Theory of flux motion with backflow current in high-κ superconductors

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Based upon the well-known normal-core model for flux line in high-κ superconductors and by correctly taking into account the backflow current due to both pinning forces and other vortices, we developed a theory for flux motion in the framework of the Bardeen-Stephen and Nozières-Vinen approaches. We derived analytically the longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} as functions of the magnetic field in the region of flux flow. Our results explain qualitatively all the essential features of recent experiments on ρ_{xy}, including, in particlar, the observed negative Hall resistivity at low magnetic field in certain high-T_c superconductors. Furthermore, a long-standing problem concerning the expressions for the viscous-drag force and the applied force on a vortex is discussed and clarified.

I. INTRODUCTION

Although flux motion in type-II superconductors is an old subject,[1–3] it currently enjoys a renaissance because of worldwide research into many aspects of the mixed state in high-T_c superconductors. The Hall effect in the mixed state has been an important problem for the understanding of flux motion in type-II superconductors.[4–9] Recently, transport measurements on some high-T_c superconductors with high critical currents have shown an anomalous behavior of Hall resistivity ρ_{xy} in ceramic, single-crystal, and epitaxial-film samples, especially an unexpected sign reversal of the Hall voltage (or ρ_{xy}) in low magnetic field and at temperatures close to but below the superconducting transition temperature T_c.[10–15] Furthermore, this behavior seems to be an intrinsic property of these superconductors and could not be explained with existing theories[5,6,8] of flux motion.

An important theory of flux motion was proposed by Bardeen and Stephen.[6] It was based upon a local normal-core model without considering the pinning effect. In that work the dissipation of the system is assumed to come from the region inside the normal core and normal-superconducting transition boundary (a closely related model was also suggested by Niessen and van Vijfeijken[7]). Let us consider a single flux core which is parallel to the direction of the magnetic field (z axis, unit vector $\mathbf{k}$), carrying a quantum of flux $\Phi_0=hc/2|e|$ (e =charge on the carrier). The drift velocity $\mathbf{v}_{\rm NC}$ of charge carriers is assumed to be constant inside the core and is described by a force-balance equation,

$$\frac{m\mathbf{v}_{\rm NC}}{\tau}=e\mathbf{E}_{\rm NC}+\frac{e}{c}\mathbf{v}_{\rm NC}\times\mathbf{H} , \tag{1.1}$$

where τ and m are, respectively, the momentum relaxation time and mass of the charge carrier, and $\mathbf{E}_{\rm NC}$ and $\mathbf{H}$ are, respectively, the effective electric and magnetic fields inside the core. In the Bardeen-Stephen (BS) theory, it has been shown that the drift velocity $\mathbf{v}_{\rm NC}$ is equal to the applied current velocity $\mathbf{v}_T$ in the absence of pinning forces. In addition, the charge carriers are assumed to be in local equilibrium with the lattice; i.e., the total chemical potential in the lattice frame of reference, $\mu_{\rm tot}$, is continuous across the boundary of the flux core and there should exist a contact electrostatic potential. Correspondingly, the tangential component of the force field is continuous, and the effective electric field is obtained as[1,6]

$$e\mathbf{E}_{\rm NC}=-(e/c)\mathbf{v}_L\times\mathbf{H}_{c2} \quad \text{for } H\ll H_{c2} . \tag{1.2}$$

Here the core radius $a\sim\xi$ with ξ as the superconducting coherence length, $H_{c2}=\Phi_0/2\pi\xi^2$ is the usual upper critical field, and $\mathbf{v}_L$ is the velocity of flux flow. Consequently, inside the core, one should have

$$\mathbf{v}_T=-\frac{e\tau}{mc}(\mathbf{v}_L\times\mathbf{H}_{c2})+\frac{e\tau}{mc}(\mathbf{v}_T\times\mathbf{H}) . \tag{1.3}$$

The measured macroscopic electric field is equal to[1,16]

$$\langle\mathbf{E}\rangle=\frac{\mathbf{B}\times\mathbf{v}_L}{c} , \tag{1.4}$$

where $\mathbf{B}$ is the magnetic induction (the average local magnetic field over the whole sample). The longitudinal resistivity ρ_{xx} and Hall resistivity ρ_{xy} are defined by

$$\rho_{xx}=\langle E_x\rangle/J_T, \quad \rho_{xy}=\langle E_y\rangle/J_T , \tag{1.5}$$

where $J_T=Ne\mathbf{v}_T$ is the applied current density with N as the charge-carrier density. Thus ρ_{xx} and the Hall angle θ_H can be obtained as

$$\rho_{xx}=\rho_n\frac{B}{H_{c2}} , \tag{1.6}$$

$$\theta_H=\tan^{-1}\left|\rho_{xy}/\rho_{xx}\right|=\tan^{-1}\left|\frac{e\tau}{mc}H\right| , \tag{1.7}$$

where $H=|\mathbf{H}|$, $B=|\mathbf{B}|$, and $\rho_n=(Ne^2\tau/m)^{-1}$ is the resistivity of the normal state.

Nozières and Vinen[8] (NV) proposed an alternative model in which the electrostatic potential is continuous and there should be no contact potential at the core boundary. The absence of the contact potential leads to an additional driving force on the charge carrier inside the core. This total force derived by them is given by

$$e\mathbf{E}_{\rm NC}=\frac{e}{c}(\mathbf{v}_T-\mathbf{v}_L)\times\mathbf{H}_{c2} \ . \tag{1.8}$$

When this result is substituted into Eq. (1.1) and again $\mathbf{v}_{\rm NC}$ is taken to be $\mathbf{v}_T$, the basic equation in NV model becomes

$$\mathbf{v}_T=\frac{e\tau}{mc}(\mathbf{v}_T-\mathbf{v}_L)\times\mathbf{H}_{c2} \quad \text{for } H \ll H_{c2} \ . \tag{1.9}$$

The longitudinal resistivity is the same as that in BS model. However, the Hall angle is

$$\theta_H=\tan^{-1}\left[\frac{e\tau}{mc}H_{c2}\right] \ . \tag{1.10}$$

Obviously, neither Eq. (1.7) nor Eq. (1.10) can give the sign reversal for the Hall effect.

Recently, there have been some suggestions that the sign reversal may result from superconducting fluctuations,[11,17] two types of charge carrier,[18] the existence of fluxon and antifluxon,[19] thermomagnetic effect,[20] and a special form for the drag force.[13,15] Although these models are interesting, we feel that they are unlikely and are based upon some artificial assumptions.

A simple explanation for the negative Hall effect based upon the existence of pinning forces in the sample has been given by us in an earlier publication.[21] In this paper we shall present a general and more detailed derivation of the theory for flux flow in the presence of pinning forces. The effective friction force acting on a moving flux is the viscous-drag force plus the pinning force. In Sec. II we shall set up the equation of motion for describing the flux motion with the backflow current due to both pinning forces and other vortices. In Sec. III the longitudinal and Hall resistivities will be derived analytically. We demonstrate that the observed negative Hall resistivity at low magnetic field in some high-T_c superconductors can be qualitatively explained by our results. In the meantime, a simple physical picture for this effect will be discussed. In Sec. IV the viscous drag force and applied force acting on a single vortex will be derived and discussed in detail. Finally, in Sec. V a short summary will be given.

