Lack of reentrance in randomly frustrated three-dimensional XY ferromagnets

Michel J. P. Gingras and Erik S. Sørensen*
Laboratoire de Physique Statistique, Ecole Normale Supérieure, 24, rue Lhomond, 75231, Paris CEDEX 05, France
(Received 20 November 1991)

A Migdal-Kadanoff real-space renormalization-group method is used to study two nearest-neighbor randomly frustrated XY ferromagnets in three dimensions. The models are an XY ferromagnet with a dilute concentration of antiferromagnetic exchange couplings and an XY ferromagnet with random Dzyaloshinskii-Moriya spin-orbit interactions. The purpose of the investigation is to examine whether these two systems exhibit *reentrance*, characterized by the loss of long-range ferromagnetic order upon cooling and subsequent appearance of either a paramagnetic or a spin-glass phase. Unlike claims in the literature concerning reentrant behavior for two-dimensional versions of the systems considered here, we do not find any evidence for reentrance for these models in three dimensions. We propose two possible mechanisms for reentrance in three-dimensional dilute magnetic vector-spin systems.

I. INTRODUCTION

Many dilute magnetic spin systems show upon lowering the temperature, and for a range of concentration of the magnetic species, x, the following sequence of two transitions:

$$\text{paramagnetic} \xrightarrow{T_{\mathrm{PF}}} \text{ferromagnetic} \xrightarrow{T_{\mathrm{FG}}} \text{spin glass} .$$

Such systems are called reentrant spin glasses since they *reenter* into a less magnetically ordered state, the spin-glass phase, as the temperature is decreased.[1] The insulating Heisenberg ferromagnetic system $Eu_xSr_{1-x}S$ is a textbook example of a reentrant spin glass for a range of concentration $0.5 < x < 0.65$. Figure 1 shows the phase diagram for this material.[2] The above sequence of phase transitions implies that the spin-glass phase has the lowest energy while the ferromagnetic phase has more entropy, which is rather counterintuitive.

The experimental discovery of reentrant spin glasses did not really come as a surprise since the replica symmetric mean-field theory of the Ising spin glass did predict such behavior for a ratio of the mean and the standard deviation of the bond distribution within a certain range.[3] However, it is now known that the correct non-replica-symmetric mean-field solution of Parisi does not show reentrance.[4,5]

Reentrant behavior has been observed in binary mixtures,[6] liquid crystals,[7] and superconducting systems.[8] More recently, a reentrant melting transition has been seen in a polymer glass[9] while the wrinkling of partially polymerized membranes upon a decrease of temperature could be interpreted as the reentrance of a crumpled phase.[10] The presence of some "hidden" interactions which are not necessary to explain the thermodynamic stability (existence) of the intermediate ordered phase are usually at the origin of a reentrant phenomenon.[11] The existence of such interactions and of a related disordering mechanism has not yet been identified in reentrant spin glasses. Hence, the physical origin of reentrance in these systems is still unclear.[12]

In a recent paper, Reger and Young[13] have used a real-space renormalization-group scheme (Migdal-Kadanoff method) and a matrix transfer technique to study the competition between paramagnetic, ferromagnetic, and spin-glass order in several two-dimensional (2D) and three-dimensional (3D) Ising spin-glass models. From both methods, for all models considered and in both dimensions, they were unable to find any reentrant transition from either a ferromagnetic to a spin glass (in 3D) or from a ferromagnet to a paramagnet (in 2D) upon a decrease of the temperature. Their results agree with

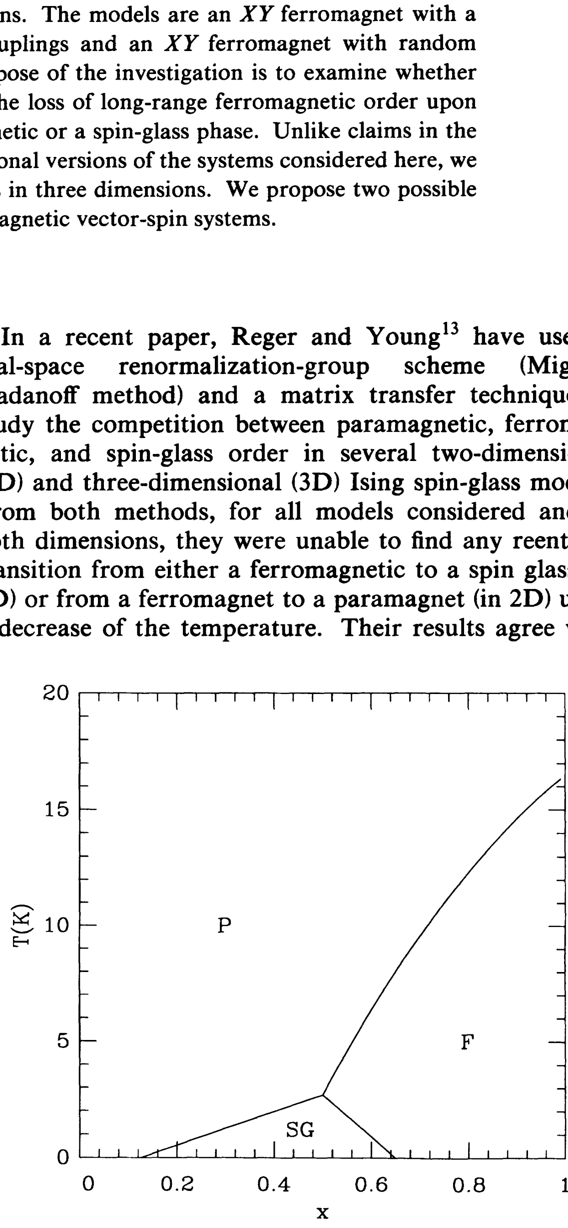

FIG. 1. Concentration, x, temperature, T, phase diagram of the insulating Heisenberg ferromagnet $Eu_xSr_{1-x}S$ which shows reentrant behavior in the range $0.5 < x < 0.65$ (adapted from Maletta and Convert in Ref. 2). $Eu_xSr_{1-x}S$ can be described by a Heisenberg spin Hamiltonian, H, with $H = \sum_{(i,j)} -J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$, where the sum runs over the nearest and second-nearest neighbors of a regular fcc lattice whose sites are randomly occupied by either the Eu or Sr atoms. The exchange couplings J_{ij} are ferromagnetic and antiferromagnetic for the nearest, J_1, and second-nearest, J_2, neighbors, respectively, with $J_2/J_1 \approx -\frac{1}{2}$.

the experimental findings that Ising systems do not show a reentrant transition.[14] Their failure to find reentrance in these Ising models suggests that some relevant ingredients present in the experimental systems showing reentrance are missing in the Ising models.[15] In particular, the experimental observation that reentrance is seen in ferromagnetic Heisenberg vector-spin systems[2,16] led them to suggest two mechanisms which could lead to reentrance in these. The first mechanism is that reentrance occurs in isotropic vector-spin systems because of the not yet understood behavior of transverse fluctuations. The second is that couplings between the longitudinal and transverse degrees of freedom, such as those present in dipole-dipole interactions, are needed to induce reentrance. It is the purpose of this paper to investigate whether or not the two mechanisms proposed by Reger and Young are sufficient to lead to reentrance in short-range vector-spin models.

