Microwave response of mesoscopic rings

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(Received 2 March 1992)

The nonlocal conductivity of a mesoscopic conductor manifests itself at nonzero frequencies. As a consequence of this, the microwave scattering intensity of a mesoscopic metal ring acquires a nontrivial frequency dependence if the frequency is comparable to the inverse of the diffusion time around the circumference of the ring.

One of the interesting transport properties of mesoscopic systems is the nonlocal character of the conductivity. Experimentally, the conductance of a multiprobe system has been found to depend sensitively on the configuration of the entire sample.[1,2] Magnetoconductance measurement on wires has revealed the nonlocal effect on the period of the magnetic-field dependence.[3] In discussing the voltage fluctuation problem it has been found that there can be an electric field in a voltage probe even though the current here is zero.[4,5] Kane, Serota, and Lee[5] point out that, to the lowest order of disorder, the ensemble-averaged dc conductivity consists of two parts: a local Drude term and a long-range diffusive term [see Eq. (11) below]. The long-range term does not contribute to the average dc conductance and so cannot be observed directly. It is nevertheless important in calculating the correlation of the conductivity tensor. Recently, it has become feasible to measure the conductance of mesoscopic samples at microwave frequencies.[6] We show below that a direct observation of the long-range part of the averaged conductivity is possible in this frequency region. This potentially provides another test of the theory. We shall concentrate on the nonlocal effects related to diffusion; the local, weak-localization-related aspects of the problem have been discussed in Ref. 6.

We consider the scattering of microwave by a mesoscopic ring. Unlike the problem of the persistent current,[7] which is an *equilibrium* property of the ring, here we study its high-frequency linear response to an external field. The incoming and scattered waves are assumed to be classical and therefore described by an amplitude and a phase. This corresponds to choosing coherent states for both the initial and final states of the scattering process. We find that due to the long-range nature of the conductivity tensor, the ensemble-averaged electric dipole ($E1$) mode of the scattered wave shows a significant frequency dependency; the other modes are affected to a lesser extent.

Throughout this paper we assume the amplitude of the electromagnetic (EM) wave to be low so that nonlinear effects may be neglected. For the same reason, any dephasing caused by the EM wave is assumed to be negligible.

We first review some properties of the electrical conductivity. To the lowest order of the disorder parameter $(k_F l)^{-1}$ (l is the mean free path) and within linear-response theory, the most important diagrams contributing to the ensemble-averaged conductivity are shown in Fig. 1. The Drude term [Fig. 1(a)] is short-ranged and extends up to a range of order l. On the other hand, the diffuson contribution [Fig. 1(b)] is long-ranged. Formally, the reason for the difference is that the impurity-averaged Green's functions themselves are short-ranged, whereas the diffuson ladder does not constrain the distance between its two end points. The cooperon contribution [Fig. 1(c)] enters in the next order in $(k_F l)^{-1}$ and is short-ranged. The long-range part does not contribute to the average conductance. This is because it is of the form of a total derivative and can therefore be converted to a boundary term. However, we show below that a direct observation of this long-range part is possible at high frequencies. To concentrate on this, we shall in our discussion neglect the cooperon part, which can be readily incorporated into the final result as a correction to the local conductivity.

Consider the elastic scattering of microwave by a mesoscopic ring as shown in Fig. 2. The z axis is chosen to be perpendicular to the ring. If the ring is sufficiently thin, one may neglect the component of $\mathbf{E}$ normal to the ring plane. We therefore assume the incoming E field $\mathbf{E}$ to be in the x direction, and the wave vector of the incoming wave to lie in the y-z plane. The corresponding vector potential is

$$\mathbf{A}(\mathbf{x})=\frac{c}{i\omega}e^{i\mathbf{k}_i\cdot\mathbf{x}-i\omega t}E\hat{\mathbf{x}}\ , \qquad (1)$$

where $\mathbf{k}_i=k(\sin\theta_i\hat{\mathbf{y}}+\cos\theta_i\hat{\mathbf{z}})$. Far away from the ring,

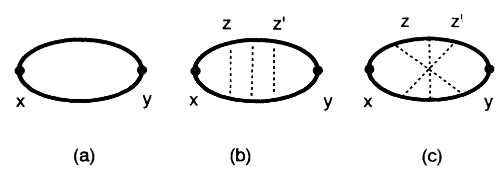

FIG. 1. Diagrams contributing to the conductivity tensor $\sigma_{\mu\nu}(x,y)$. (a) The Drude term is short-ranged because the averaged Green's function (the solid lines) are short-ranged; (b) the diffuson term is long-ranged as the distance between the impurities vertices (dashed lines) at the end points z and z is unconstrained; (c) the cooperon contribution is short-ranged: z and z' must both be within a mean free path from both x and y.

