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Quasiparticle damping due to antiparamagnons below T_c and the microwave conductivity

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The quasiparticle lifetime due to scattering from antiparamagnons is investigated in the superconducting state. In the normal state, antiparamagnon scattering gives rise to a linear resistivity with temperature. Below T_c, the antiparamagnon spectrum is suppressed as the quasiparticles, and hence the particle-hole antiparamagnons, condense out. This suppression leads to an increased quasiparticle lifetime, thereby increasing the normal fluid component of the conductivity σ_n below T_c. Apart from the very-low-temperature region, calculations of the quasiparticle lifetime and σ_n are in reasonable agreement with the values recently extracted from microwave surface impedance measurements on $YBa_2Cu_3O_7$.

Antiferromagnetic fluctuations dominate nuclear spin-lattice relaxation of copper nuclei in the cuprate oxide superconductors. The phenomenological theory developed by Millis, Monien, and Pines[1] has been successfully applied to the analysis of NMR measurements[2] on $YBa_2Cu_3O_{7-\delta}$ and $La_{2-x}Sr_xCuO_4$ in the normal state. These fluctuations also give rise to a resistivity which is linear in temperature in the normal state.[3] A dramatic departure from linearity is expected in the superconducting state. Since the antiferromagnetic excitations, antiparamagnons, correspond to excited quasiparticle-hole pairs, one would expect the spectral density of antiparamagnons to decrease as the quasiparticles condense out below T_c. Similar behavior is observed in superfluid ^3He, where the paramagnon spectral density is suppressed below T_c.[4] Recently Mason, Aeppli, and Mook[5] have observed a decrease in the antiferromagnetic spin fluctuations below T_c in $La_{1.86}Sr_{0.14}CuO_4$ by inelastic neutron scattering. This suppression of the antiparamagnon spectral density will result in an increase in the quasiparticle lifetime arising from spin fluctuations. This increase in lifetime will strongly affect the real part of the conductivity σ_n. As the superconducting state is entered, the "normal fluid" component σ_n will increase as the quasiparticle lifetime increases. As the temperature is lowered well below T_c, σ_n will peak and then fall as the quasiparticles eventually condense out. Nuss *et al.*[6] were the first to suggest that the far infrared conductivity peak was probably due to the suppression of inelastic quasiparticle scattering below T_c.

The goal of the present work is to explain the recent microwave measurements of the surface impedance of high-quality single crystals of $YBa_2Cu_3O_7$ by Bonn *et al.*[7] Their most recent result[8] for σ_n, as extracted from this data, is reproduced in Fig. 1. The conductivity rises below T_c and peaks near 40 K. We shall see that this rise (by a factor of 20 or so) is mainly due to the suppression of quasiparticle damping in the superconducting state. We first discuss the behavior of the antiparamagnon spectral density in the superconducting state assuming BCS singlet s-wave pairing. Then we calculate the quasiparticle lifetime and use this in the real part of the long wavelength, low-frequency conductivity in the London limit. We only present the simplest version of a calculation which clearly can be improved by using the full self-consistent Eliashberg formalism.[9] Our goal is to bring out the essential physics in the simplest possible manner and to exhibit the qualitative features. We find that the striking temperature-dependent results of Bonn and co-workers[7,8] can be understood quantitatively in terms of the known temperature dependence of the spin-lattice relaxation rate. Our work gives a microscopic basis to the conclusions of Bonn *et al.*[8] and focuses attention on the pivotal role of the spin response function, which can be measured by nuclear spin-relaxation as well as neutron scattering.