To investigate the proposals of Reger and Young, we have considered two three-dimensional nearest-neighbor spin-glass lattice models. In particular, we have studied an XY ferromagnet with a dilute concentration of antiferromagnetic exchange couplings and XY ferromagnetic with random Dzyaloshinskii-Moriya spin-orbit interactions. These two models are interesting since they both display transverse spin fluctuations. Further, in the second model, the random Dzyaloshinskii-Moriya interactions lead to a coupling between the longitudinal and transverse spin components.

The rest of the paper is organized as follows. Our two models are described in Sec. II. The method we use is an approximate real-space renormalization-group calculation based on the Migdal-Kadanoff method, which we describe in Sec. III. Our main result, presented in Sec. IV, is that the above two models do not show reentrance within the Migdal-Kadanoff approach. We interpret our result as preliminary evidence that transverse spin fluctuations in isotropic vector-spin systems and the presence of a longitudinal-transverse coupling, such as the one present in an XY ferromagnet with random Dzyaloshinskii-Moriya interactions, are not sufficient ingredients to lead to reentrance in spin glasses. This leads us to speculate in Sec. V about ingredients which could lead to reentrance in frustrated vector-spin systems.

II. MODELS

A. The bimodal XY spin glass

The first model we study in the isotropic m-component bimodal spin-glass model with Hamiltonian

$$H_B = \sum_{\langle i,j \rangle} -J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \ , \tag{2.1}$$

where the sum is taken over the nearest neighbors of a d-dimensional cubic lattice and $\mathbf{S}_i$ is a classical m-component vector of unit length. The exchange interactions, J_{ij}, are random with a biased bimodal probability distribution, $\mathcal{P}(J_{ij})$, given by

$$\mathcal{P}(J_{ij}) = x\delta(J_{ij}-J_0)+(1-x)\delta(J_{ij}+\lambda J_0) \ . \tag{2.2}$$

A bond between sites i and j has a probability x to be ferromagnetic and of strength J_0, and a probability $1-x$ of being antiferromagnetic and of strength $-\lambda J_0$. Here $0 \le \lambda \le 1$, since negative λ corresponds to a strictly, though random, ferromagnetic model while $\lambda > 1$ can be viewed as an antiferromagnetic system diluted by ferromagnetic bonds and whose phase diagram can be obtained from the one of the ferromagnetic system with a rescaling of the temperature axis. This is one of the simplest models that can be used to investigate the competition between paramagnetic (P), ferromagnetic (F), and spin glass (SG) order.[5]

We mention here that we do not expect the bimodal XY spin glass to exhibit a thermodynamically stable spin-glass phase at nonzero temperature in 3D, since the lower critical dimension for spin-glass order for vector-spin models is believed to be four.[17–19] As in the 2D systems studied in Ref. 13, we shall investigate whether the 3D bimodal XY model shows reentrant behavior from a ferromagnetic phase to a paramagnetic phase upon cooling.

The competition between the three types of order has been extensively studied for the Ising system ($m=1$) using both high-temperature series[20] and real-space renormalization-group calculations.[13,21] As mentioned in the Introduction, this model does not show reentrance from a ferromagnetic to paramagnetic phase in 2D, nor one from a ferromagnetic to a spin-glass phase in 3D.[13,21]

The case of the three-component ($m=3$, Heisenberg) model has attracted much attention recently, in particular, in the context of the slightly doped antiferromagnetic cuprate high-T_c superconductors.[22,23] In these systems, the localization of holes in the slightly doped insulating materials generates effective ferromagnetic bonds giving rise to random frustration[22,23] and to the spin-glass properties observed in the intermediate region between the antiferromagnetic and the superconducting portion of the phase diagram.[24–27] In a different context, Thompson *et al.*[28] have recently performed a Monte Carlo study of the Heisenberg version of model (2.1) in an attempt to understand the transverse spin-freezing transition observed in amorphous Fe_xZr_{1-x} alloys.[29]

The XY model ($m=2$) has also been extensively studied in recent years.[30–36] Results on a 2D XY model similar to (2.1), which are suggestive of a reentrant behavior from a ferromagnetic phase[37] to a paramagnetic phase, have been observed by using an interaction of local mean-field equations.[31,38] The possible existence of a chiral glass[32,39] showing a true spin-glass transition in 2D at finite temperature has recently been investigated.[36]

The 3D XY version of model (2.1) is the first of the two models that we investigate in this paper. It is an interesting model for three reasons. First, it is the simplest isotropic vector-spin model displaying a competition between randomly distributed ferromagnetic and antiferromagnetic bonds and which contains transverse spin fluctuations, thus allowing us to investigate the first proposal of Reger and Young.[13] Secondly, as mentioned above, there already exists preliminary numerical evidence that a version of this model (2.1) may show reentrant behavior in 2D,[31,38] and this may indicate that it is a good prospect to be considered for a study of reentrance in random magnetic systems. Finally, there are some experimental results from muon spin resonance (μSR),[24,25] nuclear quadrupole resonance[26] (NQR), and neutron-diffraction studies[25,27] which suggest that the high-T_c superconducting materials may also exhibit reentrant spin-glass behavior for small doping. This provides another motivation for investigating the possible occurrence of reentrance in a classical XY version of Aharony *et al.*'s model[22] which is believed to be the simplest effective model describing the insulating antiferromagnetic phase of the slightly doped high-T_c cuprates.[22,23,40,41]

B. The XY ferromagnet with random Dzyaloshinskii-Moriya interactions

The second model we consider applies to a metallic (nonmagnetic) system doped with magnetic impurities with a classical magnetic moment $\mathbf{S}_i$ whose orientation is constrained in the xy plane. The Hamiltonian is

$$H_{DM} = -J_0 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_{\langle i,j \rangle} D_{ij} \hat{\mathbf{z}} \cdot \mathbf{S}_i \times \mathbf{S}_j \ , \qquad (2.3)$$

where $\hat{\mathbf{z}}$ is a unit vector perpendicular to the xy plane and $\mathbf{S}_i$ is of unit length. As in (2.1), the sum is taken over nearest-neighbor sites of a d-dimensional cubic lattice. The presence of additional randomly positioned spin-orbit impurity scatterers are responsible for the random Dzyaloshinskii-Moriya (DM) interactions, D_{ij}.[5,12] We want to emphasize that $H_{\rm DM}$ is rotationally invariant, whereas this is not the case for the equivalent Heisenberg Hamiltonian where $D_{ij}\hat{\mathbf{z}}$ becomes a three-component vector $\mathbf{D}_{ij}$ with random orientation. We take D_{ij} to be given by a Gaussian probability distribution, $\mathcal{P}(D_{ij})$, with zero mean and width D

$$\mathcal{P}(D_{ij}) = \frac{1}{\sqrt{2\pi D^2}} \exp(-D_{ij}{}^2/2D^2) \ . \qquad (2.4)$$

If the magnetic impurities are randomly distributed, one would also expect the exchange interaction, J_0, to vary in sign due to the usual oscillatory Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction.[5,12] Here we take the average distance between the magnetic atoms to be given by the lattice spacing and adjusted such that J_0 is ferromagnetic ($J_0 > 0$).

