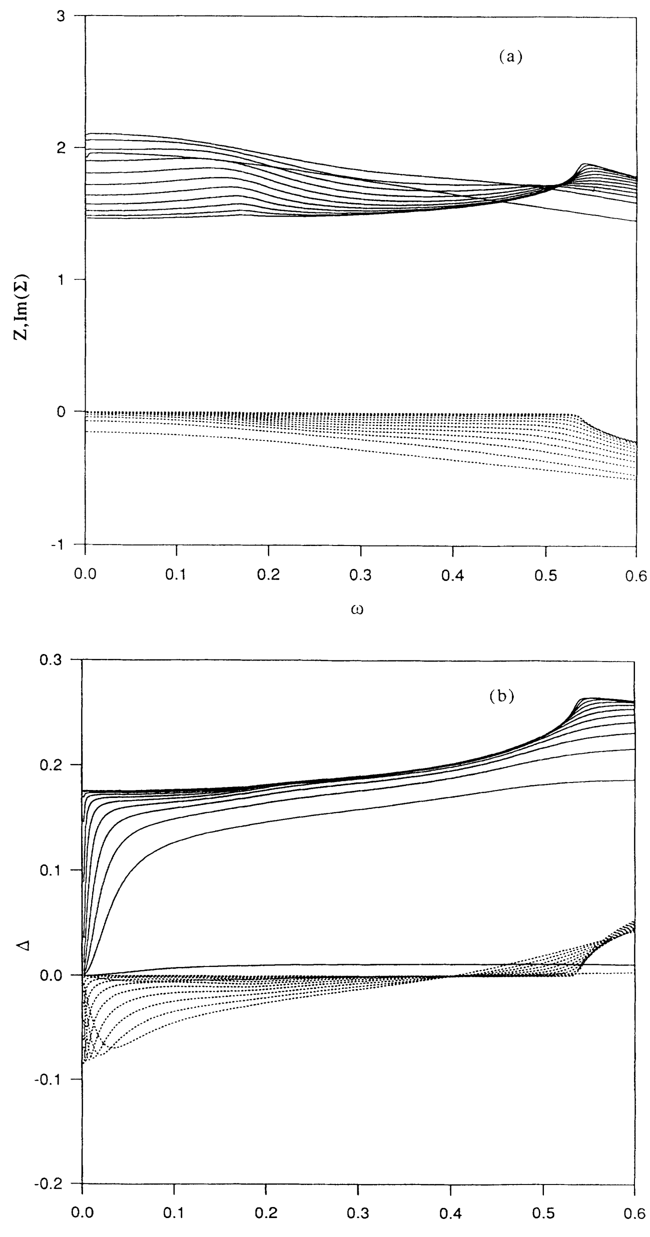

FIG. 4. (a) Mass enhancement (solid line) and $\mathrm{Im}(\Sigma)$ (dashed line); (b) $\mathrm{Re}(\Delta)$ (solid line) and $\mathrm{Im}(\Delta)$ (dashed line). Model B, $\lambda^+=\lambda^-=0.3$, same temperatures as Fig. 2, and $\omega_0=1$.

dependence of the photoemission spectra is anomalous, with the quasiparticle peak broadening approximately linearly in $|\omega-\mu|$ as $k \rightarrow k_F$. This anomalous behavior is quite consistent with marginal-Fermi-liquid character, where one finds that the quasiparticle spectral function

$$A(\mathbf{k},\omega) = -2\,\mathrm{Im}[\omega-\varepsilon_{\mathbf{k}}-\Sigma(\omega)]^{-1} \tag{19}$$

varies as $|\omega|^{-1}$ for $|\omega| > |\varepsilon_k - \mu|$ and as $|\omega|$ in the opposite limit.[24] This effect has its origin in the linear ω dependence of the damping rate $\mathrm{Im}(\Sigma)$.

From Fig. 4 it is clear that this behavior must be drastically modified in the superconducting state, and this is reflected in the spectral function (Fig. 5). At low energies, close to the gap edge, the quasiparticle states now become well defined so that $A(\mathbf{k},\omega) = Z\delta(\omega - E_{\mathbf{k}})$, with $E_{\mathbf{k}} = (\varepsilon_k^2 + \Delta^2)^{1/2}$, and $Z \sim [1+\lambda\ln(\omega_c/\Delta)]^{-1}$. Above an energy of 3Δ from the chemical potential, the shake-off excitations of particle-hole pairs begin, and there is a continuum of excitations. In contrast, model A, where no gap develops, does not show the sharpening of the quasiparticle peak below T_c. Consequently, the superconducting state restores the quasiparticle character for low-energy states.[35]

