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Transmissions through low-dimensional mesoscopic systems subject to spin-orbit scattering

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Transmissions through low-dimensional disordered mesoscopic systems, subject to a perpendicular magnetic field and random spin-orbit coupling, are studied. The transmissions are obtained from the scattering matrix, in whose derivation the role of spin-orbit coupling as a phase factor is emphasized. The scattering matrix of a system connected to several reservoirs by single-channel leads is considered; for the one-dimensional ring, its form is derived analytically. In the latter case, it is found that the point-to-point transmission is an even function of the flux only when there is no additional connection of the system to an external reservoir. Otherwise, the transmission is a general (periodic) function of the flux. Implications of this observation are discussed. Averaging over an ensemble of realizations, it is shown that the transmission of a two-dimensional sample includes only even harmonics as a function of the magentic flux per unit cell when the average over the spatial disorder alone is insufficient to extinguish the odd harmonics. The transmission through a one-dimensional ring is shown analytically to consist solely of the zeroth and the second harmonics of the threading flux.

I. INTRODUCTION AND SPIN-ORBIT COUPLING AS A PHASE FACTOR

The effect of a constant magnetic field threading a mesoscopic ring (or a thin cylinder) has been studied quite extensively.[1–4] The reason is that in such systems, at sufficiently low temperatures where the phase of the electronic wave function is not lost (due to inelastic scattering) over distances of the order of the sample size, the field appears almost solely in the form of the Aharonov-Bohm[5] phase factor. This leads to nonclassical phenomena related to quantum interference. Recently it was shown[6] that in multiply connected, single-channel systems, spin-orbit coupling can be represented as an additional phase factor, superimposed on that due to the magnetic flux. (See also Ref. 7 for a discussion of the representation of spin-orbit coupling as a Berry phase.) From this observation follow several distinct predictions concerning the behavior of spin-independent properties in the presence of both a magnetic flux and spin-orbit scattering. The present work is devoted to an elaboration of the arguments presented in Ref. 6, and, in particular, to study *transmissions* through low-dimensional mesoscopic samples subject to a constant magnetic field and including random spin-orbit coupling.

We begin by reexamining the appearance of spin-orbit coupling as a phase factor, or alternatively as an effective vector potential. We then proceed to analyze various transmissions of low-dimensional disordered mesoscopic samples. Throughout the discussion we neglect the Zeeman interaction and disregard inelastic events.

It was shown in Ref. 6 that in the presence of spin-orbit coupling and a magnetic flux, the boundary condition on the electron wave function in a ring of circumference L is

$$\psi(x+L)=\exp[i\lambda\mathbf{d}\cdot\boldsymbol{\sigma}/2+2\pi\Phi/\Phi_0]\psi(x) . \tag{1}$$

Here $\psi(x)$ is a two-component spinor, Φ is the magnetic flux threading the ring, and $\Phi_0=hc/e$ is the flux quantum unit. The spin-orbit coupling in the ring is represented by the first factor in (1). Here $\boldsymbol{\sigma}$ is the vector of the Pauli matrices and λ is the angle of rotation of the spin around the axis $\mathbf{d}$ as the electron encircles the ring. It is seen from Eq. (1) that in a one-dimensional ring, spin-orbit coupling appears as an effective flux, which is a 2×2 matrix. This form can be obtained from a different point of view, by considering a magnetic moment $\boldsymbol{\mu}$ moving with a velocity $\mathbf{v}$ in the presence of an electric field $\mathbf{E}$. In such a situation, the dipole interacts with the induced magnetic field $-\mathbf{v}\times\mathbf{E}/c$ that appears in its frame of reference. The energy of the interaction is $\boldsymbol{\mu}\cdot(\mathbf{v}\times\mathbf{E}/c)$. For an electron, $\boldsymbol{\mu}=\mu_B\boldsymbol{\sigma}/2$ (where μ_B is the Bohr magneton) and the interaction term becomes the spin-orbit coupling. One may then write for the kinetic energy

$$\frac{1}{2m}(\mathbf{p}+e\,\mathbf{A}_{\text{s.o.}}/c)^2, \quad \mathbf{A}_{\text{s.o.}}=(\hbar/4mc)\boldsymbol{\sigma}\times\mathbf{E} . \tag{2}$$

This form, also discussed in the context of Bloch electrons,[8] is valid as long as $|\mathbf{p}|\gg\boldsymbol{\mu}|\,|\mathbf{E}|/c$. The appearance of a vector potential in the presence of an electric field in the Hamiltonian of a magnetic moment was found by Aharonov and Casher.[9] The connection between the Aharonov-Casher effect and spin-orbit scattering in mesoscopic systems was recently pointed out by Mathur and Stone.[10]

