Local spin-wave excitations in ferromagnetic electron-gas superlattices

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(Received 24 July 1991; revised manuscript received 2 March 1992)

The spin-wave excitations are studied within a factorization approximation for ferromagnetic-electron-gas superlattices. An equation for determining the excitations is obtained. For a superlattice containing a defect layer whose charge (spin) density of the electrons and their effective mass, for instance, are different from those in the other layer planes of the superlattice, we find, in addition to the bulk spin-wave modes, another spin-wave mode localized by this defect layer.

Over the last decade the problem of the charge density excitations of a layered electron gas, in particular the plasma oscillations and the associate dielectric screening, has been extensively studied and the plasmon dispersion relations have been predicted theoretically and observed experimentally by many authors,[1–22] while there are fewer reports on the problem of other types of excitations of a layered electron gas.[23–28] The spin excitations in periodic, layered ferromagnetic electron gases have been studied by Gasser,[24,28] and by Zhou and Gong.[27] The purpose of this paper is to investigate the spin-wave excitations in ferromagnetic-electron-gas superlattices containing a defect layer. We will apply the equation-of-motion method[29] to determine the spin-wave excitation spectrum within a factorization approximation similar to the Hubbard approximation,[30] since this method is one of the most powerful techniques in studying the collective modes. The equation for determining the excitations is obtained and is used to calculate the dispersion relations of spin-wave excitations in the long-wavelength limit. We find, in addition to the bulk spin-wave excitations, that there exists another kind of spin-wave mode localized by the defect layer, which we call local spin-wave excitation in this paper.

The system we shall discuss consists of an infinite number of periodically arranged layers of quasi-two-dimensional (quasi-2D) ferromagnetic electron gases, but in which one layer (called defect layer) is different from all the same other layers. The electrons are free to move within the x-y plane but are subject to a potential in the z direction. Limited to the lowest miniband, the single-electron wave function may be given as

$$\psi_{\mathbf{k}l}(\mathbf{r},z-ld)=S^{-1/2}\exp(i\mathbf{k}\cdot\mathbf{r})\xi_l(z-ld)\ , \tag{1}$$

where $\mathbf{k}$ and $\mathbf{r}$ are the momentum and coordinate in the electron layer planes of area S, respectively, and $\xi_l(z-ld)$ is the Wannier function centered at the lth plane with coordinate ld along the z axis. For simplicity, we may assume $|\xi_l(l-zd)|^2$ to be a δ function located at $z=ld$, without loss of generality. Changing $\xi_l(z-ld)$ to more complicated localized functions does not change the conclusions of this paper in any qualitative fashion. Therefore, we can neglect the overlap of $\psi_{\mathbf{k}l}$'s in the z direction. The system Hamiltonian can be expressed as

$$H=\sum_{\mathbf{k},l,\sigma}\varepsilon_{\mathbf{k}l}\hat{C}^{\dagger}_{\mathbf{k}l\sigma}\hat{C}_{\mathbf{k}l\sigma}+\frac{1}{2S}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q},l,l',\sigma,\sigma'}V_{ll'}(q)\hat{C}^{\dagger}_{\mathbf{k}+\mathbf{q}l\sigma}\hat{C}_{\mathbf{k}'-\mathbf{q}l'\sigma'}\hat{C}_{\mathbf{k}'l'\sigma'}\hat{C}_{\mathbf{k}l\sigma}\ , \tag{2}$$

where

$$\varepsilon_{\mathbf{k}l}=\varepsilon_{\mathbf{k}}+\delta\varepsilon_{\mathbf{k}}\delta_{l,0}=\varepsilon_{\mathbf{k}}+(\varepsilon'_{\mathbf{k}}-\varepsilon_{\mathbf{k}})\delta_{l,0} \tag{3}$$

is the dispersion relation of electrons confined in the lth layer and

$$V_{ll'}(q)=\frac{2\pi e^2}{\epsilon q}\exp(-q|ld-l'd|) \tag{4}$$

denotes the Coulomb potential between electrons. In Eq. (2) $\hat{C}^{\dagger}_{\mathbf{k}l\sigma}$ ($\hat{C}_{\mathbf{k}l\sigma}$) is the electron creation (annihilation) operator corresponding to the single electron state $\psi_{\mathbf{k}l}$ with spin σ, and ϵ is the background dielectric constant. The defect layer is denoted by $l=0$ where some physical quantities, such as the charge (spin) density of electrons n' (M') and their effective mass m', etc., may be different from those, i.e., n (M) and m, etc., in other layers of such a system. The electron spin fluctuation operator of the system is[30]

$$\hat{S}(\mathbf{r},z)=\sum_{\mathbf{k},\mathbf{k}',l,l'}\psi^{*}_{\mathbf{k}l}(\mathbf{r},z-ld)\psi_{\mathbf{k}'l'}(\mathbf{r},z-l'd)\hat{C}^{\dagger}_{\mathbf{k}l\downarrow}\hat{C}_{\mathbf{k}'l'\uparrow}\ . \tag{5}$$

