System with time-reversal symmetry breaking

M. E. Simón
Instituto Balseiro, 8400 S.C. de Bariloche, Río Negro, Argentina

A. A. Aligia
Instituto Balseiro, 8400 S.C. de Bariloche, Río Negro, Argentina
and Centro Atómico Bariloche, 8400 S.C. de Bariloche, Río Negro, Argentina
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We consider a system composed of ferromagnetically aligned localized spins embedded in a simple metal with a spiral spin-density wave. The localized spins and the spins of the extended electrons interact via an exchange interaction. The mean-field Hamiltonian is not invariant under any symmetry operation containing time reversal. There is no spontaneous current, although it is allowed by symmetry. The ultrasonic attenuation in the superconducting phase does not have the usual exponential temperature dependence and might have a peak just below T_c, as observed in several heavy-fermion compounds.

I. INTRODUCTION

Eliashberg[1] and Tavger[2] stated that it is possible for electric current $\mathbf{j}$ to flow in thermal equilibrium if symmetry permits it. In particular, time-reversal symmetry (T) should be broken. Blount[3] argued that such a current is forbidden by the nature of equilibrium and showed that for periodic systems $\mathbf{j}=0$.

Halperin, March-Russell, and Wilczek[4] studied the consequences of T and mirror-symmetry (P) violation in models for high-T_c superconductivity. The anyon models[4,5] predict that these symmetries should be broken.

The possibility that a free-electron metal could have a spiral or linear spin-density wave has been considered by Overhauser.[6] In addition, Overhauser and Daemen[7] solved the anisotropic equations for the superconducting gap $\Delta(\mathbf{k})$. Here we consider the spiral spin-density wave (SSDW) interacting with ferromagnetically aligned localized spins. Although we do not pretend that this model represents a real system, the physics involved has certain similarities with heavy-fermion U compounds: $\Delta(\mathbf{k})$ is highly anisotropic,[7] as is the case in UPt_3,[8,9] UBe_{13},[10] and (U,Th) Be_{13} (Ref. 11) according to ultrasonic attenuation measurements. The specific heat of superconducting UBe_{13} looks very similar to the theoretical predictions for a SSDW.[7] A small localized magnetic moment is present in the superconducting phases of UPt_3 (Ref. 12) and UBe_{13}.[13] Note that in contrast to Ce systems, a magnetic ground state is expected for dilute Tm, Pr, and U systems, according to recent exact-diagonalization studies.[14,15] In addition, if the T-breaking field is strong enough, the ultrasonic attenuation should develop a peak slightly below the superconducting critical temperature T_c. This fact, which was not included in the study of Ref. 4, is explained in Sec. IV and is in qualitative agreement with the observations in the above-mentioned heavy-fermion compounds[8-11] and in high-T_c systems,[16,17] although several alternative explanations were given. In Sec. III we show that $\mathbf{j}=0$, in agreement with Ref. 3. Section V contains a short summary and discussion and we explain the model in Sec. II.

II. MODEL

We consider a free-electron gas with a SSDW with its axis along the z direction and a homogeneous ferromagnet magnetized along the same direction. The spins of both systems interact via an exchange interaction. In the Hartree-Fock approximation, the effective one-particle Hamiltonian for the itinerant electrons takes the form

$$H=\sum_{\mathbf{k},\sigma}\frac{\hbar^2k^2}{2m}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}-G\sum_{\mathbf{k}}(c^\dagger_{\mathbf{k}+\mathbf{Q}\downarrow}c_{\mathbf{k}\uparrow}+\text{H.c.})-\tfrac{1}{2}JS\sum_{\mathbf{k}}(c^\dagger_{\mathbf{k}\uparrow}c_{\mathbf{k}\uparrow}-c^\dagger_{\mathbf{k}\downarrow}c_{\mathbf{k}\downarrow})\ . \quad (1)$$

