Static dielectric susceptibility of the lowest Landau level

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(Received 8 January 1992)

The static dielectric susceptibility of a two-dimensional electron plasma in a strong perpendicular magnetic field is obtained, including the electron-electron interactions with use of a Green's-function formalism at finite temperature. Below a critical temperature, the susceptibility ceases to be negative definite, having poles related to a second-order transition into charge-density-wave states. Below yet another critical temperature, the negative sign is restored for some wavelengths and for some filling factors that correspond to unstable homogeneous states. The dielectric constant, as well as the thermodynamic density of states, can take on negative values.

I. INTRODUCTION

It is known that, due to the electron-electron interaction, the two-dimensional electron plasma in a strong perpendicular magnetic field undergoes a transition to a charge-density-wave (CDW) state.[1-4] At extremely low temperatures and for low filling fractions of the lowest Landau level, the inhomogeneous states become a Wigner solid.[5]

The stability criterion of the homogeneous states against the spontaneous appearance of a CDW with wave vector q is given by the inequalities

$$\epsilon(q)\geq 1 \tag{1.1a}$$

or

$$\epsilon(q)<0 \tag{1.1b}$$

satisfied by the static dielectric function.[6]

The random-phase approximation (RPA) for $\epsilon(q)$ of the two-dimensional electron gas in a perpendicular magnetic field had been considered in several papers.[7-13] For any temperature T and wave vector q, the inequality (1.1a) is fulfilled so that the CDW instabilities are absent in this approximation. This conclusion remains true even if the nonlinear components of the dielectric response are taken into account.[11] When the highest Landau level is only partially occupied, the RPA for the static dielectric function diverges like $1/T$ as $T\rightarrow 0$, yielding perfect screening ($\epsilon=\infty$) for any q; for finite T the screening becomes perfect only for $q\rightarrow 0$. When the highest level involved is totally occupied, long-wavelength ($q\rightarrow 0$) screening is absent ($\epsilon=1$). These properties are extensively discussed in Refs. 7 and 12.

Experimental measurements on high-mobility silicon (MOSFET's) (Refs. 14 and 15) show a negative thermodynamic density of states $\partial\langle\rho\rangle/\partial\mu$, $\langle\rho\rangle$ being the average electron density and μ the chemical potential, as another manifestation of strong electronic correlations at low temperatures (but higher than those involved in the fractional quantum Hall effect). In terms of the dielectric response, since in the long-wavelength limit[7,12,13,16]

$$\epsilon(q)=1+\frac{2\pi}{q}\frac{\partial\langle\rho\rangle}{\partial\mu}\ , \tag{1.2}$$

this means a negative dielectric constant[16] ("overscreening"), according to inequality (1.1b).[6]

The present paper is an attempt to extend the calculation of the dielectric response to include the above-mentioned effects of the electron-electron interactions. The starting point is the Hartree-Fock approximation (HFA) for the temperature-dependent Green's function.[17,18] The RPA for the dielectric susceptibility may be regained by neglecting the exchange term in the self-energy, i.e., by considering it in the Hartree approximation.[17] However, due to the very high degeneracy of the Landau level, this term yields drastic deviations from the RPA; this approximation then leads to CDW instabilities and overscreening. In Sec. II, after a brief review of the dielectric response functions of noninteracting electrons, the desired approximation is obtained. Section III is devoted to the dielectric function and the conclusions are collected in Sec. IV.

II. DIELECTRIC SUSCEPTIBILITY

A. Noninteracting electrons

In this subsection the response functions of noninteracting electrons are briefly reviewed in order to increase the self-consistency of the paper and to establish the main notations.

The two-dimensional electron system (neutralized by a uniform and positively charged background) will be considered under the action of an external electrostatic potential,

$$V_{\rm ext}(\mathbf{r})=V\cos(\mathbf{q}\cdot\mathbf{r})\ , \tag{2.1}$$

where $\mathbf{r}=(x,y)$.

Throughout this paper, magnetic units are used: the cyclotronic-resonance energy $\hbar\omega_c=\hbar eB/mc=1$ and the magnetic length $l_B=(\hbar c/eB)^{1/2}=1$. Moreover, the electronic charge e and Boltzmann's constant are also taken equal to unity.

The magnetic field will be described in the Landau gauge, i.e., the vector potential will be $\mathbf{A}=(0,x)$. The directions of the coordinate axes may be chosen such that the electrostatic potential (2.1) varies only along the x axis, as long as boundary effects are neglected in the thermodynamic limit. Therefore in the following calculations we will assume the presence of $\cos(qx)$ in Eq. (2.1), even if not explicitly mentioned.