Thus, spin-orbit coupling can be formulated as an effective vector potential, or alternatively, as a phase factor. This formulation can be carried out in general dimensions within the tight-binding picture[11] in a way similar to the one by which a constant magnetic field is treated.[12] However, the vector potential $\mathbf{A}_{\text{s.o.}}$ differs from the magnetic vector potential in two aspects: (a) $\mathbf{A}_{\text{s.o.}}$ involves non-Abelian matrices and does not violate timereversal symmetry; (b) in a disordered system $\mathbf{A}_{\mathrm{s.o.}}$ is a random quantity, unlike the vector potential of a constant magnetic field. But in a one-dimensional ring one can use the apparent analogy between the two vector potentials [see Eq. (1)] to relate the rotation angle λ to a length $l_{\mathrm{s.o.}}$ characteristic of the spin-orbit coupling. Recalling that the characteristic magnetic length l_B is defined by $(l_B/L)^2=\Phi_0/\Phi$, we obtain from (1) that $2\pi/\lambda=(l_{\mathrm{s.o.}}/L)^2/2$. Thus, $l_{\mathrm{s.o.}}$ is proportional to $\lambda^{-1/2}$ and tends to infinity as λ approaches zero.

The appearance of spin-orbit scattering as a phase factor, albeit a random one, enables us to separate exactly, in the Green's function of a disordered one-dimensional ring, the spatial disorder from the spin-orbit scattering. We present in Sec. II an analytic solution for the Green's functions and the scattering matrix of the disordered ring, when it is coupled to several one-dimensional leads (see Fig. 1). The scattering matrix is used to derive the transmissions between any pair of sites. It should be emphasized that our expression for the transmission takes into account that other sites may be connected to external reservoirs. These types of transmissions are required for obtaining, e.g., the transverse resistance[13,14] (e.g., the resistance measured between sites 2 and 4 in Fig. 1 when current is flowing from site 1 to site 3). We find that for a specific ring the point-to-point transmission, when all other sites are not connected to external reservoirs, is an even function of the flux. But when other sites are connected, the transmission is a general periodic function of the flux.

We show explicitly in Sec. II that the transmission between any pair of the ring sites, in the presence of spin-orbit coupling and a magnetic flux, takes the form

$$T(\Phi)=\tfrac{1}{2}[T_o(2\pi\Phi/\Phi_0+\lambda/2)+T_0(2\pi\Phi/\Phi_0-\lambda/2)]\ . \tag{3}$$

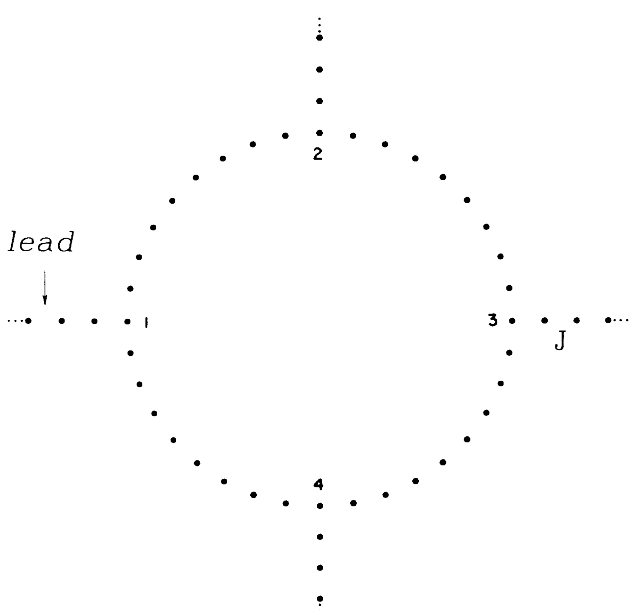

FIG. 1. A one-dimensional ring connected to several wires.

Here, $T_0(\Phi)$ is the same transmission as $T(\Phi)$ but in the absence of spin-orbit coupling. Equation (3) confirms the predictions put forward in Ref. 6. it was also noted there that Eq. (3) yields a specific form for the average of $T(\Phi)$ over spin-orbit scattering. Since $T_0(\Phi)$ is a periodic function of Φ (with period $2\pi/\Phi_0$), it may be expanded in Fourier series. The form (3) shows that the nth harmonic is multiplied by $\cos n\lambda/2$ in the presence of spin-orbit scattering. One can then average each harmonic upon the latter scattering: The rotation angle λ is distributed randomly[15] over an ensemble of rings. General arguments of group theory give that $0\le\lambda\le 2\pi$ with the weight function $(1/\pi)\sin^2(\lambda/2)$. This yields that[6,16]

$$\frac{1}{\pi}\int_0^{2\pi} d\lambda\,\sin^2(\lambda/2)\cos n\lambda/2=\delta_{n,0}-\tfrac{1}{2}\delta_{n,2}\ , \tag{4}$$

namely, only the zeroth and the second harmonics survive the averaging over the spin-orbit scattering. We test this prediction numerically in Sec. II for the longitudinal transmission (between the sites by which particles are fed in and taken out of the sample). We compute the transmission averaged over both the spatial disorder and the spin-orbit coupling disorder. The averaging procedure we employ does not assume that the two types of disorder are uncorrelated. The numerical results show clearly that all harmonics of the transmission, except the zeroth and second one, disappear upon averaging.

