Single-particle relaxation time of one-dimensional electron gases

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The single-particle relaxation time τ_s for the disordered interacting quasi-one-dimensional electron gas is calculated within a one-subband model. For interface-roughness scattering and alloy-disorder scattering, we find that the ratio between the transport relaxation time τ_t and τ_s, if they are calculated in the lowest-order Born approximation, is given by $\tau_t/\tau_s=\frac{1}{2}$. For charged-impurity scattering we derive $\tau_t/\tau_s \gg 1$ for $E_F\tau_t \gg 1$, where E_F is the Fermi energy. Multiple-scattering effects calculated with the self-consistent Born approximation are also discussed. The density of states versus energy in the presence of disorder is calculated. We present analytical results, and recent experimental and theoretical results are discussed.

I. INTRODUCTION

In an electron gas various lifetimes are introduced by a finite amount of disorder. The transport relaxation time τ_t describes the mobility of an electron gas and can be determined by conductivity measurements. The single-particle relaxation time τ_s describes the decay time of one-particle excitations and gives rise to a renormalization of the density of states. It is widely believed that the temperature dependence of the Shubnikov–de Haas oscillations in two-dimensional electron gases is determined by the single-particle relaxation time.[1–3]

In two-dimensional electron gases the ratio τ_t/τ_s can be strongly enhanced if the random potential is long ranged.[2,3] For remote impurity doping of $Al_xGa_{1-x}As/GaAs$ heterostructures with large spacer width α, one finds $\tau_t/\tau_s=1$ for $2k_F\alpha \ll 1$ and $\tau_t/\tau_s=(2k_F\alpha)^2$ for $2k_F\alpha \gg 1$.[3] k_F is the Fermi wave number. Recent experiments[4,5] on two-dimensional electron gases supported the idea that measurements of τ_t and τ_s provide important information on the scattering mechanisms. The strong enhancement of τ_t/τ_s in a two-dimensional electron gas for a long-range random potential indicates that even for high-mobility samples the effects of disorder on the density of states can be much larger than expected from the transport time. For a review, see Ref. 6.

Electronic properties of quasi-one-dimensional systems are studied intensively in experiments. For a review, see Ref. 7. We use the word "quasi" in order to indicate that the area of the wire is finite. From a theoretical point of view the density of states (DOS) of quasi-one-dimensional electron systems has attracted some attention during recent years.[8–10] Electron-electron interaction effects were *neglected* in Refs. 8–10. In recent calculations[8–10] most attention was given to see how DOS looks like if several subbands are occupied.[8–10] In one-dimensional systems the DOS exhibits a square-root divergence at every subband edge. A finite amount of disorder destroys these square-root singularities of the DOS of the free-electron gas.

Measurements[11–15] of the conductivity versus the electron density of quasi-one-dimensional systems have been performed. It was argued that the scattering rate in one-dimensional systems is increased when a new subband becomes occupied because of the square-root singularity in the DOS. Measurements have been interpreted according to this argument.[13,14] In general, however, the conductivity is not directly expressed in terms of the DOS.

In the derivation of the resistance quantization for the ballistic transport through narrow constrictions[16,17] the DOS of the one-dimensional free-electron gas is used.[16] The renormalization of the density of states due to the finite disorder could give a heuristic argument for the effects of disorder on the sharpness of the resistance steps as found in experiments.[18]

Recently, a model for a quasi-one-dimensional electron gas confined in a cylinder of radius R_0 with infinite barrier height was discussed.[19] Analytical results have been presented for the transport relaxation time. In the following we calculate τ_s and we discuss the effect of τ_s on the DOS of a quasi-one-dimensional electron gas.

The paper is organized as follows. In Sec. II we describe the results for the single-particle relaxation time. The results for the density of states are given in Sec. III. We discuss the theory and the theoretical results in Sec. IV and the experimental results in Sec. V. The conclusion is in Sec. VI.

II. THE SINGLE-PARTICLE RELAXATION TIME

A. Theory

In this paper we calculate the self-energy $\Sigma(\mathbf{q},E)$ of the Green's function $G(\mathbf{q},E)=1/[E-q^2/2m+\Sigma(\mathbf{q},E)]$ for wave vector $\mathbf{q}$ and energy E. Planck's constant $h/2\pi$ is set equal to 1. m is the electron mass. Many-body effects are neglected and we consider only the contribution of the disorder to the self-energy. $G(\mathbf{q},E)$ is the Green's function for electrons in the lowest subband. The energy is measured from the subband energy. Within the model of a cylindrical wire the one-subband approximation is fulfilled for electron density $N<2/R_0$.[19]