It is useful to introduce the angle ϕ_i that $\mathbf{S}_i$ makes with respect to a fixed axis and rewrite Eq. (2.3) in the following form:

$$H_{\rm DM} = \sum_{\langle i,j \rangle} -K_{ij} \cos(\phi_i - \phi_j - A_{ij}) \ , \qquad (2.5)$$

where $K_{ij} = \{J_0^2 + D_{ij}^2\}^{1/2}$ and $\tan(A_{ij}) = D_{ij}/J_0$. Written in this form, $H_{\rm DM}$ is now a lattice model whose Hamiltonian describes a random array of superconducting grains in a magnetic field and coupled via the Josephson coupling K_{ij}. Within this interpretation of the model, A_{ij} is the line integral of the vector potential, $\mathbf{A}$, from one grain to the other ($A_{ij} = 2\pi\Phi_0^{-1}\int_i^j \mathbf{A}(\mathbf{r})\cdot d\mathbf{l}$), and Φ_0 is the elementary flux quantum, $\Phi_0 = hc/2e$. Model (2.5) has very recently been the subject of an intensive study.[42–44] In particular, in the limit of extreme disorder, where A_{ij} undergoes large variations from one pair of grains to the other, (2.5) is believed to be an adequate model to describe the vortex-glass phase in disordered type-II superconductors.[44,45] The paramagnetic, ferromagnetic, and spin-glass phases in the magnetic problem [Eq. (2.3)] translate for the superconducting problem [Eq. (2.5)] into the normal, superconducting, and vortex-glass phases, respectively. In this paper, we concentrate on the regime of small disorder where $D < J_0$ in order to investigate the competition between ferromagnetic (superconducting) and spin-glass (vortex-glass) order and the eventual existence of a F-SG reentrance in this regime.

A 2D version of model (2.3) has been studied by Rubinstein, Shraiman, and Nelson[46] using Kosterlitz-Thouless-like renormalization-group equations.[47,48] More recently, Paczuski and Kardar[49] have extended the work of Rubinstein *et al.* by studying the effect of nonrandom symmetry-breaking fields.[48] The random 2D Josephson-junction formulation of the model, Eq. (2.5), has been investigated by Granato and Kosterlitz.[42] The main interesting result which comes out of the above studies[42,46,49] is the prediction of a reentrant transition from a ferromagnetic phase[37] to a paramagnetic phase for any, but nonzero, value of the disorder D. For D larger than some critical value, D_c, the ferromagnetic phase is absent and the system is paramagnetic at all temperatures. See Fig. 2 for a schematic representation of the predicted D-T phase diagram for model (2.3) in 2D.

The 3D version of (2.3) is the second model that we investigate in this paper. It is a very interesting model to

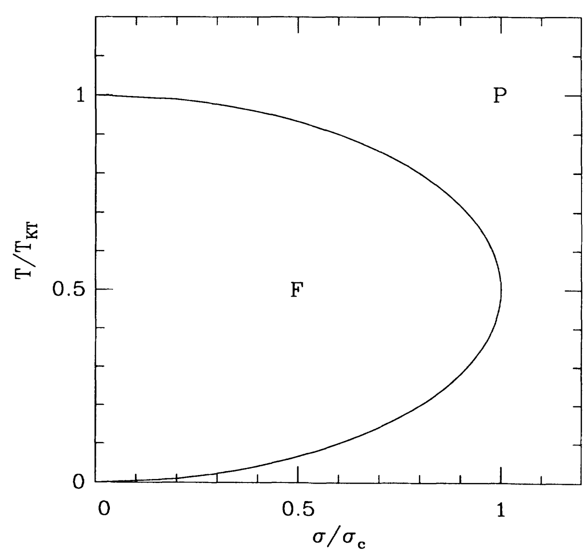

FIG. 2. Schematic representation of the disorder, D, temperature, T, phase diagram for the two-dimensional XY ferromagnet with random Dzyaloshinskii-Moriya interactions. $T_{\rm KT}$ and D_c are the Kosterlitz-Thouless transition temperature in the absence of disorder and the critical amount of disorder to observe an ordered phase. (Phase diagram adapted from Ref. 42.)

study for the following three reasons. First, model (2.3) with (2.4) contains random couplings between the longitudinal and transverse spin components due to the random D_{ij} or A_{ij} quantities. Hence, this model allows us to investigate the second proposal of Reger and Young[13] that such random $S_{ix}S_{jy}$ couplings may be responsible for reentrant behavior in vector-spin systems. Secondly, we have at our disposal a theoretical prediction of reentrance from the rather rigorous Kosterlitz-Thouless renormalization-group framework.[42,46–49] Furthermore, a $2+\epsilon$ expansion of the 2D renormalization-group equations of Ref. 46 predicts reentrance for $\epsilon>0$. This may suggest that reentrance may indeed occur for model (2.3) in 3D. Finally, and not the least, the Josephson-junction formulation of the problem is attractive since experiments on such systems with a controllable amount of disorder can be performed with the theory being checked accurately.[50]

Before presenting the method we use to investigate the phase diagram of models (2.1) and (2.3), we comment on the relevance of the type of disorder considered for the paramagnetic to ferromagnetic transition in these two systems. The continuum description of the bimodal XY model is known as the XY random-T_c model.[51] The Harris criterion[51,52] states that a small amount of disorder of this type should be irrelevant at the P-F transition, since the specific-heat exponent, α, is slightly negative for the pure XY model.[53] Thompson *et al.* have, using a Monte Carlo simulation, confirmed the previous statement for a Heisenberg model with (2.2) for which they find the same critical exponents as for the pure (nonrandom) Heisenberg ferromagnet which also has $\alpha<0$.[28] In the case of model (2.3), Hertz has shown that the kind of disorder considered in Eq. (2.4) is irrelevant near the usual Gaussian fixed point in four dimensions.[54] From these two results, we thus expect our two models to have a stable ferromagnetic phase at finite temperature in three dimensions, and the paramagnetic to ferromagnetic phase transition to lie in the universality class of the pure XY model for small disorder [$x\approx 1$ in model 2.1 and $D\ll 1$ in model (2.3)].