In Sec. III we consider small two-dimensional samples subject to a perpendicular magnetic field. We present there the energy spectrum in the presence of random spin-orbit coupling, for a specific realization. This spectrum does not show any trace of the features characteristic of the zero-spin-orbit-coupling case.[17] Nevertheless, we find that the longitudinal transmission of such a system, when averaged over spatial disorder and spin-orbit scattering, includes only even harmonics and is dominated by the zeroth and second harmonics of the magnetic flux per elementary unit cell. The numerical results at two dimensions can be explained on the basis of the exact analytical analysis carried out for the one-dimensional case in Sec. II.

II. TRANSMISSION THROUGH A ONE-DIMENSIONAL RING

Here we first derive the scattering matrix (in the site representation) of a system connected to several external reservoirs by one-dimensional leads.[13] The scattering matrix is obtained generally, in terms of the Green's function of the isolated system. We then apply the result to a one-dimensional ring by calculating analytically the Green's function of the latter. This is used to discuss the various transmissions of the ring.

The Hamiltonian of the isolated sample (without the leads) in the site representation reads

$$H=\sum_{n,n'}|n\rangle(\varepsilon_n\delta_{nn'}+V_{nn'})\langle n'|\ , \tag{5}$$

where $|n\rangle$ denotes the site and the spin state, i.e., $|n\rangle$ is a spinor and $V_{nn'}$ is a 2×2 matrix in spin space. We assume that there is no disorder on the leads and describe them by a nearest-neighbor, tight-binding Hamiltonian with zero on-site energies and a constant transfer integral J.[13] It then follows that the energy of the incident and transmitted particles, E, (assumed for simplicity to be the same on all the leads[18]) is related to their wave vector $\mathbf{q}$ by $E=2J\cos q$.

Let us denote the wave function on the ith lead by

$$\psi_i(l)=\alpha_i e^{-iql}+\alpha_i' e^{iql} , \tag{6}$$

where l denotes the site and α_i and α_i' are the incoming and outgoing (spinor) amplitudes, respectively. The scattering matrix $\hat{S}$ in this situation is given by

$$\alpha_i'=\sum_j S_{ij}\alpha_j . \tag{7}$$

Note that S_{ij}, the transition amplitude from lead j to lead i, is a 2×2 matrix in spin space.

To obtain the scattering matrix, we proceed as follows. The Schrödinger equation at the site connecting lead i to the sample site n reads

$$E(\alpha_i+\alpha_i')=J\alpha_i e^{-iq}+J\alpha_i' e^{iq}+J\phi_n , \tag{8}$$

where ϕ_n is the wave function at the nth site of the system. The Schrödinger equation at the latter site is

$$\sum_m (E\delta_{n,m}-H_{nm})\phi_m=J(\alpha_i+\alpha_i') . \tag{9}$$

Using the relations $E=2J\cos q$ and $(E-H)^{-1}=G(E)$ in Eqs. (8) and (9), it is readily obtained that

$$\hat{S}=[\hat{J}\hat{G}\hat{J}-\hat{J}e^{-iq}]^{-1}[\hat{J}e^{iq}-\hat{J}\hat{G}\hat{J}] . \tag{10}$$

In this equation, $\hat{G}$ and $\hat{J}$ are $N\times N$ matrices whose entries are 2×2 matrices in spin space (N is the number of sites in the system). $\hat{G}$ is the Green's function of the isolated system at energy E and $\hat{J}$ is the coupling matrix: It is a diagonal matrix with nonzero matrix elements (equal to J) only at the entries corresponding to the sites where the leads are connected. Thus $\hat{J}\hat{G}\hat{J}$, $\hat{J}$, and $\hat{S}$ are practically $M\times M$ matrices, where $M\leq N$ is the number of external leads connected to the sample. We note that the matrix elements of $\hat{J}\hat{G}\hat{J}$ are J^2G_{ij}, where i and j run over the system sites connected to the leads. Denoting this reduced matrix by $\hat{g}$, it is straightforward to obtain from Eq. (10) the form

$$\hat{S}=-e^{2iq}[1+2J\sin q\hat{g}(1-e^{iq}\hat{J}\hat{g})^{-1}] , \tag{11}$$

where $\hat{S}$ is now an $M\times M$ matrix (whose entries are 2×2 matrices in spin space). Note that the factor $2J\sin q$ gives the velocity of the particles in the tight-binding picture.[18]