Its Fourier transform, defined by

$$\hat{S}(Q)=\int dr\int dz\,\exp(-i\mathbf{q}\cdot\mathbf{r}-iq_z z)\hat{S}(\mathbf{r},z)\ ,$$

can be written as

$$\begin{aligned}\hat{S}(Q)&=\sum_{\mathbf{k},l}\hat{C}_{\mathbf{k}+\mathbf{q}l\downarrow}\hat{C}_{\mathbf{k}l\uparrow}\exp(-iq_z ld)\ ,\\&=\sum_{K}\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{K\uparrow}\ ,\end{aligned} \tag{6}$$

where $K\equiv(\mathbf{k},k_z)$, and the operators $\hat{C}^{\dagger}_{K\sigma}$, $\hat{C}_{K\sigma}$ are given by

$$C^{\dagger}_{\mathbf{k}l\sigma}=\frac{1}{\sqrt{N}}\sum_{k_z}\hat{C}^{\dagger}_{K\sigma}\exp(+ik_z ld),\quad \hat{C}_{\mathbf{k}l\sigma}=\frac{1}{\sqrt{N}}\sum_{k_z}\hat{C}_{K\sigma}\exp(-ik_z ld)\ , \tag{7}$$

with N the number of the layers.

We now consider the equation of the operator $\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{K\uparrow}$. One finds

$$\begin{aligned}[\hat{H}-\mu\hat{N},\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{K\uparrow}]=&(\varepsilon_{\mathbf{k+q}}-\varepsilon_{\mathbf{k}})\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{K\uparrow}+\frac{1}{N}\sum_{k'_z}(\delta\varepsilon_{\mathbf{k+q}}\hat{C}^{\dagger}_{\mathbf{k+q},k'_z\downarrow}\hat{C}_{K\uparrow}-\delta\varepsilon_{\mathbf{k}}\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{\mathbf{k},k_z\uparrow})\\&+\frac{1}{SN}\sum_{K',Q',\sigma}V(Q')(C^{\dagger}_{K+Q+Q'\downarrow}\hat{C}^{\dagger}_{K'-Q'\sigma}\hat{C}_{K'\sigma}\hat{C}_{K\uparrow}-\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}^{\dagger}_{K'-Q'\sigma}\hat{C}_{K'\sigma}\hat{C}_{K-Q'\uparrow})\ ,\end{aligned} \tag{8}$$

in which

$$V(Q)=\frac{2\pi e^2}{\epsilon q}\frac{\sinh(qd)}{\cosh(qd)-\cos(q_z d)}\ . \tag{9}$$

We have added the term $-\mu\hat{N}$ to $\hat{H}$, where μ is the chemical potential or, what is equivalent, the energy of the electrons at Fermi surface. In the random phase approximation (RPA), we then get from (8) that

$$\begin{aligned}&[\hbar\omega-(\varepsilon_{\mathbf{k+q}}-\varepsilon_{\mathbf{k}})]\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{K\uparrow}-\frac{1}{N}\sum_{k'_z}(\delta\varepsilon_{\mathbf{k+q}}\hat{C}_{\mathbf{k+q},k'_z\downarrow}\hat{C}_{K\uparrow}-\delta\varepsilon_{\mathbf{k}}\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{\mathbf{k},k'_z\uparrow})\\&\quad=\frac{1}{S}\sum_{Q'}V(Q')[(n_{\mathbf{k+q}\downarrow}-n_{\mathbf{k}\uparrow})\hat{C}^{\dagger}_{K+Q+Q'\downarrow}\hat{C}_{K+Q'\uparrow}+(n_{\mathbf{k+q'}\uparrow}-n_{\mathbf{k+q+q'}\downarrow})\hat{C}_{K+Q\downarrow}\hat{C}_{K\uparrow}]\\&\qquad+\frac{1}{SN}\sum_{Q'k'_z}V(Q')[(\delta n_{\mathbf{k+q}\downarrow}\hat{C}^{\dagger}_{\mathbf{k+q+q'},k'_z\downarrow}\hat{C}_{K+Q'\uparrow}-\delta n_{\mathbf{k}\uparrow}\hat{C}^{\dagger}_{K+Q+Q'\downarrow}\hat{C}_{\mathbf{k+q'},k'_z\uparrow}\\&\qquad+(\delta n_{\mathbf{k+q'}\uparrow}\hat{C}^{\dagger}_{K+Q\downarrow}\hat{C}_{\mathbf{k},k'_z\uparrow}-\delta n_{\mathbf{k+q+q'}\downarrow}\hat{C}^{\dagger}_{\mathbf{k+q},k'_z\downarrow}\hat{C}_{K\uparrow})]\ .\end{aligned} \tag{10}$$