In the self-consistent Born approximation $\Sigma(\mathbf{q},E)$ is expressed as[20]

$$\Sigma(\mathbf{k},E)=\sum_{\mathbf{q}}\frac{\langle|U(\mathbf{q})|^2\rangle}{\epsilon(\mathbf{q})^2}G(\mathbf{q}+\mathbf{k},E)\ . \tag{1}$$

The conduction-band edge for the free-electron gas is at energy $E=0$. $\langle|U(\mathbf{q})|^2\rangle$ is the averaged squared random potential and $\epsilon(\mathbf{q})$ is the screening function due to the Coulomb interaction of the electrons. $\epsilon(\mathbf{q})$ was calculated in the random-phase approximation.[21]

In order to get analytical results we calculate the self-energy in mass-shell approximation: $\Sigma(k=k_F,E=E_F)$. E_F is the Fermi energy. For details, see Ref. 3. The single-particle relaxation time is defined as

$$\frac{1}{\tau_{sr}}=2\,\mathrm{Im}[\Sigma(k=k_F,E=E_F)]\ . \tag{2}$$

For the lowest-order calculation we use the symbol τ_s and the r in τ_{sr} is for renormalized, which means that multiple-scattering effects are included. We restrict our calculation to low temperature $T\ll E_F$, see Eq. (10a).

B. General results: Lowest-order theory

For quasi-one-dimensional electron systems we get an analytical result for τ_s because the $\mathbf{q}$ sum in Eq. (1) corresponds to a one-dimensional q integral which can be calculated with the help of $\mathrm{Im}[G^0(\mathbf{q},E)]=\mathrm{Im}[1/(E-\mathbf{q}^2/2m+i0)]$. We find

$$\frac{1}{\tau_s}=\frac{k_F}{2E_F}\left[\frac{\langle|U(2k_F)|^2\rangle}{[\epsilon(2k_F,T)]^2}+\frac{\langle|U(0)|^2\rangle}{[\epsilon(0,0)]^2}\right]\ . \tag{3a}$$

In Eq. (3a) we have introduced the temperature-dependent screening function because of the well-known singularity of $\epsilon(q,T=0)$ for $q=2k_F$.[21] Analytical results for $\epsilon(2k_F,T\ll E_F)$ can be found in Ref. 21, see also Eq. (39) in Ref. 19. In one-dimensional systems forward-scattering [corresponding to momentum transfer $q=0$, the second term on the right-hand side (rhs) of Eq. (3a)] and backscattering [corresponding to momentum transfer $q=2k_F$, the first term on the rhs of Eq. (3a)] are possible. Forward scattering does not contribute to the scattering time:[19]

$$1/\tau_t=k_F\langle|U(2k_F)|^2\rangle/\{E_F[\epsilon(2k_F,T)]^2\}\ .$$

By comparing $1/\tau_s$ in Eq. (3a) with $1/\tau_t$ we get

$$\frac{\tau_t}{\tau_s}=\frac{1}{2}\left[1+\frac{[\epsilon(2k_F,T)]^2}{[\epsilon(0,0)]^2}\frac{\langle|U(0)|^2\rangle}{\langle|U(2k_F)|^2\rangle}\right]\ . \tag{3b}$$

On the one hand, the relation between τ_s and τ_t seems to be quite simple. On the other hand, one could argue that τ_t and τ_s are expressed in terms of Fermi's golden rule: The density of final states and the matrix element for $1/\tau_s$ are the Green's function and $\langle|U(\mathbf{q})|^2\rangle$, respectively. The density of final states and the matrix element for $1/\tau_t$ are the density-density relaxation function and $\mathbf{q}^2\langle|U(\mathbf{q})|^2\rangle$, respectively. The densities of final states and the matrix elements are quite different for τ_s and τ_t.

C. General results: Self-consistent theory

If we use[3] $G(\mathbf{q}+\mathbf{k},E)$ with $\Sigma(\mathbf{q}+\mathbf{k},E)=i/2\tau_{sr}$ on the rhs of Eq. (1) we get for the renormalized single-particle relaxation time in mass-shell approximation

$$1=\frac{2m^2}{\pi}\int_{-\infty}^{\infty}dq\frac{\langle|U(\mathbf{q})|^2\rangle}{\epsilon(\mathbf{q})^2}\times\frac{1}{(2qk_F-q^2)^2+(m/\tau_{sr})^2}\ . \tag{4a}$$

The main contribution to the integral in Eq. (4a) comes from $q\sim 2k_F$ and $q\sim 0$. To evaluate the integral in Eq. (4a) we use an approximation, which was used before for two-dimensional systems and is discussed in Ref. 3, and we find

$$4\pi=\frac{k_F}{E_F^2}\left[\frac{\langle|U(2k_F)|^2\rangle}{\epsilon(2k_F)^2}+\frac{\langle|U(0)|^2\rangle}{\epsilon(0)^2}\right]\times\int_{-\infty}^{\infty}dx\frac{1}{(2x-x^2)^2+(1/2E_F\tau_{sr})^2}\ . \tag{4b}$$