The transmission amplitude between two sites, i and j, is given by the i,j matrix element of $\hat{S}$. Thus

$$S_{ij}=-e^{2iq}2J\sin q\{[\hat{g}^{-1}-\hat{J}e^{iq}]^{-1}\}_{ij}, \quad i\neq j , \tag{12}$$

where $\hat{g}$ includes matrix elements of the Green's function, at energy E, of the isolated system. This observation enables us to further simplify the expression for S_{ij}. Imagine first that all the sample sites are connected to external leads. Then $\hat{g}$ is identical to $\hat{G}(E)$. But G^{-1} is just $E-H$ and, therefore, it follows that

$$S_{ij}=-e^{2iq}2J\sin qG_{ij}(E-Je^{iq}), \quad i\neq j . \tag{13}$$

In the general case where only part of the sites are connected, Eqs. (12) and (13) yield that $\hat{S}$ is given by $\hat{G}$, such that

$$\hat{G}=(E\hat{1}-\hat{J}e^{iq}-\hat{H})^{-1} , \tag{14}$$

where $\hat{J}$ is the diagonal matrix with nonzero matrix elements at the entries corresponding to the connected sites.

This completes the derivation of the scattering matrix in terms of the Green's function of the system. The transmission probability T_{ij} for unpolarized incident particles is

$$T_{ij}=\tfrac{1}{2}\mathrm{Tr}(S_{ij}^{+}S_{ij}) . \tag{15}$$

Here, Tr denotes a trace in spin space and S_{ij} is given (up to a phase factor) by $2J\sin qG_{ij}$, where $\hat{G}$ is calculated from Eq. (14). It should be noted that this result is not restricted to low-dimensional systems.

We next apply the general result to a disordered one-dimensional ring threaded by magnetic flux, and including spin-orbit scattering. To this end we derive the Green's function of the ring in the site representation.

Adopting a nearest-neighbor tight-binding description, the Hamiltonian of the ring can be written in the form

$$H=\sum_{n,\sigma}\varepsilon_n|n\sigma\rangle\langle n\sigma| + \left[\sum_{n,\sigma,\sigma'}|n+1\sigma\rangle V_n\langle n\sigma'|+\mathrm{H.c.}\right] , \tag{16}$$

where σ denotes the spin state. In (16), ε_n are the on-site energies, which are randomly distributed. They describe the spatial disorder. The coupling (2×2) matrices V_n include the effect of the magnetic flux threading the ring and the spin-orbit scattering

$$V_n=V\exp(i2\pi\Phi/N\Phi_0)\exp(i\lambda_n\mathbf{d}_n\cdot\boldsymbol{\sigma}/2) . \tag{17}$$

Here, λ_n is the angle of the spin rotation about the axis $\mathbf{d}_n$ as the electron is transferred from site n to site $n+1$, both being random quantities. It is assumed here that the spin-orbit coupling does not affect the on-site energies. In the following we measure energies in units of V [Eq. (17)] so that the coupling terms are of unit amplitudes.

We now calculate the Green's function (14). Noting that the factor Je^{iq} can be "swallowed" in the on-site energies of the sites connected to the external leads, it is sufficient to consider the matrix $(E-H)^{-1}$. For the one-dimensional ring this matrix can be obtained by the transfer matrix technique, and may be written as follows:

$$G_{ll}(E)=c_l(E)M_l(E) , \tag{18}$$

$$G_{lm}(E)=M_l(E)[a_{lm}(E)V_{ml}^{-1}+b_{lm}(E)V_{lm}] = [a_{lm}(E)V_{ml}^{-1}+b_{lm}(E)V_{lm}]M_m(E) , \quad l\neq m , \tag{19}$$

where the matrices M_l and V_{lm} are 2×2 matrices in spin space. The matrix M_l is given by

$$M_l(E)=\frac{1}{\mathrm{Tr}(\tau)-2\cos[(2\pi\Phi/\Phi_0)+\lambda/2]}P_l^+ + \frac{1}{\mathrm{Tr}(\tau)-2\cos[(2\pi\Phi/\Phi_0)-\lambda/2]}P_l^- , \quad (20)$$

where $\tau=\tau(E)$ is the product of the transfer matrices around the ring *in the absence of* flux and spin-orbit coupling. In (20), $P_l^\pm$ are the projection operators onto the eigenvalues of the product matrix $V_l V_{l+1}\cdots V_{l+1}$ around the ring [see Eq. (17)]. Namely,

$$P_l^\pm V_l\cdots V_{l-1}=V_l\cdots V_{l-1}P_l^\pm =\exp(i2\pi\Phi/\Phi_0)\exp(\pm i\lambda/2)P_l^\pm , \quad (21)$$

where

$$V_l\cdots V_{l-1}=\exp(i2\pi\Phi/\Phi_0)\exp(i\lambda\mathbf{d}_l\cdot\boldsymbol{\sigma}/2) \quad (22)$$