Quasiparticle excitations in high-temperature superconductors can be described by a dynamic spin susceptibility $\chi(q,\omega)$ which consists of a low frequency, long-wavelength contribution $\chi_{QP}(q,\omega)$ as well as a term $\chi_{AF}(q,\omega)$ which describes the antiferromagnetic fluctuations[1] peaked at $q=Q=(\pi/a,\pi/a)$. χ_{AF} in the normal state is apparently reasonably well-described phenomenologically by the expression[1,2]

$$\chi_{AF}^n(q,\omega)=\frac{\chi_Q}{1+\xi^2(Q-q)^2-i(\omega/\omega_{SF})}, \qquad (1)$$

where ξ is the spin fluctuation coherence length and ω_{SF} is the characteristic spin fluctuation frequency at Q. As de-

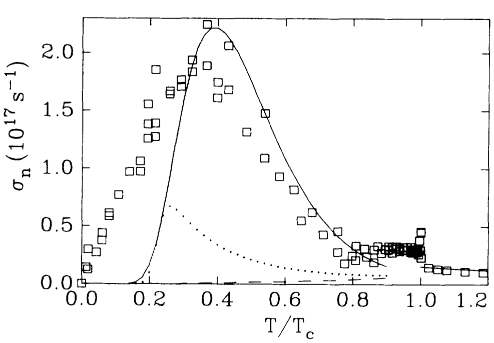

FIG. 1. Temperature dependence of σ_n. The solid line is the calculated value of σ_n given by (7). The dashed line is σ_n with $f(T)=1$ as discussed in the text. In order to illustrate the sensitivity of σ_n to the choice of $f(T)$ we show σ_n with $f(T)=Y(T)$, the dotted line. The squares are the data of Bonn *et al.* (Ref. 8).

scribed in the literature, an expression like (1) can be understood starting from a random-phase approximation theory of the antiparamagnons, where the imaginary part of the dynamic susceptibility is approximated by

$$\frac{\chi''_{0n}(q,\omega)}{[1-J\chi'_{0n}(q,\omega)]^2} . \tag{2}$$

Here $\chi_{0n}=\chi'_{0n}+i\chi''_{0n}$ is some appropriate free-particle susceptibility and J is the antiferromagnetic exchange constant.

In order to calculate the quasiparticle damping below T_c using the Eliashberg formalism, we need the antiparamagnon spectral density in the superconducting state

$$P_{\rm AF}(\omega)=N(0)J^2\int_0^{2k_F}\frac{q\,dq}{2k_F^2}4\,{\rm Im}\chi^s_{\rm AF}(q-Q,\omega) , \tag{3}$$

which involves an average over q about Q [here $N(0)$ is the density of states at the Fermi surface]. The antiparamagnon spectral density we use here is similar to that used in marginal Fermi-liquid theory[10] in the normal phase in that it is linear in frequency up to about $\hbar\omega\simeq k_BT$. Any theory based on (2) is easily generalized to the superconducting state, with $\chi_{0s}(q,\omega)$ modified by the usual[11] type-II (for spin-singlet pairing) coherence factors. Below T_c, our antiparamagnon spectral density intensity will be suppressed at all frequencies due to the decreasing number of quasiparticles. However, there is no gap at twice the quasiparticle energy gap $\Delta(T)$ in $P_{\rm AF}(\omega)$ as assumed in recent work.[10,12] Other work on $\chi^s_{\rm AF}$ has been largely based on tight-binding Hubbard-like models (see, for example, Refs. 9 and 11), which involves extensive numerical work. Since the dominant effect on $\chi_{\rm AF}$ below T_c is the suppression of the quasiparticles forming the particle-hole antiparamagnons, for our present purposes it is sufficient to use

$$\chi^s_{\rm AF}(q,\omega)=f(T)\chi^n_{\rm AF}(q,\omega) , \tag{4}$$

where $f(T)$ is some average measure of the quasiparticle (normal fluid) density.[13]

An obvious first guess for $f(T)$ would be the Yosida function $Y(T)$, which describes the temperature dependence of $\chi_{\rm QP}(q=0,\omega=0)$. A more appropriate choice can be made by recognizing that the nuclear spin-lattice relaxation rate is proportional to a sum over q of $\chi^s_{\rm AF}(q,0)$. It has been noted that[14] the spin-lattice relaxation rate, which is dominated by the short wavelength antiferromagnetic fluctuations near Q, has a temperature dependence of $\exp(T/T_0)$, where $T_0\simeq 13$ K for $YBa_2Cu_3O_7$. Hence the choice

$$f(T)=\exp[(T-T_c)/T_0] \tag{5}$$

should adequately describe the suppression of the antiparamagnon spectrum below T_c and this is what we use in (4) as input.

