Spin excitations in a two-dimensional antiferromagnet with mobile holes

I. R. Pimentel* and R. Orbach
Department of Physics, University of California, Los Angeles, California 90024
(Received 24 June 1991; revised manuscript received 29 January 1992)

The spin dynamics of a two-dimensional antiferromagnet doped with a finite concentration n of mobile holes is studied. The holes weaken the antiferromagnetic order by dilution, but more importantly by disrupting the antiferromagnetic order as they move. We calculate the renormalization of the spin excitations induced by hole motion. Spin-wave theory is applied to the t-J model and the range of small hole concentrations, $n \ll 1$, is considered. We find that the spin-wave spectrum is significantly softened upon doping, and that strong damping effects set in at a low concentration, resulting from decay of spin waves into "electron-hole" pair excitations. This implies that the spin-wave spectrum and eventually the antiferromagnetic order will collapse as the hole concentration increases. A comparison is made with measurements of spin-wave softening in doped copper oxide superconductors and other layered magnetic systems. Implications for hole motion are also briefly discussed.

I. INTRODUCTION

The discovery of superconductivity in the layered copper oxides[1] has stimulated a large number of studies on the nature of magnetism in these materials, both because of its intrinsic interest and its possible role in the mechanism for high-T_c superconductivity. The undoped parent compounds are antiferromagnetic (AF) insulators. When dopant holes are introduced, the AF order is rapidly destroyed with increasing hole concentration. Upon further doping, the system becomes superconducting. There is experimental evidence for a magnetic moment on Cu, while short-range AF correlations remain in the superconducting state.[2]

It is widely believed that the CuO_2 planes are responsible for the superconductivity. In the undoped materials, these are well described by the isotropic spin-$\frac{1}{2}$ Heisenberg model in a square lattice.[3] Doping introduces holes into the oxygen orbitals. Zhang and Rice[4] argued that, in the relevant parameter range, the oxygen holes may form a singlet with the copper moments, leading to a hole in the magnetic square lattice of copper moments. As initially suggested by Anderson,[5] the simplest model that contains the physics of the high-T_c materials is the single-band Hubbard model with strong on-site repulsion U, near half-filling. In the limit of very large U, the Hubbard model can be transformed into the t-J model Hamiltonian acting on the space with no doubly occupied sites:

$$H_{t\text{-}J} = -t \sum_{\langle i,j \rangle,\sigma} [(1-n_{i,-\sigma})c_{i\sigma}^{\dagger}c_{j\sigma}(1-n_{j,-\sigma})+\text{H.c.}] + J\sum_{\langle i,j\rangle}(\mathbf{S}_i\cdot\mathbf{S}_j - \tfrac{1}{4}n_i n_j) . \quad (1)$$

Here $\langle i,j\rangle$ indicates nearest-neighbor pairs, $n_{i,-\sigma} = c_{i,-\sigma}^{\dagger}c_{i,-\sigma}$ and $c_{i\sigma}^{\dagger}, c_{i\sigma}$ are creation and annihilation operators of an electron on site i with spin σ, the factor $(1-n_{i,-\sigma})$ enforcing the constraint of no double occupancy, $\mathbf{S}_i$ is the spin on site i, $n_i = n_{i\uparrow}+n_{i\downarrow}$, and the exchange coupling is $J=4t^2/U$.[6] At half-filling only the Heisenberg part of the Hamiltonian is relevant, and it describes an AF insulator. With doping and near half-filling, the Hamiltonian describes holes which are the charge carriers moving in an Heisenberg spin system. For the copper oxide materials the exchange coupling has been determined experimentally to be $J \sim 0.1$ eV. The hopping parameter is less well known, but is usually taken to be $t \sim 3J$.