II. EQUATION OF MOTION FOR FLUX FLOW

In order to study the transport property in the flux-flow region where the average driving-force density is larger than the pinning-force density, we need to establish the equation of motion for charge carriers. For high-κ superconductors, we shall adopt a mathematical description using a local model and approximate the flux line to have a normal core with radius $a\,(\sim\xi)$, i.e., normal-state region for $r<a$ and superconducting-state region for $r>a$.[6]

Let $\mathbf{v}_0(\mathbf{r})$ be the circular velocity distribution of the superfluid flow of a stationary flux, where $\mathbf{r}=\mathbf{x}+\mathbf{y}$ is the position vector in the x-y plane, and $\mathbf{v}_b(\mathbf{r})$ be the velocity associated with the backflow current. Outside the normal core, the total velocity of a charge carrier can be written as

$$\mathbf{v}_s=\mathbf{v}_0+\mathbf{v}_T+\mathbf{v}_b^{\rm out} \ , \tag{2.1}$$

with $\mathbf{v}_b^{\rm out}=\mathbf{v}_b(r>a)$. If we choose the gauge as

$$\mathbf{A}(\mathbf{r})=A(r)\mathbf{e}_\theta \ , \tag{2.2}$$

with

$$A(r)=(1/r)\int_0^r r'H(r')dr' \ , \tag{2.3}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{e}_\theta$ is the unit vector of the θ component, and $H(r)$ is the magnitude of the local magnetic field $\mathbf{H}$ [$=H(r)\mathbf{k}$], $\mathbf{v}_0$ can be represented as[22,23]

$$\begin{aligned}\mathbf{v}_0&=(e\Phi_0/mc)(\mathbf{e}_\theta/2\pi r)-(e/cm)\mathbf{A}\\&=[e\phi_s(r)/mc](\mathbf{e}_\theta/2\pi r) \ ,\end{aligned} \tag{2.4}$$

where

$$\phi_s(r)=[1-\pi r^2\overline{H}(r)/\Phi_0]\Phi_0 \ ,$$

with

$$\overline{H}(r)=(2/r^2)\int_0^r r'H(r')dr' \ ,$$

as the average magnetic field inside radius r. In steady state the flux is supposed to move uniformly at a constant velocity $\mathbf{v}_L$, and the charge carriers drift with a uniform velocity $\mathbf{v}_{\rm NC}$ inside the normal core. In consideration of the charge-conservation condition $\nabla\cdot\mathbf{v}_s=0$ and the London equation

$$\nabla\times\mathbf{v}_s+(e\mathbf{H}/mc)=0 \ , \tag{2.5}$$

outside the normal core we obtain

$$\nabla\cdot\mathbf{v}_b^{\rm out}=0, \quad \nabla\times\mathbf{v}_b^{\rm out}=0 \ . \tag{2.6}$$

Therefore it can be shown that the backflow velocity for $r>a$ takes the dipolar form (see Appendix A)

$$\mathbf{v}_b^{\rm out}=\nabla[(\mathbf{v}_T-\mathbf{v}_{\rm NC})\cdot\mathbf{r}(a^2/r^2)] \ . \tag{2.7}$$

Next, from the force-balance condition, we can, in principle, establish the equation of motion for charge fluid inside the core (per unit length in the z direction):

$$\mathbf{F}_{\rm NC}+\mathbf{F}_p^{\rm in}=\frac{Nm}{\tau}\pi a^2\mathbf{v}_{\rm NC} \ . \tag{2.8}$$

The term $(Nm/\tau)\pi a^2\mathbf{v}_{\rm NC}$ denotes the momentum dissipated[24] inside the normal core (per unit time). $\mathbf{F}_p^{\rm in}$ and $\mathbf{F}_{\rm NC}$ are, respectively, the effective pinning force and external driving force acting on the charge fluid inside the core. $\mathbf{F}_{\rm NC}$ can be represented as

$$\mathbf{F}_{\rm NC}=\int\int\int_{\Omega^-}N\left[e\mathbf{E}+\frac{e}{c}\mathbf{v}_{\rm NC}\times\mathbf{H}-\nabla\mu_0\right]d\Omega \ . \tag{2.9}$$

Here $\mathbf{E}$ is the local electric field, μ_0 is the chemical potential in the absence of currents and fields, and Ω^- represents the volume of the unit-length cylinder with core radius $a^-=a-0^+$ (sketched in Fig. 1). The terms inside the integration represent, respectively, the electric force, Lorentz force, and the force due to fluid pressure. From the general formula for the local electric field,

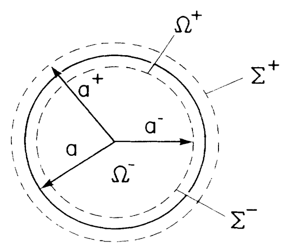

FIG. 1. Sketched profile for Ω^-, Ω^+, Σ^+, and Σ^-.

$$\mathbf{E}=-\frac{\partial \mathbf{A}}{c\,\partial t}-\nabla\phi\ , \tag{2.10}$$

with ϕ as the electrostatic potential and $\mathbf{A}=\mathbf{A}(\mathbf{r}-\mathbf{v}_L t)$.[6] It is straightforward to show that

$$\mathbf{E}=-\frac{1}{c}\mathbf{v}_L\times\mathbf{H}-\nabla V(r)\ , \tag{2.11}$$

$$V(r)=\phi-\mathbf{v}_L\cdot\mathbf{A}/c\ . \tag{2.12}$$

Consequently, we rewrite Eq. (2.9) as

$$\mathbf{F}_{\rm NC}=\int\int\int_{\Omega^-}[(-N)\nabla(eV+\mu_0)+(Ne/c)(\mathbf{v}_{\rm NC}-\mathbf{v}_L)\times\mathbf{H}]d\Omega\ . \tag{2.13}$$

By constructing two special functions in whole space ($r\ge 0$) as

$$V_1=V\quad(r<a)\quad\text{with }\nabla V_1\text{ continuous at }r=a\ , \tag{2.14}$$

$$V_2=V\quad(r>a)\quad\text{with }\nabla V_2\text{ continuous at }r=a\ , \tag{2.15}$$

Eq. (2.13) becomes

$$\mathbf{F}_{\rm NC}=\int\int\int_{\Omega^-}[(-N)\nabla(eV_2+\mu_0)+(Ne/c)(\mathbf{v}_{\rm NC}-\mathbf{v}_L)\times\mathbf{H}]d\Omega-Ne\int\int\int_{\Omega^-}\nabla(V_1-V_2)d\Omega\ . \tag{2.16}$$

The second term on the right-hand side of the above equation is the contact force acting at the interface of the core, which can also be represented as follows due to the continuity of ∇V_1 and ∇V_2:

$$\mathbf{F}_{\rm con}=Ne\int\int\int_{\Omega^+}\nabla(V_1-V_2)d\Omega$$
$$=\int\int_{\Sigma^+}Ne[V(a^-)-V(a^+)]d\mathbf{s} \tag{2.17}$$
$$=\int\int_{\Sigma^+}Ne[\phi(a^-)-\phi(a^+)]d\mathbf{s}\ , \tag{2.18}$$

where Ω^+ and Σ^+ represent, respectively, the volume and surface of the cylinder with core radius $a^+=a+0^+$ (Fig. 1). By defining $\mu_{\rm tot}$ as the total chemical potential in the lattice frame of reference,

$$\mu_{\rm tot}=\mu_{\rm tot}^{\rm out}=\mu_0+e\phi^{\rm out}+\frac{m}{2}(\mathbf{v}_0+\mathbf{v}_T+\mathbf{v}_b^{\rm out})^2\quad(r>a)\ , \tag{2.19}$$

$$\mu_{\rm tot}=\mu_{\rm tot}^{\rm in}=\mu_0+e\phi^{\rm in}+\frac{m}{2}\mathbf{v}_{\rm NC}^2\quad(r<a)\ , \tag{2.20}$$

where $\phi^{\rm out}=\phi(r>a)$ and $\phi^{\rm in}=\phi(r<a)$, the contact force can be represented as

$$\mathbf{F}_{\rm con}=\int\int_{\Sigma^+}N[\mu_{\rm tot}^{\rm in}(a^-)-\mu_{\rm tot}^{\rm out}(a^+)]d\mathbf{s}+Nm\int\int_{\Sigma^+}(\mathbf{v}_T+\mathbf{v}_b^{\rm out})\cdot\mathbf{v}_0 d\mathbf{s}$$
$$=\int\int_{\Sigma^+}N[\mu_{\rm tot}^{\rm in}(a^-)-\mu_{\rm tot}^{\rm out}(a^+)]d\mathbf{s}+\frac{Ne}{2c}(2\mathbf{v}_T-\mathbf{v}_{\rm NC})\times\boldsymbol{\phi}_0\ . \tag{2.21}$$

In the above derivation, the formulas of (B1) and (B6) in Appendix B are used.