In order to calculate the quasiparticle lifetime $\tau\equiv 1/2\Gamma$, the weak-coupling expression for the quasiparticle damping rate of Kaplan *et al.*[15] is generalized to

$$\begin{aligned}\Gamma(\omega)=\frac{\pi}{\hbar[1-f(\omega)]}\Bigg[&\int_0^{\omega-\Delta}d\Omega\,N(\omega-\Omega)\left\{\Lambda^+(\Omega)-\frac{\Lambda^-(\Omega)\Delta^2}{\omega(\omega-\Omega)}\right\}[n(\Omega)+1][1-f(\omega-\Omega)]\\&+\int_{\Delta+\omega}^{\infty}d\Omega\,N(\Omega-\omega)\left\{\Lambda^+(\Omega)+\frac{\Lambda^-(\Omega)\Delta^2}{\omega(\Omega-\omega)}\right\}[n(\Omega)+1]f(\Omega-\omega)\\&+\int_0^{\infty}d\Omega\,N(\Omega+\omega)\left\{\Lambda^+(\Omega)-\frac{\Lambda^-(\Omega)\Delta^2}{\omega(\Omega+\omega)}\right\}n(\Omega)[1-f(\Omega+\omega)]\Bigg] .\end{aligned} \tag{6}$$

Here $n(\omega)$ and $f(\omega)$ are the Bose and Fermi distributions, respectively, and $N(\omega)$ is the BCS density of states. The spectral density is $\Lambda^{\pm}(\omega)=\alpha^2F(\omega)\pm P_{\rm AF}(\omega)$, where the $\pm$ sign enters as antiparamagnon scattering breaks time-reversal invariance. For simplicity, we ignore the contribution from the electron-phonon scattering described by $\alpha^2F(\omega)$. Strictly speaking, the appropriate spectral density is the transport $\Lambda_{\rm tr}(\omega)$ if we are trying to calculate the lifetime which enters the conductivity, but we also ignore this difference here.

Our results for $\Gamma(\Delta(T))$ are plotted as a function of temperature in Fig. 2. Note that $\Gamma(\Delta(T))$ is linear in temperature above T_c and that the quasiparticle lifetime increases substantially below T_c as the quasiparticles condense out. The weak-coupling formalism used here[15] is only valid for $\Gamma\ll\Delta$; hence, our results in the temperature region $0.9T_c<T<T_c$ are not valid.

General expressions for the conductivity $\sigma(q,\omega)$ are given by Nam[16] and more recently by Lee, Rainer, and Zimmerman.[17] Taking the $q\to 0$ (London) limit, since the London penetration depth is much greater than the superconducting coherence length, one has [see Eq. (5.9) of Ref. 16]

$$\begin{aligned}\sigma_n(\omega)=\frac{\omega_P^2}{4\pi}\int_\Delta^\infty d\omega'\left[-\frac{df}{d\omega'}\right]\Bigg\{&[N(\omega'+\omega)N(\omega')+M(\omega'+\omega)M(\omega')+1]\frac{1}{\Gamma(\omega')\{N(\omega'+\omega)+N(\omega')\}}\\&+[N(\omega'+\omega)N(\omega')+M(\omega'+\omega)M(\omega')-1]\frac{\Gamma(\omega')\{N(\omega'+\omega)+N(\omega')\}}{(\{(\omega'+\omega)^2-\Delta^2\}^{1/2}+(\omega'^2-\Delta^2)^{1/2})^2+\Gamma(\omega')^2\{N(\omega'+\omega)+N(\omega')\}^2}\Bigg\}\end{aligned} \tag{7}$$