A very important feature of these systems is that the holes are strongly coupled to the antiferromagnetic spin array. The motion of holes tends to disrupt the AF order because, as it moves, the hole leaves behind a trail of overturned spins. This is a consequence of the strong constraint on the occupancy. In the case of highly anisotropic Ising spin interactions, the hole motion is hindered by the scrambling of the spin order, and the holes are localized.[7] The physics is qualitatively different if we consider Heisenberg interactions because quantum fluctuations associated with the transverse exchange interactions may "repair" a pair of wrongly pointed overturned spins, allowing the holes to be mobile. It then follows that the holes are no longer simple free charge carriers, but become dressed by a cloud of spin excitations. Spin dynamics is the essential factor in determining hole mobility, i.e., the hole effective mass or bandwidth. The spin arrangement is only a perfect antiferromagnet at half-filling. With doping, the spin order is disrupted. The excitations are no longer pure AF spin waves, but are renormalized by interaction with the holes. The dressing of the holes is therefore determined by the resulting dressed spin excitations. In order to understand the destruction of the AF order, and the transition to superconductivity, one needs to include the renormalization of both the hole and the spin dynamics.

The problem of hole motion has been studied in a variety of approaches. These include exact diagonalization of small clusters,[8,9] variational calculations,[10] and self-consistent Green's-functions techniques.[11-14] Using Green's functions, Schmitt-Rink, Varma, and Ruckenstein,[11] and Kane, Lee, and Read[12] considered the coupling of a hole to virtual spin excitations, Gros and Johnson[13] developed a technique that improves the treatment of the constraint of no double occupancy, and Su *et al.*[14] further included the effects of local spin distortions around a hole. However, most of this work treats only one hole so that the spin excitations correspond to that of a pure antiferromagnet. With a finite concentration of holes, the AF spin excitation spectrum will change, and the proper renormalized form for AF excitations should be taken into account when calculating hole motion. Indeed, numerical calculations by Szczepanski *et al.*[9] show that the shape of the hole spectrum depends significantly on the nature of the spin background. Nevertheless, spin-wave renormalization due to a finite hole concentration has largely been neglected. The work of Ko,[15] and Brenig and Kampf[16] considers the limit of localizated holes. Gan, Andrei, and Coleman[17] studied the effect of mobile holes on spin dynamics, however, followig the Schraiman and Siggia[18] hypothesis in which incommensurate helimagnetic long-range order is assumed. There is no experimental evidence for the existence of such incommensurate order in the low concentration regime considered.[19]

We study the spin dynamics of an antiferromagnet in the presence of a finite concentration of mobile holes in this work. We are particularly interested in the effects of hole motion on the spin excitations. We find as a result that the spin excitations in the two-dimensional AF planes are very sensitive to doping, and become significantly softened even upon light doping. We also find that strong damping effects start to occur at low hole concentrations. Recent experiments in copper oxides,[20,21] and also in other layered magnetic systems,[22] have revealed a combination of softening and damping in the spin fluctuations of the doped materials when compared with those in the pure materials. It has been suggested that these effects are associated with the mobility of the holes. This will be shown to be the case in this work. In addition, the softening of the spin excitations has implications for hole motion, and this will also be discussed below.

II. CALCULATION OF THE SPIN EXCITATIONS

Our starting point is the t-J Hamiltonian, Eq. (1), for a two-dimensional square lattice with spin $\frac{1}{2}$. We use a Green's-functions formalism in our calculation. Because we consider small hole concentrations, $n \ll 1$, and so wish to study states close to the pure AF state, we choose the Néel state as the vacuum. We then define hole operators (obeying Fermi statistics) $h_i = c_{i\uparrow}^\dagger$ on the spin-up sublattice and $c_{i\downarrow}^\dagger$ on the spin-down sublattice, and hard-core boson operators b_i, such that $b_i^\dagger = S_i^-$ on the spin-up sublattice and S_i^+ on the spin-down sublattice.[23] Introducing these definitions in (1), the Hamiltonian becomes

$$H_{t\text{-}J} = -t \sum_{\langle i,j\rangle} h_i h_j^\dagger (b_i^\dagger + b_j) + \tfrac{1}{2} J \sum_{\langle i,j\rangle} (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j) \times (b_i^\dagger b_i + b_j^\dagger b_j + b_i b_j + b_i^\dagger b_j^\dagger - 1) \,. \tag{2}$$

In the Heisenberg part, the factor $(1-h_i^\dagger h_i)(1-h_j^\dagger h_j)$ projects out the spin coupling between two neighboring sites if one or both are hole occupied, and the usual Holstein-Primakoff transformation has been performed keeping only linear terms. The transfer part of the Hamiltonian preserves the constraint of no double occupancy because $h_i b_i = 0$, and also properly describes the spin reversal as the hole moves. It neglects, however, the distortion of the spin directions in the vicinity of a hole. Su *et al.*[14] used a particular formalism to study the effect of spin distortion on hole motion. They found that spin distortion can generate hole hopping and hence contribute to the hole bandwidth. For small mobile hole concentrations, we do not believe this will qualitatively change our results.

