Conduction breakdown in lightly doped p-type germanium

R. N. Zitter and Xuesong Zhang
Physics Department, Southern Illinois University, Carbondale, Illinois 62901
(Received 26 September 1991; revised manuscript received 2 March 1992)

In Ge samples with net acceptor concentrations $\sim 10^{12}$ cm^{-3}, the hole concentration changes by four orders of magnitude in conduction breakdown at 4.2 K. The data are in good agreement with a hot-carrier model in which the process of impact ionization is in detailed balance with its inverse, Auger recombination.

INTRODUCTION

Semiconductors at low temperatures may have very few carriers because there is insufficient thermal energy to ionize donors or acceptors. Under an electric field, these carriers can gain enough energy to ionize impurities by impact, producing new carriers and non-Ohmic conduction. At a critical field, conduction breakdown occurs, where carrier concentration and current density increase by orders of magnitude for a relatively small increase in field, until ionization is complete.

Studies of the phenomenon span several decades.[1–8] Rate equation models have explained the major features semiquantitatively, but true quantitative analyses have been hampered by a lack of reliable expressions for the ionization and recombination coefficients. These are quite difficult to calculate from first principles, and various approximations have been employed, with results often differing from measurements by orders of magnitude. The fact that a hot-carrier distribution exists in breakdown is a further complication. In addition, the data in the literature do not give sufficient detail over the breakdown regime.

Here we report a complete set of measurements spanning the breakdown regime in lightly doped p-type germanium at 4.2 K, together with a theoretical model that is in good agreement with the data. The model is based on detailed balancing between impact ionization and its inverse, Auger recombination.

THEORY

In a p-type semiconductor with concentrations of holes p, acceptors N_A, and donors N_D, the concentration of neutral or un-ionized acceptors is N_A-N_D-p while that of ionized acceptors is N_D+p. The hole production rate can be characterized by the rate equation[1–8]

$$\frac{dp}{dt}=(A_1+A_2p)(N_A-N_D-p)-(B_1+B_2p)p(N_D+p)\ . \tag{1}$$

Here the coefficient A_1 represents the hole generation process involving phonon (or photon) absorption, A_2 represents a process where the ionization energy E_a is supplied by hole kinetic energy (impact ionization), while the recombination coefficients B_1 and B_2 represent the corresponding inverse processes; in particular, the B_2 term is known as Auger recombination. The coefficients generally depend on both carrier and lattice temperature.

In the steady state, (1) has the form

$$p(N_D+p)/(N_A-N_D-p)=(A_1+A_2p)/(B_1+B_2p) \tag{2}$$

and for thermal equilibrium at temperature T_0, detailed balancing[2,3] requires the relations

$$A_1(T_0)/B_1(T_0)=A_2(T_0)/B_2(T_0)=K(T_0)\ , \tag{3}$$

where $K(T_0)$, called the equilibrium constant, is given by

$$K(T_0)=(2\pi m^*kT_0/h^2)^{3/2}g^{-1}\exp(-E_a/kT_0)\ . \tag{4}$$

Here the first term in the preexponential factor is just the valence-band density of states, and g is the degeneracy factor of the acceptor state. It follows from (2) and (3) that

$$p_0(N_D+p_0)/(N_A-N_D-p_0)=K(T_0)\ , \tag{5}$$

a well-known expression for a p-type semiconductor in thermal equilibrium.

For nonthermal conditions such as conduction breakdown, quantitative applications of (1) or (2) have been hampered in the past by a lack of reliable expressions for the A and B coefficients. Since calculation of these terms from first principles is difficult, various approximations and alternate mechanisms have been used, and comparisons with measured values have often shown differences of orders of magnitude. The problems, however, can be simplified by noting two points. First, in conduction breakdown where the hole concentration greatly exceeds its thermal equilibrium value, the impact ionization rate A_2p must be much larger than the "thermal" rate A_1. Second, the photoconductivity experiments of Sclar and Burstein[2] show that $B_2p \gg B_1$, at least for carrier concentrations as low as 10^7 cm^{-3}, and the pulse-decay experiments of Koenig *et al.*[6] are consistent with that result. Then with $A_2p \gg A_1$ and $B_2p \gg B_1$ the steady-state relation (2) reduces to