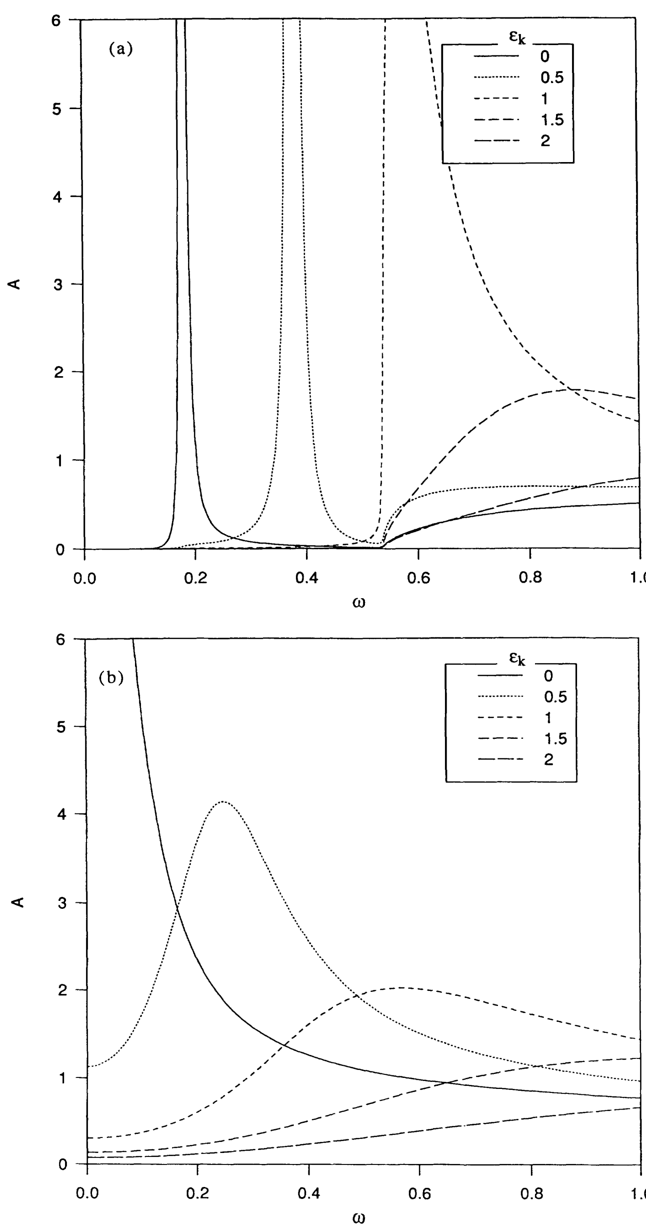

FIG. 5. Spectral function $A(\varepsilon_k,\omega)$ at (a) $T/T_c = 0.3$ and (b) T_c in the normal state. Model B, $\lambda^+ = \lambda^- = 0.3$, $\omega_0 = 1$, values of ε_k/ω_0 shown in figure.

There have been a number of experiments studying photoemission in the superconducting state[49,51,52] which have observed spectral weight transfer from below to above a gap. Recent angle-resolved measurements on $Bi_2Sr_2CaCu_2O_8$ (Refs. 53 and 54) observed in addition a dip at higher energies. The dip is isotropic in k space according to Hwu *et al.*,[54] while Dessau *et al.*[53] reported it to be absent in the Γ-X direction. This dip is qualitatively the feature expected from Fig. 5, although we see no fundamental reason to expect anisotropy in the a-b plane. On the other hand, Anderson[55] has explained the observed dip in the Γ-$\overline{M}$ direction and its absence in the Γ-X direction on the basis of his idea of superconductivity due to interlayer tunneling, where the three-dimensional band structure reappears below T_c. Our prediction is of a dip in all directions, with anisotropy arising solely from the anisotropy of the gap.

Another feature of the experiments is that the spectral weight in the superconducting peak is larger than accounted for by *low-energy states alone* above T_c. This is also a feature of our results. For example, comparing the solid curves in Figs. 5(a) and 5(b), it is clear that spectral weight even above 3Δ is reduced in comparison to the normal state; this weight is transferred into the δ-function peak. The corresponding decrease in Z (increase in spectral weight) is seen clearly also in Fig. 4(a). The quasiparticle recovers its oscillator strength from a region of energy extending up to ω_0. Of course, there does exist a sum rule when the integration is carried out to very large energies.

Quasiparticle tunneling is another probe of the single-particle spectral function.[56] The tunneling current through a barrier between two metals is[57]

$$J = \frac{4\pi e}{h} \sum_{k_1 k_2} |t(\mathbf{k}_1,\mathbf{k}_2)|^2 \times \int d\omega_1 \int d\omega_2 \mathrm{Im}G_1(\mathbf{k}_1,\omega_1)\mathrm{Im}G_2(\mathbf{k}_2,\omega_2) \times \delta(\omega_1-\omega_2-eV)[f(\omega_1)-f(\omega_2)]. \tag{20}$$

Here t is the transmission probability through the barrier, V the voltage bias, and the subscripts 1 and 2 refer to the materials on either side of the barrier. We shall consider here only the simple case of tunneling between a normal metal (say, 1) which has a uniform density of states and no unusual many-body corrections, and a MFL model of the superconductor. In this case we can simplify Eq. (20) by integrating over k_1 to yield an uninteresting constant, obtaining, for the conductance $g(V) = dJ/dV$,

$$g(V) = -\frac{e^2}{h}\int d\varepsilon_k T(\varepsilon_k)\int d\omega\, A(\mathbf{k},\omega)\frac{\partial f(\omega - eV)}{\partial\omega} \quad (21)$$

and

$$T(\varepsilon_k) = N_0(\varepsilon_k)\sum_{k_1}|t(\mathbf{k}_1,\mathbf{k})|^2 \mathrm{Im}G_1(\mathbf{k}_1,\omega_1)\ , \quad (22)$$

assumed independent of the incident quasiparticle energy ω_1, and with $N_0(\varepsilon_k)$ the bare density of states from the band structure. Note that we have here assumed that the barrier is independent of the bias potential for simplicity; dependence of T on V leads to conventional[58] corrections to the tunneling conductance at large biases. Finally, at low temperatures $\lesssim eV$, the derivative of the Fermi function yields a δ function so that the conductance is proportional to the spectral function at an energy $\omega = eV$, integrated over momenta weighted by the transfer function $T(\varepsilon_k)$.

The behavior in the normal state was discussed elsewhere,[59] but we recall that a purely energy-dependent self-energy does not modify the density of states $N(\omega)$ if the bare density of states N_0 is momentum independent.[60] Any dependence of $g(V)$ on Σ arises only via momentum dependence of either the bare density of states or the tunneling matrix element.[24] Particularly in anisotropic materials, momentum dependence of $T(\varepsilon_k)$ comes about naturally, and this was used to argue for a difference between tunneling in the c axis from the a-b plane directions. Simply put, if the barrier collimates the tunneling electron so that the momentum parallel to the barrier is small, then $T(\varepsilon_k)$ will be peaked far from k_f in the c-axis geometry and not necessarily so in the other directions. Consequently, the particle must make use of strong inelastic scattering (hidden in Σ) to reach the Fermi surface. This was pointed out by Kirtley and Scalapino,[61] although their mechanism required the scattering to be confined to the barrier, which we do not. Nevertheless, they do require the existence of a broad, flat continuum of states as in MFL, and our approach has operationally many features in common.[24]

We can qualitatively illustrate the effects of k dependence by considering the analytically tractable case of T a step function, selecting out a range of ε_k. In Fig. 6 we plot

$$g(V)_{\varepsilon_1\varepsilon_2} = \int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon_k\, A(k,eV)\ , \quad (23)$$

for the pairs $\varepsilon_1,\varepsilon_2 = (0,\infty),(5,\infty)$. In the first case, we are plotting simply the density of states, which looks BCS like at low temperatures and has a weak feature due to lifetime effects at 3Δ. For the second parameter set, we have the contribution only of states far from k_F. Above T_c there is the characteristic linear conductance, rounded out by finite temperature, although this is not obeyed for a large range of bias since we have here voltages comparable to the cutoff ω_c $(=1)$. At low temperatures there is no contribution at energies below 3Δ, since the quasiparticle lifetime is infinite. The two cases are shown here for illustration, and for a realistic barrier the spectrum would be a combination of the two figures, with features at both Δ and 3Δ. The scales in the two figures are comparable (i.e., identical weights), and the relative intensity shows the dominance of contributions for near k_F, provided they exist. Measurement of the gap by c-axis tunneling is clearly difficult in practice and exacerbated because there are undoubtedly other inelastic-scattering mechanisms leading to leakage at small biases. However, we expect the measured conductance to be influenced strongly by barrier geometry and quality as they affect the transfer function T.