With Eqs. (3a) and (4b) we receive

$$\tau_s=\tau_{sr}\frac{\cos[\arctan(1/2E_F\tau_{sr})/2]}{[1+1/(2E_F\tau_{sr})^2]^{1/4}}\ . \tag{5}$$

For weak disorder ($2E_F\tau_{sr}\gg 1$) we get

$$\tau_{sr}=\tau_s\left[1+\frac{3}{32}\frac{1}{(E_F\tau_s)^2}\right]\ . \tag{6a}$$

For strong disorder ($2E_F\tau_{sr}\ll 1$) we find

$$E_F\tau_{sr}=(E_F\tau_s)^{2/3}\ . \tag{6b}$$

Equation (6a) tells us that the lowest-order Born approximation overestimates the effects of disorder: $\tau_{sr}>\tau_s$. With $\langle|U(q)|^2\rangle\propto N_i$ (N_i is the impurity density) we get for weak disorder: $1/\tau_s\propto N_i$. With Eq. (6b) we find for strong disorder: $1/\tau_{sr}\propto N_i^{2/3}$. This result was found before for a short-range random potential where the Fourier transform of the random potential does not depend on the wave number.[9,10] For a two-dimensional electron gas we derived $1/\tau_{sr}\propto N_i^{1/2}$, see Eq. (34) of Ref. 3.

Equation (5) interpolates between weak and strong disorder. Multiple-scattering effects will decrease the transport relaxation time: $\tau_{\mathrm{tr}}<\tau_t$. Therefore, we expect that $\tau_{\mathrm{tr}}/\tau_{sr}<\tau_t/\tau_s$. A similar result was found for two-dimensional systems.[3] However, we believe that multiple-scattering effects are more important in interacting one-dimensional systems because of the singularity of the screening function for $q=2k_F$.

D. Results for interface-roughness and alloy-disorder scattering

For interface-roughness scattering[19] and alloy-disorder scattering[22] $\langle|U(\mathbf{q})|^2\rangle$ is finite for all wave numbers and $\epsilon(\mathbf{q})$ is diverging for $q=0$. With Eq. (3b) we get

$$\frac{\tau_t}{\tau_s}=\frac{1}{2} . \tag{7}$$

We conclude, that, similar to two-dimensional disordered electron gases with electron density going to zero, the ratio τ_t/τ_s is of order unity for quasi-one-dimensional electron systems in case of interface-roughness scattering or allow-disorder scattering. However, the ratio does not depend on the electron density, which is different for disordered electron systems in higher dimensions.[3]

For alloy-disorder and interface-roughness scattering in a d-dimensional electron gas ($d=1,2,3$) and E_F going to zero one gets $\tau_t/\tau_s=d/(d+1)$. The result for $d=2$ was already found in Ref. 3.

E. Results for charged-impurity scattering

For charged-impurity scattering we assume that impurities are located at a distance R from the center of the cylindrical wire. One gets

$$\langle |U(qR_0\ll 1)|^2\rangle \propto N_i[\ln(qR_0/2)]^2 f(qR)$$

and

$$\epsilon(q\ll 2k_F)\propto \rho_0(E_F)\ln(qR_0/2)$$

(Ref. 19). $f(x)$ is a form factor with $f(0)=1$. N_i is the (one-dimensional) impurity density. $\rho_0(E_F)$ is the DOS of the one-dimensional electron gas if no disorder is present: $\rho_0(E_F>0)=(2mg_v^2/\pi^2E_F)^{1/2}$ and $\rho_0(E_F<0)=0$. g_v is the valley degeneracy. With Eq. (3a) we get

$$\frac{\tau_t}{\tau_s}=\frac{1}{2}+\frac{\pi}{g_v}\frac{N_i}{N}E_F\tau_t . \tag{8a}$$

Detailed results for τ_t have been given in Ref. 19. For $N_i=N$ and $E_F\tau_t\gg 1$ one finds $\tau_t/\tau_s\gg 1$. Equation (8a) can be rewritten as

$$\frac{1}{\tau_s}=\frac{1}{2\tau_t}+\frac{\pi^3}{4g_v^3}[N_iNa^{*2}]R^* . \tag{8b}$$

a^* and R^* are the effective Bohr radius and the effective Rydberg, respectively, defined with the effective mass and the dielectric constant of the background.