By Fourier transforming, and applying the Bogoliubov transformation for the spin-wave variables

$$\beta_k = u_k b_k - v_k b_{-k}^\dagger \,,$$
$$\beta_{-k}^\dagger = -v_k b_k + u_k b_{-k}^\dagger \,,$$

where

$$u_k = \left[\frac{1 + (1-\gamma_k^2)^{1/2}}{2(1-\gamma_k^2)^{1/2}} \right]^{1/2} ,$$
$$v_k = -\mathrm{sgn}(\gamma_k) \left[\frac{1 - (1-\gamma_k^2)^{1/2}}{2(1-\gamma_k^2)^{1/2}} \right]^{1/2} ,$$

and $\gamma_k = \frac{1}{2}(\cos k_x + \cos k_y)$, the Hamiltonian (2) becomes

$$H_{t\text{-}J} = H_{\mathrm{SW}} + H_h + H_t + H_J \tag{3}$$

with

$$H_{\mathrm{SW}} = \sum_k \omega_k^0 \beta_k^\dagger \beta_k, \quad \omega_k^0 = J(1-n^2)(1-\gamma_k^2)^{1/2} \,, \tag{3a}$$

$$H_h = \varepsilon_0 \sum_k h_k^\dagger h_k, \quad \varepsilon_0 = \tfrac{1}{2} J (1-n) \,, \tag{3b}$$

$$H_t = -t \sum_{k,q} h_k h_{k-q}^\dagger [(\gamma_{k-q} v_q + \gamma_k u_q)\beta_{-q} + (\gamma_{k-q} u_q + \gamma_k v_q)\beta_q^\dagger] \,, \tag{3c}$$

$$H_J = -J(1-n) \sum_{k,p,q} h_k^\dagger h_{k-p-q} [A_{pq} \beta_{-p}^\dagger \beta_q + B_{pq}(\beta_p \beta_q + \beta_{-p}^\dagger \beta_{-q}^\dagger)] \,, \tag{3d}$$

$$A_{pq} = (1-\gamma_{p+q})(u_p u_q + v_p v_q) + (\gamma_p + \gamma_q)(u_p v_q + v_p u_q) \,,$$
$$B_{pq} = \tfrac{1}{2}[(1+\gamma_{p+q})(u_p v_q + v_p u_q) + (\gamma_p + \gamma_q)(u_p u_q + v_p v_q)] \,,$$

where now $J = zJ$, $t = zt$, z is the coordination number ($z=4$ for the square lattice), and $(1-h_i^\dagger h_i)(1-h_j^\dagger h_j)$ has been set equal to $1-(1-n)(h_i^\dagger h_i + h_j^\dagger h_j) - n^2$.[15] The interaction terms H_t and H_J renormalize the pure spin-wave Green's function. Our purpose is to study the effects of hole motion and thus we are primarily interested in the renormalization due to the transfer interaction H_t. We will neglect the interaction H_J while studying this renormalization because the effect of H_J on this effect is J/t times smaller than the effect itself.

We select terms from Eq. (3) to construct the effective Hamiltonian,

$$H = H_{\mathrm{SW}} + H_h + H_t \tag{4}$$

and calculate the spin Green's function,

$$G(\mathbf{k}, t-t') = -i\langle T\beta_k(t)\beta_k^\dagger(t')\rangle \ . \tag{5}$$