$c^\dagger_{\mathbf{k}\sigma}$ creates an electron in a plane-wave state with momentum $\mathbf{k}$ and spin σ. The first two terms are the second quantization version of Eq. (1) of Ref. 7. $2\pi/Q$ is the wavelength of the SSDW. The vector Q maps the point of the Fermi surface for $G=0$ and spinup with minimum k_z ($k_z=-k_{F\uparrow},k_x=k_y=0$) to the corresponding point for spindown and maximum k_z ($k_z=k_{F\downarrow},k_x=k_y=0$) (see Fig. 1). $2G$ is the magnitude of the SSDW gap at these two points.[6,7] S is the mean value of the z component of the localized spin and J the exchange interaction. T alone is not a symmetry of H, even if $J=0$. However, in this case,

$$[H,Tt(\hat{\mathbf{z}}\pi/Q)]=[H,TR_z(\pi)]=0,\quad J=0\ , \quad (2)$$

where $t(\mathbf{v})$ translates the system in a vector $\mathbf{v}$ and $R_z(\phi)$ is a rotation of angle ϕ around the z axis. As a consequence, no anomalous behavior in the transport properties exists for $J=0$. If $|\psi\rangle$ is an eigenstate of H, $Tt(\hat{\mathbf{z}}\pi/Q)|\psi\rangle$ is also an eigenstate with the same energy, and the contributions of both states to $\mathbf{j}$ have equal magnitude and opposite sign. For $J\neq0$, H does not commute with any symmetry operation containing T.

The energies of the one-particle eigenstates of the predominantly up spin are given by

$$E_\uparrow(\mathbf{k})=\frac{\hbar^2}{2m}(k_x^2+k_y^2)+F_\uparrow(k_z)\ , \quad (3)$$

$$F_\uparrow(k_z)=\frac{\hbar^2}{2m}(k_z^2+Qk_z+Q^2/2)-\mathrm{sgn}(k_z+k_{F\uparrow})\left\{\left[\frac{\hbar^2}{2m}(Qk_z+Q^2/2)+\tfrac{1}{2}JS\right]^2+G^2\right\}^{1/2}\ , \quad (4)$$

where

$$Q=(0,0,k_{F\uparrow}+k_{F\downarrow})\ , \quad (5)$$

$$\mu=\frac{\hbar^2k_{F\uparrow}^2}{2m}-\tfrac{1}{2}JS=\frac{\hbar^2k_{F\downarrow}^2}{2m}+\tfrac{1}{2}JS\ , \quad (6)$$

and μ is the chemical potential. The dispersion relation is discontinuous at the plane $k_z=-k_{F\uparrow}$ and the Fermi surface has a hole of radius $(2mG)^{1/2}/\hbar$ centered at $(0,0,-k_{F\uparrow})$. A similar situation occurs for states of predominantly down spin, replacing $k_{F\uparrow}$ with $-k_{F\downarrow}$. The dispersion relation of these states is

$$E_\downarrow(\mathbf{k})=\frac{\hbar^2}{2m}(k_x^2+k_y^2)+F_\downarrow(k_z)\ , \quad (7)$$

$$F_\downarrow(k_z)=\frac{\hbar^2}{2m}(k_z^2-Qk_z+Q^2/2)+\mathrm{sgn}(k_z-k_{F\downarrow})\left\{\left[\frac{\hbar^2}{2m}(Q^2/2-Qk_z)-\tfrac{1}{2}JS\right]^2+G^2\right\}^{1/2}\ . \quad (8)$$

The $E_\sigma(\mathbf{k})$ for $k_x=k_y=0$ are represented in Fig. 1.

III. ABSENCE OF A SPONTANEOUS CURRENT

It can be easily seen that

$$[H,t(\hat{\mathbf{z}}\pi/Q)R_z(\pi)]=0 \Longrightarrow j_x=j_y=0\ . \quad (9)$$

Thus, we need to concentrate on the current in the z direction:

$$j_z=-\frac{eV}{8\pi^3}\sum_\sigma\int\frac{\partial E_\sigma(\mathbf{k})}{\partial k_z}f(E_\sigma(\mathbf{k}))d^3k\ . \quad (10)$$

This expression is analogous to that given in Ref. 3. However, in this case we do not have periodicity in $\mathbf{k}$ space. Expressing the integral for each σ in cylindrical coordinates with axis k_z and changing the variable normal to k_z by $E_\sigma(\mathbf{k})$ we can write, after some algebra,