Experimental measurements of tunneling into the superconductor have yielded a wide range of behavior,[56] and one of the points we wish to make here is that wide

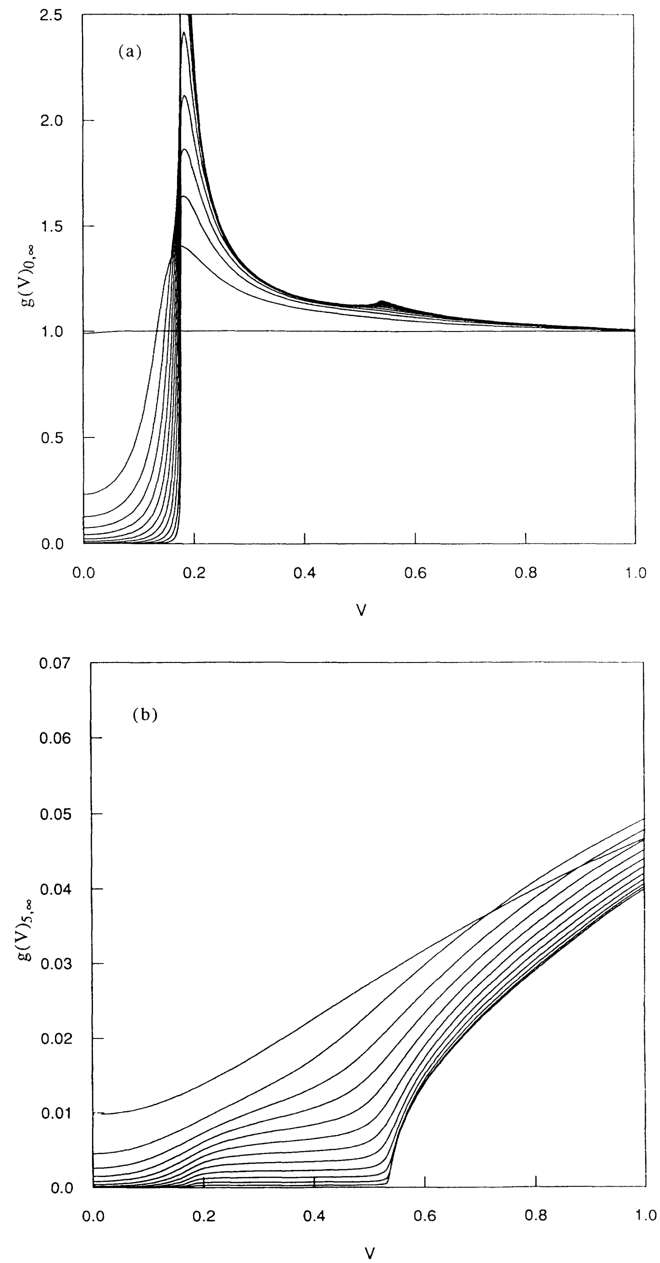

FIG. 6. Calculated tunnel conductance for a transfer function $T(\varepsilon_k) = \Theta(\varepsilon_k - \varepsilon_1)[1 - \Theta(\varepsilon_k - \varepsilon_2)]$ for $(\varepsilon_1,\varepsilon_2) =$ (a) $(0,\infty)$ and (b) $(5,\infty)$. Parameters and temperatures of Fig. 4.

variations are to be expected and are not necessarily indicative of "inferior" junctions. The existence of tunneling anisotropy has often been the subject of comment[62] and has been given as evidence for gap anisotropy. Recent measurements on $Bi_2Sr_2CaCu_2O_8$ from break-junction tunneling[63] and junctions on oriented films of $YBa_2Cu_3O_7$ (Ref. 64) have indeed shown anisotropy of the kind we describe, with a V-shaped characteristic for c-axis tunneling (and a poorly defined gap) and more "conventional" behavior for a-b plane tunneling. In addition, in $Bi_2Sr_2CaCu_2O_8$ in the superconducting state, both break-junction[63] and scanning-tunneling-microscopy (STM) (Ref. 65) measurements show a second feature at higher energies than the gap which we suggest may correspond to 3Δ.

C. Response functions: Optical conductivity and NMR

As we remarked above, if the quasiparticle lifetime τ is taken to be constant and temperature independent, then we have $K_{el} \propto \alpha_L$ and $T_1^{-1} \propto \mathrm{Re}[\sigma(\omega \to 0)]$. Furthermore, in a conventional BCS superconductor, the coherence factors govern the transport properties close to T_c. In particular, T_1^{-1} and $\mathrm{Re}[\sigma(\omega)]$ have a peak just below T_c before the rapid falloff due to the freeze out of the quasiparticles.

In Fig. 7 we calculate the local susceptibility $\chi_0(\omega) = \mathrm{Im}[\overline{\Pi}^{\mathrm{I,II}}(\omega)/\omega]$; Fig. 8 shows the optical conductivity [Eq. (13)] calculated for model B with two different values of coupling constants and, in addition, with the addition of static impurity scattering. Aside from details associated with different choices of cutoff functions (which are important only near T_c), our results appear to be in good agreement with Nicol, Carbotte, and Timusk.[37] The spectrum $B(\omega)$ which is used in model B is exactly that of Fig. 7(a) and corresponds to the same temperatures and parameters as Figs. 8(a) and 8(b). Figure 8(b) shows the effect of adding moderate impurity scattering, comparable to the inelastic-scattering rate at T_c. This indeed changes the results, but is far from the dirty limit, when $\sigma(\omega) \propto \mathrm{Im}\Pi^{\mathrm{II}}(\omega)/\omega$, as can be seen by comparing to Fig. 7(b). There is a marked difference over the whole range of frequencies.