Because $J \approx 1000$ K, we adopt the zero-temperature formalism. The Fourier transform of the spin Green's function (5) is given by

$$G(\mathbf{k},\omega) = \frac{1}{\omega - \omega_k^0 - \Sigma(\mathbf{k},\omega) + i\eta} \ , \tag{6}$$

where $\Sigma(\mathbf{k},\omega)$ is a self-energy generated by H_t, and $\eta \to 0^+$. In our calculation we sum an infinite class of "bubble" diagrams which describes decay of the spin waves into "electron-hole" pairs. This corresponds to considering only the first contribution from H_t to the self-energy, of order t^2, as illustrated in Fig. 1. Summing only the "bubble" diagrams ignores corrections to the hole-spin interaction vertex. It would seem that this diagrammatic expansion in terms of the transfer interaction would not yield a sensible result because we treat the regime where $t > J$. However, as will be shown, the actual expansion parameter is $[(t/J)\sqrt{n}\,]$. Thus, the expansion is valid even for $t > J$ as long as n is sufficiently small.[24] It is this condition which sets the limits on the magnitude of the hole concentration regime that we are allowed to study. The expression for the self-energy shown in Fig. 1 is

$$\Sigma(\mathbf{k},\omega) = -it^2 \sum_q (\gamma_q v_k + \gamma_{q-k} u_k)^2 \times \int \frac{d\omega_q}{2\pi} \bar{G}(\mathbf{q}-\mathbf{k}, \omega_q - \omega)\bar{G}(\mathbf{q},\omega_q) \ , \tag{7}$$

where $\bar{G}$ is the hole propagator.

To calculate this self-energy, one needs to know the hole propagator. Physically each hole can gain energy by transfer, for $t > J$, and will do so by carrying a cloud of flipped spins as discussed earlier. In the effective Hamiltonian Eq. (4), however, the holes have a flat band if no renormalization effects induced by the transfer interaction are included. Thus, in order to treat the holes correctly, it is first necessary to perform a self-consistent treatment of the one-hole problem to produce a quasiparticle band allowing hole dispersion.[25] Besides providing a proper description of the hole propagation, this avoids artificial effects that can later arise in the determination of the spin-excitation spectrum. Using a self-consistent perturbation theory, Kane, Lee, and Read, among others,[11,13,14] have studied the renormalization of one-hole hopping in an AF background. They found that the hole spectrum is strongly renormalized by the interaction with the spin excitations, and that the hole can be described by a narrow quasiparticle band with a bandwidth of order J, because it is J that limits the transfer rate. They also identified the position of the band minimum as being located at the points $(\pm\pi/2, \pm\pi/2)$ in the Brillouin zone. Similar results have also been obtained in numerical calculations.[8,9]

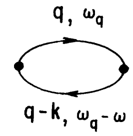

FIG. 1. Self-energy diagram considered in the approximation for the propagator of spin excitations. The solid line is the hole propagator and the shaded circle is a hole-spin vertex, which is of order t.

We shall assume the following. The hole quasiparticles form a weakly interacting Fermi gas described by a single-hole dispersion relation.[26] The holes have a quadratic dispersion with effective mass $m \sim 1/J$, and are located in "pockets" at $(\pm\pi/2, \pm\pi/2)$ in the Brillouin zone for an AF ordered system. Figure 2 illustrates the Fermi surface for the system with a small concentration of holes.[27] Although it has been found that the band is anisotropic, being relatively flat along the zone boundary, in our calculation we shall consider the dispersion $\varepsilon_k \approx (1/2m)(k-k_i)^2$ in each of the four "pockets" $k_i = (\pm\pi/2, \pm\pi/2)$ for simplicity. Performing the frequency integration in (7), we obtain the following expression for the self-energy:

$$\Sigma(\mathbf{k},\omega) = t^2 \sum_q (\gamma_q v_k + \gamma_{q-k} u_k)^2 \left[\frac{\left[1 - \sum_i \Theta(k_F - |\mathbf{q}-\mathbf{k}_i|)\right]\left[\sum_i \Theta(k_F - |\mathbf{q}-\mathbf{k}-\mathbf{k}_i|)\right]}{\omega + \varepsilon_{q-k} - \varepsilon_q + i\eta} - \frac{\left[1 - \sum_i \Theta(k_F - |\mathbf{q}-\mathbf{k}-\mathbf{k}_i|)\right]\left[\sum_i \Theta(k_F - |\mathbf{q}-\mathbf{k}_i|)\right]}{\omega + \varepsilon_{q-k} - \varepsilon_q - i\eta} \right] , \tag{8}$$