and

$$P_l^\pm=(1\pm\mathbf{d}_l\cdot\boldsymbol{\sigma})/2 . \quad (23)$$

Thus, the diagonal matrix element [Eq. (18)] of the Green's function (in the site representation) is given by the product of the coupling matrices around the ring. The coefficient $c_l(E)$ in Eq. (18) is[19]

$$c_l(E)=\partial\,\mathrm{Tr}(\tau)/\partial\varepsilon_l , \quad (24)$$

and is independent of the magnetic flux and the spin-orbit scattering. The nondiagonal matrix element, Eq. (19), consists of two contributions: the part $b_{lm}(E)V_{lm}$, which describes the "path" going from site l to site m, where

$$V_{lm}=V_l V_{l+1}\cdots V_{m-1} , \quad (25)$$

with a coefficient independent of the magnetic flux and the spin-orbit scattering

$$b_{lm}(E)=\partial^{|l-m+1|}\mathrm{Tr}(\tau)/\partial\varepsilon_l\partial\varepsilon_{l+1}\cdots\partial\varepsilon_m ; \quad (26)$$

the part $a_{lm}(E)V_{ml}^{-1}$ which describes the complementary path that goes from m to l, with

$$a_{lm}(E)=\partial^{|l-m+1|}\mathrm{Tr}(\tau)/\partial\varepsilon_m\partial\varepsilon_{m+1}\cdots\partial\varepsilon_l . \quad (27)$$

The explicit expressions for the Green's-function matrix elements manifest clearly the appearance of spin-orbit coupling as a phase factor. They are obtained from the corresponding matrix elements in the absence of flux and spin-orbit coupling by introducing two modifications: (i) Each path acquires a phase factor due to the magnetic flux and due to the spin-orbit coupling along the path (the latter being a 2×2 matrix); (ii) the denominator of the matrix element [i.e., the denominators in Eq. (20)] is modified into

$$\mathrm{Tr}(\tau)-2\cos(2\pi\Phi/\Phi_0\pm\lambda/2) . \quad (28)$$

Recalling that the eigenenergies of the isolated ring are given by the poles of $G_{ll}(E)$, it is seen that the eigenenergies in the presence of spin-orbit coupling are obtained from those in the absence of this coupling, by changing $E_\mu(2\pi\Phi/\Phi_0)$ into $E_\mu(2\pi\Phi/\Phi_0\pm\lambda/2)$. This reproduces the result of Ref. 6 under less restrictive conditions.

Having determined the matrix elements of the Green's function of the ring, we now use them to discuss various transmissions. Let us first consider the case where only two sites of the ring are connected to external reservoirs, say sites 1 and 3 (see Fig. 1). One then needs G_{13} where the on-site energies ε_1 and ε_3 are modified into ε_1+Je^{iq} and ε_3+Je^{iq} [see Eq. (14)]. We note[19] that in this case a_{13} and b_{13} [cf. Eqs. (26) and (27)] are *independent of* q. It follows that

$$\mathrm{Tr}[G_{13}(q)G_{13}^+(q)]=\frac{1}{|f_+|^2}[a_{13}^2+b_{13}^2+2a_{13}b_{13}\cos(2\pi\Phi/\Phi_0+\lambda/2)] + \frac{1}{|f_-|^2}[a_{13}^2+b_{13}^2+2a_{13}b_{13}\cos(2\pi\Phi/\Phi_0-\lambda/2)] , \quad (29)$$

where

$$f_\pm=\mathrm{Tr}(\tau)-2\cos(2\pi\Phi/\Phi_0\pm\lambda/2) , \quad (30)$$

and the only q dependence (i.e., the dependence upon the energy of the transmitted particle) is within $\mathrm{Tr}(\tau)$. In this case the transmission T_{13} is a function of $\cos(2\pi\Phi\pm\lambda/2)$ and is even in the flux. Its Fourier series will include cosine harmonics alone.

This is not the case for the transmission calculated between two sites when some other sites are also connected to external leads. Consider, for example, the transmission T_{13} when site 2 is also connected to an external reservoir. In this situation we need G_{13} with ε_1, ε_2, and ε_3 modified into ε_1+Je^{iq}, ε_2+Je^{iq}, and ε_3+Je^{iq}. In this case[19] [see Eqs. (26) and (27)] either a_{13} or b_{13} will include the factor Je^{iq}, depending upon the relative positions of the three sites. Thus, in general, the transmission T_{13} when sites other than 1 and 2 are connected to external leads is given by

$$\mathrm{Tr}[G_{13}(q)G_{13}^+(q)=\frac{1}{|f_+|^2}[|a_{13}(q)|^2+|b_{13}(q)|^2+2\,\mathrm{Re}a_{13}^*(q)b_{13}(q)\exp i(2\pi\Phi/\Phi_0+\lambda/2)] + \frac{1}{|f_-|^2}[|a_{13}(q)|^2+|b_{13}(q)|^2+2\,\mathrm{Re}a_{13}^*(q)b_{13}(q)\exp i(2\pi\Phi/\Phi_0-\lambda/2)] . \quad (31)$$