(i) First, $\sigma(\omega)$ falls off as ω^{-1} at high frequencies on account of the anomalous self-energy as discussed earlier. This shows up in reflectivity measurements as a reflectivity decreasing roughly linearly with ω, instead of the constant expected in the relaxation regime of a metal. This is the most salient feature of the ir reflectivity in the normal state and has been discussed elsewhere.[41]

(ii) Second, the gap in the two spectra differs by a factor of 2 at low temperatures, with the gap in the optical conductivity predicted to be 4Δ in the clean limit. This follows naturally because there can be no absorption by the excited carriers unless the lifetime is finite; at low temperatures this requires that at least one of the quasiparticles be at an energy $> 2\Delta$ above the band edge. An equivalent explanation of this was given by Orenstein, Schmitt-Rink, and Ruckenstein.[66] Conservation of energy and momentum requires the creation of two particle-hole pairs for every photon absorbed. This argument[66] also makes it clear that if there are final-state attractive interactions between the excited pairs, the threshold could be lower as a result of an excitonic shift. Such processes are neglected in our treatment.

This phenomenon is another form of the "Holstein effect"[67] familiar from the interaction of quasiparticles with phonons. An Einstein mode at a frequency ω_0 leads to an electron self-energy whose imaginary part is nonzero only for energies $|\varepsilon_k - \mu| > \omega_0$; there are corresponding features in the optical conductivity at frequencies near ω_0. In the superconducting state, these features shift up to $\omega_0 + 2\Delta$ on account of the gap. In our picture the situation is very similar, except that $\omega_0 \sim 2\Delta$, so that

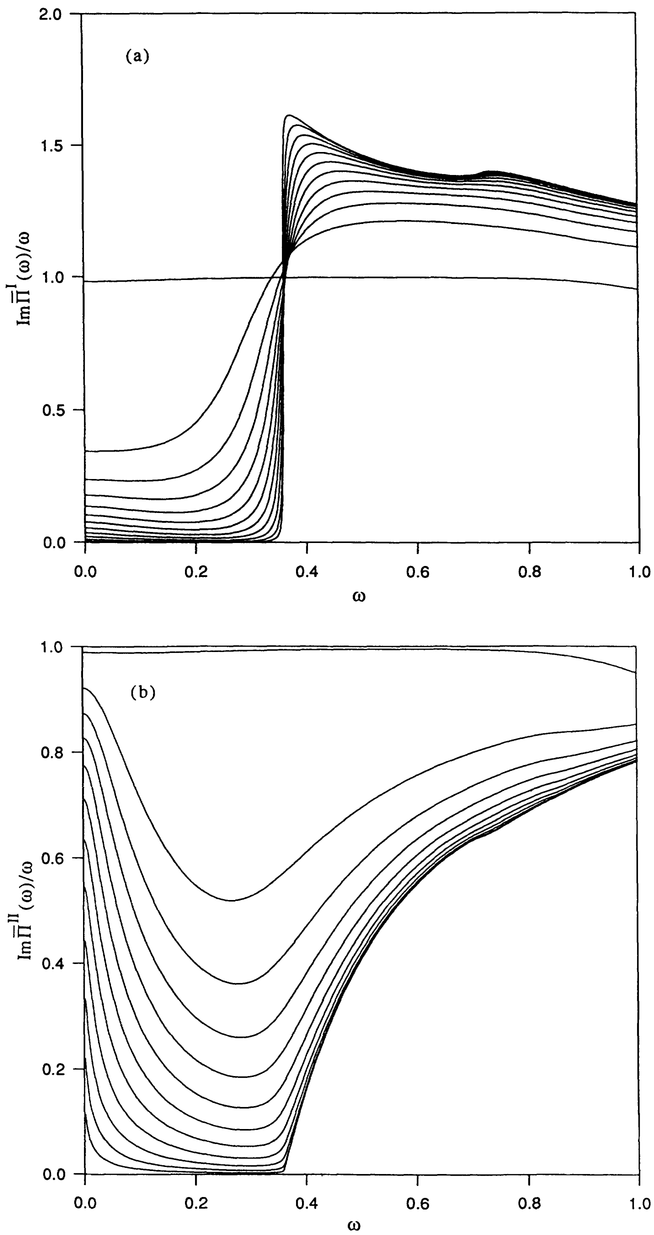

FIG. 7. Local susceptibilities $\mathrm{Im}\overline{\Pi}^{\mathrm{I,II}}(\omega)/\omega$ for the same parameters and temperatures as Fig. 4. (a) Type I coherence factors and (b) type II.

there is no threshold in the normal state.[68] A significant difference between MFL and conventional electron-phonon coupling theory is that the conductivity in the superconductor is lower at all frequencies than in the normal state—this will no longer be true if the impurity scattering is strong enough.

This property is also very sensitive to our assumption about the form of $\Gamma(\omega)$ and may not survive in a better theory. In particular, if the MFL boson itself carries a dipole moment (as it may do if "interband" transitions are involved), the absorption onset will stay near 2Δ. Such an additional contribution was suggested earlier for a sample of $YBa_2Cu_3O_{7-x}$ with a reduced transition temperature of 80 K.[24]

(iii) Adding impurity scattering induces a double threshold, with an onset at 2Δ with strength dependent on the impurity-scattering rate τ_0^{-1}.

