Thermoelectric power of a narrow constriction in the adiabatic approximation

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The thermoelectric power of a narrow constriction is calculated in the adiabatic approximation with the transmission probability, which includes the quantum effects. The peak values of the thermoelectric power of a constriction depend on not only the radius of curvature of the constriction but also the temperature of the system. Streda's prediction as to the peak values should be modified at low temperatures.

It is well known experimentally[1,2] that the conductance G of the ballistic electrons in a narrow constriction in the two-dimensional electron gas (2DEG) realized in $Al_xGa_{1-x}As/GaAs$ heterojunctions reveals quantized values:

$$G=\frac{2e^2}{h}n \ , \tag{1}$$

where $n(=1,2,\ldots)$ is the number of the occupied subbands. Theoretically this quantization can be easily explained within the adiabatic approximation,[3-5] which assumes a smooth (on the scale of the Fermi wavelength λ_F) variation of the width of the constriction. Under this approximation, the electron transport through the constriction is regarded as the 1D Wentzel-Kramers-Brillouin (WKB) scattering problem in which a semiclassical transmission of the electron through the potential barrier is expected. The studies under this approximation have been done by many theorists.[6-10] Although there is increasing realization that the adiabatic approximation is not a good approximation for the experimental systems,[11] it is still useful as a qualitative guide.

Recently the peak structure of the thermoelectric power S of a constriction has been theoretically clarified by Streda.[12] The predicted quantized values of the peaks are independent of the temperature of the system and are given by

$$S_n^{\text{peak}}=-\frac{k_B}{|e|}\frac{\ln 2}{n-1/2}\simeq-\frac{59.73}{n-1/2}\ \mu\text{V/K} \ . \tag{2}$$

The nth peak S_n^{peak} occurs at the width of the constriction where the number of the occupied subbands at the narrowest point of the constriction changes from $n-1$ to n and the height of the peak is independent of the temperature.[13] (The first peak S_1^{peak} is not defined because the thermoelectric power approaches negative infinity when the number of occupied subbands is zero.) This prediction has been confirmed experimentally[14,15] as regards the period of the peaks. As to the heights of the peaks, however, the experimental values are smaller than the predicted ones. It is very interesting to study the condition under which the predicted peak values are obtained.

In this paper we investigate how Streda's prediction is modified in the low-temperature regime due to the effect of the curvature of the constriction and the temperature of the system. A similar investigation has recently been worked out by Proetto.[16]

The thermoelectric power in a narrow constriction within the linear-response approximation is given by[12]

$$S=-\frac{k_B}{|e|}\frac{\sum_{n=1}^{\infty}\int_0^{\infty}\left[-\frac{\partial f(E)}{\partial E}\right]T_n(E)\frac{E-\mu}{k_BT}dE}{\sum_{n=1}^{\infty}\int_0^{\infty}\left[-\frac{\partial f(E)}{\partial E}\right]T_n(E)dE} \ , \tag{3}$$

where the infinite square-well lateral confinement potential for the constriction is assumed for simplicity and $f(E)$ is the Fermi-Dirac distribution function, and $T_n(E)$ is the transmission probability of the electron at energy E through the nth subband. Streda has approximated $T_n(E)$ by a step function of energy:

$$T_n(E)=\theta(E-E_n) \ , \tag{4}$$

where E_n is the lowest energy of the nth subband. This expression is correct in the adiabatic limit.[3] Substitution of Eq. (4) into Eq. (3) yields

$$S=-\frac{k_B}{|e|}\frac{\sum_{n=1}^{\infty}\int_{E_n}^{\infty}\left[-\frac{\partial f(E)}{\partial E}\right]\frac{E-\mu}{k_BT}dE}{\sum_{n=1}^{\infty}f(E_n)} \ . \tag{5}$$

From this expression, Streda has concluded that the thermoelectric power exhibits a peak given by Eq. (2) when μ equals E_n.

In the real sample the shape of the constriction is not so smooth, as we can neglect the tunneling and above-barrier reflection at the potential barrier. Taking account of these quantum effects, we get the following expression for the transmission probability in the adiabatic approximation:[3]

$$T_n(E)=\left\{1+\exp\left[-\pi^2\left[\frac{2R}{W}\right]^{1/2}\left[\frac{kW}{\pi}-n\right]\right]\right\}^{-1} , \tag{6}$$

where W is the narrowest width of the constriction and R is the radius of curvature of the constriction at the nar-

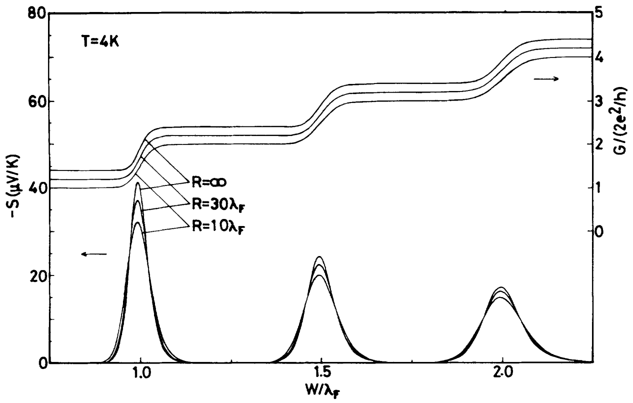

FIG. 1. S and G for $R=\infty$, $30\lambda_F$, and $10\lambda_F$ vs W, the width of the narrowest point of the constriction, at $T=4$ K. G for $R=\infty$ and $30\lambda_F$ are offset to the above by 0.4 and 0.2, respectively, relative to that for $R=10\lambda_F$.