where $k_F = \sqrt{\pi n}$. The function $\Theta(x)$ restricts the summation in $\mathbf{q}$ to values such that x lies in the "pockets" illustrated in Fig. 2. The spin-excitation energies are obtained from the poles of the dressed Green's function (6), with $\Sigma(\mathbf{k},\omega)$ given by (8). Performing the summation over $\mathbf{q}$, neglecting terms of $O(k_F^4) \sim O(n^2)$, one obtains

$$\mathrm{Re}\Sigma(\mathbf{k},\omega) = -t^2 \frac{k_F^2}{8\pi} \frac{1}{w - (k^2/2m)^2} (\sin^2 k_x + \sin^2 k_y) \times [w - (k^2/2m)(1-\gamma_k^2)^{-1/2}] \tag{9}$$

with

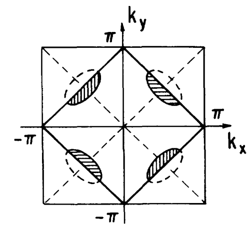

FIG. 2. The reduced Brillouin zone appropriated for an AF ordered system and the four "pockets" of the Fermi sea at $(\pm\pi/2, \pm\pi/2)$.

$$\mathrm{Im}\Sigma(\mathbf{k},\omega)\neq 0$$
$$\text{for } [-kk_F/m + k^2/(2m)] < \omega < [kk_F/m + k^2/(2m)] \ . \tag{10}$$

The actual form for the self-energy $\Sigma(\mathbf{k},\omega)$ is very sensitive to the position of the holes in the Brillouin zone. We introduce Eq. (9) in (6) and study the poles of $G(\mathbf{k},\omega)$. We find that, for $k < (t/J)k_F$, the spin dispersion is given by the following expression:

$$\omega = \tfrac{1}{3}\omega_k^0[1 + (r + s^{1/2})^{1/3} + (r - s^{1/2})^{1/3}] \tag{11}$$

with

$$r = \left[1 - 9\frac{(k^2/2m)^2}{\omega_k^{02}}\right] - 9t^2\frac{k_F^2}{16\pi}\frac{(\sin^2 k_x + \sin^2 k_y)}{\omega_k^{02}} \times \left[1 - 3J\frac{(k^2/2m)}{\omega_k^{02}}\right]$$

$$s = -3^3\frac{(k^2/2m)^2}{\omega_k^{02}}\left[1 - \frac{(k^2/2m)^2}{\omega_k^{02}}\right]^2 + 3^3 t^2\frac{k_F^2}{8\pi}\frac{(\sin^2 k_x + \sin^2 k_y)}{\omega_k^{02}} \times \left[J\frac{(k^2/2m)}{\omega_k^{02}} + 5\frac{(k^2/2m)^2}{\omega_k^{02}} - 9J\frac{(k^2/2m)^3}{\omega_k^{04}} + 3\frac{(k^2/2m)^4}{\omega_k^{04}}\right] .$$

In the limit $k \ll 1$, this reduces to

$$\omega = Zck \ ,$$

where $c = (1/\sqrt{2})J$ is the spin-wave velocity for the pure antiferromagnet, and $Z = Z[(t/J)\sqrt{n}\,]$ is the renormalization factor, to first order in the doping n,

$$Z = \frac{1}{3}\left\{1 + \left[1 + \frac{3\sqrt{3}}{4}\frac{t}{J}\sqrt{n} - \frac{9}{32}\left[\frac{t}{J}\sqrt{n}\right]^2\right]^{1/3} + \left[1 - \frac{3\sqrt{3}}{4}\frac{t}{J}\sqrt{n} - \frac{9}{32}\left[\frac{t}{J}\sqrt{n}\right]^2\right]^{1/3}\right\} . \tag{12}$$

It follows from (11), and more obviously from (12), that the perturbation of the dispersion with respect to that for a pure antiferromagnet is determined by the parameter $[(t/J)\sqrt{n}\,]$. It turns out that, for finite hole concentrations, the renormalization factor is $Z<1$, implying softening of the spin excitations upon doping. The results obtained here are discussed in the next section.