(iv) We noted that the density of states curves in Fig. 2 showed that the temperature dependence of the gap was very abrupt, with an onset at more or less the full value. The same feature can be seen in Figs. 7 and 8—at intermediate energies the spectral strength is continuously removed as temperature is lowered. This is quite different from the behavior of a BCS superconductor, where the gap would increase with lowering temperature. This is not true at low energies, where there is a contribution from thermally excited quasiparticles, but conventional ir spectra (extending down to about a tenth of the gap) will

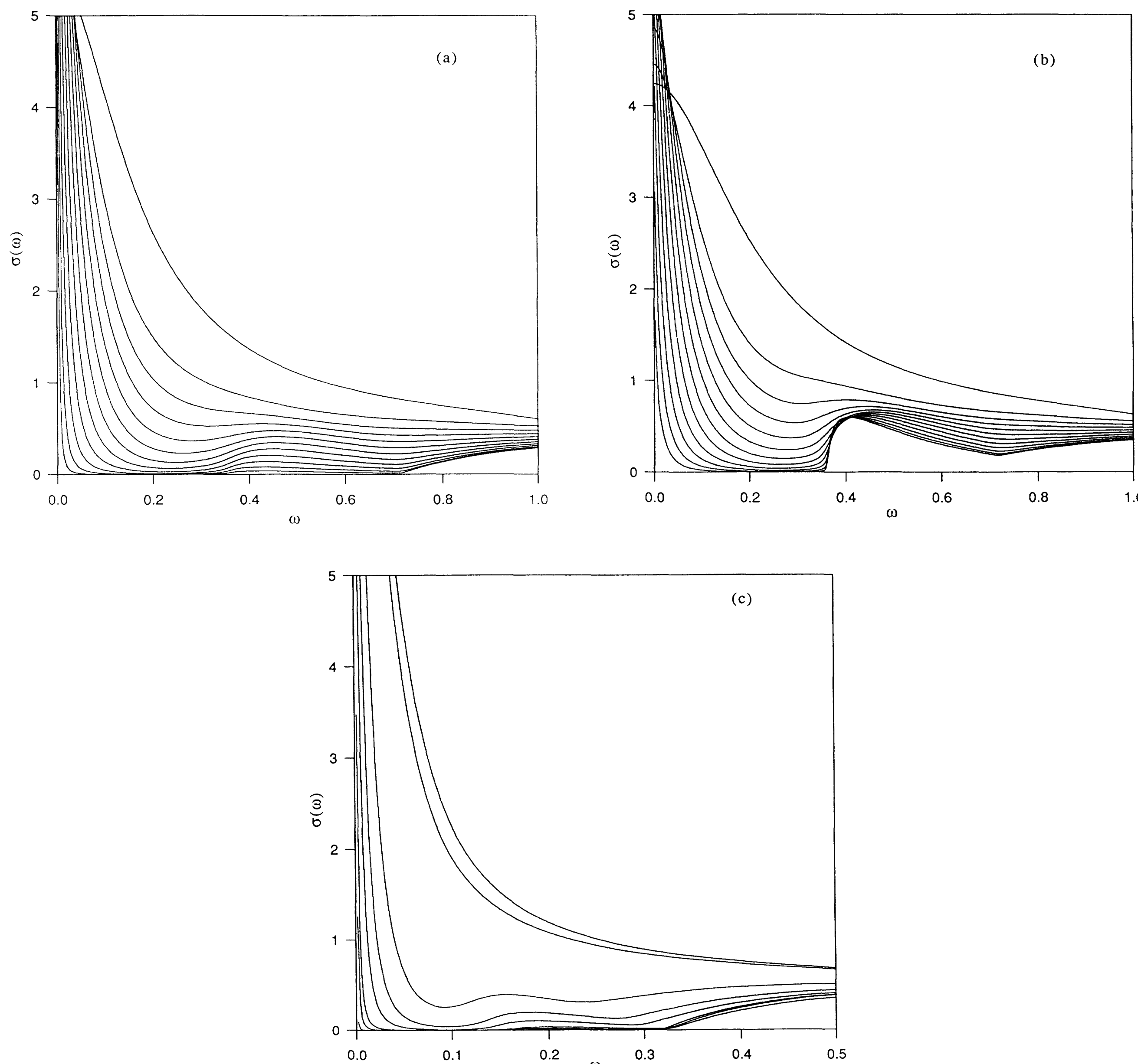

FIG. 8. Conductivity $\sigma(\omega)$ for model B. (a) and (b) $\lambda^+=\lambda^-=0.3$; $\tau_0^{-1}=$ (a) 0 and (b) 0.1; (c) $\lambda^+=0.6$, $\lambda^-=0.3$, $\tau_0^{-1}=0$. Temperatures are equally spaced between $T/T_c=0.35$ and 1.

not be sensitive to this feature.

(v) At low frequencies $\omega \ll 2\Delta$, σ rises above the normal-state conductivity at temperatures just below T_c, a feature not seen in χ_0. In a conventional BCS superconductor, a "coherence" peak is expected at low frequencies because of the combination of type-II coherence factors with the singular density of states. This is not the origin of the behavior close to T_c because the density of states is quite rounded, and the "gap" is already large. The strong pair breaking close to T_c is exactly what gives rise to the rounding out of the low-frequency peak in $\mathrm{Im}\overline{\Pi}^{\mathrm{II}}(\omega)$ [Fig. 7(b)]. The enhancement in σ comes from the temperature dependence of the quasiparticle lifetime, which becomes very long at low temperatures. Since $T_1^{-1} \propto \lim_{\omega \to 0} \mathrm{Im}\overline{\Pi}^{\mathrm{II}}(\omega)/\omega$, we thus expect to have a peak in the low-frequency optical conductivity that will be absent, or reduced, in the NMR relaxation rate.[38,42] Figures 9 and 10 give a comparison between T_1^{-1} and the optical conductivity at low frequency. This feature is a general property of a reduced relaxation rate below T_c and should survive in a better theory. Our results are thus consistent with the observation of a peak in the microwave conductivity[42] but not in the mid-ir conductivity.[69,70] We especially note that at higher frequencies the falloff in the optical conductivity with temperature approximately tracks T_1^{-1}, as has been noted experimentally.[70]

Strong inelastic scattering from any source will suppress the coherence peaks,[36] and recent work has demonstrated this carefully within strong-coupling Eliashberg theory for the electron-phonon interaction.[71–73] However, for electron-phonon coupling, *both* the conductivity *and* the NMR relaxation rate remain proportional[71,73] so that coherence features would be suppressed in both quantities. The effect in the MFL model arises principally from quasiparticle lifetimes.