When the temperature T is close to T_c and $a\sim\xi>l_0$ (mean free path of the charge carrier), the normal charge carrier should be in local equilibrium with the lattice. Correspondingly, the total chemical potential $\mu_{\rm tot}$ should be continuous at the normal-superconducting boundary,[1,6] i.e., $\mu_{\rm tot}^{\rm in}(a^-)=\mu_{\rm tot}^{\rm out}(a^+)$. Under this condition, $\mathbf{F}_{\rm con}$ is given by the second term on the right-hand side of Eq. (2.21). When $T\ll T_c$ and $a\sim\xi\ll l_0$, it is hard to imagine how one can achieve an equilibrium distribution which varies rapidly over a scale ξ. According to the arguments by Nozières and Vinen,[8] $\mu_{\rm tot}$ is not defined over a scale $\xi\ll l_0$ ($T\ll T_c$); thus ϕ should be continuous at the normal-superconducting boundary $[\phi(a^-)=\phi(a^+)]$ and $\mathbf{F}_{\rm con}=\mathbf{0}$ from Eq. (2.18). In the intermediate region $[\xi(T)\sim l_0]$, it is reasonable to believe that the contact force should be in between the above two limiting cases. For simplicity, we introduce a parameter γ to represent the contact force in general:

$$\mathbf{F}_{\rm con}=\gamma\frac{Ne}{2c}(2\mathbf{v}_T-\mathbf{v}_{\rm NC})\times\boldsymbol{\phi}_0\ . \tag{2.22}$$

The parameter γ ($0\le\gamma\le 1$) should be determined by the ratio ξ/l_0 and is insensitive to the magnetic field and pinning force. From the above discussion, we expect that $\gamma\simeq 0$ as $\xi\ll l_0$ when ϕ is continuous at the normal-superconducting boundary and $\gamma\simeq 1$ as $\xi\ge l_0$ when $\mu_{\rm tot}$ is continuous. In order to treat our problem conveniently, it is reasonable to consider γ increasing continuously and rapidly from 0 to 1 as T is increased from below T_0 to above T_0, where T_0 is defined as $\xi(T_0)=l_0$.

On the other hand, a charge fluid outside the normal core is governed by the Euler equation

$$\frac{d\mathbf{v}_s}{dt}=\frac{\partial\mathbf{v}_s}{\partial t}+\mathbf{v}_s\cdot\nabla\mathbf{v}_s=(\mathbf{v}_s-\mathbf{v}_L)\cdot\nabla\mathbf{v}_s$$
$$=\tfrac{1}{2}\nabla(\mathbf{v}_s-\mathbf{v}_L)^2-(\mathbf{v}_s-\mathbf{v}_L)\times\nabla\times\mathbf{v}_s$$
$$=(e/m)[\mathbf{E}+\mathbf{v}_s\times\mathbf{H}/c]-\nabla\mu_0/m\ . \tag{2.23}$$

Combining Eq. (2.11) and the London equation [Eq. (2.5)] with the above equation, an important relation for $r>a$ is obtained:

$$\mu_0+eV+(m/2)(\mathbf{v}_s-\mathbf{v}_L)^2=\text{const} . \quad (2.24)$$

Considering the continuity of ∇V_2 and substituting Eq. (2.24) into Eq. (2.16), $\mathbf{F}_{\rm NC}$ becomes

$$\mathbf{F}_{\rm NC}=\int\int_{\Sigma^+}(Nm/2)(\mathbf{v}_s-\mathbf{v}_L)^2 d\mathbf{s} +(Ne/c)\int\int\int_{\Omega^-}(\mathbf{v}_{\rm NC}-\mathbf{v}_L)\times\mathbf{H}\,d\Omega-\mathbf{F}_{\rm con} . \quad (2.25)$$

Inserting Eq. (2.1) into the first term of the above equation, this term can be written as

$$\int\int_{\Sigma^+}(\mathbf{v}_s-\mathbf{v}_L)^2 d\mathbf{s} =\int\int_{\Sigma^+}[(\mathbf{v}_T+\mathbf{v}_b^{\rm out}-\mathbf{v}_L)^2 +2(\mathbf{v}_T-\mathbf{v}_L+\mathbf{v}_b^{\rm out})\cdot\mathbf{v}_0+(\mathbf{v}_0)^2]d\mathbf{s} . \quad (2.26)$$

In consideration of the symmetry condition $[\mathbf{v}_0(\mathbf{r})]^2=[\mathbf{v}_0(-\mathbf{r})]^2$, $\mathbf{v}_b^{\rm out}(\mathbf{r})=\mathbf{v}_b^{\rm out}(-\mathbf{r})$ and the antisymmetry condition $d\mathbf{s}(\mathbf{r})=-d\mathbf{s}(-\mathbf{r})$, the contributions from the first and third terms in the brackets of the above equation are individually zero. Using the formulas of (B1) and (B6) in Appendix B, the integration over the second term in Eq. (2.26) can be carried out. Substituting this result into the Eq. (2.25), we obtain

$$\mathbf{F}_{\rm NC}=\frac{Ne}{2c}(2\mathbf{v}_T-\mathbf{v}_{\rm NC}-\mathbf{v}_L)\times\boldsymbol{\phi}_0 +\frac{Ne}{2c}\frac{\bar{H}}{h_{c2}}(\mathbf{v}_{\rm NC}-\mathbf{v}_L)\times\boldsymbol{\phi}_0-\mathbf{F}_{\rm con} , \quad (2.27)$$

where $\boldsymbol{\phi}_0=\phi_0\mathbf{k}$, in which $\phi_0=\phi_s(a)=(1-\bar{H}/2\bar{H}_{c2})\Phi_0$, with $\bar{H}_{c2}=\Phi_0/2\pi a^2$, $\bar{H}=\bar{H}(a)$ being the average magnetic field over the core, and $h_{c2}=\phi_0/2\pi a^2$.

In order to solve Eq. (2.8), it is necessary to know the form of the effective pinning force. Since the asymmetry of the backflow current due to the effective pinning force is polarized in the direction of $\mathbf{F}_p^{\rm in}$, i.e., $\mathbf{F}_p^{\rm in}\cdot\mathbf{v}_{bp}^{\rm in}=0$, we can write

$$\mathbf{F}_p^{\rm in}=\alpha(Ne/c)\mathbf{v}_{bp}^{\rm in}\times\boldsymbol{\phi}_0 , \quad (2.28)$$

and set $\mathbf{F}_p^{\rm in}=\alpha_0\mathbf{F}_p$, where $\mathbf{v}_{bp}^{\rm in}$ is the velocity of the backflow current due to the pinning force inside the normal core. $\mathbf{F}_p$ is the pinning force acting on a flux. α and α_0 are coefficients to be determined in accordance with certain physical considerations which will be presented below. When the intrinsic backflow current with velocity $\mathbf{v}_{b0}^{\rm in}$ induced by the other vortices is taken into account,[25] the total backflow-current velocity is written as

$$\mathbf{v}_b^{\rm in}=\mathbf{v}_{bp}^{\rm in}+\mathbf{v}_{b0}^{\rm in}=\mathbf{v}_{\rm NC}-\mathbf{v}_T . \quad (2.29)$$

For simplicity, the backflow current due to the other vortices is assumed to be uniform. It should be smaller than J_T and in the opposite direction of the applied current in ordinary cases,[25] i.e., $\mathbf{v}_{b0}^{\rm in}=-\delta\mathbf{v}_T$ ($0\le\delta<1$, and δ is not close to 1). Thus Eq. (2.28) becomes

$$\mathbf{F}_p^{\rm in}=\alpha(Ne/c)[\mathbf{v}_{\rm NC}-(1-\delta)\mathbf{v}_T]\times\boldsymbol{\phi}_0 . \quad (2.30)$$