In the absence of the external potential (2.1) the wave functions of the free electrons in the lowest Landau level are oscillatorlike:

$$f_k(\mathbf{r})=L_y^{-1/2}\pi^{-1/4}e^{iky}e^{-(x-k)^2/2} . \tag{2.2}$$

The corresponding energy will be taken as the origin of the energy scale. The system is assumed to be enclosed in a large rectangular box, i.e., $|x|<L_x/2$, $|y|<L_y/2$, these giving $|k|<L_x/2$, and $k=2\pi/L_y\times$(integer) (from periodic boundary conditions along the y-axis direction).

Taking into account the external potential (2.1), the new single-particle states are described by the wave functions $\varphi_k(\mathbf{r})$ which may be found using, e.g., the perturbation theory with respect to V_{ext}. The energies constitute an energy band, E_k. Starting from the density of particles (all in the lowest Landau level),

$$\rho(\mathbf{r})=\sum_k |\varphi_k(\mathbf{r})|^2\mathcal{F}((E_k-\mu)/T) , \tag{2.3}$$

$\mathcal{F}(x)=(e^x+1)^{-1}$ being the Fermi function, one can find the isothermal density-response function:[13]

$$\chi^{(0)}\cos(\mathbf{q}\cdot\mathbf{r})=\left.\frac{\partial\rho(\mathbf{r})}{\partial V}\right|_{V=0} , \tag{2.4}$$

the label (0) denoting the absence of the electron-electron interaction. It has the form

$$\chi^{(0)}=\nu\chi_1^{(0)}+\chi_2^{(0)} , \tag{2.5}$$

where the temperature is taken to be sufficiently low so that the Fermi function in (2.3) reduces to the filling factor ν when the external perturbation vanishes.

The first term of Eq. (2.5) comes from the action of the electrostatic potential on the wave functions and has the expression[7,9,10,13]

$$\chi_1^{(0)}(q)=-\frac{q^2}{2\pi}e^{-q^2/2}\sum_{n\geq 1}\frac{1}{n!\,n}\left[\frac{q^2}{2}\right]^{n-1} , \tag{2.6}$$

which is the susceptibility of the lowest Landau level totally occupied ($\nu=1$). The second term of Eq. (2.5) is due to the influence of the external potential on the occupation numbers of the single-particle states in the energy band E_k and is given by[7,11,13]

$$\chi_2^{(0)}(q)=-\frac{\nu(1-\nu)}{2\pi T}e^{-q^2/2} . \tag{2.7}$$

Equations (2.5)–(2.7) yield the random-phase approximation of the dielectric function, i.e.,

$$\epsilon(q)\equiv\left[1+\frac{2\pi}{q}\chi(q)\right]^{-1}\approx 1-\frac{2\pi}{q}\chi^{(0)}(q) , \tag{2.8}$$

which diverges for $T\to 0$ and satisfies (1.1a). This means that the CDW instabilities are absent in the RPA.[11]

B. Hartree-Fock approximation

Taking into account the electron-electron interaction $U(\mathbf{r})=1/r$, the many-body system may be described with the temperature-dependent Green's function.[17–19] In the HFA it has the form

$$\mathcal{G}_\sigma^{(\text{HFA})}(\mathbf{r},\mathbf{r}';\omega_j)=\sum_{n,k}\frac{\psi_{nk}(\mathbf{r})\psi_{nk}^*(\mathbf{r}')}{i\omega_j-(\varepsilon_{nk,\sigma}-\mu)} , \tag{2.9}$$

where σ is the spin index, n denotes the Landau band, $\omega_j=(2j+1)\pi T$ (j integer); $\varepsilon_{nk,\sigma}$ together with $\psi_{nk}(\mathbf{r})$ are the single-particle energies and wave functions given by the T-dependent Hartree-Fock equations. They give the lowest band ($n=0$ and $\sigma=+1$) as

$$\varepsilon_k=\langle\psi_k|V_{\text{ext}}|\psi_k\rangle+\Sigma_k , \tag{2.10}$$

with the self energy

$$\begin{aligned}\Sigma_k&=\sum_{k'}\mathcal{F}_{k'}\langle\psi_k\psi_{k'}|U|\psi_k\psi_{k'}\rangle-\sum_{k'}\mathcal{F}_{k'}\langle\psi_k\psi_{k'}|U|\psi_{k'}\psi_k\rangle\\&\equiv T_k^D+T_k^X ,\end{aligned} \tag{2.11}$$

where T_k^D and T_k^X denote the direct and the exchange contributions and $\mathcal{F}_{k'}\equiv\mathcal{F}((\varepsilon_{k'}-\mu)/T)$ (all the electrons are again assumed to be in the lowest band).

