RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 46, NUMBER 5 1 AUGUST 1992-I

Nonlocal Josephson electrodynamics and pinning in superconductors

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(Received 18 February 1992; revised manuscript received 8 April 1992)

Local Josephson electrodynamics based on the sine-Gordon equation is generalized to the nonlocal case of high critical current density j_s across a contact for which the Josephson penetration depth is smaller than the London magnetic penetration depth. Magnetic flux is shown to penetrate the contact in the form of Abrikosov vortices having highly anisotropic cores much larger than the coherence length. An exact solution describing such a vortex is found; the lower critical field, vortex mass, and flux-flow resistivity are calculated. It is argued that vortices localized on planar crystalline defects are weakly pinned, therefore any weak links with j_s smaller than the depairing current density form a dissipative network which essentially reduces the critical current and facilitates a possibility of quantum flux creep.

Mechanisms determining current-carrying capacity of high-T_c superconductors, especially anomalous dependences of critical currents I_c upon magnetic field H and temperature T, and also a high sensitivity of I_c to even weak crystalline disorder, due to the short coherence length ξ and high anisotropy,[1-3] have attracted considerable interest since the discovery of these materials. These features, as well as significant thermal fluctuations of pinned fluxons,[4] are believed to be the most important factors limiting I_c of high-T_c oxides. For instance, grain boundaries that cause local suppression of the order parameter $\psi=\Delta\exp(i\varphi)$ divide a sample into superconducting grains coupled by a weak Josephson interaction.[1] Magnetic field penetrates the grain boundaries in the form of Josephson (J) vortices having sizes of order

$$\lambda_J=(c\phi_0/16\pi^2\lambda j_s)^{1/2}, \quad (1)$$

where ϕ_0 is the flux quantum, c is the velocity of light, λ is the bulk London penetration depth, and j_s is the critical current density across the contact.[5] Usually the Josephson penetration depth λ_J is assumed to be much larger than λ because j_s is regarded as much smaller than either the intragranular critical current density j_c caused by bulk pinning or the depairing current density $j_d=c\phi_0/12\sqrt{3}\pi^2\lambda^2\xi$. Due to their larger sizes, the J vortices are pinned much more weakly than the intragrain Abrikosov (A) fluxons (see, e.g., Ref. 6). This causes the decoupling of superconducting grains, since the grain boundaries form a dissipative network[7] along which the magnetic flux can move through a superconductor at much smaller I_c than that determined by bulk pinning.

This model of superconducting decoupling cannot be directly applied to good thin films or single crystals, where high-angle grain boundaries and other incoherent defects are absent. Nevertheless, these superconductors still contain coherent planar defects such as twins, stacking faults, low-angle grain boundaries, etc.,[3] which do not cause strong lattice distortions, but can lead to a local reduction of superconducting gap 2Δ due to the small value of ξ.[1] In this case one could expect a moderate local decrease of j_d at the defects to some value j_s which can still be larger than the critical current density j_c determined by the intragrain pinning of fluxons. Hereafter, such defects will be called "hidden" weak links, since they cannot affect I_c directly due to the decoupling of superconducting grains and related effects of magnetic granularity. However, it will be shown that these weak links change the structure of the vortex core, which can lead to a significant reduction of longitudinal pinning force for vortices localized at the defects. This results in the appearance of weakly pinned A vortices which can move along a dissipative network[7] similar to the Josephson network in granular materials and thereby limit the I_c of high-current superconductors despite having $j_s>j_c$. An analogous situation may occur in conventional low-T_c superconductors as well, for example, in optimized high-j_c Nb-Ti alloys, where the strong pinning is caused by a dense network of thin $[d\sim(0.1/2)\xi]$ α-Ti ribbons,[8] or in Nb_3Sn, where the pinning is due to grain boundaries.[3]

As an illustration, we consider a vortex situated at a planar defect with $j_s \ll j_d$ (Fig. 1). Since the current density $j(r)$ at the core ($r\sim\xi$) of the A fluxon is of order j_d,[9] the length l of the core along the defect becomes much larger than the core size ξ in the transversal direction due to that $j(r)$ across the defect cannot exceed j_s. The