$$p(N_D+p)/(N_A-N_D-p)=A_2/B_2\ . \tag{6}$$

This represents a detailed balance between the rates of

impact ionization (rate coefficient A_2) and its inverse, Auger recombination (rate coefficient B_2), which can be symbolized as a reversible reaction:

$$p + N_A^0 \underset{B_2}{\overset{A_2}{\rightleftarrows}} 2p + N_A^* , \tag{7}$$

where N_A^0 and N_A^* denote, respectively, neutral and ionized acceptors. By the same detailed balance considerations which apply to the equilibrium constant $K(T_0)$, the ratio A_2/B_2 should also be an "equilibrium constant," determined by the average carrier energy or effective temperature. Accordingly, we make the following suppositions.

The first is that the effective carrier temperature T_c is given in terms of the mean kinetic energy U by the relation $U=3kT_c/2$. While this suggests the presumption of a Maxwellian velocity distribution, we point out that U has been shown to be insensitive to the form of the distribution function, except for a factor of order unity.[9,10] As a second assumption we take $A_2/B_2=K(T_c)$, where the "equilibrium constant" K has the functional form given by (4), so that (6) now becomes

$$p(N_D+p)/(N_A-N_D-p) = (2\pi m^* kT_c/h^2)^{3/2} g^{-1} \exp(-E_a/kT_c) . \tag{8}$$

Obviously, when the carrier temperature T_c approaches the lattice temperature T_0, (8) reduces to (4) and (5).

To relate T_c with observable quantities such as applied field E and drift velocity v_d we use a relaxation-time formalism[9,11] for the difference between mean kinetic energy U and the equilibrium energy $U_0=3kT_0/2$:

$$(U-U_0)/\tau_e = eEv_d , \tag{9}$$

where τ_e is the energy relaxation time, characterizing inelastic collision processes, and the right side of (9) represents the power supplied by the field. Since $E=v_d/\mu$, and the mobility μ is given by $e\tau_m/m^*$, where τ_m represents the momentum relaxation time, we obtain from (9) that

$$T_c = T_0 + (4\tau_e/3k\tau_m)(m^* v_d^2/2) \tag{10}$$

or, alternately,

$$T_c = T_0 + (2e^2\tau_e\tau_m/3km^*)E^2 . \tag{11}$$

Therefore, (8) together with (10) or (11) are the equations to be compared with experimental determinations of $p(v_d)$ or $p(E)$.

Mobility measurements give the momentum scattering time τ_m, which has both elastic and inelastic contributions, but the inelastic-scattering time τ_e is unknown. In the regime of strong impact ionization, however, it is probable that inelastic collisions are dominant in momentum relaxation, so that the ration τ_e/τ_m is expected to be of order unity.

EXPERIMENTS

Samples measuring $1\times1\times10$ mm^3 were cut from two single-crystal slices supplied by the Tennelec Corporation (Oak Ridge, TN). After etching of the samples, indium-soldered contacts for conductivity and Hall measurements were applied in a standard configuration, with conductivity contacts spaced by 8 mm along one sample side and Hall contacts on opposing sides at sample center. The soldered contacts are rectifying at low temperatures, but this does not affect the validity of the potential measurements between conductivity and Hall contacts, taken with high impedance (10^{10} Ω) meters, nor the measurements of sample current.

According to the supplier, the Ge slices were gallium doped with net acceptor concentrations of 1.1×10^{12} and 4.1×10^{12} cm^{-3}. Measurements on our samples at 77 K (where all acceptors are thermally ionized) confirmed these values, and hereafter we shall refer to the samples as either S-1.1 or S-4.1. Except as noted, all measurements reported below were made with samples immersed in liquid helium and shielded from external thermal radiation. The relevant quantities obtained from the measurements are current density J, longitudinal electric field E, mobility μ, drift velocity v_d, and carrier concentration p.

LOW-FIELD RESULTS

The samples at 4.2 K show Ohmic conduction for fields $E\lesssim0.1$ V/cm, with carrier concentrations $\lesssim10^6$ cm^{-3}. The S-4.1 mobility of 3.2×10^6 cm^2/V sec matches other reported values for pure Ge and is quite close to the calculated lattice mobility value. The S-1.1 mobility is 13% smaller, and since the samples were not oriented before cutting from slices, the discrepancy could be due to a difference in crystallographic orientation and the known dependence of effective mass on orientation. Alternatively, dislocations introduced during sample fabrication could be responsible. This point was not checked further.