Combining Eqs. (2.22), (2.27), and (2.30) with (2.8), the drift velocity of the charge carriers inside the core can be shown to have the form

$$\mathbf{v}_{\rm NC}=\beta\{2[1-\gamma-\alpha(1-\delta)]\mathbf{v}_T\times\mathbf{k} +(2\alpha+\gamma-1+\bar{H}/h_{c2})\mathbf{v}_{\rm NC}\times\mathbf{k} -(1+\bar{H}/h_{c2})\mathbf{v}_L\times\mathbf{k}\} , \quad (2.31)$$

where $\beta=\mu_m h_{c2}=(1-\bar{H}/2\bar{H}_{c2})\bar{\beta}$, with $\bar{\beta}=\mu_m\bar{H}_{c2}$, $\mu_m=\tau e/mc$ being the mobility of the charge carrier. Inserting the above equation into Eq. (2.30) and eliminating $\mathbf{v}_{\rm NC}$, we obtain the following equation of motion after some procedure of vector algebra:

$$a_1\mathbf{v}_T\times\mathbf{k}+a_2\beta^2\mathbf{v}_L\times\mathbf{k}+a_3\beta\mathbf{v}_T+a_4\beta\mathbf{v}_L+\tilde{\mathbf{f}}_p=\mathbf{0} , \quad (2.32)$$

with

$$\begin{aligned} a_1&=1-\delta+\beta^2(2\alpha-1+\gamma+\bar{H}/h_{c2})\times[1-\gamma+\bar{H}/h_{c2}+\delta(1-\gamma-\bar{H}/h_{c2})] ,\\ a_2&=-(2\alpha-1+\gamma+\bar{H}/h_{c2})(1+\bar{H}/h_{c2}) ,\\ a_3&=2[1-\alpha(1-\delta)]-2\gamma ,\\ a_4&=-(1+\bar{H}/h_{c2}) ,\\ \tilde{\mathbf{f}}&=[\mathbf{F}_p^{\rm in}/(\alpha Ne\phi_0/c)][1+\beta^2(2\alpha-1+\gamma+\bar{H}/h_{c2})^2] , \end{aligned} \quad (2.33)$$

Up to now, we have established the equation of motion for a single vortex. In the following section, we shall solve Eq. (2.32) to obtain $\mathbf{v}_L$ in terms of $\mathbf{v}_T$ and $\mathbf{F}_p^{\rm in}$.

III. LONGITUDINAL AND HALL RESISTIVITIES

It has generally been accepted that the pinning force $\mathbf{F}_p$ should be antiparallel to the direction of the flux-flow velocity $\mathbf{v}_L$. After solving Eq. (2.32) by choosing $\mathbf{v}_T$ along the x direction, we obtain x and y components of $\mathbf{v}_L$ as

$$\begin{aligned} v_{Lx}&=(-1/a_4)\{[1-\gamma+\bar{H}/h_{c2}+\delta(1-\gamma-\bar{H}/h_{c2})]v_T -D_xF_p^{\rm in}/(\alpha\phi_0Ne/c)\} ,\\ v_{Ly}&=(1/a_4\beta)[(1-\delta)v_T-D_yF_p^{\rm in}/(\alpha\phi_0Ne/c)] , \end{aligned} \quad (3.1)$$

where

$$D_x=\sin\theta_H/\beta+\cos\theta_H(2\alpha-1+\gamma+\bar{H}/h_{c2}) ,$$
$$D_y=\cos\theta_H+\beta\sin\theta_H(2\alpha-1+\gamma+\bar{H}/h_{c2}) .$$

The longitudinal resistivity in the flux-flow region has the form

$$\rho_{xx}=\left[\frac{\rho_n(\tau)}{(1+H/2\bar{H}_{c2})\bar{H}_{c2}}\right]\times\left[B(1-\delta)-\frac{D_y(\Phi_0\alpha_0/\phi_0\alpha)\bar{F}_p}{J_T/c}\right] , \quad (3.2)$$

where $\bar{F}_p=(B/\Phi_0)|\mathbf{F}_p|$ is the average pinning-force density. Considering the fact $\theta_H\ll1$, we neglect the higher-order term in θ_H (i.e., $\sin\theta_H\simeq\theta_H$, $\cos\theta_H\simeq1$) and have $D_y \simeq 1$. From the physical consideration that flux lines will flow ($\rho_{xx} \geq 0$) when the average driving Lorentz-force density exceeds the pinning-force density $[J_T B(1-\delta)/c \geq \bar{F}_p]$, we obtain the relation between α and α_0:

$$\frac{\alpha_0}{\alpha} = \frac{\phi_0}{\Phi_0} = \left[1 - \frac{\bar{H}}{2\bar{H}_{c2}} \right] . \tag{3.3}$$

By setting $\chi = \xi^2/a^2$, Eq. (3.2) can be rewritten as

$$\rho_{xx} = \rho_n(\tau) \frac{B(1-\delta) - \bar{F}_p/(J_T/c)}{\chi H_{c2} + \bar{H}/2} . \tag{3.4}$$

Here the longitudinal resistivity is independent of the parameter γ. Meanwhile, the Hall angle satisfies the equation

$$\begin{aligned} \theta_H \simeq \frac{\rho_{xy}}{\rho_{xx}} = -\frac{v_{Lx}}{v_{Ly}} \\ &= \beta\{ [1-\gamma+\bar{H}/h_{c2}+\delta(1-\gamma-\bar{H}/h_{c2})] - [\bar{F}_p/(J_T B/c)](\theta_H/\beta) \\ &\quad -(2\alpha-1+\gamma+\bar{H}/h_{c2})\bar{F}_p/(J_T B/c)\}/[1-\delta-\bar{F}_p/(J_T B/c)] . \end{aligned} \tag{3.5}$$

Therefore the Hall resistivity is derived as

$$\begin{aligned} \rho_{xy} = \rho_{xx}\beta_0([\chi - \bar{H}/2H_{c2})(1-\gamma)(1+\delta)/(1-\delta) + (\bar{H}/H_{c2})]\{1-\bar{F}_p/[J_T B(1-\delta)/c)]\} \\ -2\chi(1-\bar{H}/2\bar{H}_{c2})[\alpha-(1-\gamma)/(1-\delta)]\bar{F}_p/[J_T B(1-\delta)/c]) , \end{aligned} \tag{3.6}$$

with $\beta_0 = \mu_m H_{c2}$. Now let us examine the asymptotic behaviors of ρ_{xx} and ρ_{xy}. When $B \to H_{c2}$ ($\bar{F}_p \to 0$, $\delta \to 0$), the normal cores of the flux lines will touch each other and $B \simeq \Phi_0/\pi a^2 \simeq H_{c2}$, which leads to $\chi \simeq \frac{1}{2}$. Consequently, both the longitudinal and Hall resistivities in Eqs. (3.4) and (3.6) will reduce to their normal-state values as $B \to H_{c2}$, i.e., $\rho_{xx} \to \rho_n$ and $\rho_{xy} \to \rho_n \mu_m H_{c2} = H_{c2}/Nec$.

Next, from Eq. (3.6), it is apparent that there exists a region near $B \sim \bar{F}_p/[J_T(1-\delta)/c]$ in which $\rho_{xy}/\beta_0 < 0$ so long as $[\alpha-(1-\gamma)/(1-\delta)] > 0$. Intuitively, the pinning force should almost totally act on the bulk of the flux core, i.e., $\alpha_0 \simeq 1$. Therefore, so long as γ is close to $1 (T \sim T_0)$ and the effect of the pinning force on ρ_{xy} is not negligible, the negative Hall resistivity ($\rho_{xy}/\beta_0 < 0$) should be observable in experiments.