III. COMPARISON WITH EXPERIMENT AND DISCUSSION

We have treated a two-dimensional antiferromagnet doped with a small concentration of mobile holes, and have calculated the renormalization of the spin excitations due to hole hopping. We find that the long-wavelength, $k \ll 1$, excitations have a dispersion of the form $\omega = Zck$, where Z is a renormalization factor. The fact that $Z<1$ implies that the spin excitations are softened upon doping. We find that, for $t/J=3$, and a concentration of $n=0.01$ hole per site, the spin-wave energies are renormalized by a factor $Z=0.98$. For $n=0.05$, the renormalization factor takes the value $Z=0.78$, showing substantial softening. The renormalization down of the spin-wave velocity would imply instability of the AF long-range order for a concentration n such that $Z \sim 0$. However, important damping effects start to occur at a lower concentration, which we now discuss.

Damping of spin waves occurs in the region where the spin-wave spectrum crosses the pair excitation continuum, defined in Eq. (10) as the region where $\mathrm{Im}\Sigma(\mathbf{k},\omega)\neq 0$. For sufficiently small doping, long-wavelength spin waves remain well defined. The reason is that decay of these spin waves into electron-hole pairs is not possible because, in this case, the spin-wave velocity is larger than the Fermi velocity. However, some of the short-wavelength spin waves are heavily damped by decay into "electron-hole" pairs. For concentrations above a certain threshold n^*, such that the spin-wave velocity equals the Fermi velocity, $Z^*c/(k_F^*/m)=1$, the spin-wave spectrum lies entirely in the pair excitation continuum, and, in this case, even the long-wave-length spin waves are overdamped. We find from our calculation that $n^*=0.32$ (for which $Z=0.36$). The decay of spin waves into "electron-hole" pairs gives rise to a broadening of the spectral density of the spin-wave modes. Ramakrishnan[28] has suggested that this implies an increase in the zero-point fluctuations which leads to a dramatic reduction and eventual destruction of the magnetic order. Hence, one expects that long-range AF order could collapse at a concentration n_c ($n_c < n^*$), above which a considerable fraction of spin waves is overdamped. In real materials, true long-range AF order disappears at rather low concentrations, for example, in $La_{2-n}Sr(Ba)_nCuO_4$ at a concentration $n \simeq 0.02$. However, neutron-scattering experiments[2] have revealed that, above such concentrations, there are AF correlated regions present in the system corresponding to the magnetic correlation lengths ξ. These regions can sustain spin excitations with wavelengths up to the size of ξ.[29] Aeppli *et al.*[20] and Hayden *et al.*[21] investigated the spin dynamics of pure La_2CuO_4 and doped $La_{2-0.05}Ba_{0.05}CuO_4$. Their data shows that, for the dynamics of the doped material, excitations with energies above a certain value, corresponding to excitations within the AF correlated regions, exhibit a combination of softening and damping effects with respect to the corresponding ones in the pure material. More specifically, for the doped material, Aeppli *et al.* find that the spin-wave velocity is renormalized by a factor $Z = 0.74(\pm 0.08)$, while Hayden *et al.* find a renormalization factor $Z = 0.60$. $La_{2-n}Ba_nCuO_4$ is insulating at very low concentrations but undergoes a metal-insulator transition at $n \simeq 0.05$, before finally becoming a superconductor at $n \simeq 0.15$. Based on electrical resistivity measurements in the doped material, Aeppli *et al.*[20] and Hayden *et al.*[21] suggest that the softening and damping effects observed in the spin-wave excitations are associated with the appearance of (weakly localized) carriers rather than to simple disorder as found in insulating random magnets. Spin-wave softening has also been observed in $YBa_2Cu_3O_{6+n}$,[30] though the authors remark that, in this case, the experiments were performed in a regime where the holes are well localized (for a concentration of $n = 1.8\%$ they find a spin-wave velocity renormalized by a factor $Z = 0.50$). Besides these cases, strong spin-wave softening associated with an increase in metallic conductivity has also been found in the layered magnetic material La_2NiO_{4+n}.[22] The spin-wave velocity is reduced by at least a factor of 2 in this case, which implies a renormalization factor $Z < 0.50$ for a concentration of $n = 0.05$. Our results are roughly in agreement with all of the above examples. Thus, our calculation predicts a significant softening of the spin excitations resulting from hole motion at low concentrations, and this is in agreement with experiments. As one would expect, the spin-wave softening effects that we find caused by mobile holes are stronger than the ones found by Brenig and Kampf[16] for static holes. However, the work of Ko,[15] which also considers static holes, but utilizes a different formulation from that of Brenig and Kampf, yields a reduction of the spin-wave energies not only larger than theirs, but even larger than we find allowing for hole transport. We are unable to explain this discrepancy. Finally, the work of Gan, Andrei, and Colman,[17] which considers mobile holes in an incommensurate ordered background, reveals much stronger softening effects than we have found, or than exhibited by experiments but, as we have said before, there is no experimental evidence for the existence of such incommensurate order in the low concentration regime considered.