Using (2.9) the particle density may be written as in Eq. (2.3) with the Hartree-Fock wave functions and energies replacing the noninteracting ones. The interacting density-response function can now be calculated just like in Eq. (2.4). One gets[20]

$$\begin{aligned}\chi(q)\cos(\mathbf{q}\cdot\mathbf{r})=&\nu\left[\frac{\partial}{\partial V}\sum_k|\psi_k(\mathbf{r})|^2\right]_{V=0}\\&-\frac{\nu(1-\nu)}{T}\sum_k|f_k(\mathbf{r})|^2\left[\frac{\partial\varepsilon_k}{\partial V}-\frac{\partial\mu}{\partial V}\right]_{V=0} .\end{aligned} \tag{2.12}$$

Let us focus on the first term of Eq. (2.12). If the electron-electron interaction is ignored in this term, which means $\psi_k(\mathbf{r})=\varphi_k(\mathbf{r})$, then it becomes [see Eq. (2.5)]

$$\nu\chi_1^{(0)}(q)\cos(\mathbf{q}\cdot\mathbf{r}) .$$

If only the dependence of the Hartree-Fock wave functions on ν is ignored then it may be written as

$$\nu(\chi(q))_{\nu=1}\cos(\mathbf{q}\cdot\mathbf{r}) . \tag{2.13}$$

Assuming now that for a totally occupied Landau level the electron-electron interactions have no dramatic effect, these will be taken into account in the first term of Eq. (2.12) through the random-phase approximation [Eq. (2.8)],

$$\chi(q)|_{\nu=1}\approx\chi^{(\text{RPA})}(q)|_{\nu=1}=\chi_1^{(\text{RPA})}(q) \tag{2.14}$$

and, as will be seen later, this term will behave like a small correction in the susceptibility $\chi(q)$ unless for $\nu\approx 1$.

Now we shall proceed with the evaluation of the second term of Eq. (2.12). To first order in V, the Hartree-Fock energies (2.10) must have the form

$$\varepsilon_k = V\eta(q)\cos(qk) , \tag{2.15}$$

where $\eta(q)$ will be determined later.

For the moment we can see that

$$\left.\frac{\partial\mu}{\partial V}\right|_{V=0} = 0 \tag{2.16}$$

in Eq. (2.12); taking the derivative of $\nu \sim \sum_k \mathcal{F}((\varepsilon_k-\mu)/T)$ and using Eq. (2.15) one gets $(\partial\mu/\partial V)_{V=0} \sim L_x^{-1}\int dk\cos(qk)$ and then Eq. (2.16) in the thermodynamic limit ($2\pi L_y^{-1}\sum_k \to \int dk$ has been used).

C. From HFA to RPA and beyond

To first order in V, the matrix elements of the electrostatic potential in Eq. (2.10) are[20]

$$V\cos(qk)e^{-q^2/4} . \tag{2.17}$$

The direct term in the self-energy (2.11) may be written as

$$T_k^D = \int d\mathbf{r}\, d\mathbf{r}'\rho(\mathbf{r})U(\mathbf{r}-\mathbf{r}')|\psi_k(\mathbf{r}')|^2 \tag{2.18}$$

and can be expressed with the help of the susceptibility $\chi(q)$ replacing $\rho(\mathbf{r}')$ by $\delta\rho(\mathbf{r}')\equiv\rho(\mathbf{r}')-\langle\rho\rangle = \chi(q)V\cos(\mathbf{q}\cdot\mathbf{r})$, with V assumed to be small. This is possible because the average density $\langle\rho\rangle$ gives no contribution in Eq. (2.18) since the electrical neutrality condition can be taken into account as $U(\mathbf{q}=0)=0$. Then using Eq. (2.2) we get, to leading order in V,[20,21]

$$T_k^D = \frac{2\pi}{q}\chi(q)V\cos(qk)e^{-q^2/4} . \tag{2.19}$$

Before starting to calculate the exchange term in the self-energy, suppose we neglect it. Then, the Hartree-Fock energies (2.10) become, in the linear approximation in V,

$$\varepsilon_k = V\cos(qk)e^{-q^2/4}\left[1+\frac{2\pi}{q}\chi(q)\right] . \tag{2.20}$$