III. NUMERICAL METHOD: THE MIGDAL-KADANOFF RENORMALIZATION-GROUP SCHEME

The Migdal-Kadanoff bond-moving scheme is a simple real-space renormalization-group method which, despite its simplicity, has proved to be a useful tool in investigating spin glasses. For example, it can correctly predict the lower critical dimension and the spin-stiffness exponent of Ising[55] and XY spin glasses.[17] It also predicts that gauge glasses are most likely to be in a different universality class than other isotropic m-component spin glasses.[56] The method has limitations, as it cannot give good estimates of the critical temperature in spin glasses and can give inaccurate critical exponents. It is generally accepted, however, that it can give the correct topology of the phase diagram. The Migdal-Kadanoff scheme follows the distribution of couplings under the renormalization-group iterations and is able to locate the paramagnetic, ferromagnetic, and spin-glass phases. It is therefore a well-suited tool to examine reentrant behavior.

We concentrate on 3D systems since it is known that the Migdal-Kadanoff method cannot adequately reproduce a low-temperature phase with quasi-long-range ferromagnetic order for the 2D XY model.[37,48] Hence, one must be careful in interpreting the numerical results obtained from a Migdal-Kadanoff calculation for 2D XY models such as those described in Sec. II. This problem does not exist in 3D where the method correctly predicts the existence of an ordinary ferromagnetic phase with true long-rang order.

The first choice we have when implementing the Migdal-Kadanoff scheme is the rescaling parameter, b, which is the ratio of the lattice spacing of the new rescaled lattice to the old spacing. In a nonrandom system, one can obtain differential renormalization-group equations by taking the limit $b\to 1$. In a random system, this approach is difficult to implement since one must follow the whole distribution of couplings under renormalization, and integer values for b are used. We choose $b=2$, since it is generally believed that the smaller b gives the best results.[13,21]

Having chosen $b=2$, the second variable is the number of bonds moved in parallel and combined with the two nondisplaced bonds i—n and n—j. The three possible choices, the x, y, and z recursions, are schematically illustrated in Fig. 3. We have found the y recursion convenient for computational purposes [Fig. 3(b)]. In this case, the renormalized coupling H' is given by

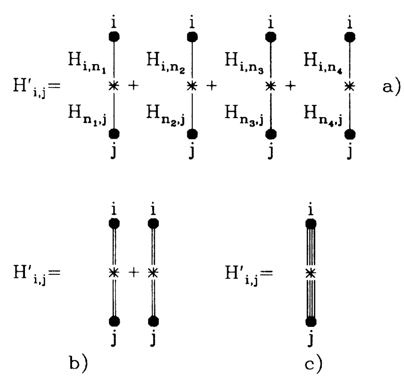

FIG. 3. Schematic representation of the Migdal-Kadanoff decimation scheme of the cubic lattice. (a), (b), and (c) correspond to the x, y, and z recursions. The solid circles represent the spins $\mathbf{S}_i$ and $\mathbf{S}_j$ and the star is the spin $\mathbf{S}_n$ to be integrated out. Each solid line represents a different coupling H_{ij} taken from the pool of random bonds at a given step of the iteration. For example, (b) shows that the couplings H_{in_1}, H_{in_2}, H_{n_1j}, and H_{n_2j} are combined together to give a first intermediate new coupling $H_{ij}^{\{\mathrm{int},1\}}$ [first integral over ϕ_n in (3.1)]. The couplings H_{in_3}, H_{in_4}, H_{n_3j}, and H_{n_4j} are combined together to give the second intermediate couplings, $H_{ij}^{\{\mathrm{int},2\}}$ [second integral in (3.1)]. The final new coupling, $H'_{ij}(\phi_i-\phi_j)$, is $H'_{ij}=H_{ij}^{\{\mathrm{int},2\}}+H_{ij}^{\{\mathrm{int},2\}}$.

$$\exp\{\beta H'_{ij}(\phi_i-\phi_j)\}=\int_0^{2\pi}\frac{d\phi_n}{2\pi}\exp\left[\sum_{\alpha=1}^{2}\beta H_{in_\alpha}(\phi_i-\phi_n)+\beta H_{n_\alpha j}(\phi_n-\phi_j)\right]$$
$$\times\int_0^{2\pi}\frac{d\phi_n}{2\pi}\exp\left[\sum_{\alpha=3}^{4}\beta H_{in_\alpha}(\phi_i-\phi_n)+\beta H_{n_\alpha j}(\phi_n-\phi_j)\right], \quad (3.1)$$

where $\beta=1/k_B T$.

To iterate Eq. (3.1) numerically, one first constructs an original pool of N couplings $H_{ij}(\phi_i-\phi_j)$, drawn from the desired original distribution [Eq. (2.2) or (2.4)]. One step of the recursion is to combine eight such interactions, $(H_{in_1},H_{n_1j},H_{in_2},H_{n_2j})$ and $(H_{in_3},H_{n_3j},H_{in_4},H_{n_4j})$, according to (3.1) to obtain a new interaction, H'_{ij}. This step is repeated N times to get a new pool of N renormalized couplings. In order to make this procedure numerically tractable, some kind of discretization scheme must be introduced to perform the iteration (integration) of the renormalization-group equation (3.1). We now describe the two methods that we have used to do so.

The most straightforward way to iterate (3.1) is to assume that we are not dealing with models with continuous symmetry, but with Q-state clock models, where the spins can only point along one of the Q angles separated from each other by $2\pi/Q$, and where we take Q "large." From a strictly mathematical point of view, this is equivalent to performing the integrals in (3.1) by using the simple rectangle method. This method has recently been used in an attempt to estimate the lower critical dimension of XY-like spin glasses without using the usual zero-temperature Gaussian approximation of the Hamiltonian.[57] For $Q=2$, we recover the Ising spin-glass model.[13,21] This means that when we pick a coupling H_{in_α} in the distribution pool, we take, in fact, Q numbers which are the Q-discretized values of the interaction for bond i—n_α. The details of the algebraic manipulation are given in the Appendix. We checked for independence of the results on the choice of Q. To provide an independent check of the Q-state clock results, we used the Fourier analysis method employed by José *et al.* in their study of the two-dimensional ferromagnetic XY model.[48]

The basic idea of the Fourier method is to transform the integral in (3.1) into convolutions of the exponential function which are given by simple, and quick to perform, sets of algebraic sums (see the Appendix). This method is *not* equivalent to working in the Fourier representation of the clock method since we are using a Fourier decomposition of the exponential of the coupling, $\exp\{H_{ij}(\phi_i-\phi_j)\}$. In this way, we always preserve a full continuous symmetry description of the interactions, which is not the case in the above Q-state clock decomposition.