This transmission involves terms such as

$$\cos(2\pi\Phi/\Phi_0 \pm \lambda/2 \pm q)$$

and is not an even function of the flux. This means that its Fourier series includes cosine and sine harmonics. Each such harmonic will be multiplied by $\cos n\lambda/2$ as predicted in Ref. 6 and discussed above [see Eqs. (3) and (4)], but T_{13} will be sensitive to the sign of the flux. It is exactly this property which allows for the calculation of the "Hall resistance" according to Buttiker's four-probe expression.[14] That expression involves transmissions between two sites when other sites of the system are "open."

As discussed in Ref. 6 [see also Eqs. (3) and (4)] one expects that only the zeroth and second harmonics of the transmission, as a function of Φ, remain upon averaging over random spin-orbit scattering. We have tested this prediction numerically, by computing the transmission T_{13} (see Fig. 1) in the situation where only these sites are open. A ring of 36 sites was considered, in which the on-site energies are distributed uniformly in a band of width W. It is assumed for simplicity that $J=V$ [see Eq. (17)], which sets the energy scale. The energy of the incident particle is taken to be $E=-0.5$. The spin-orbit coupling parameters are chosen as follows: To each bond we assign a phase factor $\exp(i\lambda_n \mathbf{d}_n \cdot \sigma/2)$, where $\mathbf{d}_n$ is a unit vector distributed uniformly over the unit sphere and $0 \le \lambda_n \le 2\pi$ with the weight factor $[\sin^2(\lambda_n/2)]/\pi$. Figure 2 depicts the Fourier analysis of the transmission (averaged over 500 realizations) when the on-site energies are distributed in a band of width 0.5. It shows clearly that in the presence of random spin-orbit scattering there remain only the zeroth and second harmonics, the latter being reduced by the factor -0.5 compared to its value in the absence of spin-orbit scattering [cf. Eq. (4)]. Inspection of the harmonics when no spin-orbit coupling is present (Fig. 2) shows that our result includes even and odd harmonics (of decreasing amplitudes). However, it was predicted[4] that the periodicity of the transmission is doubled upon averaging over spatial disorder, i.e., all odd harmonics should average to zero. To investigate this point we have computed the averaged transmission as a function of the number of realizations (here the on-site energies are distributed in a band of width $W=2$). The Fourier analysis is exhibited in Fig. 3 for the zeroth, first, second, and fourth harmonics. One sees that the amplitude of the zeroth harmonic, for samples with and without spin-orbit scattering, tends to a common value as the number of realizations is increased. The amplitude of the first harmonic tends to zero for both kinds of samples (the situation for higher odd harmonics is similar). The amplitude of the second harmonic of samples including spin-orbit scattering is -0.5 that of samples without it. Finally, the amplitude of the fourth harmonic for samples with spin-orbit scattering tends to zero while that of samples without it approaches some finite value. The greater efficiency of averaging over spin-orbit scattering as compared to spatial-disorder averaging may be due to the following possibility. In the strong spin-orbit scattering limit, the total angle λ along the ring may be of order unity even when the spin rotation per bond is small. By choosing λ_n, the single-bond rotation, from the broad distribution (4), we may be enhancing spin-orbit scattering to some ultrastrong limit, which makes it very effective. However, Figs. 2 and 3 are clearly in accordance with the theoretical predictions.

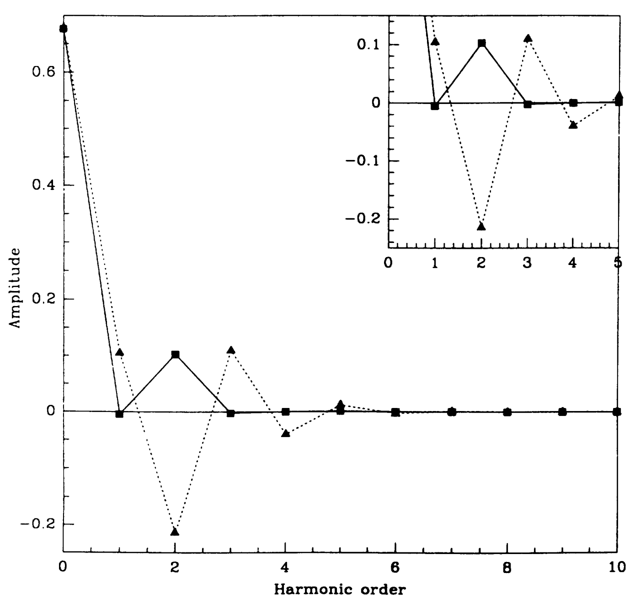

FIG. 2. Fourier analysis of the averaged transmission of a one-dimensional ring, in the absence of spin-orbit coupling (dotted line) and in the presence of random spin-orbit scattering (solid line). The inset shows a zoom of the first five harmonics.