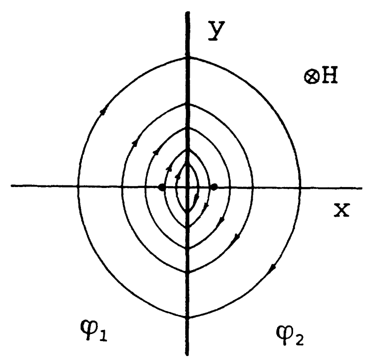

FIG. 1. Vortex localized on a planar defect being in the yz plane. Current lines described by Eqs. (12)-(14) are two sets of arcs centered at the point $x=l$ for the half plane $x<0$ and $x=-l$ for the half plane $x>0$. The points on the x axis ($x=l$ and $x=-l$) indicate positions of fictitious A fluxons determining the field distribution $H(x,y)$ at $x<0$ and $x>0$, respectively.

length l can be estimated from the continuity of currents flowing parallel and perpendicular to the defect within the core, $j_d\xi \sim j_s l$, whence

$$l \sim j_d\xi/j_s \gg \xi \,. \tag{2}$$

Thus the vortex core becomes highly anisotropic, its size l along the defect being smaller than λ provided that $j_s \gtrsim j_d\xi/\lambda$. As follows from Eq. (1), the latter occurs in the case of strong Josephson coupling for which $\lambda_J(j_s) < \lambda$, i.e.,

$$j_s > j_l = c\phi_0/16\pi^2\lambda^3 \sim j_d/\kappa \,, \tag{3}$$

with $\kappa = \lambda/\xi$ the Ginzburg-Landau parameter. Assuming $\lambda(T) = \lambda(0)[1-(T/T_c)^2]^{-1/2}$, one gets for $YBa_2Cu_3O_7$ [$\lambda_{ab}(0) = 1500$ Å, $T_c = 92$ K] that $j_l(0) = 1.1\times10^7$ A/cm^2 and $j_l(77\text{ K}) = 1.6\times10^6$ A/cm^2. In thin films of thickness $d_f < \lambda$ the condition $\lambda_J(j_s) < \lambda$ can hold at smaller j_s because now the magnetic field penetrates the defect over the length $\lambda_j \sim \lambda_J^{\text{bulk}}\lambda^{1/2}/d_f^{1/2}$,[5] whereas the effective London penetration depth becomes λ^2/d_f.[9] Hence it follows that $j_l^{\text{film}} \sim (d_f/\lambda) j_l^{\text{bulk}} \ll j_l^{\text{bulk}}$.

If a superconductor contains only defects with $j_s > j_l$ the J vortices ($\lambda_J \gg \lambda$) are absent. However, there appear Abrikosov-like vortices having highly anisotropic Josephson cores ($l \gg \xi$) (Fig. 1) which are pinned more weakly than fluxons within grains. Indeed, consider the most effective core pinning by normal precipitates when the maximum elementary pinning force per unit vortex length $f \sim \alpha(\phi_0/4\pi\lambda)^2/l$ is inversely proportional to the core size l, in the optimum case of pins having sizes $\sim l$ (α is a volume fraction occupied by pins).[10] Hence it follows that the longitudinal pinning force f_s for vortices localized on planar defects is much smaller than the intragrain pinning force f_b if current flows perpendicular to the defect plane yz. This result is independent of details of pinning potential $U(\mathbf{r})$ determined by both randomly distributed pins within the grains and inhomogeneities of the grain boundaries. An additional mechanism leading to $f_s \ll f_b$ may occur if the potential $U(\mathbf{r})$ is not optimum for the both types of vortices simultaneously. For instance, if $U(\mathbf{r})$ is optimized for intragrain fluxons only [$U(\mathbf{r})$ varies over scales of order of ξ] it cannot lead to the effective pinning of vortices at the grain boundaries, where the core size is much bigger than ξ. Thus both the difference in vortex core sizes ($l \gg \xi$) and the mismatch of the pinning potential $U(\mathbf{r})$ yield $f_s \ll f_b$. As a result, the pinning force $\mathbf{f}$ turns out to be highly anisotropic with respect to the orientation of j: The force f_s is maximum ($f_s \sim f_b$) for $\mathbf{j} \parallel yz$ and minimum ($f_s \ll f_b$) for $\mathbf{j} \perp yz$. Thereby the planar defect can become a channel along which the vortices begin moving at much smaller j_c, as compared with intragrain fluxons. This model differs from the intrinsic pinning model[11] in which a stack of planar defects is regarded to result in an additional pinning force perpendicular to the defect plane. Both mechanisms result in the similar anisotropy of $\mathbf{f}(\mathbf{j})$, however, here only the contribution coming from the change of the vortex core structure at hidden weak links is considered.