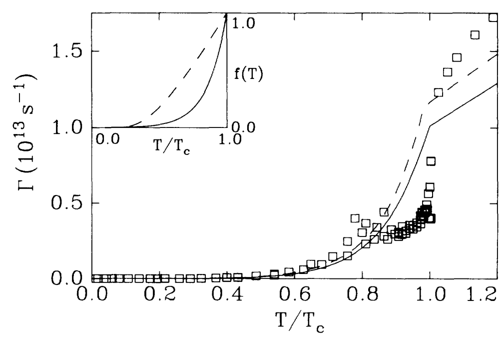

FIG. 2. Quasiparticle damping rate $\Gamma(\Delta(T))$ given by (6), solid line. Parameters used are $J=0.136$ eV and $N(0)=2.63$ states/eV Cu. The squares are the values of $1/2\tau$ extracted from the data of Bonn *et al.* (Ref. 8) as described in the text. The dashed line is the extracted value of $\Gamma(\Delta(T))$ using our calculated $\sigma_n(T)$ as input. The inset shows $\exp[(T-T_c)/T_0]$ (solid line) and the Yosida function (dashed line).

in addition to a small contribution from an integral over the region $(\Delta-\omega,\Delta)$. In writing (7), we have made use of the fact that $\omega \ll \Delta$ and $\omega' \geq \Delta$. The second contribution in (7) is much smaller than the first at low temperatures. $\omega_p^2=4\pi ne^2/m$ is the usual normal bulk plasma frequency, which we take to be equal to 1.4 eV. Note that for elastic impurity scattering $2\Gamma(\omega)N(\omega)=1/\tau_n$. As the damping is quite strong it is also included in the density of states in the square brackets in (7) through the imaginary part of the gap Δ_2 (i.e., $\Delta_2 > \omega$). At all but the lowest temperatures, the penetration depth in $YBa_2Cu_3O_7$ is well approximated by the usual BCS expression with a weak-coupling BCS gap $\Delta(T)$. This fact has been emphasized in Ref. 8. In our numerical calculations, the temperature dependence of the gap is approximated by the well-known analytic form

$$\Delta(T)=\Delta(0)\tanh\left[\left(-\frac{d\Delta^2}{dT}\right)_{T_c}\frac{(T_c/T-1)^{1/2}}{\Delta(0)}\right] \quad (8)$$

where we take $2\Delta(0)/k_BT_c=3.5$. Note that we assume a form for $\Delta(T)$; we are not assuming that $\Lambda^{\pm}(\omega)$ is responsible for the pairing although it may play some role.

Our results for the microwave conductivity σ_n at 2 GHz is shown in Fig. 1. For comparison, we show σ_n calculated with $f(T)=1$ in (4), i.e., no suppression of the antiparamagnon spectrum. In this case the conductivity immediately decreases below T_c. In contrast, σ_n calculated for the suppressed antiparamagnon scattering rises below T_c. It continues to rise down to a temperature of $0.4T_c$ and then falls as the quasiparticles condense out. This rise in σ_n *well below* T_c is indicative of a substantial increase in the quasiparticle lifetime below T_c. Note that this behavior is similar to that seen in the data of Bonn and coworkers[7,8] (see Fig. 1). No coherence effects are seen in the data as strong coupling effects eliminate them.[9,12] The rise in σ_n can easily be distinguished from coherence effects, which are apparent only in a small temperature region just below T_c. In Fig. 1, for comparison, we also show our results for σ_n calculated with $f(T)=Y(T)$ in (4).

The antiparamagnon parameters in (1) are taken from the NMR analysis[2] on $YBa_2Cu_3O_7$. Data from various samples[8] indicate that the temperature at which σ_n peaks is fairly constant, which is consistent with our model. In particular, this indicates that ordinary impurity scattering cannot be very important in the samples used in Ref. 8.

Our calculated quasiparticle damping rate can be compared to that extracted from the microwave data using the two-fluid model extension of the Drude expression

$$\sigma_n(T)=x_n(T)\frac{ne^2}{m}\tau(T)\,, \quad (9)$$

where τ is the lifetime of quasiparticles with thermal energies above the gap Δ and $x_n(T)$ is the normal fluid fraction. Keeping only the first term (dominant at temperatures $T \leq 0.9T_c$), we find (7) indeed reduces to an expression like (9) with

$$x_n(T)=\int_\Delta^\infty d\omega'\left(-\frac{df}{d\omega'}\right)$$

$$\times\frac{N(\omega'+\omega)N(\omega')+M(\omega'+\omega)M(\omega')+1}{N(\omega')}\,. \quad (10)$$

We note that this expression for $x_n(T)$ reduces to the Yosida function $Y(T)$ in the zero frequency limit $\omega \rightarrow 0$. This result is consistent with the fact that the BCS superfluid density fraction determined from the penetration depth is given by $x_s(T)=1-Y(T)$.