The interpretation of optical data in the superconductor has been controversial,[74] largely because reflectivity is

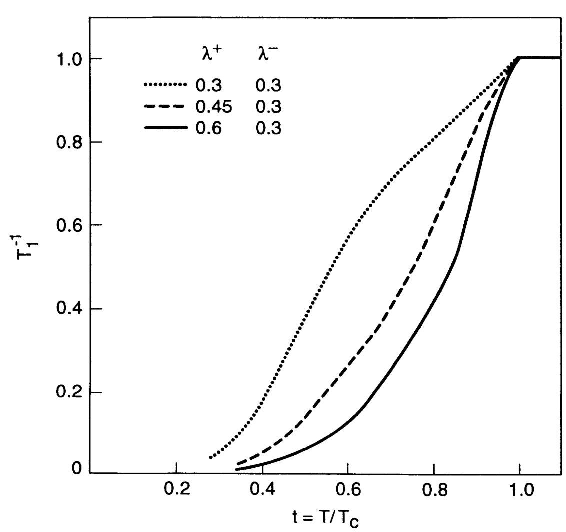

FIG. 9. T_1^{-1} calculated from Eq. (14) for model B.

insensitive to the transition between a good metal and a superconductor. In both $YBa_2Cu_3O_7$ (Refs. 7 and 75) and $Bi_2Sr_2CaCu_2O_8$,[10] there is an absorption threshold near 500 cm^{-1}, but there are persistent reports of onsets at lower frequencies, both in $Bi_2Sr_2CaCu_2O_8$ (Refs. 10, 11, and 76) and $YBa_2Cu_3O_7$.[7] In transmission, a clear gap onset has not been seen in absorption on $YBa_2Cu_3O_7$ (Ref. 77) or in transmission on $Bi_2Sr_2CaCu_2O_8$.[12] The results are further complicated by anisotropy[78,77] in the *a*-*b* plane and the fact that a feature near 500 cm^{-1} exists *above* T_c in $YBa_2Cu_3O_7$ samples with reduced transition temperatures, and perhaps also in $Bi_2Sr_2CaCu_2O_8$. If we tentatively assign the $\sim$500-cm^{-1} onset to a 4Δ threshold (i.e., the clean limit), then the lower-energy features could be disorder induced or direct excitation of the MFL boson. We then also have a ratio $2\Delta/kT_c \sim 4-5$, a little smaller than that estimated from phonon linewidth measurements,[79] but larger than that from neutron scattering in $YBa_2Cu_3O_7$ (Ref. 20) or $La_{2-x}Sr_xCuO_4$.[80] These values appear to be in conflict with electronic Raman scattering in both $YBa_2Cu_3O_7$ (Ref. 81) and $Bi_2Sr_2CaCu_2O_8$,[82] which has yielded widely varying estimates (2Δ between $2kT_c$ and $8kT_c$), with variations between different symmetries, as well as a persistence of scattering below the gap. Tunneling and photoemission measurements in $Bi_2Sr_2CaCu_2O_8$ have generally also given larger ratios $2\Delta/T_c \sim 7$. The experimental assignment must thus remain tentative at present, and we anyway repeat our earlier caution about reading too much into the numerical ratio of $2\Delta/T_c$.

The evolution of this gap feature with temperature has been noted to be anomalous.[69,70] The frequency of the gap feature is T independent, whereas the superfluid fraction falls close to a two-fluid behavior, a situation modeled carefully by van der Marel *et al.*[69] Our results (see Figs. 3 and 8) have precisely these qualitative characteristics.

The absence of a coherence peak in the NMR relaxation rate[14,15] has long been a puzzle for the high-T_c superconductors. Measurements of the low-frequency conductivity should give complementary information, but only recently have they been reported, using time-resolved spectroscopy on $YBa_2Cu_3O_7$ films[42] or microwave measurements on $Bi_2Sr_2CaCu_2O_8$ (Ref. 83) and $YBa_2Cu_3O_7$.[84,85] In all cases a peak was found in $\mathrm{Re}[\sigma(T)]$ at low frequencies close to T_c, but measured to be much broader (and higher) by Nuss *et al.*[42] and Bonn *et al.*[84] than by Holczer *et al.*[83] Indications of a rise in the quasiparticle lifetime close to T_c were seen also by Tanner *et al.*[86] As there are differences in material, preparation, frequency, and technique between these measurements, it is possible that the results may indeed be consistent. Nevertheless, the existence of a peak is not disputed. We would interpret these results as indicative of a rapidly growing quasiparticle lifetime below T_c, a position independently arrived at by others.[86] Such a sharp feature as reported by Holczer *et al.* is difficult to achieve within our model. We note, however, that their reported width is roughly consistent with simple arguments for fluctuation effects,[87] and we should not rule out the possibility of two features, with different physical origins.

The magnitude of the peaks shown in Fig. 10 is in fact smaller than in an earlier calculation.[42] These discrepancies are a result of fine details of the introduction of the gap into the spectrum, with our present model having more spectral weight at low energies close to T_c—note that in Fig. 7(a) there is an upturn at low energies. This feature enhances pair breaking and slows the rate of growth of the quasiparticle lifetime. The previous model showed a tendency to induce first-order behavior in strong coupling,[41] which we do not find in the present case. Increasing the ratio λ^+/λ^- also suppresses the peaks when evaluated at the same relative frequencies ω/Δ [Fig. 10(c)]; however, at low enough frequency there will always be a peak in $\mathrm{Re}(\sigma)$. That the origin of the peaks is principally a lifetime effect can be seen by the effect of added impurity scattering, in Fig. 10(b), which also suppresses the peak. Nicol and Carbotte[40] have noted how this effect can be modeled as a contribution to the Drude conductivity within a two-fluid model (i.e., in addition to the δ function from the superfluid):

$$\sigma_n = \frac{n_n^S(\Delta,T)e^2\tau_s(\Delta,T)}{m} , \tag{24}$$

where n_n^S is the "normal" fraction (quite well modeled by the two-fluid formula at moderate T/T_c) and τ_s the