The softening of the spin-wave spectrum has implications for the nuclear spin-lattice relaxation rate $(1/T_1)$. Chakravaty *et al.*[31] calculated the relaxation rates in La_2CuO_4 and found that they are inversely proportional to the spin-wave velocity raised to different powers depending on the site, Cu, O, or La, and temperature range considered. Consequently, the softening of the spin waves implies an increase in the $1/T_1$ rates in the doped material compared with those in the pure material.

Furthermore, the softening of the spin waves has important implications for the motion of the holes. The decrease in the spin-wave velocity can be seen as a decrease in the effective coupling between the spins, so the misalignment generated by the motion of the hole now costs less energy. This implies an increase in the hole mobility. Also, the reduction of the spin-wave velocity gives rise to an increase in the density of low-energy excitations that can couple to the hole. This may eventually lead to a significant increase in the scattering of the hole, even for very long wavelengths, and, consequently, give rise to a finite lifetime for the hole quasiparticle.

IV. CONCLUSIONS

We have considered a two-dimensional antiferromagnet doped with a small concentration of mobile holes, and have studied the modification of the spin excitations induced by hole motion. We applied spin-wave theory to the t-J model and evaluated the renormalization of the spin-wave propagator caused by the transfer interaction. We calculated the self-energy in the "bubble" approximation, describing the decay of spin waves into "electron-hole" pairs. We find that, as a result of the strong spin-hole correlation, the spin excitations are very sensitive to doping, and become significantly softened even for light doping. We also find that strong damping effects set in at a low concentration because of decay of spin waves into "electron-hole" pairs. This implies that the spin-wave spectrum and eventually the antiferromagnetic order will collapse as the hole concentration increases. We compared our results with experiments on copper oxides, and other layered magnetic materials. These experiments show a combination of softening and damping in the spin fluctuations of the doped materials, and it has been suggested that these are associated with the mobility of the holes. Our results support this view. We also predict an increase in the nuclear spin-lattice relaxation rates in the doped materials when compared with those in the pure material, as a consequence of the softening of the spin excitations. Finally, we discussed implications of the softening of the spin waves for hole motion. We note that it leads to an increase in hole mobility, and may eventually give rise to a finite lifetime for the hole quasiparticle.

Future work is planned to extend our investigation to the very low doping regime where holes are *localized*. We shall study the modification of the spin-wave spectrum in the presence of vacancies, as well as in the presence of local frustration caused by ferromagnetic bonds randomly distributed in the AF planes. We hope that this will allow us to understand results of recent nuclear spin-lattice relaxation measurements[32] performed in the insulating phase of copper oxide superconductors.

ACKNOWLEDGMENTS

We would like to thank S. Chakravarty, S. Kivelson, D. Scalapino, and B. Spivak for most helpful discussions. We particularly thank D. Scalapino for a discussion on hole dispersion, and for bringing to our attention the importance of considering a proper renormalized hole bandwidth in order to avoid a artificial effect in the spin wave excitation spectrum. We especially thank S. Kivelson for calling our attention to the importance of considering the proper Fermi surface. One of us (I.R.P.) also thanks S. Sondhi for useful discussions. This work was supported by the National Science Foundation under Grant No. DMR-88-05443 and by the U.S. Office of Naval Research under Contract No. Nonr N00014-88-K-0058. Also, I.R.P. gratefully acknowledges support by Fundação Luso-Americana and Junta Nacional de Investigação Científica (Portugal).

---

*Permanent address: Departamento de Fisica, Universidade de Lisboa, Campo Grande Ed. C1, 1700 Lisboa, Portugal.

[1]J. G. Bednorz and K. A. Müller, Z. Phys. B **64**, 189 (1986).

[2]R.J. Birgeneau and G. Shirane, in *Physical Properties of High Temperature Superconductors,* edited by D. M. Ginzberg (World Scientific, New Jersey, 1990), Vol. I.