In obtaining (10), the following relation has been used:

$$\langle\hat{C}^{\dagger}_{\mathbf{k}l\sigma}\hat{C}_{\mathbf{k}l'\sigma}\rangle=n_{l\sigma}(\mathbf{k})\delta_{l,l'}=[n_{\mathbf{k}\sigma}+(n'_{\mathbf{k}\sigma}-n_{\mathbf{k}\sigma})\delta_{l,0}]\delta_{l,l'}\ . \tag{11}$$

Now we further use a factorization approximation similar to the Hubbard approximation.[30] The Hubbard approximation has been widely used in studies of dielectric screening and plasmons of electron gases and subsequently used by Zhou and Gong[27] to consider the spin excitations of a multilayered ferromagnetic electron gas. It should be mentioned that since certain terms in Wick's theorem have been neglected in this approximation, the influence of the anisotropy of the Coulomb interaction on the spin-wave spectra is overestimated and the results of the dynamical local-field theories[31] should be more reliable, as pointed out by Gasser.[28] But for our present problem it is difficult to get an analytical result from the method of the dynamical local-field theories, and so we use the simple factorization approximation, i.e., the factor $V(Q')$ in the summation of Eq. (10) is replaced by a screened potential $U(Q)$. On the basis of general arguments, we know that $U(Q)$ must tend to a constant as $Q\rightarrow O$ and be vanishingly small as $Q\rightarrow\infty$. Supposing that $U(Q)$ takes the following form:

$$\begin{aligned}U(Q)&=\sum_{l-l'=-\infty}^{\infty}V_{ll'}(q+q_{\parallel})\exp[-(q_{\perp}-q_{\parallel})|ld-l'd|]\exp[-iq_z(ld-l'd)]\ ,\\&=\frac{2\pi e^2}{\epsilon(q+q_{\parallel})}\frac{\sinh(qd+q_{\perp}d)}{\cosh(qd+q_{\perp}d)-\cos(q_z d)}\ ,\end{aligned} \tag{12}$$

with $q_{\parallel}$ and $q_{\perp}$ the nonzero screening wave numbers in the directions parallel and perpendicular to the two-dimensional charge layer planes, we can then solve our problem analytically. After a lengthy but straightforward standard calculation, we finally arrive at the following equation:

$$\sum_{l'}\left|\{[\hbar\omega-(\varepsilon_{\mathbf{k+q}l}-\varepsilon_{\mathbf{k}l})]\delta_{l,l'}-U_{ll'}(q)M_{l'}\}\hat{S}_{l'}(\mathbf{k},\mathbf{q})-U_{ll'}(q)[n_{l'\downarrow}(\mathbf{k+q})-n_{l'\uparrow}(\mathbf{k})]\frac{1}{S}\sum_{\mathbf{k}}\hat{S}_{l'}(\mathbf{k},\mathbf{q})\right|=0\ , \tag{13}$$

where

$$U_{ll'}(q)=\frac{1}{N}\sum_{q_z}U(Q)\exp[+iq_z(ld-l'd)] ,$$

$$\hat{S}_l(\mathbf{k},\mathbf{q})=\sum_{k_z,q_z}\hat{C}^{\dagger}_{K+Q\uparrow}\hat{C}_{K\uparrow}\exp(-iq_zld) ,$$

and

$$M_l=\frac{1}{S}\sum_{\mathbf{k}}[n_{l\uparrow}(\mathbf{k})-n_{l\downarrow}(\mathbf{k})]=M+(M'-M)\delta_{l,0} \quad (14)$$

is the spin density in the lth layer. We should point out here that, if ε_l, n_l, and M_l are regarded as the relevant physical quantities of the electrons in the lth layer, the expression (13) is applicable to layered-ferromagnetic-electron-gas systems in a general sense.