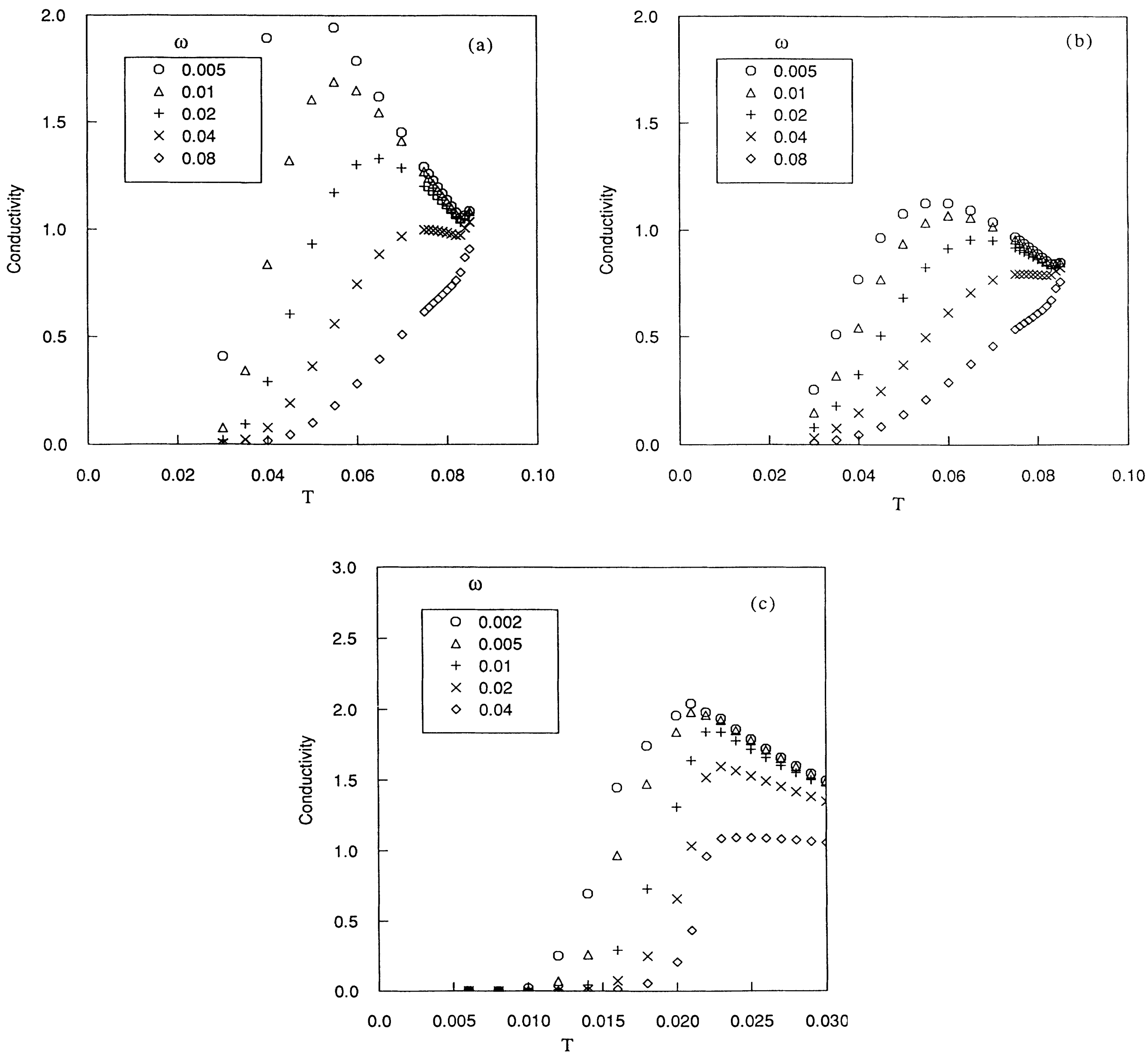

FIG. 10. Conductivity calculated at several frequencies (see legend) for model B ($\omega_0=1$). (a) $\lambda^+=0.3$, $\lambda^-=0.3$, $\tau_0^{-1}=0$; (b) $\lambda^+=0.3$, $\lambda^-=0.3$, $\tau_0^{-1}=0.1$; (c) $\lambda^+=0.6$, $\lambda^-=0.3$, $\tau_0^{-1}=0$.

quasiparticle lifetime. At low temperatures we shall have $n_n^S \propto \exp(-\Delta/T)$ and $\tau_s^{-1} \propto \tau_0^{-1} + A\exp(-2\Delta/T)$. Noting the different numerical factors in the exponents, we see that in the clean limit $(\tau_0^{-1} \rightarrow 0)$ the contribution to σ_n is *infinite*. The situation is actually more delicate, because these formulas are predicated on the existence of a clean gap Δ at nonzero T, whereas the evolution of the gap is continuous. This nevertheless explains the underlying physics and is a useful approximate form, particularly since a microscopic theory for the spectrum is lacking.

As we have stressed above, the difference between the behavior of T_1^{-1} and the low-frequency conductivity σ_1 [recall the difference between Eqs. (11) and (13)] is caused by the strong temperature dependence of the relaxation time and is not associated with coherence factors. The same comparison occurs between thermal conductivity K and (longitudinal) ultrasonic attenuation rate α for a short-wavelength mode $(q > \omega/v_F)$. Both have type-I coherence factors (so on coherence peak is expected), but K depends on a suitably averaged scattering rate. Figure 11 compares calculations of the two quantities; our model predicts a peak in K, which arises solely from the temperature dependence of the lifetime. It is well established that there is indeed a peak in the measured thermal conductivity.[88] This has often been ascribed to the phonon thermal conductivity (arising from the increased phonon mean free path in the superconductor), but our analysis suggests that this assumption is questionable. The important point is that the optical conductivity requires a decreased electron-electron scattering rate below T_c. The same variation in scattering rate must also contribute to a bump in the thermal conductivity as well.

We should note that the present model does not produce the nearly constant contribution to T_1^{-1} in the normal state, which has been attributed either to additional contributions to the polarizability[24,36] or to features in the band structure[28–30] possibly enhanced by magnetic couplings.[27,89,90] Models in which this contribution is found also predict that it rapidly decreases below T_c. Inclusion of such contributions in the calculations here would give a sharper decrease in T_1^{-1} below T_c than represented in Fig. 9; they would not alter the results for other (non-spin-dependent) correlation functions calculated here.