We differentiate between the paramagnetic, ferromagnetic, and spin-glass phases by looking at the height of the potential, h_{ij}, defined as

$$h_{ij}=H_{ij}(0)-H_{ij}(\pi/2)\ . \quad (3.2)$$

The different phases are located by following the behavior of the distribution of heights h_{ij} under the iteration of (3.1). We thus calculate the absolute average height, $\bar{h}$, and width, Δh, of this distribution. $\bar{h}$ and Δh evolve in the three phases as[13,21]

$$\bar{h}\to 0\ ,\ \Delta h\to 0:\ \text{paramagnetic phase}\ , \quad (3.3a)$$

$$\bar{h}\to\infty\ ,\ \Delta h/\bar{h}\to 0:\ \text{ferromagnetic phase}\ , \quad (3.3b)$$

$$\Delta h\to\infty\ ,\ \bar{h}/\Delta h\to 0:\ \text{spin-glass phase}\ . \quad (3.3c)$$

The phase boundaries are located by stepping through the phase diagram in an optimal way zeroing in on the boundaries. Obviously the choice of criteria of convergence for the different phases is somewhat subjective and can slightly shift the phase boundaries. This last remark is especially relevant to the detection of the spin-glass phase. We now present the numerical results and the phase diagrams we have obtained for our two spin-glass models.

IV. RESULTS

The phases exhibited by our two models are easily detected upon iteration of Eq. (3.1) using the clock and Fourier methods, and by monitoring the evolution of the distribution of barrier heights, h_{ij}, according to the classification given in (3.3). We took typically between 2000 and 5000 bonds, and $Q=100$ and $s_{\max}=50$. For the two models considered, the clock and the Fourier methods gave almost identical results, except at low temperatures, where the Fourier method was numerically ill behaved (see the comment in the Appendix). For $T<0.5$, the results presented below for model (2.3) are obtained with the clock method only.

A. The bimodal XY spin glass

The concentration-temperature (x-T) phase diagram for this model is shown in Fig. 4. The error on the position of the boundaries is of the order of the size of the symbols. The ferromagnetic transition temperature is plotted as a function of the concentration x of the ferromagnetic bonds and for different values of the relative strength of the randomly diluted antiferromagnetic bonds, λ. As found by others,[17,18] we do not find a spin-glass phase at nonzero temperature for this XY model. The phase boundary is vertical at low temperatures and no ferromagnetic to paramagnetic reentrance is observed.

We mention here the very recent results of Thompson *et al.* who have performed a Monte Carlo simulation of a three-dimensional Heisenberg version of model (2.1) with $\lambda=1$.[28] Their results show a true thermodynamic transition from a paramagnetic phase to a ferromagnetic phase at a temperature $T_{\rm PF}$, and a *dynamical* transverse spin-freezing transition at a temperature T_{XY} less than T_{PF}. The ferromagnetic phase disappears altogether for a concentration x less than 0.75. Finally, they find no sign of a reduction of the longitudinal magnetization upon a decrease of temperature and they conclude that *no* reentrance is present in their model. We expect that the competition between ferromagnetic and antiferromagnetic couplings at play in their Heisenberg and in our XY model should be very similar. Therefore, as expected, their phase diagram has the same topology as ours, provided that we disregard the transverse spin-freezing transition that we cannot detect with our Migdal-Kadanoff method.

As discussed in Sec. III, the Migdal-Kadanoff scheme cannot reproduce exactly the existence of a line of fixed points for the pure ferromagnetic 2D XY model. However, for several iterations of the decimation scheme, it behaves as if it could.[48] With this caveat in mind, we have done a Migdal-Kadanoff calculation for a 2D version of model (2.1) to locate the phase boundaries between a paramagnetic phase and a ferromagnetic phase.[37] The phase diagram of the 2D version of model (2.1) has the same topology as in Fig. 4, with the ferromagnetic phase being replaced by a "Migdal-Kadanoff version" of a phase with quasi-long-range ferromagnetic order. Due to the above-mentioned problem with the Migdal-Kadanoff method for 2D XY models, we cannot comment at present whether the disagreement between our results and the observation of reentrance in a 2D XY model studied in Ref. 31 is due to either the weaknesses of the Migdal-Kadanoff scheme, the use of iterated local mean-field equations in Ref. 31, or to the slight difference in the models considered here and in Ref. 31.[38] We also note that José has found from a set of Kosterlitz-Thouless renormalization-group equations[47,48] that there is "some evidence" for a low-temperature instability in a 2D version of model (2.1), whose interpretation is unclear.[30] One may think that a low-temperature proliferation of multipolelike topological defects[35] could destroy the ordered phase[37] in a way similar to the behavior described in Ref. 46 for model (2.3) in 2D.

B. The XY ferromagnet with random Dzyaloshinskii-Moriya interactions

The phase diagram for this model is shown in Fig. 5 where now, unlike the bimodal XY model, a spin-glass phase at nonzero temperature is observed.[44,57] The determination of the phase boundaries for this model is not as precise as for the bimodal case, as the convergence of height distribution close to the boundaries according to the classification (3.3) was poor. This is the reason we ascribe somewhat broader phase boundaries in this case than in Fig. 4. The slope of the phase boundary between the ferromagnetic and spin-glass phase is, within its width, slightly negative, suggesting a possible transition from a spin glass to a ferromagnetic phase upon decreasing the temperature. We do not want to put too much emphasis on this result and prefer to restrict ourselves to the observation that no evidence for wide F-SG reentrance is found. The reentrant transition predicted for this model in 2D is expected to occur for any nonzero D.[42,46,49] Our results are certainly not compatible with such behavior.

We are not aware of Monte Carlo results on this 3D model. However, experimental results[50] and Monte Carlo simulations[43] on the two-dimensional Josephson-

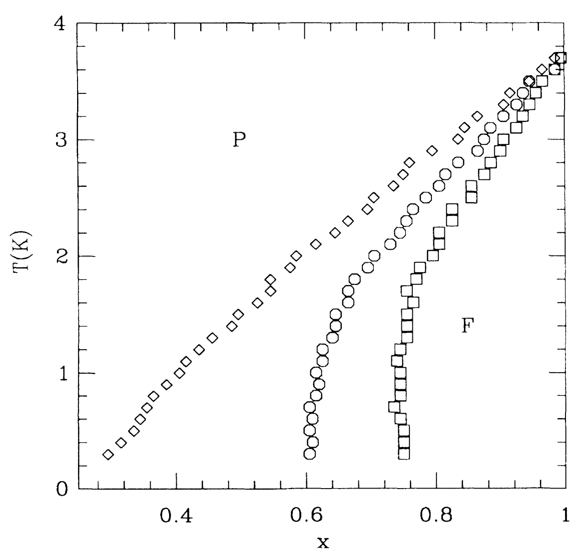

FIG. 4. Temperature (T) vs the impurity concentration (x) phase diagram of the 3D bimodal XY model [Eqs. (2.1) and (2.2)]. The temperature, T, is measured in units of J_0/k_B. The boundaries separate the paramagnetic phase (P) at high temperatures from the low-temperature ferromagnetic phase (F). The diamonds, circles, and squares are for $\lambda=0.0$, 0.3, and 0.7, respectively. No spin-glass phase at nonzero temperature occurs in this 3D model.