Combining now Eq. (2.12) with Eq. (2.14) and with Eqs. (2.16), (2.20), and (2.2) we obtain

$$\chi(q) = \nu\chi_1^{(\mathrm{RPA})}(q) - \frac{\nu(1-\nu)}{2\pi T}e^{-q^2/2}\left[1+\frac{2\pi}{q}\chi(q)\right] . \tag{2.21}$$

For low enough temperatures and for $\nu\neq 1$, $\chi_1^{(\mathrm{RPA})}(q)$ may be neglected in Eq. (2.21), which leads to the random-phase approximation of the susceptibility $\chi(q)$, given also by Eq. (2.8).

A question then arises as to whether the RPA can be retrieved in the same way, in the absence of the magnetic field. The answer is negative. In this case, the unperturbed Hartree-Fock wave functions are plane waves, $f_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}/(L_xL_y)^{1/2}$, $\mathbf{k}=(k_x,k_y)$. The corresponding energies, $\varepsilon_{\mathbf{k}} = \hbar^2k^2/2m + \Sigma_{\mathbf{k}}$, may be represented in a Brillouin-zone scheme by repeated translations with arbitrary q along k_x. Since all the matrix elements of the external potential $V_{\mathrm{ext}}(\mathbf{r}) = V\cos(qx)$ vanish except the off-diagonal (in k_x) ones $(V_{\mathrm{ext}})_{\pm q/2,\mp q/2}$, the only effect of the perturbation, to first order in V, is a small splitting at the crossing of the nearest-neighboring energy branches, thereby opening an energy gap. This means the perturbation gives $(\partial\varepsilon_{\mathbf{k}}/\partial V)_{V=0}=0$ for any $\mathbf{k}$, except along the lines $\mathbf{k}_\pm = (\pm q/2, k_y)$, which is a set of measure zero. Consequently the second term of Eq. (2.12) vanishes in the thermodynamic limit.

A natural step to extend the calculation of the dielectric susceptibility of $2D$ electron plasma under a magnetic field would therefore be to take into account the exchange term in Eqs. (2.10)–(2.12). In order to avoid explicit calculations of the Hartree-Fock wave functions $\psi_k(\mathbf{r})$, we shall approximate them in T_k^X with $\varphi_k(\mathbf{r})$, i.e., the wave functions for the noninteracting problem with the external potential (2.1). In fact, this should be the zeroth-order approximation for the Hartree-Fock wave functions in an iterative scheme of solving the Hartree-Fock equations (2.10) and (2.11). Note that as $V\to 0$, both $\psi_k(\mathbf{r})$ and $\varphi_k(\mathbf{r})$ tend to $f_k(\mathbf{r})$.[20]

For small V, the functions $\varphi_k(\mathbf{r})$ may be expanded in standard perturbation theory. However, since this would imply the manipulation of infinite series, this wave function will be used in a simpler, variational, oscillatorlike form:[22]

$$\varphi_k(\mathbf{r}) = \frac{e^{iky}}{L_y^{1/2}}\left[\frac{\omega}{\pi}\right]^{1/4}\exp[-\omega(x-\xi)^2/2] . \tag{2.22}$$

The variational parameters $\omega=\omega(k)$ and $\xi=\xi(k)$ must satisfy the system of nonlinear equations

$$\begin{aligned} \omega^2 &= 1 - q^2V\cos(q\xi)\exp(-q^2/4\omega) , \\ \xi &= 1 + qV\sin(q\xi)\exp(-q^2/4\omega) . \end{aligned} \tag{2.23}$$

The exchange-term contribution is in fact given by its derivative with respect to V included in $(\partial\varepsilon_k/\partial V)_{V=0}$ in Eq. (2.12). This contribution is evaluated in the Appendix [Eq. (A4)] and depends on the Hartree-Fock energies through $\eta(q)$, which can now be calculated from the equation obtained by identifying the derivative of Eq. (2.10) [see (2.17), (2.11), (2.19), and (A4)], i.e.,

$$\left.\frac{\partial\varepsilon_k}{\partial V}\right|_{V=0} = \cos(qk)e^{-q^2/4}\left[1+\frac{2\pi}{q}\chi(q)\right] + \left.\frac{\partial T_k^X}{\partial V}\right|_{V=0} , \tag{2.24}$$