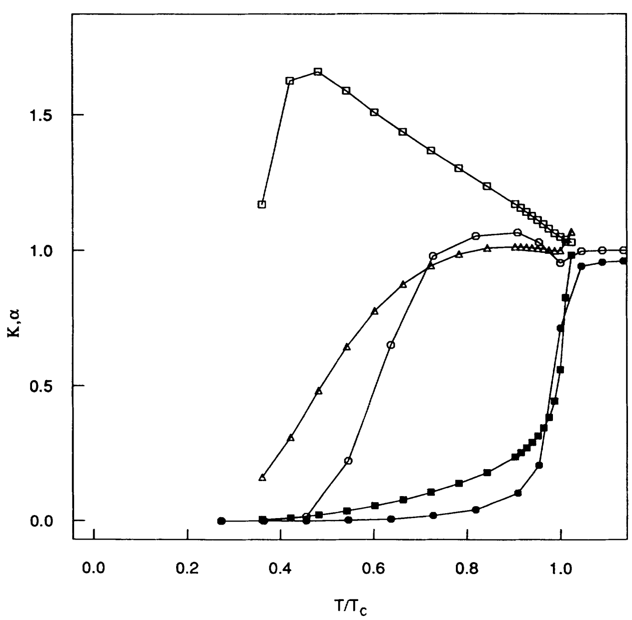

FIG. 11. Thermal conductivity (open symbols) and ultrasonic attenuation (solid symbols) calculated for model B ($\omega_0 = 1$). $\lambda^+ = 0.3$, $\lambda^- = 0.3$, $\tau_0^{-1} = 0$ (squares); $\lambda^+ = 0.3$, $\lambda^- = 0.3$, $\tau_0^{-1} = 0.1$ (triangles); $\lambda^+ = 0.6$, $\lambda^- = 0.3$, $\tau_0^{-1} = 0$ (circles).

In the Introduction we remarked that our neglect of vertex corrections could be justified only for response function near $q = 0$, although the noninteracting "bubble" could give the dominant contribution in regions of q space selected by band structure. The analysis of this paper has completely neglected band-structure effects, which can be very important in two dimensions. As noted elsewhere,[30] the band-structure susceptibility $\mathrm{Im}[\chi_0(q,\omega \rightarrow 0)]$ is strongly q dependent in two dimensions, even when $\mathrm{Re}[\chi_0(q,0)]$ is flat (i.e., there are no strong nesting features). It turns out that the inclusion of trivial band-structure effects, along with the anomalous self-energy corrections, is consistent with the measured neutron structure factor and NMR relaxation rates in the normal state of $La_{2-x}Sr_xCuO_4$.[30] Although the neutron scattering shows sharp incommensurate peaks, these are not necessarily indicative of strong magnetic interactions, and the correlation length may be short. The enhancements found for T_1^{-1} in the normal state are closely associated with contributions within $\sim T$ of the Fermi surface and will certainly be reduced below T_c; quantitative calculations are currently in progress and are planned to be reported elsewhere.

We also remarked that it is hard to imagine a theory for the self-energy in which the four-point vertex function is not singular for most momenta. Thus there will be an additional contribution to the finite-q correlation function of the form $(\omega/T)\mathrm{Re}[\chi_0(q)]$, which is, however, not a strong function of momentum. Such a contribution would be more prominent in local probes (e.g., NMR) than in neutron scattering. The contribution of this term to T_1^{-1} is then constant in the normal state,[24] but rapidly reduced in the superconductor when there is a gap.[36] All this may be summarized by stating that the actual T_1^{-1} in a better theory will almost certainly fall off more rapidly than in the calculations presented here.

Other approaches[90,27] have used antiferromagnetic fluctuations to obtain the enhanced NMR relaxation rates above T_c. To the extent that $\mathrm{Re}[\chi_0(q)]$ is changed on entering the superconducting state, this enhancement will be reduced because the Stoner factor is changed. Since, however, the effects of superconductivity on $\mathrm{Re}[\chi_0(q)]$ are limited to $q < \xi^{-1}$, where ξ is the superconducting coherence length, this effect will only be important for the q-averaged response if ξ is very small.

V. CONCLUSIONS

We have sought to extend the MFL phenomenology from the normal to a superconducting state of s-wave symmetry, by self-consistently incorporating a gap of 2Δ into the scattering spectrum. While details depend upon the exact specification, there are some general features which are robust and which would be true for any model where low-energy pair breaking is suppressed in the superconductor and the quasiparticle relaxation rate decreases.

By assuming $\lambda^+ > \lambda^-$, we have tacitly assumed that the pairing arises from the charge channel. Of course, if large momentum spin fluctuations were indeed dominant, one would expect to have higher angular momentum pairing and, therefore, no complete gap in the spectrum. This would suppress many of the effects due to increased quasiparticle lifetime. An overall comparison of the calculations with experiments appears to favor model B of Sec. II, with $\lambda^+ \approx 0.6$, $\lambda^- \approx 0.3$, and a gap ratio $2\Delta/k_B T_c \approx 6$.

The principal qualitatively different features of the superconducting state in the cuprates come from the sharp reduction of the electron-electron scattering rate in the passage from the normal to the superconductive state. This introduces features not seen in BCS *s*-wave electron-phonon-induced superconductors.

In the single-particle spectrum, the quasiparticle character is restored for states whose energies $|E_k - E_F| < 3\Delta$, and this leads to features at both Δ and 3Δ in single-particle spectroscopy (i.e., tunneling and photoemission). Furthermore, because of the strong anisotropy of the material, tunneling measurements in different orientations can have very different characteristics, with apparently different "gaps" depending upon orientation and momentum transfer.

In the optical conductivity the onset of "free-carrier" absorption will be 4Δ at low temperature (unless there are strong final-state effects), with a 2Δ feature induced by disorder and other scattering mechanisms. Strong pair breaking near T_c suppresses the coherence peak in the NMR relaxation rate T_1^{-1}. However, the increase in the quasiparticle lifetime below T_c may induce a peak in the conductivity at low frequencies $\omega \ll 2\Delta$, which is not due to coherence factors; a similar mean-free-path effect would also enhance the electronic thermal conductivity below T_c. On a larger scale, the "gap" sets in at an almost temperature-independent value. For the same reason that the microwave conductivity has a peak whereas T_1^{-1} does not, we also predict a peak in the electronic thermal conductivity below T_c, arising solely from lifetime effects.

Finally, we stress that all of these features are generic and are not particularly contingent on the details of the marginal-Fermi-liquid phenomenology. Provided the quasiparticle lifetime in the normal state is dominated by electronic scattering processes and the superconducting state has a gap, these qualitative results are expected. As far as possible, we have tried to stress phenomenology and to avoid microscopic perambulations. When a microscopic theory becomes available, some of these results may need to be revisited.

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