At low temperatures $\pi k_BT\ll 2^3e^cE_F$ we can derive analytical results. k_B is the Boltzmann constant and $c=0.577$ is Euler's constant. We get for $2k_FR_0\ll 1$ and $R\ll R_0$

$$\frac{\tau_t}{\tau_s}=\frac{1}{2}+\frac{1}{8}[1-G(2k_F)]^2\ln^2\left[\frac{2^3e^cE_F}{\pi k_BT}\right]>1/2 . \tag{9a}$$

For $2k_FR_0<1$ and $2k_FR\gg 1$ ($R\gg R_0$) we find

$$\frac{\tau_t}{\tau_s}=\frac{1}{2}+\frac{k_FR}{2\pi}\exp[4k_FR\,][1-G(2k_F)]^2\ln^2[k_FR_0]\times\ln^2\left[\frac{2^3e^cE_F}{\pi k_BT}\right]\gg 1 . \tag{9b}$$

$G(q)$ is the local-field correction. In the Hubbard approximation[19] we get for $2k_FR_0\ll 1$

$$G(2k_F)=1/2g_v . \tag{9c}$$

We conclude from Eq. (9) that local-field corrections are very important for $2k_FR_0\ll 1$ and reduce τ_t/τ_s. For $g_v=1$ the reduction factor is 4. In silicon wires with $g_v=2$ the local-field correction is less important: The reduction factor is $\frac{16}{9}$. The conditions $\pi k_BT\ll 2^3e^cE_F$ and $2k_FR_0\ll 1$ correspond to

$$[g_v^2k_BT/2\pi e^cR^*]^{1/2}\ll Na^*\ll g_va^*/\pi R_0 . \tag{10a}$$

Numerical results for τ_t/τ_s versus electron density for GaAs wires ($g_v=1$, $a^*=100$ Å, and $R^*=5.6$ meV) and for charged-impurity scattering are shown in Fig. 1. τ_t/τ_s increases very rapidly with increasing density due to the $q=2k_F$ dependence of the random potential [$\epsilon(2k_F,T=0.02R^*)\approx 1$ for $N>5\times 10^5$ cm^{-1}]. The results shown in Fig. 1 are for $T=0.02R^*=1.3$ K. Condition (10a) transforms for $T=0.02R^*$ and $R_0=a^*=100$ Å to

$$4.2\times 10^4\ll N\ll 3.2\times 10^5 \text{ cm}^{-1} \tag{10b}$$

and the range of validity of Eq. (9) is small. However, for smaller wire radii ($R_0<a^*$) or (and) lower temperatures ($T<0.02R^*$) Eq. (9) is useful.

F. Homogeneous background doping

We assume that charged impurities are distributed homogeneously either inside the wire ($B1$) or outside the wire ($B2$). For homogeneous background doping one finds the following for the random potential $\langle |U(\mathbf{q})|^2\rangle$:[19]

$$\langle |U(\mathbf{q})|^2\rangle=N_{B1(B2)}R_0^2\left[\frac{e^2}{2\epsilon_L}\right]^2F_{B1(B2)}(\mathbf{q}) . \tag{11a}$$

N_{B1} and N_{B2} are the (three-dimensional) background doping densities. ϵ_L is the dielectric constant of the background. $F_{B1}(\mathbf{q})$ and $F_{B2}(\mathbf{q})$ are form factors. For vanish-

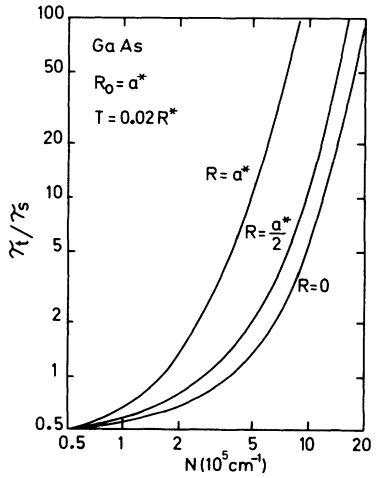

FIG. 1. Ratio of the transport relaxation time τ_t and the single-particle relaxation time τ_s vs electron density N for charged-impurity scattering according to Eq. (8a). The radius of the wire is R_0 and impurities are located at the distance R from the center of the wire.

ing wave numbers we find

$$F_{B1}(qR_0 \ll 1) = 8 \ln^2[qR_0/2] \tag{11b}$$

and

$$F_{B2}(qR_0 \ll 1) = 8/[qR_0]^2 . \tag{11c}$$

For background doping in the wire ($B1$) we derive

$$\frac{\tau_t}{\tau_s} = \frac{1}{2} + \frac{\pi}{2g_v} \frac{N_{B1} R_0^2}{N} E_F \tau_t . \tag{12a}$$

Compare Eq. (12a) with Eq. (8a). We get for $2k_F R_0 \ll 1$

$$\frac{\tau_t}{\tau_s} = \frac{1}{2} + \frac{1}{8}[1 - G(2k_F)]^2 \ln^2 \left[\frac{2^3 e^c E_F}{\pi k_B T} \right] > 1/2 , \tag{12b}$$

which is identical to Eq. (9a). With Eq. (12a) and the results for τ_t given in Ref. 19 the ratio τ_t/τ_s can be calculated.