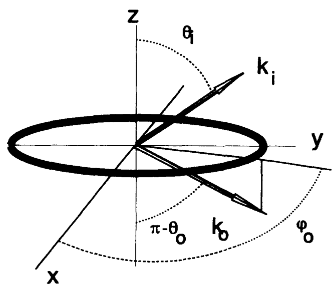

FIG. 2. Geometry of the setup.

the outgoing wave is determined by the current density in the ring

$$\mathbf{A}_0(\mathbf{R})=\frac{e^{ikR}}{cR}e^{-i\omega t}\int_{\text{ring}} d^3x\, \mathbf{j}(\mathbf{x})e^{-i\mathbf{k}_0\cdot\mathbf{x}} , \tag{2}$$

where $\mathbf{k}_0=k(\sin\theta_0\cos\phi_0\hat{\mathbf{x}}+\sin\theta_0\sin\phi_0\hat{\mathbf{y}}+\cos\theta_0\hat{\mathbf{z}})$. Typically in an experiment like this, the radius ρ_0 of the ring is about 10^{-6} m whereas both the height b and the width a of the cross section are less than 10^{-7} m. At microwave frequencies, we then expect $R \gg \lambda \gg \rho_0 > a,b$. This justifies the use of the multipole expansion in (2). The current is, to a good approximation, purely azimuthal:

$$\mathbf{j}(\mathbf{x})=\sum_{n=-\infty}^{\infty} j_n(\rho,z)e^{in\phi}\hat{\mathbf{e}}_\phi . \tag{3}$$

Here j_0 corresponds to a uniform current and so contributes to the magnetic dipole ($M1$) radiation, j_1 and j_{-1} correspond to a dipolar flow and contribute to the $E1$ mode, and the other j_n's can be similarly interpreted. Using (2) and (3), we get

$$\mathbf{A}_{M1}(\mathbf{R})=i\alpha(k_{0y}\hat{\mathbf{x}}-k_{0x}\hat{\mathbf{y}})\rho_0 I_0 , \tag{4}$$

$$\mathbf{A}_{E1}(\mathbf{R})=\alpha\{(i\hat{\mathbf{x}}+\hat{\mathbf{y}})I_{-1}+(-i\hat{\mathbf{x}}+\hat{\mathbf{y}})I_1\} , \tag{5}$$

and similarly for higher modes. We have introduced $\alpha\equiv\pi(e^{ikR}/cR)e^{-i\omega t}$ and

$$I_n\equiv\int d\rho\int dz\,\rho j_n(\rho,z)$$
$$=\frac{1}{2\pi}\int d^3x\, e^{-in\phi}j_\phi(\mathbf{x}) . \tag{6}$$

The experimentally measured E and H fields are

$$H=\frac{1}{c}\dot{\mathbf{A}}\times\hat{\mathbf{k}}_0 ,\quad \mathbf{E}=\mathbf{H}\times\hat{\mathbf{k}}_0 . \tag{7}$$

It can be seen that for each mode, $\mathbf{E}$ and $\mathbf{H}$ are linear combinations of I_n and I_{-n}. For example, for the $E1$ mode, in which we are particularly interested, we have

$$\mathbf{H}_{E1}=-i\alpha k(\cos\theta_0\hat{\mathbf{x}}-i\cos\theta_0\hat{\mathbf{y}}-\sin\theta_0 e^{-i\phi_0}\hat{\mathbf{z}})I_{-1}-i\alpha k(\cos\theta_0\hat{\mathbf{x}}+i\cos\theta_0\hat{\mathbf{y}}-\sin\theta_0 e^{i\phi_0}\hat{\mathbf{z}})I_1 , \tag{8}$$

$$\mathbf{E}_{E1}=\alpha k\{(-1+\sin^2\theta_0\cos\phi_0 e^{-i\phi_0})\hat{\mathbf{x}}+(i+\sin^2\theta_0\sin\phi_0 e^{-i\phi_0})\hat{\mathbf{y}}+\tfrac{1}{2}\sin(2\theta_0)e^{-i\phi_0}\hat{\mathbf{z}}\}I_{-1}$$
$$+\alpha k\{(1-\sin^2\theta_0\cos\phi_0 e^{i\phi_0})\hat{\mathbf{x}}+(i-\sin^2\theta_0\sin\phi_0 e^{i\phi_0})\hat{\mathbf{y}}+\tfrac{1}{2}\sin(2\theta_0)e^{-i\phi_0}\hat{\mathbf{z}}\}I_1 . \tag{9}$$