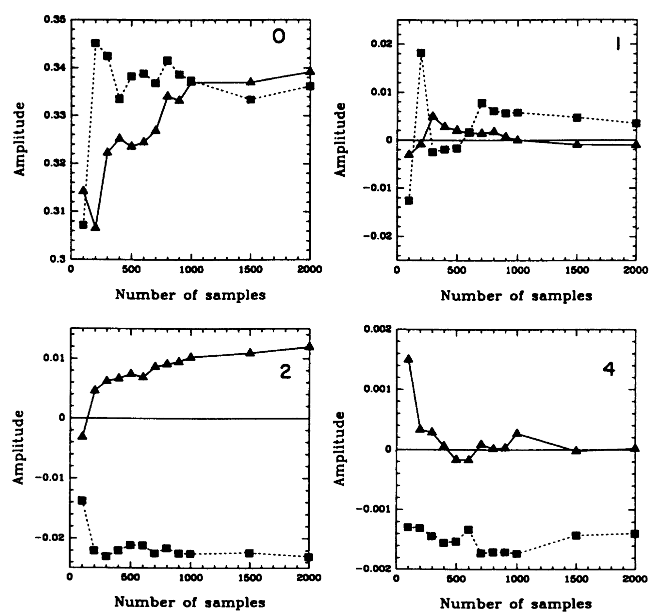

FIG. 3. Fourier analysis of the zeroth, first, second, and fourth harmonics of the averaged transmission as a function of the number of samples in the ensemble. In the absence of spin-orbit coupling (dotted lines), and in the presence of random spin-orbit scattering (solid lines).

III. SMALL TWO-DIMENSIONAL SAMPLES

Here we consider a small disordered two-dimensional system subject to a perpendicular magnetic field. The system is described by the nearest-neighbor, tight-binding Hamiltonian, in which each bond carries a phase factor due to the magnetic field and a (2×2 matrix) random-phase factor, which represents random spin-orbit scattering. (The choice of the latter is explained in the preceding section.) It is interesting to examine the energy spectrum of such a system. Figure 4 portrays the first 100 levels of a 10×10 system, for a random realization of the spin-orbit coupling parameters. Comparing this figure with Fig. 1 of Ref. 17, it is seen that the spectrum contains no features typical of those of a system without spin-orbit coupling (see discussion in Ref. 17).

The transmission through the two-dimensional system (connected at the diagonal ends to external reservoirs by single-channel leads) is again given by the relevant matrix element of its Green's function, as discussed above. However, in this case we are not able to derive the Green's function analytically. One can imagine G_{ij} to consist of contributions from all the paths connecting sites i and j, in analogy with expression (19) above. In the two-dimensional system these paths enclose different numbers of elementary unit cells. As a result, the nth harmonic of the transmission (of the flux per unit cell) arises from paths encircling the unit cell n times, those encircling the unit cell $n-2$ times and other two-unit cells once, etc. Introducing the random spin-orbit scattering, the averaged transmission should then include *all* even harmonics. For example, the average over the random spin-orbit coupling will leave in the fourth harmonic the contribution of paths enclosing two-unit cells. Due to this intricacy, one does not expect that the amplitudes of the harmonics in the presence of spin-orbit scattering will be -0.5 the corresponding ones in the absence of this scattering.

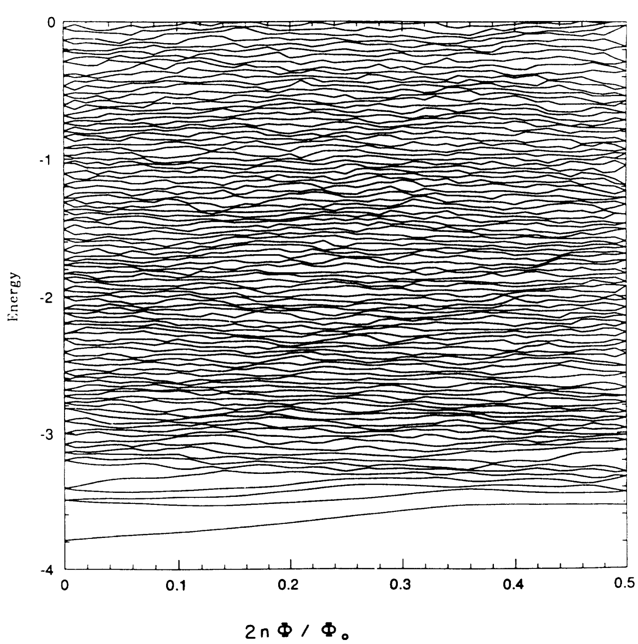

FIG. 4. The lowest energy levels of a 10×10 tight-binding Hamiltonian including random spin-orbit coupling as a function of the flux per elementary unit cell. The on-site energies are distributed uniformly in the range ± 1.