Furthermore, we shall show $\alpha_0 \simeq 1$ from the consideration of energy conservation. Since the flux flows steadily, the force-balance equation for a flux can be written as

$$(Nm/\tau)\pi a^2 \mathbf{v}_{\rm NC} + \mathbf{F}_{\rm conv} + \mathbf{F}_{\rm con} + \mathbf{f}_{\rm drag} = \mathbf{0} , \tag{3.7}$$

where $\mathbf{f}_{\rm drag}$ is the viscous-drag force acting on the flux due to the surrounding superfluid and $\mathbf{F}_{\rm conv}$ is the total momentum dissipated in the transition layer ($a^- < r < a^+$) per unit time,

$$\begin{aligned} \mathbf{F}_{\rm conv} &= Nm \left[\int \int_{\Sigma^+} \mathbf{v}_s(\mathbf{v}_L - \mathbf{v}_s)\cdot d\mathbf{s} - \int \int_{\Sigma^-} \mathbf{v}_{\rm NC}(\mathbf{v}_L - \mathbf{v}_{\rm NC})\cdot d\mathbf{s} \right] \\ &= Nm \int \int_{\Sigma^+} [\mathbf{v}_0(\mathbf{v}_L - \mathbf{v}_T) - \mathbf{v}_0 \mathbf{v}_b^{\rm out}]\cdot d\mathbf{s} . \end{aligned} \tag{3.8}$$

In the above derivation, Eq. (2.1) has been employed. Using formulas (B7) and (B8) in Appendix B, the integrations in Eq. (3.8) can be carried out; $\mathbf{F}_{\rm conv}$ becomes

$$\mathbf{F}_{\rm conv} = (Ne/2c)(\mathbf{v}_{\rm NC} - \mathbf{v}_L) \times \boldsymbol{\phi}_0 . \tag{3.9}$$

Since the effective frictional force acting on the flux is $\mathbf{f}_{\rm drag} + \mathbf{F}_p$, the total power dissipated to the lattice is $w = -(\mathbf{f}_{\rm drag} + \mathbf{F}_p)\cdot \mathbf{v}_L$ and it should be equal to $w_{\rm NC} + w_{\rm TL}$, where $w_{\rm NC} = \mathbf{F}_{\rm NC}\cdot \mathbf{v}_{\rm NC}$ is the Joule power dissipated in the bulk of the normal core and $w_{\rm TL}$ is the power dissipated in the transition layer, which is obtained as

$$\begin{aligned} w_{\rm TL} &= (Nm/2) \left[\int \int_{\Sigma^+} \mathbf{v}_s^2(\mathbf{v}_L - \mathbf{v}_s)\cdot d\mathbf{s} - \int \int_{\Sigma^-} \mathbf{v}_{\rm NC}^2(\mathbf{v}_L - \mathbf{v}_{\rm NC})\cdot d\mathbf{s} \right] \\ &= Nm \int \int_{\Sigma^+} \mathbf{v}_0\cdot[\mathbf{v}_T(\mathbf{v}_L - \mathbf{v}_T) + \mathbf{v}_T \mathbf{v}_b^{\rm out} + \mathbf{v}_b^{\rm out}(\mathbf{v}_L - \mathbf{v}_T)]\cdot d\mathbf{s} \\ &= (Ne/2c)\{[(\mathbf{v}_{\rm NC} - \mathbf{v}_L) \times \boldsymbol{\phi}_0]\cdot \mathbf{v}_{\rm NC} + 2[(\mathbf{v}_{\rm NC} - \mathbf{v}_T) \times \boldsymbol{\phi}_0]\cdot(\mathbf{v}_{\rm NC} - \mathbf{v}_L)\} . \end{aligned} \tag{3.10}$$

In the third step of the above derivation, Eqs. (B7), (B8), and (B6) in Appendix B have been employed. From Eqs. (2.27), (3.9), and (3.10) and after some algebra, we obtain

$$\begin{aligned} (\mathbf{F}_{\rm NC} + \mathbf{F}_{\rm conv} + \mathbf{F}_{\rm con})\cdot \mathbf{v}_L &= w_{\rm TL} + w_{\rm NC} - \gamma(Ne/c)[(\mathbf{v}_{\rm NC} - \mathbf{v}_T) \times \boldsymbol{\phi}_0]\cdot \mathbf{v}_T] \\ &= w_{\rm TL} + w_{\rm NC} - \gamma(1 - \bar{H}/2\bar{H}_{c2})\mathbf{F}_p\cdot \mathbf{v}_T . \end{aligned} \tag{3.11}$$

On the other hand, from Eqs. (3.7) and (2.8) and by requiring $w = w_{\rm NC} + w_{\rm TL}$, we should have

$$(\mathbf{F}_{\rm NC}+\mathbf{F}_{\rm conv}+\mathbf{F}_{\rm con})\cdot\mathbf{v}_L = -(\mathbf{f}_{\rm drag}+\mathbf{F}_p^{\rm in})\cdot\mathbf{v}_L = w_{\rm TL}+w_{\rm NC}+(1-\alpha_0)\mathbf{F}_p\cdot\mathbf{v}_L \ . \tag{3.12}$$

Since $\mathbf{F}_p\cdot\mathbf{v}_T = F_p v_T \sin\theta_H$ (where $\theta_H \ll 1$ and $\beta \ll 1$), $\mathbf{F}_p\cdot\mathbf{v}_L = -F_p v_L$, and $v_T \sim \beta v_L$ [see Eq. (3.1)] and comparing Eq. (3.12) and Eq. (3.11), we obtain the relation $\alpha_0 \simeq 1$. The Hall resistivity then becomes

$$\rho_{xy} = \rho_{xx}\beta_0\{ [(\chi - \overline{H}/2H_{c2})(1-\gamma)(1+\delta)/(1-\delta) + (\overline{H}/H_{c2})](1-H_p/B) - [(1-\gamma)\overline{H}/H_{c2} + 2\chi(\gamma-\delta)][H_p/B(1-\delta)]\} \ , \tag{3.13}$$

The above equation is valid only for $B \ge H_p = \overline{F}_p/[J_T(1-\delta)/c]$, where $\overline{F}_p$ and δ depend on B and temperature T. From Eq. (3.4), it shows $\rho_{xy}=0$ at $B=H_p$. It also indicates clearly that there is a region in which Hall resistivity has its sign reversed in the Bardeen-Stephen limit ($\gamma \simeq 1$), when B is slightly larger than H_p. Moreover, the value of the Hall angle is of the same order of magnitude as (or less than) that of the normal-state Hall angle $\theta_n = \tan^{-1}[(e\tau/mc)H]$, and it is certainly very small as a result of θ_n being very small.

In experimental measurements the negative Hall signal could only be observed for both γ and $\overline{F}_p/(J_T/c)$ being not too small. It should be noted that we will recover the Bardeen-Stephen result by setting $\delta=0$, $\gamma=1$, and $\overline{F}_p=0$ and recover Nozières-Vinen result by setting $\delta=0$, $\gamma=0$, $\overline{F}_p=0$, and neglecting the term $\overline{H}/H_{c2}$.[1,6,8]

A physical picture for the negative Hall effect can be understood as follows: As $\gamma \simeq 1$, $\mathbf{F}_{\rm NC}$ in Eq. (2.27) becomes

$$\mathbf{F}_{\rm NC} \sim -(Ne/2c)\mathbf{v}_L \times \mathbf{\Phi}_0 + (Ne/2c)(\overline{H}/\overline{H}_{c2})\mathbf{v}_{\rm NC}\times\mathbf{\Phi}_0 \ ,$$

the charge carrier feels the Lorentz-like force density $\mathbf{f}_l \sim B/\Phi_0$ times the second term in the above equation, and the pinning-force density $\overline{\mathbf{f}}_p \sim \mathbf{j}_{bp}^{\rm in}\times\mathbf{B}/c$, with $\mathbf{j}_{bp}^{\rm in} = Ne\mathbf{v}_{bp}^{\rm in}$. So long as $\overline{H}/\overline{H}_{c2}$ is small enough to enable the inequality $|\overline{f}_{py}| > |f_{ly}|$ to hold, the charge carrier will have a tendency to move along the $\mathbf{y}$ direction. In order to balance the above tendency, a macroscopic electric field ($\propto$ the first term of the above equation) will have to be induced along the negative-$\mathbf{y}$ direction (i.e., $\rho_{xy}<0$) by flux motion with $v_{Lx}<0$. In Fig. 2 we show all the effective forces acting on the charge carrier for both the negative Hall effect ($|\overline{f}_{py}| > |f_{ly}|$) and positive Hall effect ($|\overline{f}_{py}| < |f_{ly}|$). In other words, if the pinning effect is neglected, the Hall resistivity should always be positive.