In a ring-array experiment the number of rings within an area of the size of microwave wavelength is large, and the overall response is given by the disorder-averaged conductivity. We shall therefore not address the question of sample-to-sample variations. Unlike the problem of persistent current in mesoscopic rings,[7] here the response does not depend sensitively on the total number of electrons on the ring, and we are allowed to use the grand canonical ensemble. The averaged conductivity tensor is

$$\langle\sigma_{\mu,\nu}(\mathbf{x},\mathbf{x}',\omega)\rangle=\frac{e^2}{8\pi m^2}\int dE\frac{f(E+\omega)-f(E)}{\omega}\lim_{\mathbf{x}_{1,2}\to\mathbf{x};\mathbf{x}'_{1,2}\to\mathbf{x}'}\left[\frac{\partial}{\partial x_{1\mu}}-\frac{\partial}{\partial x_{2\mu}}\right]$$
$$\times\left[\frac{\partial}{\partial x'_{2\nu}}-\frac{\partial}{\partial x'_{1\nu}}\right]\langle G^+(\mathbf{x}_1,\mathbf{x}'_1,E+\omega)G^-(\mathbf{x}'_2,\mathbf{x}_2,E)\rangle , \tag{10}$$

where f is the Fermi function and $\langle\cdots\rangle$ denotes the impurity average. As a result, at zero temperature,

$$\langle\sigma_{\mu\nu}(\mathbf{x},\mathbf{x}',\omega)\rangle=\frac{\sigma_0}{1-i\omega\tau}\delta_{\mu\nu}\bar{\delta}(\mathbf{x}-\mathbf{x}')$$
$$-\frac{\sigma_0 D}{(1-i\omega\tau)^4}\partial_{x_\mu}\partial_{x'_\nu}d(\mathbf{x},\mathbf{x}',\omega) , \tag{11}$$

where $\bar{\delta}(\mathbf{x}-\mathbf{x}')$ is a modified δ function that extends up to the mean free path[8] and $d(\mathbf{x},\mathbf{x}',\omega)$ is the rescaled diffuson propagator, τ is the mean free time, and $\sigma_0=e^2DN(\epsilon_F)$ with $N(\epsilon_F)$ the density of state at the Fermi energy and D the diffusion constant. The diffuson propagator satisfies the diffusion equation

$$\{-i\omega-D\partial_x^2\}d(\mathbf{x},\mathbf{x}',\omega)=\bar{\delta}(\mathbf{x}-\mathbf{x}') . \tag{12}$$

Since the mean free path is small compared with the dimension of the ring, we may take the $\bar{\delta}$ function to be the true δ function. Hence, $\langle j_\phi\rangle=\langle j_\phi^0\rangle+\langle j_\phi^d\rangle$ is given by

$$\langle j_\phi^0(\mathbf{x})\rangle=-\sin\phi\frac{\sigma_0 E}{1-i\omega\tau} , \tag{13}$$

$$\langle j_\phi^d(\mathbf{x})\rangle=\frac{\sigma_0 ED}{\rho_0^2(1-i\omega\tau)^4}\frac{\partial}{\partial\phi}\int_{\mathbf{x}'}e^{i\mathbf{k}_1\cdot\mathbf{x}'}\sin\phi'\frac{\partial}{\partial\phi}d(\mathbf{x},\mathbf{x}') . \tag{14}$$