Obviously, if there are no defect layers, i.e., $\varepsilon'_{\mathbf{k}}-\varepsilon_{\mathbf{k}}=n'_{\mathbf{k}\sigma}-n_{\mathbf{k}\sigma}=M'-M=0$, Eq. (13) reduces exactly to the results obtained for a perfect periodically arranged ferromagnetic superlattice. The spin excitations are given from (13)

$$1-U(Q)\frac{1}{S}\sum_{\mathbf{k}}\frac{n_{\mathbf{k}+\mathbf{q}\downarrow}-n_{\mathbf{k}\uparrow}}{\hbar\omega-(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})-U(Q)M}=0 . \quad (15)$$

Clearly, the spin-wave excitations $\omega_s(Q)$ exist for $\omega\leq\omega_{\min}$, where $\omega_{\min}$ is the minimum excitation energy of the Stoner excitations. Detailed investigation of the dispersion relations indicates that the interlayer Coulomb interaction modifies the simple two-dimensional spin-wave spectrum significantly. The whole spectrum forms a spectral band restricted to lie between a highest branch ω_H $[\equiv\omega_s(q_z=0)]$ and a lowest branch ω_L $[\equiv\omega_s(q_z=\pm\pi/d)]$ which correspond, respectively, to the in-phase and out-of-phase oscillations of spins in adjacent planes. In the limit of small q and ω, or more specifically

$$\hbar\omega, |\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}| \ll MU(Q) , \quad (16)$$

we obtain from (15)

$$\hbar\omega_s(Q)=\left[n-\frac{2\Delta}{U(Q)}\right]\frac{\varepsilon_q}{M} , \quad (17)$$

where n and Δ are given by

$$n_l=\frac{1}{S}\sum_{\mathbf{k}}[n_{l\uparrow}(\mathbf{k})+n_{l\downarrow}(\mathbf{k})]=n+(n'-n)\delta_{l,0} , \quad (18)$$

$$\Delta_l=\frac{1}{SM_l}\sum_{\mathbf{k}}[n_{l\uparrow}(\mathbf{k})-n_{l\downarrow}(\mathbf{k})]\varepsilon_{\mathbf{k}l}=\Delta+(\Delta'-\Delta)\delta_{l,0} . \quad (19)$$

Equation (17) shows clearly that the spin waves display appropriate crossover behavior, i.e., from two- to three-dimensional behavior, as the coupling between adjacent layers is increased by decreasing the parameter qd, which is analogous to the plasmon spectral band of the layered electron gases.[14]

In the case of a layered ferromagnetic system containing a defect layer, Eq. (13) can be written as

$$\hat{S}_l(\mathbf{k},\mathbf{q})-\sum_{l''}g_{ll''}(\mathbf{k},\mathbf{q})\sum_{l'}U_{l''l'}(q)[n_{l'\downarrow}(\mathbf{k}+\mathbf{q})-n_{l'\uparrow}(\mathbf{k})]\frac{1}{S}\sum_{\mathbf{k}}\hat{S}_{l'}(\mathbf{k},\mathbf{q})=0 , \quad (20)$$

where $g_{ll'}(\mathbf{k},\mathbf{q})$ satisfies

$$\sum_{l'}\{[\hbar\omega-(\varepsilon_{\mathbf{k}+\mathbf{q}l}-\varepsilon_{\mathbf{k}})]\delta_{l,l'}-U_{ll'}(q)M_{l'}\}g_{l'l''}(\mathbf{k},q)=\delta_{l,l''} , \quad (21)$$

which can be easily solved together with Eqs. (3) and (14):

$$g_{ll'}=\frac{1}{N}\sum_{q_z}\frac{f_1(\mathbf{k},Q)}{\hbar\omega-(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})-MU(Q)}\exp[+iq_z(ld-l'd)] , \quad (22)$$

with

$$f_l(\mathbf{k},Q)=1+\frac{\exp(-iq_zld)\frac{1}{N}\sum_{q_z}\frac{\delta\varepsilon_{\mathbf{k}+\mathbf{q}}-\delta\varepsilon_{\mathbf{k}}+U(Q)(M'-M)}{\hbar\omega-(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})-U(Q)M}\exp(+iq_zld)}{1-\frac{1}{N}\sum_{q_z}\frac{\delta\varepsilon_{\mathbf{k}+\mathbf{q}}-\delta\varepsilon_{\mathbf{k}}+U(Q)(M'-M)}{\hbar\omega-(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})-U(Q)M}} . \quad (23)$$