Based upon Eq. (3.13), by solving the equations $\rho_{xy}=0$ and $d\rho_{xy}/dB=0$, we can obtain the sign-reversal point $B_{\rm SR}$ and minimum point B_m for ρ_{xy} vs B, respectively. However, the quantitative results should depend on the form of $H_p(B)$ and $\delta(B)$. In the following we shall discuss some simple cases by setting $\delta=0$ within the approximation $\overline{H}\sim B$ and the assumption that H_p is independent of B in the negative-Hall-effect region.

(i) When T is well below T_0 ($\chi \sim 1$, $\gamma \simeq 0$) and $B > B_0 = H_p(B_0)$, the equation for solving $B_{\rm SR}$ is

$$\chi + \frac{B_{\rm SR}}{2H_{c2}} - \chi\frac{H_p}{B_{\rm SR}} - \frac{3H_p}{2H_{c2}} = 0 \ . \tag{3.14}$$

Hence

$$B_{\rm SR} = \frac{\chi H_p}{\chi - 3H_p/(2H_{c2})} \simeq \left[1 + \frac{3H_p}{2\chi H_{c2}}\right] H_p \ . \tag{3.15}$$

Since $H_p \ll H_{c2}$, there exists a narrow region $B_0 < B < B_{\rm SR}$, in which the Hall resistivity is negative, but its magnitude is estimated to be very small $[\sim\rho_n\beta(\overline{H}/2H_{c2})^2]$. Meanwhile, ρ_{xy} has an approximately linear B dependence with slope $1/Nec$ as $B > B_{\rm SR}$.

(ii) $T \sim T_0$ ($\xi \simeq l_0$, $\gamma \simeq 1$) and T_0 is not very close to T_c; we have the following two equations to determine $B_{\rm SR}$ and B_m:

$$\frac{B_{\rm SR}}{H_{c2}} - \frac{H_p}{H_{c2}} - 2\chi\frac{H_p}{B_{\rm SR}} = 0 \ , \tag{3.16}$$

$$\frac{H_p}{H_{c2}}\left[\frac{B_m}{H_p} - 1\right] - \chi\left[\frac{H_p}{B_m}\right]^2 = 0 \ . \tag{3.17}$$

In obtaining Eq. (3.17), we have approximated $(\chi + B/2H_{c2})$ as χ in the denominator of ρ_{xx} because $B/2H_{c2} \ll 1$. $B_{\rm SR}$ and B_m can be solved as

$$B_{\rm SR} \simeq \tfrac{1}{2}(1+\sqrt{1+2\chi H_{c2}/H_p})H_p \ , \tag{3.18}$$

$$B_m \simeq \left[\frac{\chi H_{c2}}{H_p}\right]^{1/3} H_p \ . \tag{3.19}$$

Meanwhile, the maximum negative Hall signal is estimated to be $\rho_m \sim -2\rho_n\mu_m B_0 = -2B_0/Nec$, which is in the same order of magnitude as the normal Hall signal and is in the observable region for experimental measurements.[14] It is worthwhile to point out here that even if

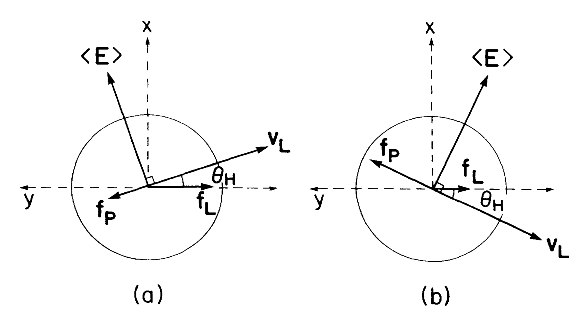

FIG. 2. Effective forces on the charge carriers (a) for the positive Hall effect (the Hall angle $\theta_H > 0$) and (b) for the negative Hall effect ($\theta_H < 0$).

the pinning-force density depends on the magnetic field, formula (3.18) still holds so long as H_p is represented as a function of B_{SR}, i.e., $H_p(B_{SR})$.

(iii) For fixed B and when T is well below T_0, we expect $\xi \ll l_0$ and $\gamma \simeq 0$; the Hall resistivity as a function of temperature is mainly affected by the factor $\sim B[1-\bar{F}_p/(J_T B/c)]^2/Nec$, and it will increase as T increases because the value of $\bar{F}_p$ decreases. When $T \sim T_0$, γ increases rapidly from 0 to ~ 1; therefore there is an apparent reduction of ρ_{xy} near T_0.[15]

(iv) When $B \gg \bar{F}_p/(J_T/c)$, we always have $\rho_{xy} \sim B/Nec$.[26]

All these results are in qualitative agreement with recent experimental measurements[11–14] on ρ_{xy}. A schematic diagram for ρ_{xy} as function of B is shown in Fig. 3.

It is interesting to note that, prior to flux flow ($B < B_0$), the longitudinal resistivity may become finite because of flux creep at finite temperature, while the Hall resistivity ρ_{xy} is still zero and not observable. This is because the flux lines are pinned, and they are only able to creep along the direction ($\mathbf{J}_T \times \mathbf{B}$) assisted by the thermal activation. This process only induces the longitudinal resistivity. Thus $\theta_H = 0$ and $\rho_{xy} = 0$ for $B \leq B_0$. Consequently, the magnetic field for the observed longitudinal resistivity starting to become finite is always lower than that for the Hall resistivity. This phenomenon is also consistent with the experimental observation.[13,14] Moreover, measuring the threshold field B_0 provides a direct method to probe the pinning force for experiment. Here we wish to emphasize that the experimentally measured resistivity ρ_{xx}^E doe snot equal ρ_{xx} in Eq. (3.13) near the flux-creep–to–flux-flow transition region. However, we may expect that $\rho_{xx} \sim \rho_{xx}^E$ when the system is well inside the flux-flow region.

We also would like to point out that in the present paper only the cases $\xi < l_0$ and $\xi \simeq l_0$ are considered. However, the negative Hall resistivity was also observed a long time ago in some conventional low-temperature dirty superconductors ($l_0 \ll \xi$).[27] We speculate that there may exist some kind of link between negative ρ_{xy} in the dirty case and the present work. This point can be seen from the physical picture for negative Hall resistivity given earlier. Qualitatively speaking, the dirty superconductors might also be roughly described by a BS normal-core model to some extent. Thus the negative Hall effect observed in some dirty superconductors ($\gamma = 1$) should originate from the existence of large pinning forces in those samples.

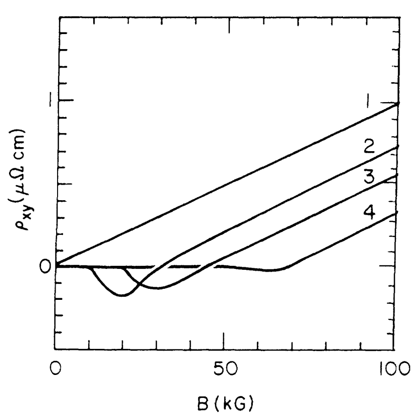

FIG. 3. Schematic drawings for ρ_{xy} as a function of B at four typical temperatures (choosing $\chi H_{c2} \sim 10 B_0$). (1) $T = T_c$. (2) $T = T_0$, $\gamma = 1$. (3) $T < T_0$, $\gamma = 0.6$. (4) $T \ll T_c$, $\gamma = 0$. In drawing, the measured data for $(1/Nec)$, B_0, and ρ_{xx} ($B > B_{SR}$) in Ref. 14 are employed approximately.