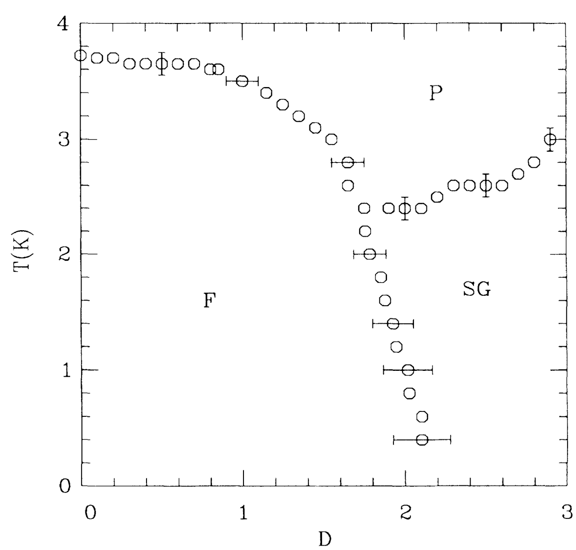

FIG. 5. Temperature (T) vs Dzyaloshinkii-Moriya interactions of the width D phase diagram of the 3D XY ferromagnet with random Gaussian Dzyaloshinkii-Moriya interactions [Eqs. (2.3) and (2.4)]. The temperature, T, is measured in units of J_0, k_B. The uncertainty on the location of the phase boundaries is shown by the error bars. Paramagnetic (P), ferromagnetic (F), and spin-glass (SG) phases are found.

junction systems have been obtained. Both experiments and simulations have failed to observe the reentrant behavior predicated in the theoretical studies.[42,46,49] Thus, the 2D Monte carlo phase diagram has the same topology as in Fig. 5, with the spin-glass phase absent. Although explanations for the absence of reentrance based on too small system sizes in the simulations and on disorder-induced vortex pinning in both the experiments and simulations have been proposed,[43] a convincing argument for this discrepancy is lacking. It has recently been proposed that the 2D version of model (2.3) may be paramagnetic at all temperatures, except for D strictly equal to zero.[58]

In the same spirit as for the bimodal XY model, we have also looked at the 2D version of the XY model with random Dzyaloshinskii-Moriya interactions. We have found that our discretized Migdal-Kadanoff scheme leads to erratic behavior in this case. In particular, it could not converge either towards a paramagnetic phase or to a "Midgal-Kadanoff version" of a phase with quasi-long-range ferromagnetic order. Truly deterministic chaotic behavior of the Migdal-Kadanoff scheme has been observed for spin-glass Potts models in five dimensions and also for other models.[59] We do not know at present whether the behavior observed in the present case is an example of deterministic chaos intrinsic to the rescaling equation (3.1) or if it is due to the discretization schemes used to solve them. In either case, we believe that this erratic behavior is an artifact of the Migdal-Kadanoff approximation rather than a true physical effect.

We conclude that there does not appear at present to be any experimental or numerical evidence supporting the existence of a reentrant behavior in the 3D version of an XY ferromagnet with random Dzyaloshinskii-Moriya interactions. In two dimensions the situation is less clear.

V. DISCUSSION

In conclusion, we find that transverse spin fluctuations in isotropic vector-spin systems and the presence of a longitudinal-transverse coupling, such as that present in a random XY ferromagnet with random Dzyaloshinskii-Moriya interactions, are not sufficient ingredients to lead to reentrance in spin glasses.[60]

Our results, and those of others,[13,28,43] lead us to propose two possibilities which could lead to reentrant behavior in dilute magnetic vector-spin systems and which are missing in the above XY models. The first is the existence of "disordered" spins located at randomly frustrated sites. The second is the presence of off-diagonal random couplings between the spin components which are not rotationally invariant for a given realization of the disorder.

Randomly frustrated sites occur when a spin is connected to its neighbors by competing ferromagnetic and antiferromagnetic bonds such that the total local field at that site would be zero if the neighborhood was perfectly ferromagnetically (or antiferromagnetically) ordered. Randomly frustrated sites will occur, for example, in the above randomly chosen antiferromagnetic bond model (2.1) with (2.2) for $\lambda=1$. However, the Migdal-Kadanoff method, which is a useful tool to study random bond problems, may not be so well suited for systems with randomly frustrated sites.

A 2D XY ferromagnetic model with randomly frustrated sites has been studied by Saslow and Parker using iterated local mean-field equations.[31] They found strong evidence for reentrance in a model with randomly frustrated sites while no convincing evidence for such behavior was found for model (2.1) with randomly distributed antiferromagnetic bonds such as in Eq. (2.2) above. It would therefore appear as a possibility that 3D systems with randomly frustrated sites could show reentrance. However, in the light of our results for $\lambda=1$ (these are not shown, but are almost identical to the ones obtained with $\lambda=0.7$ in Fig. 4) and those from a recent Monte Carlo showing no reentrance in a randomly frustrated 3D Heisenberg model,[28] where randomly frustrated sites "accidentally" occur probabilistically, we believe that this possibility is unlikely.

The second effect which we think could lead to a reentrant behavior is the existence of random off-diagonal couplings. Two common examples of such interactions which are very often present in magnetic spin systems are dipole-dipole interactions and Dzyaloshinskii-Moriya spin-orbit interactions for Heisenberg spin systems.[5,12,61] The importance of such couplings in otherwise isotropic random ferromagnets is twofold. It leads to a destruction of true long-range ferromagnetic order in spatial dimension less than four and is expected to induce Ising spin-glass critical behavior.

First, as pointed out by Aharony[62] and Binder,[63] the presence of random off-diagonal interactions implies a lack of long-range ferromagnetic order in spatial dimension less than four when crystalline anisotropy is absent.[51,62] Aharony makes the observation that random off-diagonal couplings lead to infrared-divergent transverse spin fluctuations which contradict the initial assumption of ferromagnetic order. Aharony's proof is thus very similar to the well-known Imry and Ma argument[64] excluding true long-rang ferromagnetic order for isotropic vector-spin systems with random fields in spatial dimensions less than four. The same conclusion is reached by noticing that the lack of global rotational symmetry in these systems implies that random on-site anisotropies are generated under renormalization, for which the lower critical dimension for long-range ferromagnetic order is four.[46,51] In practical terms, this means that, in the absence of crystalline anisotropy, the transition between the paramagnetic and the ferromagnetic phases observed in reentrant spin-glass systems is not a true thermodynamical transition with divergent spin-spin correlation length, ξ, but is rather a *crossover.* Hence, in many instances, the system will behave very close to a "good" ferromagnet. It is interesting to note that neutron-diffraction results for the Heisenberg ferromagnets $Eu_xSr_{1-x}S^2$, Fe-Mn alloys,[16] and in other systems[12] show that, for some range of dilution x, ξ does not diverge at the P-F transition. Such an absence of long-range order in these systems may be due to the presence of weak random off-diagonal couplings.