Then from Eq. (20) we have

$$\det\left[\delta_{l,l'}-\frac{1}{S}\sum_{\mathbf{k}}\sum_{l''}g_{ll''}(\mathbf{k},\mathbf{q})U_{l''l'}(q)[n_{l'\downarrow}(\mathbf{k}+\mathbf{q})-n_{l'\uparrow}(k)]\right]=0 . \quad (24)$$

Using (22) in (24) we obtain

$$\det\left[\delta_{l,l'}-\frac{1}{SN}\sum_{\mathbf{k}q_z}f_l(\mathbf{k},Q)U(Q)\exp[+iq_z(l-l')d]\frac{n_{l'\downarrow}(\mathbf{k}+\mathbf{q})-n_{l'\uparrow}(k)}{\hbar\omega-(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})-U(Q)M}\right]=0 . \quad (25)$$

Equation (25) may have two kinds of solutions corresponding to the bulk and local spin-wave excitations, respectively. Obviously, for an infinite volume of the system, $\hbar\omega=\hbar\omega_s(Q)$ is actually the solution of Eq. (25). These modes are the bulk spin-wave excitations discussed above. On the other hand, drawn on experience of studying the density excitation spectrum of the electron gases, it is clear that a general discussion of Eq. (25) is a very difficult task for both kinds of excitations, especially for local modes. However, in the limit of small q and ω, some conclusions may be drawn for the local spin-wave excitation spectrum (the bulk modes have been discussed above). In this limit, Eq. (25) becomes

$$\frac{1}{N}\sum_{q_z}\frac{\omega_s'(Q)-\omega_s(Q)}{\omega-\omega_s(Q)}=1\ , \tag{26}$$

where

$$\hbar\omega_s'(Q)=\left[n'-\frac{2\Delta'}{U(Q)}\right]\frac{\varepsilon_q'}{M'}\ , \tag{27}$$

The solutions of Eq. (26) are schematically shown in Fig. 1. The shaded part in Fig. 1 represents the region of bulk spin-wave excitations which formed a continuous spectral band. The lowest and highest branches of the bulk spin-wave excitation spectral band ω_L and ω_H are given in the small q and ω limit, respectively, by

$$\hbar\omega_L(\mathbf{q})=\left[n-\frac{\epsilon\Delta(q+q_\parallel)\cosh(qd+q_\perp d)/2}{\pi e^2\sinh(qd+q_\perp d)/2}\right]\frac{\varepsilon_q}{M}\ , \tag{28}$$

$$\hbar\omega_H(q)=\left[n-\frac{\epsilon\Delta(q+q_\parallel)\sinh(qd+q_\perp d)/2}{\pi e^2\cosh(qd+q_\perp d)/2}\right]\frac{\varepsilon_\mu}{M}\ . \tag{29}$$

We note that the lowest branch ω_L and the highest branch ω_H correspond to the out-of-phase ($q_z=\pm\pi/d$) and in-phase ($q_z=0$) oscillations of spins in adjacent planes of the perfect systems (discussed above), respectively. In the weak-coupling limit, i.e., $qd\gg1$, one has the same limit for ω_L and ω_H,

$$\hbar\omega_L,\hbar\omega_H\to\hbar\omega_s(qd\to\infty,q_z=0)=\left[n-\frac{\epsilon\Delta q}{\pi e^2}\right]\frac{\varepsilon_q}{M}\ , \tag{30}$$

showing a two-dimensional behavior. In the intermediate coupling limit, i.e., $qd\ll1$, one has the highest branch