As follows from the above qualitative arguments, the hidden weak links with $j_s > j_l$ strongly deform the cores of A fluxons. However, the field distribution outside the core ($r \gg l$), where circular screening currents decay exponentially over the length λ, remains the same as that of the A fluxon in uniform superconductor. In this paper such a vortex is considered in detail. At $j_s \ll j_d$ the current density within the vortex is much smaller than j_d, therefore the defect can be treated as a long high-current Josephson contact for which the Josephson penetration depth λ_J is smaller than λ, regardless of specific mechanisims causing the local decrease of j_d across the defect. At $\kappa \gg 1$ such a situation occurs in a wide region $j_l < j_s < j_d$, with $j_l \sim j_d/\kappa$ given by Eq. (3). In the case $\lambda_J(j_s) < \lambda$ the conventional Josephson electrodynamics based on the sine-Gordon equation[5] is invalid because the equation for the phase difference $\varphi = \varphi_1 - \varphi_2$ across the contact becomes nonlocal, as will be shown. The corresponding nonlinear integral equation for φ enables one to trace a crossover between the Josephson and Abrikosov vortices when increasing j_s from $j_s \ll j_l$ to $j_s \sim j_d$. An exact solution describing the vortex at $j_l \ll j_s < j_d$ can then be obtained.

In order to derive the equation for $\varphi(y,t)$ in the case of the infinite planar Josephson contact shown in Fig. 1, we use the following relation:[5]

$$\varphi'(y) = 8\pi^2\lambda^2[j_y(x=+0,y) - j_y(x=-0,y)]/c\phi_0 \,, \tag{4}$$

where the prime denotes differentiation with respect to y. The x and y axes are perpendicular and parallel to the contact, respectively, with magnetic field H along the z axis. It is assumed that $\varphi(\mathbf{r})$ depends only on y, and the thickness of the contact d is negligible compared with λ, which enables one to neglect a contribution from the vector potential $\mathbf{A}$ in Eq. (4). The values $j_y(\pm 0)$ are determined by the London equation describing $H(x,y)$ outside the contact

$$\lambda^2\nabla^2 H - H = 0 \tag{5}$$

(for simplicity an anisotropy of λ is not taken into account). Equation (5) can be solved by a Fourier transformation in y, which gives the Fourier transform $H(x,k)$ vanishing at $x = \pm\infty$ in the form

$$H(x,k) = H_0(k)\exp[-|x|(1+\lambda^2k^2)^{1/2}/\lambda] \,. \tag{6}$$

Here $H_0(k)$ is the Fourier transform of $H(x=0,y)$ related to $j_x(0,k)$ and $j_y(\pm 0,k)$ via the Maxwell equations $j_x(0,k) = ikcH_0(k)/4\pi$ and $j_y(\pm 0,k) = cH_0(k)(1+\lambda^2k^2)^{1/2}\text{sgn}(x)/4\pi\lambda$. Using these formulas and Eq. (4), one can exclude H_0 and express $j_x(0,k)$ via the Fourier component $\varphi(k)$:

$$j_x(k) = -\frac{c\phi_0 k^2\varphi(k)}{16\pi^2\lambda(1+\lambda^2k^2)^{1/2}} \,. \tag{7}$$