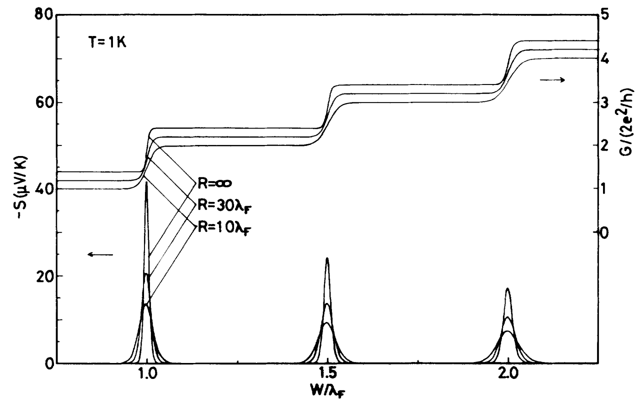

FIG. 2. S and G for $R=\infty$, $30\lambda_F$, and $10\lambda_F$ vs W at $T=1$ K. G for $R=\infty$ and $30\lambda_F$ are offset in the same way as in Fig. 1.

rowest point, and $k=(2m^*E)^{1/2}/\hbar$ [$m^*(=0.067m_0)$ is the effective mass of the electron in GaAs]. A similar expression has been derived in the case of a saddle-point constriction[17] and a more general case.[18] From the point of view of Eq. (6), Eq. (5) is considered to be "$R\rightarrow\infty$ limit," where the variation of the width of the constriction is unlimitedly smooth.

We calculate Eq. (3) with Eq. (6) as a function of W at the several values of R. In the calculation we have assumed the electron concentration $N_S=3.5\times10^{11}$ cm^{-2}, which corresponds to $\lambda_F=42.4$ nm for the 2DEG of $Al_xGa_{1-x}As/GaAs$ heterojunctions. Figure 1 shows the thermoelectric power and the conductance corresponding to three different R's ($10\lambda_F, 30\lambda_F, \infty$) at 4 K. In this figure $S_n^{\rm peak}$ ($n=2-4$) are shown. In the $R\rightarrow\infty$ limit the peak values of the thermoelectric power and their positions are slightly different from those predicted by Streda. But at $W=(\lambda_F/2)\times$integer, the thermoelectric power has surely the values given by Eq. (2). This discrepancy is due to the rough estimation of the peak values of Eq. (5). Although the numerator of Eq. (5) has the maxima at $W=(\lambda_F/2)\times$integer, the existence of the denominator (whose behavior is the same as that of the conductance) shifts the positions of the peaks of Eq. (5) to those corresponding to a narrower W. Furthermore, the heights of the peaks with the finite R are reduced in comparison with the corresponding values in the $R\rightarrow\infty$ limit. This result shows that the thermoelectric power depends on the characteristic property of the constriction. Figure 2 shows the result obtained at 1 K. The reduction of the peak values of the thermoelectric power is more remarkable at this temperature. As pointed out by Glazman *et al.*,[3] the value of R determines the effective width of the integrand of Eq. (3) at low temperatures, but at high temperatures the temperature dominates in the determination of the effective width. The critical temperature T_c is given by[3]

$$k_BT_c=\frac{n\hbar^2}{m^*(2RW^3)^{1/2}} . \tag{7}$$

For example, T_c for $n=2-4$ at $R=10\lambda_F$ and $30\lambda_F$ are 3.29 K ($10\lambda_F$) and 1.90 K ($30\lambda_F$) for $n=2$; 2.68 K ($10\lambda_F$) and 1.55 K ($30\lambda_F$) for $n=3$; 2.32 K ($10\lambda_F$) and 1.34 K ($30\lambda_F$) for $n=4$. Therefore, Fig. 1 corresponds to the high-temperature regime, where the peak values depend on R weakly. On the other hand, Fig. 2 corresponds to the low-temperature regime, where the peak values depend on R considerably. These results lead us to the conclusion that the peak values of the thermoelectric power depend on not only the characteristic property (the curvature and the width) of the constriction, but also on the temperature of the system in the temperature regime considered. As claimed by Streda, his result is expected to be valid at higher temperatures. At very high temperatures, however, a ballistic transport will break down. Therefore, his prediction on the peak structure can be realized under the conditions of an adequately high temperature and/or a very smooth constriction. Under the condition imposed in our calculation, the conductance exhibits the quantized values. The effect of the curvature of the constriction and the temperature on the conductance is seen in the slopes between the plateaus. It is reasonable that the thermoelectric power depends on the constriction geometry and the temperature, since the thermoelectric power is proportional to the energy derivative of the logarithm of the conductance.

In summary, we have calculated the thermoelectric power of the narrow constriction by using an expression plausible for the transmission probability. It is found that the prediction by Streda should be modified in the low-temperature regime, and the heights of the peaks of the thermoelectric power depend on the geometry of the constriction and the temperature of the system.

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