$$j_z=-\frac{eV}{4\pi^2}\int dE\, f(E)g(E)\ , \quad (11)$$

with

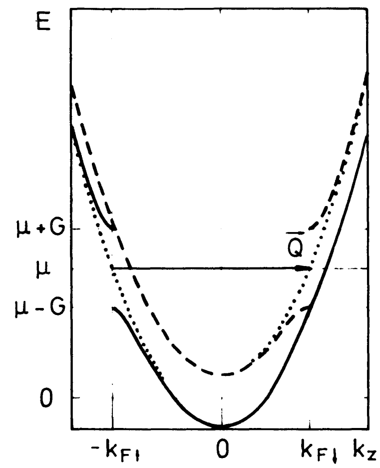

FIG. 1. Dispersion relations for the one-particle eigenstates of the system in the normal phase [Eq. (1)] for $k_x=k_y=0$. Full line: energies of eigenstates of positive mean value of the z projection of the spin s_z [Eqs. (3), (4), and (13)]. Dashed line: the same for $s_z<0$ [Eqs. (7), (8), and (14)]. Dotted lines: energies for $G=0$ and the same chemical potential μ. The wave vector of the spiral spin-density wave $\mathbf{Q}$ is also shown. Thin lines are guides to the eye. E and k_z are given in arbitrary units. Parameters are $G=0.3\mu$, $JS=0.4\mu$.

$$g(E)=\sum_\sigma\int\frac{\partial E_\sigma(\mathbf{k})}{\partial k_z}dk_z=\sum_\sigma[F_\sigma(k_{g\sigma})-F_\sigma(k_{s\sigma})]+2G[\Theta(-k_{F\uparrow}-k_{s\uparrow})-\Theta(k_{g\downarrow}-k_{F\downarrow})]\ , \quad (12)$$

where Eqs. (3) and (7) were used and $k_{g\sigma}$ ($k_{s\sigma}$) is the greatest (smallest) value of k_z for which the equation $E_\sigma(\mathbf{k})=E$ can be satisfied [if $E_\sigma(\mathbf{k})=E$ cannot be satisfied for any value of $\mathbf{k}$, one can take $k_{g\sigma}=k_{s\sigma}=0$]. The discontinuities of $F_\sigma(k_z)$ originate the last term in Eq. (12). $\Theta(x)$ is 1 for $x>0$ and 0 otherwise. Using Eqs. (3)–(8), one sees that the problem can be separated into three cases according to the value of E (see Fig. 1).

(a) $E<\mu-G$. In this case $|k_{s\uparrow}|<k_{F\uparrow}$, $k_{g\downarrow}<k_{F\downarrow}$, and $F_\sigma(k_{g\sigma})=F_\sigma(k_{s\sigma})=E$. Thus, $g(E)=0$.

(b) $\mu-G\le E\le\mu+G$. In this region $k_{s\uparrow}=-k_{F\uparrow}$, $k_{g\downarrow}=k_{F\downarrow}$. Thus, $F_\uparrow(k_{s\uparrow})=F_\downarrow(k_{g\downarrow})=\mu-G$, while $F_\uparrow(k_{g\uparrow})=F_\downarrow(k_{s\downarrow})=E$. The contributions for both σ cancel each other and again $g(E)=0$.

(c) $E>\mu+G\Longrightarrow -k_{s\uparrow}>k_{F\uparrow}$, $k_{g\downarrow}>k_{F\downarrow}$. Then $F_\sigma(k_{g\sigma})=F_\sigma(k_{s\sigma})=E$ and $g(E)=0$. Therefore, for all energies $g(E)=0$ and then $j_z=0$.

IV. THE SUPERCONDUCTING PHASE

Theoretically, it has been found that ferromagnetism can coexist with superconductivity in some cases.[18] For dilute magnetic impurities, it has been argued that the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction vanishes in the superconducting phase in the absence of impurity scattering, because in the simplest BCS case the spin susceptibility vanishes.[19] This is not the case for our system since, as it will become clear later, the superconducting phase is spin polarized for $JS\neq 0$. Also, superconductivity can coexist with other types of magnetic order that spontaneously break T and T combined with primitive translations. For these cases, our discussion is qualitatively valid. For $J=0$, the superconducting phase has been studied in Ref. 7. Extending these results for $J\neq 0$, the eigenstate of Eq. (1) with energy $E_\uparrow(\mathbf{k})$ is paired with the eigenstate of energy $E_\downarrow(-\mathbf{k})$. The operators that create an electron in these states can be written in the form

$$a^\dagger_{\mathbf{k}\uparrow}=(\cos\theta_{\mathbf{k}\uparrow})c^\dagger_{\mathbf{k}\uparrow}+(\sin\theta_{\mathbf{k}\uparrow})c^\dagger_{\mathbf{k}+\mathbf{Q}\downarrow}\ , \tag{13}$$

$$a^\dagger_{-\mathbf{k}\downarrow}=(\cos\theta_{-\mathbf{k}\downarrow})c^\dagger_{-\mathbf{k}\downarrow}+(\sin\theta_{-\mathbf{k}\downarrow})c^\dagger_{-\mathbf{k}-Q\uparrow}\ . \tag{14}$$