For ($B2$) the backscattering contribution to the single-particle relaxation time is divergent within the mass-shell approximation because of $F_{B2}(qR_0 \ll 1) \propto 1/q^2$. In $d=2$ a divergence was found for homogeneous background doping in the lowest-order theory. This infrared divergence could be treated within the self-consistent theory in mass-shell approximation.[3] This strategy does not work for ($B2$) in $d=1$ and a more sophisticated theory must be used to get meaningful results.

III. THE DENSITY OF STATES

The DOS $\rho(E)$ at the energy E is given by[20]

$$\rho(E) = -\frac{2}{\pi} \sum_{\mathbf{q}} \mathrm{Im}[G(\mathbf{q}, E + i0)] . \tag{13}$$

The factor 2 in Eq. (13) accounts for the spin degeneracy. With $G(\mathbf{q},E) = 1/(E - \mathbf{q}^2/2m + i/2\tau_{sr})$ we get

$$\rho(E>0) = \rho_0(E) \frac{\cos[\arctan(1/2E\tau_{sr})/2]}{[1 + 1/(2E\tau_{sr})^2]^{1/4}} . \tag{14}$$

The weak-disorder result ($2E\tau_s \gg 1$) is written as

$$\rho(E>0) = \rho_0(E) \left[1 - \frac{3}{32} \frac{1}{(E\tau_s)^2} \right] . \tag{15a}$$

For $E\tau_{sr} = 1$ we get $\rho(E>0)/\rho_0(E) = 0.92$. The strong-disorder result ($2E\tau_{sr} \ll 1$) is expressed as

$$\rho(E>0) = \rho_0(E)[E\tau_{sr}]^{1/2} . \tag{15b}$$

In the strong-disorder regime the DOS does not explicitly depend on the energy [$\rho(E>0) \propto \tau_{sr}^{1/2}$] and this was also found in numerical calculations, see the dashed line in Fig. 1 of Ref. 9.

$\rho(E)/\rho_0(E)$ versus $E\tau_{sr}$ is shown in Fig. 2. For $E\tau_{sr} > 1$ ($E\tau_s > 0.92$) the effects of disorder on the DOS are small and the weak-disorder result Eq. (15a) is in very good agreement with Eq. (14). For $E\tau_{sr} < 0.4$ ($E\tau_s < 0.28$) the strong-disorder result Eq. (15b) is in reasonable agreement with Eq. (14). With Eq. (14) and Eq. (5) we conclude that $\rho(E_F)/\rho_0(E_F) = \tau_s/\tau_{sr}$, see Fig. 2. Our *analytical* results for the DOS are also in qualitative agreement with the *numerical* results of Ref. 9 for the lowest subband.

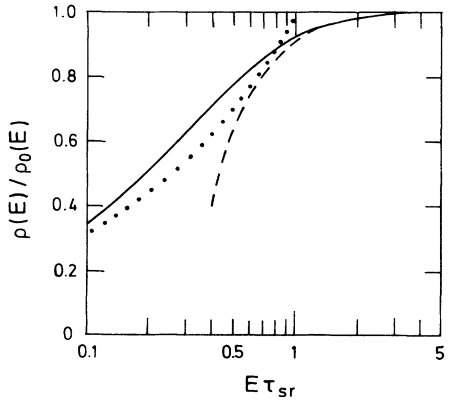

FIG. 2. Density of states $\rho(E)$ in units of the free-electron density of states $\rho_0(E)$ vs energy E in units of τ_{sr}, the renormalized single-particle relaxation time. The solid line represents Eq. (14). The dashed and dotted lines represent the weak-disorder result, Eq. (15a), and the strong-disorder result, Eq. (15b), respectively.

With Eq. (14) we conclude that for $E>0$ the DOS of the disordered system is smaller than the DOS of the clean system. This effect is explained as follows. For the free-electron gas the conduction edge E_C is at $E_C = 0$. Due to disorder the conduction-band edge is shifted to lower energies: $E_C < 0$. Therefore, for $E>0$ the density of states of the disordered system is smaller than for the clean system. A similar argument was given for two-dimensional systems.[3]

For two-dimensional systems we found for $2E\tau_s \gg 1$: $\rho(E) = \rho_0[1 - 1/(2\pi E\tau_s)]$, see Eq. (49) in Ref. 3. $\rho_0 = m/\pi$ is the DOS of the free-electron gas in two dimensions. We conclude that the effects of disorder on the DOS are larger for two-dimensional systems than for one-dimensional systems, see Eq. (15a).