By carrying out the inverse Fourier transformation of Eqs. (6) and (7) and using a standard representation of j_x as a sum of Josephson, resistive, and displacement currents,[5] one gets the following equations for $\varphi(y,t)$ and $H(x,y)$:

$$\frac{\partial^2\varphi}{\partial\tau^2} + \eta\frac{\partial\varphi}{\partial\tau} = \frac{\lambda_J^2}{\pi\lambda}\int_{-\infty}^{\infty} K_0\left(\frac{|y-u|}{\lambda}\right)\frac{\partial^2\varphi}{\partial u^2}du - \sin\varphi \,, \tag{8}$$

$$H(x,y)=\frac{\phi_0}{4\pi^2\lambda^2}\int_{-\infty}^{\infty}K_0\left[\frac{[x^2+(y-u)^2]^{1/2}}{\lambda}\right]\frac{\partial\varphi}{\partial u}du\,. \tag{9}$$

Here $K_0(x)$ is a modified Bessel function, $\eta=1/RC\omega_J$, $\tau=\omega_J t$ is a dimensionless time, $\omega_J=(2ej_s/\hbar C)^{1/2}$, $-e$ is the electron charge, and C and R are specific capacitance and resistance of the contact, respectively. Notice that unlike the sine-Gordon equation, the dynamic integral equation (8) at $\eta=0$ is no longer Lorentz invariant, with the maximum group velocity of electromagnetic waves given by $c_s=\lambda_J\omega_J$.[5,12]

Equations (8) and (9) are valid for any relation between j_s and j_l. For instance, at $\lambda_J\gg\lambda$ ($j_s\ll j_l$) the phase $\varphi(y)$ varies along the contact ($x=0$) over the length $\sim\lambda_J$; that is, $\varphi(u)$ varies much more slowly than $K_0(u)$. This enables one to replace $K_0(u)=\pi\delta(u)$, obtaining the sine-Gordon equation for $\varphi(y,t)$ and the conventional formula $H_0(y)=\phi_0\varphi'(y)/4\pi\lambda$ of local Josephson electrodynamics[5] instead of Eqs. (8) and (9). In the opposite case $\lambda_J\ll\lambda$ or $j_s\gg j_l$ Eqs. (8) and (9) are nonlocal; that is, the kernel $K_0(x,y-u)$ varies much slowly than the phase gradient $\varphi'(u)$ which changes sharply over the length $l\ll\lambda$. For instance, in the case of a single vortex, the field $H(x,y)$ outside the core $|y|\gg l$ can be obtained from Eq. (9) if one puts $\varphi'(y)=2\pi\delta(y)$, which gives the field distribution in the A fluxon.[9]

Explicit single-vortex solutions of Eq. (8) can be found both in the local ($\lambda\ll\lambda_J$, $j_s\ll j_l$) and the nonlocal ($\lambda\gg\lambda_J$, $j_s\gg j_l$) limits. For instance, in the local case the integral equation (8) reduces to the sine-Gordon equation which has the known solution $\varphi(y)=4\tan^{-1}[\exp(y/\lambda_J)]$ (Ref. 5) describing the J vortex with an accuracy to $(\lambda/\lambda_J)^2\ll 1$. In the opposite nonlocal limit the function $\varphi''(u)$ in Eq. (8) sharply decays over the length $\sim l$ therefore the Bessel function $K_0(z)$ can be replaced by its expansion at small argument, $K_0(z)=\ln(2/z)-\mathrm{C}$, where $\mathrm{C}=0.577$ is the Euler constant. In this case the nonlinear integral equation (8) has the following asymptotically exact solution describing a stationary vortex at $\lambda\gg\lambda_J$,

$$\varphi(y)=\pi+2\tan^{-1}(y/l)\,, \tag{10}$$

$$l=\frac{\lambda_J^2}{\lambda}=\frac{3\sqrt{3}}{4}\frac{j_d}{j_s}\xi\,. \tag{11}$$