Finally, it is worthwhile to point out that, when T_0 is very close to T_c and $(\gamma - \delta)$ has significant positive value only in a very narrow temperature region $|T - T_0|/T_0 \ll 1$, there should be no observable sign reversal of the Hall effect in experimental measurements. This may be the case for some superconductors.[28]

IV. DRAG FORCE AND MAGNUS FORCE

The viscous-drag force acting on a flux line is a very important physical quantity for describing flux flow in type-II superconductors. However, there seems to exist a controversy about this problem. Two different viewpoints have existed so far. One of them is the drag force proportional to the applied current, i.e., $\mathbf{f}_{\text{drag}} = -a_{NV}\mathbf{v}_T$, which was proposed by Nozières and Vinen with a_{NV} as a constant. By neglecting the second term $[(Ne/c)(\mathbf{v}_{NC} - \mathbf{v}_L) \times \mathbf{H}]$ on the right-hand side of Eq. (2.13) and replacing ϕ_0 by Φ_0, the authors in Ref. 8 showed that the flux line is subjected to a similar Magnus force found in liquid helium,[4] which should be balanced by the viscous-drag force. The other viewpoint[6] is the drag force proportional to the velocity of flux flow, for example, $\mathbf{f}_{\text{drag}} = -\eta \mathbf{v}_L$, with η as the coefficient of viscosity. However, if the applied force on the flux line is still a Magnus force, it should lead to the Hall angle approaching $\pi/2$, in sharp disagreement with experimental observations.[1] In view of this difficulty, Bardeen argued that the applied force should be Lorentz force $(Ne/c)\mathbf{v}_T \times \mathbf{\Phi}_0$.[1,5] In this section we shall present a unified treatment and description for this problem.

We shall show below that the applied force acting on the flux line is exactly the Magnus force in the absence of the backflow current. The applied force should be equal to

$$\mathbf{F} = \mathbf{F}_{NC} + \mathbf{F}_{\text{conv}} + \mathbf{F}_{\text{con}} \ . \tag{4.1}$$

Substituting Eqs. (2.27) and (3.9) with the condition $\mathbf{v}_T = \mathbf{v}_{NC}$ into the above equation, we easily obtain

$$\mathbf{F} = \frac{Ne}{c}(\mathbf{v}_T - \mathbf{v}_L) \times \mathbf{\Phi}_0 \ , \tag{4.2}$$

which is exactly the Magnus force. Next, let us discuss the drag force in the absence of the backflow.

In the NV limit, $\gamma = 0$. From Eq. (2.31) we can write

$$\mathbf{v}_T = \beta(1 + \bar{H}/h_{c2})(\mathbf{v}_T - \mathbf{v}_L) \times \mathbf{k} \ . \tag{4.3}$$

Combining the above equation with Eqs. (2.8), (3.7), (4.1), and (4.2), we immediately obtain the drag force in the NV approximation,

$$\mathbf{f}_{\rm drag}^{\rm NV}=-\frac{\Phi_0 Ne/c}{\beta(1+\bar{H}/h_{c2})}\mathbf{v}_T=-\frac{\Phi_0 Ne/c}{\beta_0(\chi+\bar{H}/2H_{c2})}\mathbf{v}_T \quad (4.4)$$

$$\simeq -(N^2e^2\Phi_0\rho_n/H_{c2})\mathbf{v}_T \quad (\bar{H}/H_{c2}\ll 1) . \quad (4.5)$$

(ii) In the BS limit, $\gamma=1$. Also, from Eq. (2.31), we have

$$\mathbf{v}_T=\beta(\bar{H}/h_{c2})\mathbf{v}_T\times\mathbf{k}-\beta(1+\bar{H}/h_{c2})\mathbf{v}_L\times\mathbf{k} ; \quad (4.6)$$

therefore the drag force in the BS approximation becomes

$$\mathbf{f}_{\rm drag}^{\rm BS}=(Ne\Phi_0/c)[\mathbf{v}_L\times\mathbf{k}-\beta_0(\chi+\bar{H}/2H_{c2})\mathbf{v}_L +\beta_0(\bar{H}/H_{c2})\mathbf{v}_T] \quad (4.7)$$

$$\simeq(Ne/c)\mathbf{v}_L\times\mathbf{\Phi}_0-(Ne\Phi_0\beta_0/c)\mathbf{v}_L \quad (\bar{H}/H_c\ll 1) . \quad (4.8)$$

The real drag force here is not proportional to $\mathbf{v}_L$, and the additional part $(Ne/c)\mathbf{v}_L\times\mathbf{\Phi}_0$ should be included even in the case $\bar{H}/H_{c2}\ll 1$. However, this additional force has no contribution to the power dissipation $-\mathbf{f}_{\rm drag}\cdot\mathbf{v}_L$. It is interesting to note that, if we set $\hat{\mathbf{f}}_{\rm drag}^{\rm BS}=-\eta\mathbf{v}_L$, with $\eta=Ne\Phi_0\beta_0=\Phi_0H_{c2}/\rho_nc^2$, we can rewrite the force-balance equation $\mathbf{F}+\mathbf{f}_{\rm drag}^{\rm BS}=\mathbf{0}$ for the flux line as

$$\frac{Ne}{c}\mathbf{v}_T\times\mathbf{\Phi}_0+\hat{\mathbf{f}}_{\rm drag}^{\rm BS}=\mathbf{0} . \quad (4.9)$$

So long as we refer to $\hat{\mathbf{f}}_{\rm drag}^{\rm BS}$ as the "effective force" for balancing the Lorentz force, we recover Bardeen's argument mentioned before.

(iii) In general case $0<\gamma<1$, the drag force takes the form

$$\mathbf{f}_{\rm drag}=-(Ne\Phi_0/c)\times\left[\gamma\beta\mathbf{v}_L+\frac{1-\gamma\beta^2(1-\gamma+\bar{H}/h_{c2})}{\beta(1+\bar{H}/h_{c2})}\mathbf{v}_T\right] . \quad (4.10)$$

When $\gamma=1$, the above equation reduces to Eq. (4.8) in consideration of Eq. (4.6) and neglecting the term $(\bar{H}/h_{c2})$. The form of $\mathbf{f}_{\rm drag}$ is similar to the expression proposed in Ref. 15 except for the prefactor of $\mathbf{v}_T$ and $\mathbf{v}_L$. However, it should be emphasized here that no negative Hall effect can be obtained in the absence of pinning forces.

When the backflow current only due to the pinning force is taken into account ($\delta=0$), the applied force becomes

$$\mathbf{F}=\frac{Ne}{c}(\mathbf{v}_T-\mathbf{v}_L)\times\mathbf{\Phi}_0+(\bar{H}/2\chi H_{c2})\mathbf{F}_p . \quad (4.11)$$

The second term on the right-hand side of the above equation comes from the difference between the current velocities inside and outside the flux core. On the other hand, the expression for the drag force becomes pinning-force dependent and is more complicated by the fact $\mathbf{v}_{\rm NC}\neq\mathbf{v}_T$. In the following we only present the results for two limiting cases.

(i) $\gamma=0$,

$$\mathbf{f}_{\rm drag}=-\frac{Ne\Phi_0/c}{\beta_0(\chi+\bar{H}/2H_{c2})}\mathbf{v}_T +(\bar{H}/2\chi H_{c2})\frac{\chi-\bar{H}/2H_{c2}}{\chi+\bar{H}/2H_{c2}}\mathbf{F}_p +\frac{\mathbf{F}_p\times\mathbf{k}}{\beta_0(\chi+\bar{H}/2H_{c2})} . \quad (4.12)$$

(ii) $\gamma=1$,

$$\mathbf{f}_{\rm drag}=(Ne/c)\mathbf{v}_L\times\mathbf{\Phi}_0 -(Ne\Phi_0\beta_0/c)[(\chi+\bar{H}/2H_{c2})\mathbf{v}_L+(\bar{H}/H_{c2})\mathbf{v}_T] -\beta_0(2\chi+\bar{H}/H_{c2})\mathbf{F}_p\times\mathbf{k}-(\bar{H}/2\chi H_{c2})\mathbf{F}_p . \quad (4.13)$$

Although the term which contains $(\mathbf{F}_p\times\mathbf{k})$ has no contribution to the energy dissipation, it has a significant effect on the Hall resistivity. Meanwhile, we should keep in mind that the total friction force acting on a flux equals the drag force plus the pinning force.