Bonn *et al.*[8] first noted that in the superconducting state the temperature dependence of $1/\tau$, as extracted from fitting the experimental σ_n data to the Drude formula (9), could be described by (5). We have shown that this result is reproduced by a microscopic calculation of σ_n, assuming that the quasiparticles scatter from antiparamagnons whose spectral density is given by (4) and (5). Our choice of $f(T)$ in (4) strongly affects the rate at which $\Gamma(T)$ decreases just below T_c (see inset of Fig. 2). If we were to use $f(T)=Y(T)$, this would essentially cancel the factor of $x_n(T)$ in (9) yielding a much weaker temperature dependent $\sigma_n(T)$ as shown in Fig. 1. Above T_c, our calculated value of $\Gamma(1.2T_c)$ is 1.29×10^{13} s^{-1}, whereas the extracted value using (9) and $x_n=1$ is 1.76×10^{13} s^{-1}. The dashed line in Fig. 2 shows $\Gamma(T)$ extracted from our calculated values of $\sigma_n(T)$ using the same procedure. Note that it yields larger values for $\Gamma(T)$ at all temperatures. This is due to the energy dependence of Γ which is ignored in (9) and (10) where we use $\Gamma(\Delta(T))$ instead of $\Gamma(\omega')$. This disregard of the energy dependence of $\Gamma(\omega')$ may account, in part, for the larger value of Γ extracted from the microwave data.

Our calculation is seen to give the absolute value of σ_n at T_c quite well and also produces a large peak in σ_n of the correct magnitude and at the observed temperature. These results are strong evidence that our antiparamagnon scattering mechanism is correct. We might note that our results shown in Fig. 1 for σ_n are quite dependent on our choice of the zero temperature gap $2\Delta(0)/k_BT_c=3.5$. Using a larger value of $\Delta(0)$ gives a smaller peak in σ_n at higher temperatures, mainly because $x_n(T)=Y(T)$ decreases much more rapidly with temperature.

Tanner *et al.*[18] have analyzed the frequency dependence of the infrared conductivity data to extract a quasiparticle lifetime, assuming a phenomenological Drude model. They found a similar increase in the lifetime below T_c.

At low temperature, σ_n drops off exponentially for an s-wave gap function. If the gap function had nodes, then the drop would presumably follow a power law in T but σ_n would nevertheless approach zero with T, since the quasiparticles are condensing out. The near linear T dependence of the low-temperature $\sigma_n(T)$ data suggests that an s-wave gap is incompatible with the microwave measurements.

In addition to spin fluctuations, however, the quasiparticles can also be scattered by charge-fluctuation modes[19] (plasmons). While these plasmons shift from being particle-hole modes to oscillations of the charged Cooper pair condensate as the temperature is lowered below T_c, there is little change in their spectral weight or energy.[20] In the layered oxide materials, these condensate plasmons can have energy less than the $2\Delta(T)$ needed to break up a Cooper pair.[21] Their contribution to the microwave absorption at low temperatures has not been included here.

In conclusion, the continued rise in σ_n down to a temperature of $0.4T_c$ is indicative of a rapid decrease in the quasiparticle damping rate. Using the observed temperature dependence of the nuclear spin-lattice relaxation rate to determine $\chi^s_{\rm AF}$, our calculation of σ_n shows that there is quantitative agreement with what is expected from antiparamagnon scattering down to about $T=0.3T_c$. The exponential temperature dependence of $\chi^s_{\rm AF}$ is different than that of $\chi^s_{\rm QP}$ because $\chi^s_{\rm AF}(q,\omega)$ involves fluctuations predominantly at Q and not at $q=0$ where $\chi^s_{\rm QP}$ follows the Yosida function. The linear behavior of the microwave conductivity at very low temperatures is not understood but appears to be inconsistent with an s-wave gap.

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