To obtain the compensating donor concentration N_D we used Eqs. (4) and (5) with the following parameter values:[12-14] $E_a=11$ meV for the gallium acceptors, hole effective mass (density-of-states average) $m^*=0.35$ m, and acceptor degeneracy factor $g=4$. The values of N_D obtained are 1.0×10^{10} and 2.9×10^{10} cm^{-3}, respectively, for the samples S-1.1 and S-4.1.

At temperatures rising above 4.2 K, activation energy measurements confirm the value $E_a=11$ meV quoted above. Thermal ionization is virtually complete at ~20 K.

For fields E between 0.1 and 1.0 V/cm, conduction is non-Ohmic. Here, carrier concentration increases due to impact ionization while the mobility falls by a factor of 3, but overall there is an increase of conductivity over the low-field value. The differential increase in carrier concentration varies approximately as the square of the field. We note that this is in agreement with Eqs. (8) and (11) of the present model when the last term in (11) is small but non-negligible and when (8) is calculated to first order in E^2, with $p\ll N_D$ and $p\ll N_A-N_D$.

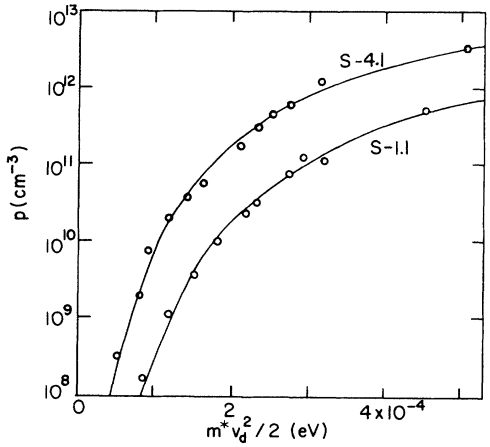

FIG. 1. Hole concentration p as a function of carrier drift kinetic energy. The curves are calculated from the model as described in the text.

BREAKDOWN

Conduction breakdown occurs in both samples at $E \sim 1.0$ V/cm. Carrier concentrations and current densities increase by four orders of magnitude for a relatively small increase in E, while mobilities remain nearly constant ($\pm 10\%$) over the breakdown regime. Again, sample S-1.1 mobilities are lower than S-4.1, by 10–20 %.

The data are plotted in Fig. 1 in the form of hole concentration p as a function of the drift kinetic energy $m^* V_d^2/2$. The curves in this figure are calculated from the model equations (8) and (10), using values for τ_e/τ_m of 1.7 and 1.4, respectively, for samples S-4.1 and S-1.1. The choice of these values is explained below. The difference in the values for the two samples is probably due to the same factors mentioned earlier in connection with the difference in mobilities. In any case, Fig. 1 demonstrates good agreement between the present theoretical model and experiment. It may be noted that the drift kinetic energies are of the order of kT_0 but are much less than the acceptor ionization energy.

Figure 2 presents the data in a different format. Here, for each measured carrier concentration p, the corresponding carrier temperature T_c is obtained from (8), and then T_c is plotted as a function of the experimental value of drift kinetic energy. If (10) is also correct, the plots should be linear. The display in Fig. 2 confirms this, and from the slopes of the lines in the figure we obtain the values of τ_e/τ_m (1.7 and 1.4) quoted above and used in the calculation of the curves in Fig. 1.

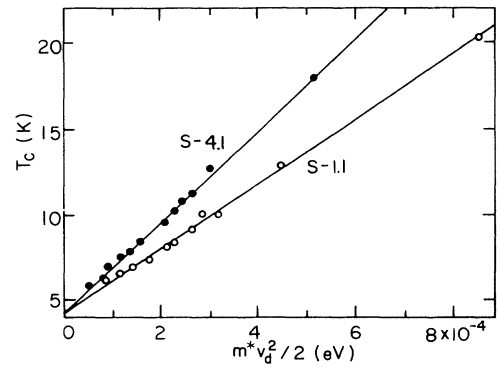

FIG. 2. Effective hot-carrier temperature T_c as a function of the carrier drift kinetic energy. The temperature values are obtained from the data by using Eq. (8) of the model.