The second relevant effect of random dipolar or random Dzyaloshinskii-Moriya interactions is to modify the critical behavior expected for an otherwise isotropic vector-spin-glass system by inducing a crossover to Ising behavior.[65] This observation is very important since it is now generally believed that the 3D Ising spin-glass exhibits a true thermodynamical spin-glass transition,[5,12,55] as opposed to a dynamical one, while isotropic vector-spin glasses are expected to have a lower critical dimension for a spin-glass transition greater than or equal to four.[17–19]

One possible scenario for reentrance could therefore be the following: Without crystal-field effects, the presence of random couplings between the longitudinal and transverse spin components in otherwise isotropic vector-spin systems destroys the long-range order in ferromagnets such that the P-F transition is, formally speaking, not a true transition, but only a crossover to what one may call a "quasiferromagnet" with a very long, but finite, spin-spin correlation length ξ. In the absence of the above perturbative random off-diagonal anisotropic couplings, the otherwise diluted ferromagnet will cross, upon decreasing the temperature, a dynamical transverse spin-freezing transition line.[28] Once the random anisotropies are considered, we expect this dynamical transverse spin-freezing transition to convert into a true spin-glass transition.[65] In this sense, only "pseudoreentrant" behavior is seen, as the ferromagnetic phase never really existed. Only a transition from a paramagnetic to a spin-glass phase occurs. The qualitative phase diagram that we suggest for 3D reentrant Heisenberg ferromagnets is reproduced in Fig. 6. It suggests that the P-F phase boundary is not a true transition, while the F-SG boundary, which is, in fact, a P-SG boundary, corresponds to a true thermodynamical Ising spin-glass phase transition.[5,12,55,65]

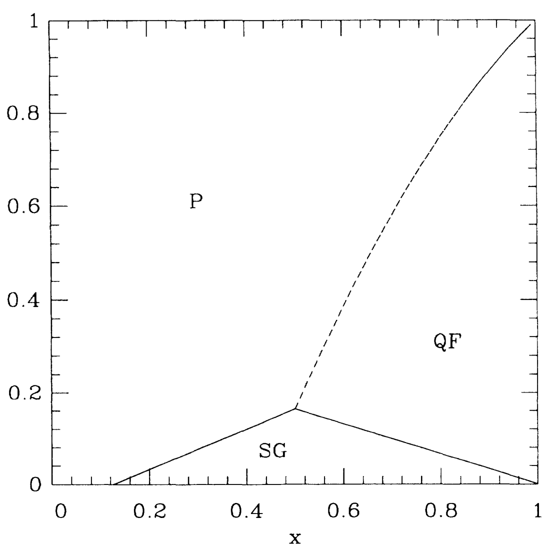

FIG. 6. Schematic x-T phase diagram proposed for reentrant spin glasses in the absence of crystalline anisotropy. T_c is the transition temperature for the pure case, $x=0$. Solid lines are true phase transitions, dashed ones are not. The phases are paramagnetic (P), spin glass (SG), and quasi-ferromagnetic (QF) with a large, but finite, correlation length ξ.

We believe that our phase diagram in Fig. 6 should be reconsidered when crystalline anisotropy is present. Indeed, the situation is much more complicated in this case and one would need to investigate the effects of the competition between the crystal-field and the random off-diagonal couplings. For example, one may expect that, in sufficiently strong cubic anisotropy, a portion of the P-F phase line may become a first-order phase-transition boundary to a ferromagnetic phase with true long-range order.[51,62] In such a case, one may also expect the universality class of the spin-glass transition to change from Ising[65] to the one of the three-state Potts model, which is believed to have a lower critical dimension equal to three.[66] Clearly, at this time we can only be very speculative about the phase diagram of a disordered vector-spin-glass system with competing anisotropy and random off-diagonal couplings. We refer the reader to Ref. 67 where several possible phase diagrams for reentrant spin glasses are proposed.

We conclude by mentioning some questions that we think would be interesting to investigate in the context of reentrant spin glasses. It would be useful to study with a Monte Carlo simulation a 3D *XY* or Heisenberg model with randomly frustrated sites to see whether reentrance occurs due to the spin-freezing mechanism discussed by Saslow and Parker.[31] It would be desirable to understand the effect of crystalline anisotropy in stabilizing ferromagnetic order and removing a reentrant behavior in otherwise random isotropic vector-spin glasses in the presence of random off-diagonal interactions. Finally, a convincing explanation for the experimental observation of the absence of reentrance in antiferromagnets[68,69] and one resolving the controversy in two-dimensions between renormalization-group calculations and Monte Carlo results for model (2.3) with (2.4) are still lacking.

Note added in proof. We have become aware of recent neutron diffraction experiments done on amorphous Fe_x-Mn_{1-x},[70,71] and other amorphous metallic spin-glass compounds.[72] These experiments show that in most compounds investigated, there is no breakdown of the ferromagnetic domains in smaller "subdomains" in the reentrant portion of the phase diagram sufficiently far from the para-ferro-spin-glass multicritical point. The drop in zero-field cooled magnetization is shown to result from a slowing down in the mobility of the domains below a transverse spin-freezing temperature[28,29] rather than a decrease of the size of the magnetization within the domains. In other words, no intradomain reentrance is found in these studies.[70–72] This is in contrast to what is suggested by the results of Ref. 16. We are grateful to M. Hennion and I. Mirebeau for telling us about Refs. 70–72.

ACKNOWLEDGMENTS

We thank G. Aeppli, A. Aharony, D. Bensimon, R. J. Birgeneau, M. Cieplak, M. Grant, M. Kardar, J. M.

Kosterlitz, D. R. Nelson, F. Nori, A.-M. Tremblay, J. Vannimenus, and A. P. Young for useful discussions or correspondence. We thank J. M. Kosterlitz for drawing our attention to Korshunov's work (Ref. 58). We also wish to thank H. Orland for a helpful and stimulating discussion. We are grateful to the Laboratoire de Physique Théorique de l'Ecole Normale Supérieure for a generous CPU time allocation on their workstations. M. G. acknowledges financial support from the Natural Sciences and Engineering Research Council of Canada, and the French Direction des Etudes et Recherches Techniques. E. S. acknowledges financial support from the Danish Natural Research Science council as well as partial support from NSF grant Nos. DMR-8721673 and DMR-9111576.

APPENDIX

1. Fourier method

José *et al.*[48] suggested using the Fourier decomposition of the exponential of the coupling, $\exp\{H_{ij}(\phi_i-\phi_j)\}$, and rewrite (3.1) in Fourier space. The basic equations relating Fourier and real space are

$$\exp\{\beta H_{ij}(\phi_{ij})\}=\sum_{s=-\infty}^{\infty}\tilde{E}_{ij}(s)\exp\{is\phi_{ij}\}\ , \tag{A1a}$$

$$\tilde{E}_{ij}(s)=\int_0^{2\pi}\frac{d\phi}{2\pi}\exp\{-is\phi_{ij}+\beta H_{ij}(\phi_{ij})\}\ , \tag{A1b}$$

where $\phi_{ij}=\phi_i-\phi_j$.