IV. DISCUSSION: THEORY AND RESULTS

A. Theory

In this paper we have chosen the strategy to calculate the DOS within a simple approach but with a realistic model for the disorder and the interaction potential. Our calculations have been performed in the self-consistent Born approximation. The additional approximation, which was performed by going from Eq. (4a) to Eq. (4b), is justified by deriving the correct weak-disorder result and the correct strong-disorder result, as discussed in Sec. II C. Impurity bands cannot be described within the Born approximation. Therefore, we think that our calculation provides a good estimate of the density of states if the Fermi energy is located in the conduction band. In the calculation of the self-energy and the DOS we used the mass-shell approximation in order to get analytical results. A more complete theory, the self-consistent t-matrix approximation, will be described in Ref. 23.

Multiple-scattering effects were calculated by neglect-

ing the effects of disorder on the dielectric function. A complete self-consistent calculation is difficult because the knowledge of the effects of disorder on the dielectric function implies that the localization problem has been solved for an *interacting* electron gas. However, the localization problem for interacting electrons is not yet solved.[24] Therefore, we think that our result given in Eq. (5) only represents a first step to the full solution.

B. Results: Single-particle relaxation time

In our paper we presented results for the single-particle relaxation time in quasi-one-dimensional systems. We compared the single-particle relaxation time with the transport time, calculated in lowest-order Born approximation. It is well known that the transport time is sensitive to "weak-localization" effects, which are stronger in one than in two dimensions.[24] Therefore, it might turn out that the measured transport time is smaller than the transport time calculated in Born approximation, see our discussion in Ref. 19. However, the single-particle relaxation time, which determines the density of states, is insensitive to "weak-localization" effects. It remains to be seen whether the arguments on "weak localization" really can account for the transport properties of interacting electrons in wires. Many more experiments are needed before this question can be answered.

We do not discuss the implications of the localization theory in this paper because we are considering the single-particle relaxation time. One argument should be given, however: Our theory on τ_s is expected to be valid for all temperatures $T \ll E_F$ and for E_F in the band-tail regime. The band-tail regime can only be estimated from a more complete theory, which can describe impurity bands.[23]

Between the localized regime and the ballistic regime a parameter space should exist where diffusive transport is possible. In this regime the transport relaxation time τ_t, as calculated in Ref. 19, should have a physical meaning. Due to the lack of a complete localization theory for interacting electrons this parameter range cannot presently be specified. It remains to see more experimental results to get an idea where the different, however incomplete, theories, can be applied. Nevertheless, we believe that our analytical results are helpful for the experimentalists to determine the parameter range of the different regimes.

An effect of a structure factor $S(q)$ for the impurities has recently been discussed in Ref. 25 for two-dimensional systems. We assumed that the impurities are distributed randomly: $S(q)=1$. However, a structure factor could easily be introduced in our theory by replacing N_i by $N_i S(q)$.

V. DISCUSSION: EXPERIMENTS

A. Diffusive transport

Subband-energy distances in quasi-one-dimensional structures have been estimated from measurements of the conductivity versus the electron density.[11–15] It was argued that when a new subband becomes occupied, the scattering rate is enhanced due to the large DOS at the subband edge. Our calculation provides a better understanding of this heuristic argument. In conductivity measurements the density-density relaxation function is probed (via the density of final states). Because of $\tau_t=\tau_s/2$ for interface-roughness scattering and alloy-disorder scattering one could argue that the density of states is probed. However, a more complete analysis must include intersubband scattering.

We calculated the single-particle relaxation time and the density of states in the lowest subband of quasi-one-dimensional electron systems. If intersubband scattering and screening effects are neglected the results for the τ_t/τ_s and the DOS also hold for the second, third, etc. subband with corresponding shifts of the subband edges. In recent experiments[26–28] on Shubnikov–de Haas oscillations in $Al_xGa_{1-x}As/GaAs$ heterostructure wires with many occupied subbands it was found that $\tau_t/\tau_s \gg 1$. The experimental results presented in Refs. 26 and 28 indicate that the scattering mechanism in the $Al_xGa_{1-x}As/GaAs$ heterostructure wires is due to charged-impurity scattering. It was also noted that the ratio τ_t/τ_s in the most narrow structure was higher than in the wider structures. From our analysis we conclude that the additional confinement by going from two-dimensional systems to quasi-one-dimensional systems should increase the ratio τ_t/τ_s in case of charged-impurity scattering: (i) $\tau_t/\tau_s=(2k_F\alpha)^2$ for $d=2$ and $2k_F\alpha \gg 1$ and (ii) $\tau_t/\tau_s \propto (k_F R)\exp(4k_F R)$ for $d=1$ and $2k_F R \gg 1$ (with $2k_F R_0 \ll 1$). However, we would like to note that for a real understanding of the magnetotransport experiments a magnetotransport theory is necessary. Such a theory was not the topic of this paper.