For our purpose, we take $e^{i\mathbf{k}_i\cdot\mathbf{x}'}\simeq 1$. In a ring geometry, d is given by

$$d(\mathbf{x},\mathbf{x}')=\frac{1}{\Omega}\sum_{m=-\infty}^{\infty}\frac{1}{-i\omega+D\left[\dfrac{m}{\rho_0}\right]^2}e^{im(\phi-\phi')} , \qquad (15)$$

where Ω is the volume of the ring. Writing $I_n=I_n^0+I_n^d$, we finally get

$$\langle I_n\rangle=\frac{-\sigma_0E}{2\pi(1-i\omega\tau)}\int_{\mathbf{x}}e^{-in\phi}\sin\phi+\frac{\sigma_0ED}{2\pi\rho_0^2(1-i\omega\tau)^4}\int_{\mathbf{x},\mathbf{x}'}e^{-in\phi}\sin\phi'\times\frac{\partial^2}{\partial\phi\,\partial\phi'}d(\phi,\phi') , \qquad (16)$$

or

$$\langle I_n\rangle=0 , \quad n\neq 1,-1 , \qquad (17)$$

$$\langle I_{\pm1}\rangle=\frac{\mp\sigma_0E\Omega}{4\pi i(1-i\omega\tau)}\left[1-\frac{1}{(-i\omega\tau_D+1)(1-i\omega\tau)^3}\right] . \qquad (18)$$

Here $\tau_D\equiv\rho_0^2/D$ is of the order of the diffusion time around the circumference of the ring. Without making the $\exp(i\mathbf{k}_i\cdot\mathbf{x}')=1$ approximation, the nonlocal term contributes to I_n for $n\neq\pm1$ as well; these lead to higher-multipole scatterings and are small since the size of the ring is typically much smaller than the wavelength. In a typical sample at microwave frequencies, $\omega\tau\ll 1$, and the Drude frequency dependency can be neglected. Hence

$$\langle I_{\pm1}\rangle=\pm\frac{\sigma_0E\Omega}{4\pi}\frac{\omega\tau_D}{-i\omega\tau_D+1} . \qquad (19)$$

On the other hand, a frequency $\omega\sim\tau_D^{-1}$ can be readily achieved and thus Eq. (19) is amenable to experimental verification: The scattering intensity depends on ω and is proportional to $\omega^2[\omega^2+\tau_D^{-2}]^{-1}$. This is in addition to any frequency dependency coming from the weak-localization effect which, as mentioned above, is largely local. The cooperon and diffuson contributions can be further distinguished from each other by their different magnetic-field dependence: The cooperon term should exhibit the familiar $\Phi_0/2$ periodicity ($\Phi_0=hc/e$ is the flux quantum) whereas the diffuson term is essentially unaffected by a magnetic field.

We thank Professor J. C. Price for discussions of his experimental work which prompted this investigation. This work was supported in part by the ONR. Portions of the work were completed at the Massachusetts Institute of Technology; we are grateful to Professor P. A. Lee for hospitality.

[1]W. J. Skocpol, P. M. Mankiewich, R. E. Howard, L. D. Jackel, D. M. Tennant, and A. D. Stone, Phys. Rev. Lett. **58**, 2347 (1987).

[2]A. Benoit, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, Phys. Rev. Lett. **58**, 2343 (1987).

[3]C. P. Umbach, P. Santhanam, C. van Haesendonck, and R. A. Webb, Appl. Phys. Lett. **50**, 1289 (1987).

[4]M. Büttiker, Phys. Rev. B **35**, 4123 (1987).

[5]C. L. Kane, R. A. Serota, and P. A. Lee, Phys. Rev. B **37**, 6701 (1988); C. L. Kane, P. A. Lee, and D. P. DiVincenzo, *ibid.* **38**, 2995 (1988); see also S. Maekawa, Y. Isana, and H. Ebisawa, J. Phys. Soc. Jpn. **56**, 25 (1987).

[6]J. B. Pieper, J. C. Price, and J. M. Martinis, Phys. Rev. B **45**, 3857 (1992).

[7]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983). For a review of the use of the grand canonical ensemble in mesoscopic systems, and in particular on how to extract the thermodynamical properties of a system with a fixed number of particles, see B. Altshuler, in *Nanostructure and Mesoscopic Systems,* Proceedings of the International Symposium, Santa Fe, edited by W. P. Kirk and M. A. Reed (Academic, Boston, 1992).

[8]A quasi-δ function here ensures current conservation at zero frequency. In the literature a true δ function is often used even though the summation of the diffusion diagrams only provides a quasi-δ function. This causes no significant error as $d(x,y)$ is mainly used for $|x-y|\gg l$.