Finally, we would like to point out that since the drag force cannot naively be written in the form $-\eta\mathbf{v}_L$, the thermomagnetic effects in most cases will become very interesting. This problem will constitute a subject for future study.

V. CONCLUSIONS

Based upon the local normal-core model of Bardeen and Stephen[6] and in the light of work by Nozières and Vinen,[8] a general treatment of flux flow with backflow current in high-κ superconductors has been developed. In the calculations presented, we have established the equation of motion for a flux line moving in the mixed state of type-II superconductors. The longitudinal and Hall resistivities have been derived analytically as a function of the magnetic field in the region of flux flow. The results qualitatively explain all the essential features of recent experimental measurements on the Hall effect in high-T_c superconductors. In particular, we demonstrated that the observed negative Hall resistivity at low magnetic field in some high-T_c superconductors can be explained in terms of the existence of pinning forces in the sample, which have solved a controversy about the Hall effect in the mixed state. Meanwhile, a simple physical picture for the negative Hall effect was given. We also calculated the viscous-drag force and elucidated the relationship between the applied and drag forces acting on a flux. The correct expressions obtained for all those forces should be useful for further study on the thermomagnetic properties in the mixed state of type-II superconductors.

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APPENDIX A

According to the second equation in Eq. (2.6), a scale potential ϕ_b can be introduced to describe the backflow velocity $\mathbf{v}_b^{\text{out}}$ outside the core:

$$\mathbf{v}_b^{\text{out}} = \nabla \phi_b \ . \tag{A1}$$

From the first equation in Eq. (2.6), we obtain the Poisson equation for ϕ_b:

$$\nabla^2 \phi_b = 0 \ . \tag{A2}$$

The general solution of Eq. (A2) in two dimensions is given by the expression

$$\phi_0 = C_0 + D_0 \ln(r) + \sum_{m=1}^{\infty} [A_m \cos(m\theta) + B_m \sin(m\theta)] \left[C_m r^m + D_m \frac{1}{r^m} \right] \tag{A3}$$

When $r \to \infty$, ϕ_b should vanish. This boundary condition leads to $C_m = 0$ ($m = 0, 1, 2, \ldots$) and $D_0 = 0$. Thus Eq. (A3) reduces to

$$\phi_b = \sum_{m=1}^{\infty} [A_m \cos(m\theta) + B_m \sin(m\theta)] \frac{1}{r^m} \ . \tag{A4}$$

On the other hand, considering the condition $\nabla \cdot \mathbf{v}_s = 0$, we have the boundary condition at the surface of the core,

$$\mathbf{v}_{\text{NC}} \cdot \mathbf{e}_r = [\mathbf{v}_T + \mathbf{v}_b^{\text{out}}(a) + \mathbf{v}_0(a)] \cdot \mathbf{e}_r \ , \tag{A5}$$

where $\mathbf{e}_r$ is the unit vector along the $\mathbf{r}$ direction. Since $\mathbf{v}_0 \cdot \mathbf{e}_r = 0$ [see Eq. (2.4)], it follows that

$$\mathbf{v}_b^{\text{out}} \cdot \mathbf{e}_r |_a = (\mathbf{v}_{\text{NC}} - \mathbf{v}_T) \cdot \mathbf{e}_r \ . \tag{A6}$$

This leads to

$$(\mathbf{v}_{\text{NC}} - \mathbf{v}_T) \cdot \mathbf{e}_r = \left. \frac{\partial \phi_0}{\partial r} \right|_a = \sum_{m=1}^{\infty} [A_m \cos(m\theta) + B_m \sin(m\theta)] \left[\frac{-m}{a^{m+1}} \right] . \tag{A7}$$

From the above equation, we easily obtain $B_m = 0$, $A_m = 0 (m \neq 1)$, and

$$A_1 \cos\theta = -a^2 (\mathbf{v}_{\text{NC}} - \mathbf{v}_T) \cdot \mathbf{e}_r \ . \tag{A8}$$

Therefore

$$\mathbf{v}_b^{\text{out}} = \nabla \left[(\mathbf{v}_T - \mathbf{v}_{\text{NC}}) \cdot \mathbf{r} \left[\frac{a^2}{r^2} \right] \right] . \tag{A9}$$

APPENDIX B

Here we shall derive four formulas which are used in our text.

(1) Let us assume $\mathbf{D}$ to be a constant vector and choose the unit vector $\mathbf{e}_x$ along the direction of the vector $\mathbf{D}$; we obtain

$$\begin{aligned} Nm \int \int_{\Sigma^+} \mathbf{D} \cdot \mathbf{v}_0 d\mathbf{s} &= (Ne\phi_0/c)(D/2\pi) \\ &\quad \times \int \int_{\Sigma^+} (-\sin\theta)(\cos\theta\, \mathbf{e}_x + \sin\theta\, \mathbf{e}_y) d\theta \\ &= \frac{Ne}{2c} \mathbf{D} \times \boldsymbol{\phi}_0 \ . \end{aligned} \tag{B1}$$

In the above derivation, Eq. (2.4) and the following standard coordinate transformation have been employed:

$$\mathbf{e}_r = \cos\theta\, \mathbf{e}_x + \sin\theta\, \mathbf{e}_y \ , \tag{B2}$$

$$\mathbf{e}_\theta = -\sin\theta\, \mathbf{e}_x + \cos\theta\, \mathbf{e}_y \ , \tag{B3}$$

$$d\mathbf{s} = r\, d\theta\, \mathbf{e}_r \ . \tag{B4}$$

(2) From (A9) and by setting $\mathbf{v}_T - \mathbf{v}_{\text{NC}} = \Delta v\, \mathbf{e}_x$, it is straightforward to show

$$\mathbf{v}_b^{\text{out}} = \Delta v \left[\frac{1}{r^2} \mathbf{e}_x - \frac{2x}{r^3} \mathbf{e}_r \right] \tag{B5}$$

and

$$\begin{aligned} Nm \int \int_{\Sigma^+} \mathbf{v}_b^{\text{out}} \cdot \mathbf{v}_0 d\mathbf{s} &= Nm\, \Delta v \int \int_{\Sigma^+} \mathbf{e}_x \cdot \mathbf{v}_0 d\mathbf{s} \\ &= \frac{Ne}{2c} (\mathbf{v}_T - \mathbf{v}_{\text{NC}}) \times \boldsymbol{\phi}_0 \ . \end{aligned} \tag{B6}$$

(3) It is also easy to carry out the following integration and obtain

$$\begin{aligned} Nm \int \int_{\Sigma^+} \mathbf{v}_0 \mathbf{D} \cdot d\mathbf{s} &= (Ne\phi_0/c)(D/2\pi) \\ &\quad \times \int \int_{\Sigma^+} (-\sin\theta\, \mathbf{e}_x + \cos\theta\, \mathbf{e}_y) \cos\theta\, d\theta \\ &= -\frac{Ne}{2c} \mathbf{D} \times \boldsymbol{\phi}_0 \ . \end{aligned} \tag{B7}$$

(4) Finally, we have

$$\begin{aligned} Nm \int \int_{\Sigma^+} \mathbf{v}_0 \mathbf{v}_b^{\text{out}} \cdot d\mathbf{s} &= (Ne\phi_0/c)(\Delta v/2\pi) \\ &\quad \times \int \int_{\Sigma^+} (-\sin\theta\, \mathbf{e}_x + \cos\theta\, \mathbf{e}_y)(-\cos\theta) d\theta \\ &= -\frac{Ne}{2c} (\mathbf{v}_T - \mathbf{v}_{\text{NC}}) \times \boldsymbol{\phi}_0 \ . \end{aligned} \tag{B8}$$

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