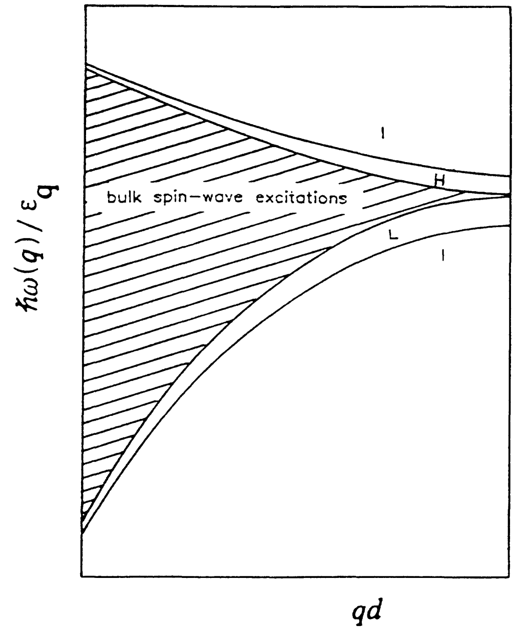

FIG. 1. Schematic drawing of the spin-wave excitations in a ferromagnetic electron-gas superlattice containing a defect layer for large M and M'. L, H, and l signify $\omega_L(q)$, $\omega_H(q)$, and $\omega_l(q)$, respectively. The hatched region corresponds to the spectrum of bulk spin-wave excitations restricted to lie between the lowest branch ω_L and the highest branch ω_H. The local spin-wave mode ω_l lies either below or above the band for $\omega_L'(q)<\omega_L(q)$ or for $\omega_H'(q)>\omega_H(q)$, respectively.

$$\hbar\omega_H\to\hbar\omega_s(qd\to0,q_z=0)=\left[n-\frac{\epsilon\Delta q_\parallel\sinh(q_\perp d/2)}{\pi e^2\cosh(q_\perp d/2)}\right]\frac{\varepsilon_q}{M}\ , \tag{31}$$

and the lowest branch

$$\hbar\omega_L\to\hbar\omega_s(qd\to0,q_z=\pm\pi/d)=\left[n-\frac{\epsilon\Delta q_\parallel\cosh(q_\perp d/2)}{\pi e^2\sinh(q_\perp d/2)}\right]\frac{\varepsilon_q}{M}\ . \tag{32}$$

In addition, we note that in Fig. 1 there is an isolated solution ω_l either below the band or above the band, which describes a spin-wave excitation localized by the defect layer. For small q and ω, this local spin-wave mode is given by

$$\hbar\omega_l(q)=\left[\frac{(\Delta'mM-\Delta m'M')n'+\Delta'm'M'n}{2\Delta'mM-\Delta m'M'}-\frac{\epsilon\Delta'(q+q_\parallel)\cosh(qd+q_\perp d)}{\pi e^2\sinh(qd+q_\perp d)}\times\left\{\frac{\Delta'mm}{2\Delta'mM-\Delta m'M'}\pm\left[\left[\frac{\dfrac{\pi e^2\sinh(qd+q_\perp d)}{\epsilon(q+q_\parallel)\cosh(qd+q_\perp d)}(mMn'-m'M'n)-\Delta'mM+\Delta m'M'}{2\Delta'mM-\Delta m'M'}\right]^2+\frac{\Delta m'M'}{(2\Delta'mM-\Delta m'M')\cosh^2(qd+q_\perp d)}\right]^{1/2}\right\}\right]\frac{\varepsilon_q'}{M'}\ . \tag{33}$$

In obtaining Eq. (33) we have let $\varepsilon_{\mathbf{k}}/\varepsilon'_{\mathbf{k}}=m'/m$ for simplicity. For $\omega'_L(q)<\omega_L(q)$ the mode $\omega_l(<\omega_L)$ is located below the spectral band of the bulk modes, whereas for $\omega'_H(q)>\omega_H(q)$ the mode $\omega_l(>\omega_H)$ is located above the bulk modes.

Equation (33) gives the local mode in the small-q and -ω limit, but with qd as an arbitrary parameter. In the following, we extend our discussions for the two extreme cases. In the weak-coupling limit, i.e., $qd\gg1$, taking $qd\to\infty$, from (33) one immediately finds

$$\hbar\omega_l(qd\to\infty)=\hbar\omega'_s(qd\to\infty) .$$

Here we obtain the very satisfying intuitive results that in the weak-coupling limit the mode ω_l is simply the two-dimensional spin-wave mode of the defect layer. In the intermediate limit, i.e., $qd\gg1$, for $\omega'_H(q)>\omega_H(q)$, we find from (33) the local mode above the mode $\hbar\omega_H(qd\to0)$, and for $\omega'_L(q)<\omega_L(q)$ the local mode below the mode $\hbar\omega_L(qd\to0)$.

In summary, we have studied the spin-wave excitations of a ferromagnetic electron-gas superlattice within the factorization approximation and have found, for a ferromagnetic superlattice containing a defect layer, a spin-wave mode localized by the defect layer, which exists along with the bulk spin-wave spectral band. An experimental investigation is expected to get a better understanding of the nature of the spin-wave excitations in such a system. We hope our work will generate more interest in this topic.

This work was supported in part by grants by the China National Science Foundation.

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