One can now obtain recursion relations for $\tilde{E}'_{ij}(s)$ by rewriting (3.1) in the following form:

$$\tilde{E}'_{ij}(s)=\sum_{s_1}\left[\sum_{s_2}\{\tilde{E}_{in_1}(s_2)\tilde{E}_{in_2}(s_1-s_2)\}\right]\left[\sum_{s_3}\{\tilde{E}_{n_1j}(s_3)\tilde{E}_{n_2j}(s_1-s_3)\}\right]\times\left[\sum_{s_4}\{\tilde{E}_{in_3}(s_4)\tilde{E}_{in_4}(s-s_1-s_4)\}\right]\left[\sum_{s_5}\{\tilde{E}_{n_3j}(s_5)\tilde{E}_{n_4j}(s-s_1-s_5)\}\right]\ . \tag{A2}$$

This is convenient since, in the case of the bimodal XY model, the initial set of $\tilde{E}_{ij}(s)$ are simply modified Bessel functions:

$$\tilde{E}_{ij}(s)=\int_0^{2\pi}\frac{d\phi_{ij}}{2\pi}\exp\{-is\phi_{ij}+\beta J_{ij}\cos(\phi_{ij})\}=I_s(\beta J_{ij})\ . \tag{A3}$$

Using the property $I_s(-x)=(-1)^sI_s(x)$, both ferromagnetic and antiferromagnetic bonds can be incorporated into the formalism. In the case of the XY model, the potential remains symmetric around $\phi=\pi$ and one needs only to work with positive s. When the random Dzyaloshinskii-Moriya interactions are introduced as in (2.5), the $\tilde{E}'_{ij}(s)$ become complex numbers and are thus no longer symmetric, otherwise the above equations remain unchanged, except for (A3) where we get

$$\tilde{E}_{ij}(s)=\int_0^{2\pi}\frac{d\phi_{ij}}{2\pi}\exp\{-is\phi_{ij}+\beta K_{ij}\cos(\phi_{ij}-A_{ij})\}=\exp\{-isA_{ij}\}I_s(\beta K_{ij})\ , \tag{A4}$$

where $K_{ij}=\{J_0^2+D_{ij}^2\}^2$ and $\tan(A_{ij})=D_{ij}/J_0$.

In order to control any "dc" offset, we shift the potential to zero at $\phi=\pi/2$, thus treating ferromagnetic and antiferromagnetic terms on an equal footing. When all the $\tilde{E}'_{ij}(s)$ are generated for a given bond, we therefore rescale them all so the following equality is fulfilled:

$$\sum_{s=-\infty}^{\infty}\tilde{E}_{ij}(s)\exp\{is\pi/2\}=1\ . \tag{A5}$$

In this way $H_{ij}(\phi=\pi/2)$ is always zero.

A finite $s_{\max}$ has to be introduced which for high temperatures can be chosen as low as 10. For lower temperatures, the Bessel functions become ill behaved for large arguments and $s_{\max}$ has to be increased significantly.

The height difference $h_{ij}=H_{ij}(0)-H_{ij}(\pi/2)$ can now be evaluated by transforming back into real space using (A1a). For some values of ϕ this is an alternating sum that for low temperatures becomes a source of additional numerical errors and limits the application of the technique to $\beta J\sim0.2$ for the bimodal XY model and $\beta J\sim0.5$ when the random Dzyaloshinskii-Moriya interactions are introduced. One could argue that it is possible for the potential to diverge at one point while staying finite between 0 and $\pi/2$ thus making h_{ij} a bad measure. The only stringent way to avoid potential trouble is to locate the maximum and minimum of each value of $H_{ij}(\phi_i-\phi_j)$ and define $h_{ij}=H_{ij}^{\max}-H_{ij}^{\min}$. This method is more time consuming and, when averaging over the bonds, procedure (3.2) is adequate

2. Clock method

The other method we use to perform the integrals in (3.1) is to discretize the angles ϕ_i by a finite set of Q states with the same angle $2\pi/Q$ between each state of the set, as proposed by Gienlak *et al.*[44] The Hamiltonian $H_{ij}(\phi_i-\phi_j)$ becomes, in this discretized form,

$$H_{ij}(2\pi\{m_i-m_j\}/Q)\ , \tag{A6}$$

where m_i is an integer that takes the values $0,1,2,\ldots,Q-1$. Since models (2.1) and (2.3) both have global continuous symmetry, we can decompose each coupling H_{ij} of the pool into Q interactions which are the Q-discretized values of the Q possible differences $\phi_i-\phi_j$, with $\phi_i-\phi_j=2\pi\nu_{ij}/Q$ and $\nu_{ij}=0,1,2,\ldots,Q-1$. Hence, Eq. (3.1) can be written as

$$E'_{ij}(\nu_{ij})=\left[\sum_{l=0}^{l=Q-1}[E_{in_1}(\mathrm{mod}\{\nu_{ij}-l+Q,Q\})E_{in_2}(\mathrm{mod}\{\nu_{ij}-l+Q,Q\})\times E_{n_1j}(l)E_{n_2j}(l)]\right]\times\left[\sum_{m=0}^{m=Q-1}[E_{in_3}(\mathrm{mod}\{\nu_{ij}-m+Q,Q\})E_{in_4}(\mathrm{mod}\{\nu_{ij}-m+Q,Q\})E_{n_3j}(m)E_{n_4j}(m)]\right], \quad (A7)$$

where we have defined $E_{ij}(\nu_{ij})=\exp\{\beta H_{ij}(\nu_{ij})\}$ and $\mathrm{mod}\{m,n\}$ is $m\,(\mathrm{mod}n)$. As in the Fourier method, we need to implement a normalization procedure to prevent the divergence of the dc offset. We have used the one in (A5) and also one where the average of each $H_{ij}(\phi_i-\phi_j)$ over $\{\phi_i-\phi_j\}$ is set to zero. The latter choice amounts to subtracting the

$$\frac{1}{Q}\left[\sum_{\nu_{ij}=0}^{\nu_{ij}=Q-1}H_{ij}(\nu_{ij})\right]=0 \quad (A8)$$

from each H_{ij} and to rescale the E_{ij} above accordingly before calculating the distribution of barrier heights h_{ij}. Both rescaling procedures give almost identical results.

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*Present address: Department of Physics, University of California, Santa Cruz, CA 95064.

[1]The terminology "reentrant spin glasses" is somewhat misleading since the system does not reenter a spin-glass phase at low temperatures, as it was a paramagnet, not a spin glass, at high temperatures. However, the system does reenter into a less magnetically ordered state at low temperatures, the spin-glass phase.

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