In such a vortex the phase $\varphi(y)$ changes from 0 to 2π over the length $\sim l$ which coincides with the qualitative estimation (2) with an accuracy to a numerical coefficient. Both for the local and the nonlocal cases the corrections to these solutions coming from the above-discussed approximations of the kernel $K_0(u)$ are negligible. For instance, at $|y|>\lambda$ the asymptotes of $\varphi(y)$ differ from those given by Eq. (10) due to a weak effect of London screening on the structure of the vortex core ($|y|\ll\lambda$). This results in exponential decay of $\varphi'(y)$ instead of the power one in Eq. (10) at $|y|>\lambda$, which do not affect the physical characteristics of the vortex considered below. For instance, by integrating Eq. (9) with $K_0(z)=\ln(2/z)-\mathrm{C}$ and $\varphi(y)$ given by Eq. (10), one gets the field and current distributions in the vortex at $x^2+y^2\ll\lambda^2$ in the form

$$H(x,y)=\frac{\phi_0}{4\pi\lambda^2}\left[\ln\frac{4\lambda^2}{y^2+(l+|x|)^2}-2\mathrm{C}\right], \tag{12}$$

$$j_x(0,y)=\frac{c}{4\pi}\frac{\partial H}{\partial y}=-\frac{c\phi_0}{8\pi\lambda^2}\frac{y}{y^2+l^2}\,, \tag{13}$$

$$j_y(\pm 0,y)=-\frac{c}{4\pi}\frac{\partial H}{\partial x}=\frac{c\phi_0}{8\pi\lambda^2}\frac{l\,\mathrm{sgn}x}{y^2+l^2}\,. \tag{14}$$

Formulas (12)–(14) describe the crossover between the A and J vortices when increasing l from $l\sim\xi$ to $l\sim\lambda$, respectively. As follows from Eq. (12), the distribution $H(x,y)$ allows a clear geometrical interpretation, namely, the current lines in the domain $x>0$ coincide with those of an A fluxon placed in the point $x=-l$ (or $x=l$ for the domain $x<0$) (Fig. 1). At $x^2+y^2\gg l^2$, such a field configuration reduces to the A fluxon in uniform superconductor, the structure of the core being inessential and one can put $l=0$ in Eqs. (12)–(14). However the field in the center of the vortex $H(0)=(\phi_0/2\pi\lambda^2)[\ln(2\lambda/l)-\mathrm{C}]$ is finite and depends on the core size l. In addition, there is a discontinuity in $j_y(x)$ at $x=0$ because the magnetic field penetrating the contact induces antiparallel screening currents at $x>0$ and $x<0$.

Within the region $|y|<l$, $|x|<\xi$ the phase $\varphi(y)$ varies significantly, with $j_x(y)$ decreasing from the maximum value $j_x(l)=j_s$ at $y=l$ to zero at $y=0$. This region plays the role of a vortex core, although it differs qualitatively from the normal core of the A fluxon resulting from a reduction of the superconducting gap $2\Delta(r)$ in its center, since $j(r)$ given by Eqs. (12)–(14) at $l=0$ becomes of order of j_d at $r\sim\xi$. By contrast, the core of the vortex localized on the defect is due to weak Josephson coupling thereby $j(x,y)$ is always much smaller than j_d. As a result, the gap 2Δ within the core coincides with the bulk value $2\Delta(\infty)$, and the low-energy ($E\ll\Delta$) electron excitations are absent if the defect can be treated a thin ($d<\xi$) insulating or normal layer.

Making use of Eq. (10), one can calculate the lower critical field H_{c1}, the vortex mass m and the viscous drag coefficient η_0 in the nonlocal limit by analogy with those of J vortices.[5,12–17] To find H_{c1}, we present the free energy F of the contact in the form

$$F=\left[\frac{\phi_0}{8\pi^2\lambda}\right]^2\int_{-\infty}^{\infty}K_0\left[\frac{|y_1-y_2|}{\lambda}\right]\frac{\partial\varphi}{\partial y_1}\frac{\partial\varphi}{\partial y_2}dy_1dy_2+\frac{\hbar j_s}{2e}\int_{-\infty}^{\infty}(1-\cos\varphi)dy\,, \tag{15}$$

where the first term is the energy of magnetic fields and superconducting currents $F_m=\int(\mathbf{H}^2+\lambda^2|\mathrm{curl}\mathbf{H}|^2)dx\,dy/8\pi$ expressed via $\varphi'(y)$ by Eqs. (6) and (9), and the second term is the energy of the Josephson coupling F_J. By substituting Eq. (10) into Eq. (15) and performing integrations,[14] one finds that $H_{c1}=4\pi F/\phi_0$ is given by

$$H_{c1}=\frac{\phi_0}{4\pi\lambda^2}\left[\ln\frac{\lambda}{l}+\gamma\right], \tag{16}$$

where $\gamma=\ln 2+1-\mathrm{C}=1.116$. The hidden weak links thus decrease the bulk H_{c1} due to the change of the vortex core energy.