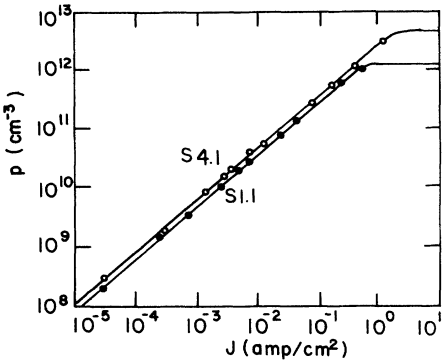

FIG. 3. Hole concentration p as a function of current density J.

In still another format, Fig. 3 shows a striking correlation between carrier concentration p and current density J, the two quantities that are changing by orders of magnitude over the breakdown regime. Saturation of impact ionization is evident at the upper ends of the data, where the carrier concentrations approach their respective net acceptor concentrations of 4.1×10^{12} and 1.1×10^{12} cm^{-3}. The "presaturation" data obviously show a power-law relation, $p \propto J^c$, and for either sample a best line fit to the data gives $C = 0.89 \pm 0.2$. To relate this to our model, we note that the current density is given by the relation $J = epv_d$, and accordingly we can plot $p(v_d)$, calculated from (8) and (10), as a function of epv_d. The results are indistinguishable from straight lines on the scale of Fig. 3, with slopes that match the experimental value of c precisely.

SUMMARY

For conduction breakdown in semiconductors, a theoretical model has been proposed in which the dominant rate processes are impact ionization and its inverse, Auger recombination. For the steady state, detailed balancing suggests that the ratio of those rates have the functional form of the thermal equilibrium constant, evaluated at an effective hot-carrier temperature. The latter is given in terms of observables (electric field or drift velocity) in a manner that is rather standard in the literature. Overall, the approach used avoids the necessity of calculating individual rate coefficients.

Detailed measurements over the breakdown regime have been made for lightly doped p-type Ge at 4.2 K. Comparisons of the model with the data show good agreement and consistency. Values of the relaxation-time ratio derived from the data are of the order of unity, as may be expected for this regime. The values of carrier

temperature obtained range up to 20 K, where experiments show that impact ionization of acceptors is nearly saturated; this is consistent with low-field (Ohmic) measurements of thermal ionization as a function of temperature.

ACKNOWLEDGMENTS

This work was supported under grants by the Research Corporation of Tuscon, AZ, and by the Materials Technology Center and the Office of Research and Development of Southern Illinois University at Carbondale.

[1]N. Sclar, E. Burstein, W. Turner, and J. W. Davisson, Phys. Rev. **92**, 858 (1954).
[2]N. Sclar and E. Burstein, Phys. Rev. **98**, 1757 (1955).
[3]N. Sclar and E. Burstein, J. Phys. Chem. Solids **2**, 1 (1957).
[4]S. H. Koenig and G. R. Gunther-Mohr, J. Phys. Chem. Solids **2**, 268 (1957).
[5]S. H. Koenig, Phys. Rev. **110**, 986 (1958).
[6]S. H. Koenig, R. D. Brown, and W. Schillinger, Phys. Rev. **128**, 1668 (1962).
[7]L. M. Lambert, J. Phys. Chem. Solids **23**, 1481 (1962).
[8]S. W. Teitsworth and R. M. Westervelt, Physica **23D**, 181 (1986).
[9]K. Hess, in *Physics of Nonlinear Transport in Semiconductors,* edited by D. K. Ferry, J. R. Barker, and C. Jacobini (Plenum, New York, 1980).
[10]W. Shockley, Bell Syst. Technol. J. **30**, 990 (1951).
[11]See, for example, K. Seeger, *Semiconductor Physics* (Springer-Verlag, Berlin, 1989), p. 103.
[12]S. M. Sze, *Physics of Semiconductor Devices* (Wiley, New York, 1981).
[13]L. S. Darkin, J. Appl. Phys. **53**, 3754 (1982).
[14]D. M. Brown and R. Bray, Phys. Rev. **123**, 1593 (1962).