Interface-roughness scattering[19] was discussed to describe experimental results in $Al_xGa_{1-x}As/GaAs$ quantum wires.[29] Alloy-disorder scattering[22] is the dominant scattering mechanism in wires made from $In_{0.53}Ga_{0.47}As/InP$.[30] Experimental results on the single-particle relaxation time in these structures are highly desirable in order to test the importance of interface-roughness scattering and alloy-disorder scattering.

B. Ballistic transport

The structures, where the resistance quantization for ballistic transport in constricted geometries was found in experiment,[16,17] were made from $Al_xGa_{1-x}As/GaAs$ heterostructures. In these structures $\tau_t/\tau_s \gg 1$ due to remote doping or homogeneous background doping.[6] It was argued in Ref. 16 that the large mean free path l_t is the reason for observing the steps. We argue that the necessary condition for observing the steps is

$$l_s=k_F\tau_s/m \gg L_c \ . \tag{16}$$

L_c is the length of the constriction. Resistance steps occur when a new subband becomes occupied. The Fermi energy in the new subband is then very small. Therefore, the renormalized single-particle relaxation time could be important: $l_{sr}=k_F\tau_{sr}/m$. The mean free path l_i

for inelastic collisions must also be large: $l_i \gg l_s$. Of course, Eq. (16) represents only a heuristic argument.

Experimentally it was observed that the sharpness of the resistance steps is not strongly correlated with the mobility values of the samples.[31] This could be an indication of the importance of the single-particle relaxation time. The single-particle relaxation time for two dimensions was discussed in connection with the resistance quantization in Ref. 32. However, the two-dimensional single-particle relaxation time was used.

In the two-dimensional electron gas in silicon metal-oxide-semiconductor structures one finds $\tau_t/\tau_s < 2$.[33] We suggest that if τ_t is the relevant parameter to observe the steps then one will not find these steps in presently available silicon structures. If τ_s is the relevant parameter then it should be possible to observe the resistance quantization also in constrictions of silicon material.

VI. CONCLUSION

A disordered *interacting* quasi-one-dimensional electron gas was studied in this paper. The aim of the paper was to present *analytical results* for the single-particle relaxation time τ_{sr} and the density of states $\rho(E)$. For weak disorder we derived $\tau_{sr} \approx \tau_s$ and we found for interface-roughness scattering and alloy-disorder scattering $\tau_t = \tau_s/2$. For charged-impurity scattering we got $\tau_t \gg \tau_s$ for $E_F \tau_t \gg 1$. Multiple-scattering effects for strong disorder have been calculated. We discussed analytical results for τ_{sr} [Eq. (5)] and $\rho(E)$ [Eq. (14)] which interpolate between weak and strong disorder.

Present experiments[11–15,26–30] are performed on systems where many subbands are occupied. We hope that our analytical results will be useful for the yet to be realized (experimental) situation that the diffusive transport properties of a one-subband system are studied. Recent experimental results[16–18,32,34,35] suggest that experimental studies of one-subband systems are possible.