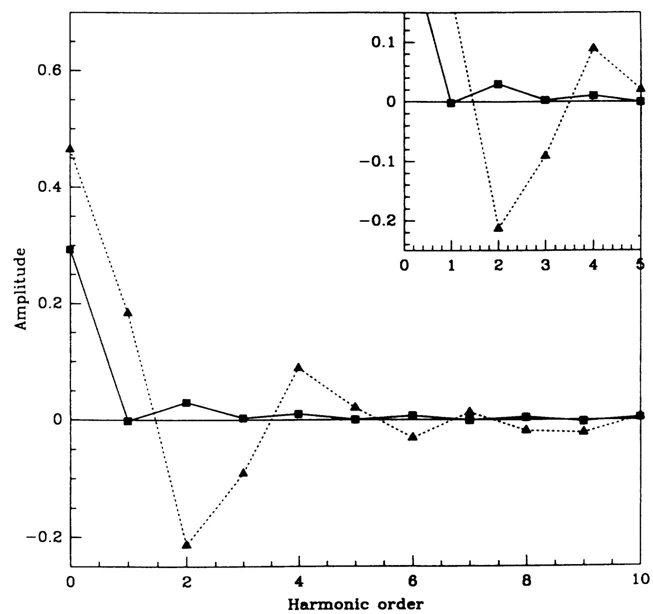

FIG. 5. Fourier analysis of the averaged transmission through the opposite corners of a 10×10 sample, in the absence of spin-orbit coupling (dotted line) and in the presence of random spin-orbit scattering (solid line). The inset shows a zoom of the first five harmonics.

Figure 5 shows the Fourier analysis of the averaged transmission through a 10×10 system. (It is the result of averaging over 500 realizations when the two opposite corners of the square are connected to external reservoirs; the on-site energies are distributed in a band of width 2, the energy of the incident particle is $E=-2$, measured in units of the transfer integral; the spin-orbit parameters are chosen as in Sec. II.) Comparing the Fourier components of the averaged transmission with and without spin-orbit scattering, we see the following. In the absence of spin-orbit coupling, the transmission includes even and odd harmonics of decreasing amplitudes; this is similar to the behavior found for the ring (cf. Fig. 2), where it has been shown that the odd harmonics average to zero upon increasing the number of realizations. In the presence of random spin-orbit coupling there are only even harmonics. One may conclude from the results shown in Figs. 2 and 5 that the "true" averaged transmission is achieved "faster" in the presence of spin-orbit scattering than in its absence. The reason may be again the one presented at the end of the previous section. It may be of interest to examine the behavior of the averaged transmission in the case where the spin rotations along *paths* connecting the contacts (rather than along single bonds) are distributed according to the distribution (4).

IV. DISCUSSION

This paper is devoted to studying effects of spin-orbit scattering upon transmissions through low-dimensional small disordered systems. The motivation for this work is the observation that spin-orbit coupling can be described as an effective phase factor, multiplying the transfer integral in the tight-binding picture. Since this is reminiscent of the phase factor due to an external magnetic field, one hopes to obtain results related to quantum interference phenomena. As is pointed out above, the spin-orbit phase factor differs from that of the magnetic field in being a random, non-Abelian phase factor.

We have developed an explicit expression for the scattering matrix in terms of the Green's-function matrix elements in the site representation. We have used it to obtain the transmission, and analyzed the latter. In the numerical simulations, we have averaged the transmission by choosing the spin-orbit coupling parameters randomly for each realization of the on-site energies, i.e., for each spatial-disorder configuration. In this way we have avoided the assumption that the two types of disorders are uncorrelated.

Our results for the one-dimensional ring confirm clearly the predictions of Ref. 6. They show that upon averaging over the random spin-orbit coupling, only the zeroth and second harmonics of the flux are left. For two-dimensional samples we find that the average over the spin-orbit scattering leaves only even harmonics (of the flux per unit cell) even in the situation where the ensemble of realizations is not sufficiently large to extinguish the odd harmonics by averaging over the spatial disorder alone.

We have neglected in this work the Zeeman interaction and inelastic scattering. Thus the results are limited to magnetic fields not too high compared with the spin-orbit coupling strength[20] and to low temperatures.

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[19]Equations (24), (26), and (27) assume no degeneracy in the on-site energies. If such a degeneracy exists, the relevant energies should be modified (i.e., $\varepsilon_i \rightarrow \varepsilon_i'$) before the differentiation process, and then remodified to assume their original value. Also, it should be noted that the structure of $\mathrm{Tr}(\tau)$ as a function of the on-site energies is such that $\partial\,\mathrm{Tr}(\tau)/\partial\varepsilon_i$ is independent of ε_i.

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