The absence of the normal core leads to a reduction of the vortex mass m, compared with the A fluxons for which m is mainly determined by electrons localized in the core.[13] However for the vortex pinned at a hidden weak link, there is only much smaller electromagnetic contribution to m caused by the vortex motion.[13-16] At small velocities v of the vortex ($v \ll l\omega_J$) and $\eta = 0$, its energy increases by $\frac{1}{2}mv^2 = \frac{1}{2}C\int V^2(y)dy$, where $V = (\hbar/2e)\partial\varphi/\partial t$ is the voltage across the contact. Hence

$$m = \frac{C\hbar^2}{4e^2}\int_{-\infty}^{\infty}\left(\frac{\partial\varphi}{\partial y}\right)^2 dy = \frac{\pi C\hbar^2}{2e^2 l}, \qquad (17)$$

where $\varphi(y)$ is given by Eq. (10). Likewise, one can find the viscous drag coefficient η_0 in the overdamping regime ($RC\omega_J \ll 1$) by equating $\eta_0 v^2$ to the total dissipation rate $R^{-1}v^2(\hbar/2e)^2\int\varphi'^2 dy$ (Refs. 12 and 17), which yields $\eta_0 = m/RC$. This allows one to get the flux-flow resistivity $\rho_f = H\phi_0/c^2\eta_0$ provided that the vortex cores in the contact do not overlap, i.e., $H < \phi_0/2l\lambda$. Then

$$\rho_f = 2\pi RlH/\phi_0 . \qquad (18)$$

It is interesting to note that similar formulas for $\varphi(y)$, H_{c1}, and η_0 have been obtained before for layered superconductors in the weak-coupling limit ($j_s \ll j_l$) within the framework of local Josephson electrodynamics.[16,17] For a stack of planar Josephson contacts the model proposed in Refs. 16 and 17 predicts the Josephson-like vortex core due to a collective many-layer interaction, although in the single-layer case considered above it reduces to the description of the J vortex. Nevertheless, both the local many-layer model and the nonlocal single-layer model result in formulas (16)–(18) in which the length l turns out to be qualitatively different in the local and the nonlocal cases. For instance, for the stack of Josephson contacts the length l is of order an interlayer spacing,[16,17] unlike the case of the strong coupling for a single contact considered in this paper, for which l depends on j_s and can be much larger than ξ.

In summary, the coherent crystalline defects with $j_l < j_s < j_d$ result in neither the appearance of J vortices in a superconductor, nor the pronounced effects of granularity and magnetic decoupling. However, they strongly deform the cores of bulk A fluxons (Fig. 1), which is a manifestation of nonlocal effects in pinning similar to those in elasticity theory of vortex lattice.[18] This reduces H_{c1} [see Eq. (16)] if the defects form a percolating network through which the magnetic flux can penetrate the superconductor. Such a network of hidden weak links forms a natural path for flux motion, since the elementary pinning forces along the network turn out to be much smaller than the intragrain f_b for the most effective core pinning. In this case the vortices localized at planar defects are mainly pinned by the weaker collective interaction,[6] which may determine the total critical current of high-j_c superconductors. This dissipative network can also affect I-V curves at $j < j_c$ and dependences of flux creep parameters on T and H.[19] For instance, the reduction of vortex mass and viscosity due to the absence of normal core considerably facilitates a possibility of quantum vortex tunneling in bulk superconductors which has recently been proposed to account for a temperature-independent flux creep in high-T_c oxides at low T.[20,21] These effects become more pronounced in granular materials, where the existence of grain boundaries with $j_s \ll j_c$ give rise to weakly pinned J vortices for which $l \sim \lambda_J$.

I am grateful to J. R. Clem, L. D. Cooley, D. C. Larbalestier, and G. Stejic for useful discussions. The work was supported by DARPA and Electric Power Research Institute.

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