$\theta_{\mathbf{k}\uparrow}$ ($\theta_{\mathbf{k}\downarrow}$) is discontinuous at the plane $k_z=-k_{F\uparrow}(k_z=k_{F\downarrow})$. All angles lie in the interval $-\pi/4\leq\theta\leq\pi/4$ and

$$\theta_{\mathbf{k}+\mathbf{Q}\downarrow}=-\theta_{\mathbf{k}\uparrow}\ . \tag{15}$$

We remind the reader that the spin index has meaning only as a label in $a^\dagger_{\mathbf{k}\sigma}$, k_z is the quasimomentum projection in the z direction, while k_x and k_y are real momentum projections. Thus, each pair has total momentum equal to zero in the x and y directions, but not in the z direction. The z component of the total spin is also different from zero. The eigenstates of Eq. (1) that form the pair are not degenerate states [compare Eqs. (3) and (4) with (7) and (8)]. This indicates that JS should be smaller or of the order of magnitude of the superconducting order parameter $\Delta(\mathbf{k})$ in order to have superconductivity. Also, the excitation energies that are obtained after the usual Bogoliubov transformation to the mean-field pairing Hamiltonian[20] are rather unusual:

$$\lambda(\mathbf{k})=\left|\frac{E_\uparrow(\mathbf{k})-E_\downarrow(-\mathbf{k})}{2}\pm\left[\frac{[E_\uparrow(\mathbf{k})+E_\downarrow(-\mathbf{k})-2\mu]^2}{4}+\Delta^2(\mathbf{k})\right]^{1/2}\right|\ . \tag{16}$$

If $\Delta(\mathbf{k})=0$, then one $\lambda(\mathbf{k})=0$ if also $E_\uparrow(\mathbf{k})=\mu$ or $E_\downarrow(-\mathbf{k})=\mu$. It is also possible to have low-energy excitations even when $\Delta(\mathbf{k})\neq 0$. This fact has important consequences for the temperature dependence of the specific heat and ultrasonic attenuation. Using Eqs. (13)–(15), the reduced BCS interaction[20]

$$H_{\text{int}}=-V\sum_{\mathbf{k},\mathbf{k}'}c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow}c_{-\mathbf{k}'\downarrow}c_{\mathbf{k}'\uparrow} \tag{17}$$

takes the form [neglecting states of the form $a^\dagger_{\mathbf{k}-\sigma\mathbf{Q},\sigma}a^\dagger_{-\mathbf{k}\sigma}$, which are irrelevant for $G\gg\Delta(\mathbf{k})$]

$$H_{\text{int}}=-V\sum_{k,k'}\cos(\theta_{\mathbf{k}\uparrow}+\theta_{-\mathbf{k}\downarrow})\cos(\theta_{\mathbf{k}'\uparrow}+\theta_{-\mathbf{k}'\downarrow})\times a^\dagger_{\mathbf{k}\uparrow}a^\dagger_{-\mathbf{k}\downarrow}a_{-\mathbf{k}'\downarrow}a_{\mathbf{k}'\uparrow} \tag{18}$$

The interaction takes a factorized form as for $J=0$. In this case $\theta_{\mathbf{k}\uparrow}=\theta_{-\mathbf{k}\downarrow}$ and the result of Ref. 7 is repeated. $\Delta(\mathbf{k})$ vanishes when $\theta_{\mathbf{k}\uparrow}+\theta_{-\mathbf{k}\downarrow}=\pm\pi/2$. Since the absolute value of both angles is at most $\pi/4$, and the limiting values are obtained for different values of k_z if $J\neq 0,\Delta(\mathbf{k})\neq 0$ for all $\mathbf{k}$ in this case.