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[1] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[2] J. P. Harrang, R. J. Higgins, R. K. Goodall, P. R. Jay, M. Laviron, and P. Delescluse, Phys. Rev. B **32**, 8126 (1985); S. Das Sarma and F. Stern, *ibid.* **32**, 8442 (1985).
[3] A. Gold, Phys. Rev. B **38**, 10 798 (1988).
[4] M. A. Paalanen, D. C. Tsui, and J. C. M. Hwang, Phys. Rev. Lett. **51**, 2226 (1983).
[5] U. Bockelmann, G. Abstreiter, G. Weimann, and W. Schlapp, Phys. Rev. B **41**, 7864 (1990); S. Ben Amor, L. Dmowski, J. C. Portal, K. P. Martin, R. J. Higgins, and M. Razeghi, Appl. Phys. Lett. **57**, 2925 (1990).
[6] A. Gold, in *Condensed Systems of Low Dimensionality,* Vol. 253 of *NATO Advanced Study Institute, Series B: Physics,* edited by J. L. Bebby (Plenum, New York, 1991), p. 283.
[7] J. Kotthaus, in *Interfaces, Quantum Wells, and Superlattices,* Vol. 179 of *NATO Advanced Study Institute, Series B: Physics,* edited by C. L. Leavens and R. Taylor (Plenum, New York, 1988), p. 95; W. Hansen, in *Quantum Coherence in Mesoscopic Systems,* Vol. 254 of *NATO Advanced Study Institute, Series B: Physics,* edited by B. Kramer (Plenum, New York, 1991), p. 23.
[8] M. J. Kearney and P. N. Butcher, J. Phys. C **20**, 47 (1987).
[9] S. Das Sarma and X. C. Xie, Phys. Rev. B **35**, 9875 (1987).
[10] P. Vasilopoulos and F. M. Peeters, Phys. Rev. B **40**, 10 079 (1989).
[11] A. C. Warren, D. A. Antoniadis, and H. I. Smith, Phys. Rev. Lett. **56**, 1858 (1986); K. F. Berggren, T. F. Thornton, D. J. Newson, and M. Pepper, *ibid.* **57**, 1769 (1986).
[12] F. Brinkop, W. Hansen, J. P. Kotthaus, and K. Ploog, Phys. Rev. B **37**, 6547 (1988).
[13] K. Ismail, D. A. Antoniadis, and H. I. Smith, Appl. Phys. Lett. **54**, 1130 (1989).
[14] J. R. Gao, C. de Graaf, J. Caro, S. Radelaar, M. Offenberg, M. Lauer, J. Singleton, T. J. Janssen, and J. A. Perenboom, Phys. Rev. B **41**, 12 315 (1990).
[15] F. Hirter, J. Smoliner, E. Gornik, G. Weimann, and W. Schlapp, Appl. Phys. Lett. **57**, 261 (1990).
[16] D. A. Wharam, J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L209 (1988); M. Pepper, C. G. Smith, R. J. Brown, D. A. Wharam, M. J. Kelly, R. Newbury, H. Ahmed, D. G. Hasko, D. C. Peacock, J. E. F. Frost, D. A. Ritchie, and G. A. C. Jones, Semicond. Sci. Technol. **5**, 1185 (1990).
[17] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).
[18] J. Faist, P. Guéret, and H. Rothuizen, Phys. Rev. B **42**, 3217 (1990).
[19] A. Gold and A. Ghazali, Phys. Rev. B **41**, 7626 (1990).
[20] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963), Sec. 39.
[21] M. J. Rice, in *Low-Dimensional Cooperative Phenomena,* edited by H. J. Keller (Plenum, New York, 1975), p. 23.
[22] P. K. Basu and C. K. Sarkar, Surf. Sci. **174**, 454 (1986); A. Gold and A. Ghazali (unpublished).
[23] A. Ghazali, A. Gold, and J. Serre (unpublished).
[24] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 2897 (1985).
[25] P. T. Coleridge, Phys. Rev. B **44**, 3793 (1991).
[26] A. D. C. Grassie, K. M. Hutchings, M. Lakrimi, C. T. Foxon, and J. J. Harris, Phys. Rev. B **36**, 4551 (1987); M. Lakrimi, A. D. C. Grassie, K. M. Hutchings, J. J. Harris, and C. T. Foxon, Semicond. Sci. Technol. **4**, 313 (1989).
[27] J. P. Bird, A. D. C. Grassie, M. Lakrimi, K. M. Hutchings, P. Meeson, J. J. Harris, and C. T. Foxon, J. Phys. Condens. Matter **3**, 2897 (1991).
[28] K. Ishibashi, Y. Aoyagi, S. Nambu, Y. Ochiai, M. Kawabe, and K. Gamo, Superlatt. Microstruct. **9**, 457 (1991).
[29] T. J. Thorton, L. M. Roukes, A. Scherer, and B. P. van de Gaag, Phys. Rev. Lett. **63**, 2128 (1989).
[30] A. Menschig, A. Forchel, B. Roos, R. Germann, K. Pressel,

W. Heuring, and D. Grützmacher, Appl. Phys. Lett. **57**, 1757 (1990).
[31]D. A. Wharam (private communication).
[32]G. Timp, R. E. Behringer, and J. E. Cunningham, Phys. Rev. B **42**, 9259 (1990).
[33]F. F. Fang, A. B. Fowler, and A. Hartstein, Phys. Rev. B **16**, 4446 (1977).
[34]S. B. Field, M. A. Kastner, U. Meirav, J. H. F. Scott-Thomas, D. A. Antoniadis, H. I. Smith, and S. J. Wind, Phys. Rev. B **42**, 3523 (1990).
[35]A. R. Goñi, A. Pinczuk, J. S. Weiner, J. M. Calleja, B. S. Dennis, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **67**, 3298 (1991).