[3]S. Chakravarty, in *High Temperature Superconductivity,* Proceedings of the Los Alamos Symposium, edited by K. S. Bedell, D. Coffey, D. E. Meltzer, D. Pines, and J. R. Schrieffer (Addison-Wesley, Redwood City, CA, 1990).

[4]F. C. Zhang and T. M. Rice, Phys. Rev. B **37**, 3759 (1988).

[5]P. W. Anderson, Science **235**, 1196 (1987); in *Frontiers and Borderlines in Many Particle Physics,* Proceedings of the International School of Physics "Enrico Fermi," Varenna, Italy, 1987, edited by R. A. Broglia and J. R. Schrieffer (North-Holland, Amsterdam, 1988).

[6]Note that Eq. (1) is obtained by retaining only the first terms in the expansion of the Hamiltonian in powers of t/U. In this expansion there are other terms of the same order as J, which correspond to hopping between second neighbors. These are often neglected and will also be neglected here.

[7]W. F. Brinkman and T. M. Rice, Phys. Rev. B **2**, 1324 (1970).

[8]J. Bronca, P. Prelovsek, and I. Sega, Phys. Rev. B **39**, 7074 (1989); E. Dagotto, A. Moreo, and T. Barnes, *ibid.* **40**, 6721 (1989); V. Elser, D. Huse, B. Shraiman, and E. Siggia, *ibid.* **41**, 6715 (1990); Y. Hasegawa and D. Poilblanc, *ibid.* **40**, 9035 (1989).

[9]K. J. Szczepanski *et al.*, Phys. Rev. B **41**, 2017 (1990).

[10]S. A. Trugman, Phys. Rev. B **37**, 1597 (1988); S. Sachdev, *ibid.* **39**, 12 232 (1989).

[11]S. Schmitt-Rink, C. M. Varma, and A. E. Ruckenstein, Phys. Rev. Lett. **60**, 2793 (1988).

[12]C. L. Kane, P. A. Lee, and N. Read, Phys. Rev. B **39**, 6880 (1989).

[13]C. Gros and M. D. Johnson, Phys. Rev. B **40**, 9423 (1989).

[14]Z. B. Su, Y. M. Li, W. Y. Lai, and L. Yu, Phys. Rev. Lett. **63**, 1318 (1989).

[15]D. Ko, Phys. Rev. Lett. **65**, 116 (1990).

[16]W. Brenig and A. P. Kampf, Phys. Rev. B **43**, 13 383 (1991).

[17]J. Gan, N. Andrei, and P. Coleman, J. Phys.: Condens. Matter **3**, 3537 (1991).

[18]B. I. Shraiman and E. D. Siggia, Phys. Rev. Lett. **61**, 467 (1989).

[19]T. R. Thurston, Phys. Rev. B **40**, 4585 (1989).

[20]G. Aeppli *et al.*, Phys. Rev. Lett. **62**, 2052 (1989).

[21]S. M. Hayden *et al.*, Phys. Rev. Lett. **66**, 821 (1991).

[22]G. Aeppli and D. Buttrey, Phys. Rev. Lett. **61**, 203 (1988).

[23]This is the same representation as used by Schmitt-Rink, Varma, and Ruckenstein, and Su *et al.*, see Refs. 11 and 14.

[24]We thank S. Chakravarty for clarifying this point.

[25]P. A. Lee, in *High Temperature Superconductivity* (Ref. 3).

[26]S. A. Trugman, Phys. Rev. Lett. **65**, 500 (1990).

[27]From B. I. Shraiman and E. D. Siggia, Phys. Rev. B **40**, 9162 (1989).

[28]T. V. Ramakrishnan, Physica B **163**, 4 (1990).

[29]S. Chakravarty, B. I. Halperin, and D. R. Nelson, Phys. Rev. B **39**, 2344 (1989).

[30]J. Rossat-Mignod *et al.*, Physica B **163**, 4 (1990).

[31]S. Chakravarty and R. Orbach, Phys. Rev. Lett. **64**, 224 (1990); S. Chakravarty *et al.*, Phys. Rev. B **43**, 2796 (1991).

[32]P. Mendels *et al.*, Physica C **171**, 429 (1990).