The electron-phonon interaction for plane waves (the eigenstates of H for $G=0$) can be written in the form[20]

$$H_{e\text{-ph}}=\sum_{\mathbf{k},\mathbf{q},\sigma}g^\sigma_{\mathbf{kq}}c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}(b_\mathbf{q}+b^\dagger_{-\mathbf{q}})\ . \tag{19}$$

Note that the interaction for opposite spins is different, since the Fermi surfaces are different.[20] This is a consequence of T breaking.[21] In terms of the eigenstates of H, $H_{e\text{-ph}}$ has a similar form for $\mathbf{q}\to 0$.

$$H_{e\text{-ph}}=\sum_{\mathbf{k},\mathbf{q},\sigma}\tilde{g}^\sigma_{\mathbf{kq}}a^\dagger_{\mathbf{k}+\mathbf{q}\sigma}a_{\mathbf{k}\sigma}(b_\mathbf{q}+b^\dagger_{-\mathbf{q}}),\quad \mathbf{q}\to 0\ , \tag{20}$$

with

$$\tilde{g}^\sigma_{\mathbf{kq}}=\cos^2\theta_{\mathbf{k}\sigma}g^\sigma_{\mathbf{kq}}+\sin^2\theta_{\mathbf{k}\sigma}g^{-\sigma}_{\mathbf{k}+\sigma Q,\mathbf{q}}\ . \tag{21}$$

We have neglected terms in Eq. (20) that connect electronic states of different energies for $\mathbf{q}\to 0$. The $\mathbf{q}$ dependence is retained in Eq. (21) for $\mathbf{q}\to 0$, because $g^\sigma_{\mathbf{kq}}$ is proportional to $\sqrt{q}$.[20,21] It would be more correct to derive the electron-phonon interaction directly from the eigenstates of H instead of using Eq. (19). However, only the symmetry properties of $\tilde{g}^\sigma_{\mathbf{kq}}$ are of importance in the present discussion.

It can be shown that the contribution of wave vector $\mathbf{k}$ to the attenuation of a longitudinal sound wave of wave vector $\mathbf{q}\to 0\,\alpha_\mathbf{q}$ is proportional to the square of matrix elements of the form[21]

$$M^\sigma_\mathbf{k}=\tilde{g}^\sigma_{\mathbf{k},\mathbf{q}}|u_\mathbf{k}|^2-\tilde{g}^{-\sigma}_{-\mathbf{k},\mathbf{q}}|v_\mathbf{k}|^2\ , \tag{22}$$

where $u_\mathbf{k}$ and $v_\mathbf{k}$ are the coefficients that relate the eigenoperators of $H+H_{\text{int}}$ [Eqs. (1) and (18)] in the mean-field approximation, with those of H.[21] As a result of the fact that $\tilde{g}^\uparrow_{\mathbf{k},\mathbf{q}}\neq\tilde{g}^\downarrow_{-\mathbf{k},\mathbf{q}}$ for $\mathbf{q}\to 0$, $M^\sigma_\mathbf{k}$ is not proportional to the coherence factor $|u_\mathbf{k}|^2-|v_\mathbf{k}|^2$ as is usual in T-symmetric systems.[22] If the change in the coherence factor is large enough and the last term of Eq. (16) dominates (as for $\mathbf{q}\cdot\mathbf{Q}=0$), $\alpha_\mathbf{q}$ has a peak below the superconducting critical temperature.[21] We have not attempted a numerical calculation, because it is difficult to perform the sums over wave vectors even for $J=0$. $\alpha_\mathbf{q}$ for $J=0$ is shown in Ref. 23. However, it is clear that for $J\neq 0$, not only a departure of the usual overall exponential behavior of $\alpha_\mathbf{q}$, but also a peculiar temperature dependence near T_c is expected.

V. SUMMARY AND DISCUSSION

We have studied a simple system in which time-reversal symmetry combined with any translation is broken. The difference between this system and a system with SSDW in the presence of a magnetic induction B is that in the former case, the orbital angular momentum is not coupled to the symmetry-breaking field and no shielding currents flow in the superconducting phase for $B=0$. Although a spontaneous current is allowed by symmetry, it does not occur, in agreement with the assessment of Blount,[3] that such a current is forbidden by the nature of equilibrium. In Ref. 4 a Hall conductance even for $B=0$ is predicted for a system with T and P broken symmetries. For our system the Hall conductance vanishes for $B=0$ because the system remains invariant under a mirror symmetry through a plane parallel to the SSDW axis acting on orbital variables only. This symmetry is broken if spin-orbit coupling is included.

For large enough electron-phonon interaction (or another pairing mechanism), the system becomes superconducting in the presence of magnetic order, and as a consequence of time-reversal symmetry breaking, the usual coherence factor entering in the expression of the longitudinal ultrasonic attenuation is changed, and a structure or a peak in the attenuation slightly below T_c, similar to that observed in some heavy-fermion[8,11] and high-T_c systems,[16,17] is expected.

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