with the derivative of Eq. (2.15). Taking the Fourier transform with respect to $\cos(qk)$ one obtains $\eta(q)$ as a linear function of $\chi(q)$. Now, introducing Eq. (2.15) in Eq. (2.12), together with Eq. (2.16), a new closed-form equation is obtained for the susceptibility $\chi(q)$ [compared to Eq. (2.21)]. Using (2.14) in the first term of Eq. (2.12), the susceptibility is obtained as

$$\chi(q)=\frac{\nu\chi_1^{(\mathrm{RPA})}(q)\left[1+(2\pi^3)^{1/2}\chi_2^{(0)}(q)e^{q^2/4}I_0\left[\frac{q^2}{4}\right]\right]+\chi_2^{(0)}(q)\left[1+\frac{\nu}{\pi}q^2\zeta(q)\right]}{1-\chi_2^{(0)}(q)\left[\frac{2\pi}{q}-(2\pi^3)^{1/2}e^{q^2/4}I_0\left[\frac{q^2}{4}\right]\right]} \tag{2.25}$$

the functions ζ and I_0 being defined in the Appendix.

This approximation is identical with the RPA only for $q \ll 1$, when all the terms introduced by the exchange integral, i.e., those $\sim\zeta$ and $\sim I_0$, as well as $\chi_1^{(0)}(q)$, can be neglected. Both approximations give then $\chi(q)\approx -q/2\pi$. If in the exchange term, Eq. (2.11), the wave functions (2.2) of unperturbed electrons would be considered instead of those of noninteracting electrons, Eqs. (2.22) and (2.23), the term $\sim\zeta$ would be absent.

The susceptibility (2.25) has poles as well as changes of sign. The poles are given by the (at most) two roots of the second-degree polynomial in ν in the denominator [see Eq. (2.7)].

For any $T > T_c \approx 0.139$ (in units of $\hbar\omega_c$) the susceptibility (2.25) has no singularity and is always negative. For $T=T_c$, a single pole, $\chi\to-\infty$, appears at $\nu=\frac{1}{2}$ and $q=q_0\approx 1.57$, showing the onset of CDW instabilities. For lower T, an instability domain of positive dielectric susceptibility [see (1.1) and (2.8)] grows between the two roots $\nu_1(q)<\nu_2(q)$ as shown in Fig. 1(a). The change of sign is made discontinuously, from $-\infty$ to $+\infty$, which means the CDW will appear even for vanishing external potential. Accordingly, these boundary curves obey the electron-hole symmetry condition,

$$\nu_1(q)+\nu_2(q)=1\ , \tag{2.26}$$

imposed by the many-body Hamiltonian of the electrons in the lowest Landau level without external field,

$$\mathcal{H}=\sum_{i\neq j}U(r_i-r_j)\ . \tag{2.27}$$

For any fixed temperature the extreme critical filling factors, i.e., the minimum of $\nu_1(q)$ and the maximum of $\nu_2(q)$ are the roots of the equation

$$1+\frac{\nu(1-\nu)}{2\pi T}\left\{e^{-q_0^2/2}\left[\frac{2\pi}{q_0}-(2\pi^3)^{1/2}e^{q_0^2/4}I_0\left[\frac{q_0^2}{4}\right]\right]\right\}=0\ , \tag{2.28}$$

where $q_0\approx 1.57$ corresponds to the minimum of the expression contained in the curly brackets. This may be interpreted as the equation of the critical temperature $T=T(\nu)$ of the second-order spontaneous CDW transition discussed in Refs. 1 and 3.

For $T<T_c'\approx 0.0754$, the sign of the susceptibility (2.25) is no longer determined only by its denominator. Due to the term $\sim\chi_1^{(\mathrm{RPA})}(q)$, an "island" with $\chi<0$ develops inside the instability region with $\chi>0$, Fig. 1(b), and it becomes wider for lower T. This boundary is crossed continuously by the susceptibility and encloses another domain of stability of the electron system modulated by a weak external electrostatic potential. The electron-hole symmetry no longer holds, the relevant Hamiltonian being now the full one, i.e., the kinetic-energy term [generating the higher Landau levels necessary to describe, e.g., $\chi_1^{(0)}$] plus (2.27) plus (2.1). If V_{ext} would vanish a spontaneous CDW should also appear for any filling factor contained in the smaller closed curve of Fig. 1(b), as shown in Refs. 1 and 3. However, under the influence of an external potential having a wave vector corresponding to this domain and an amplitude V large enough, the spontaneous CDW can be destroyed, with the external modulation imposed on the particle density. Then V may be lowered until the linear regime (with

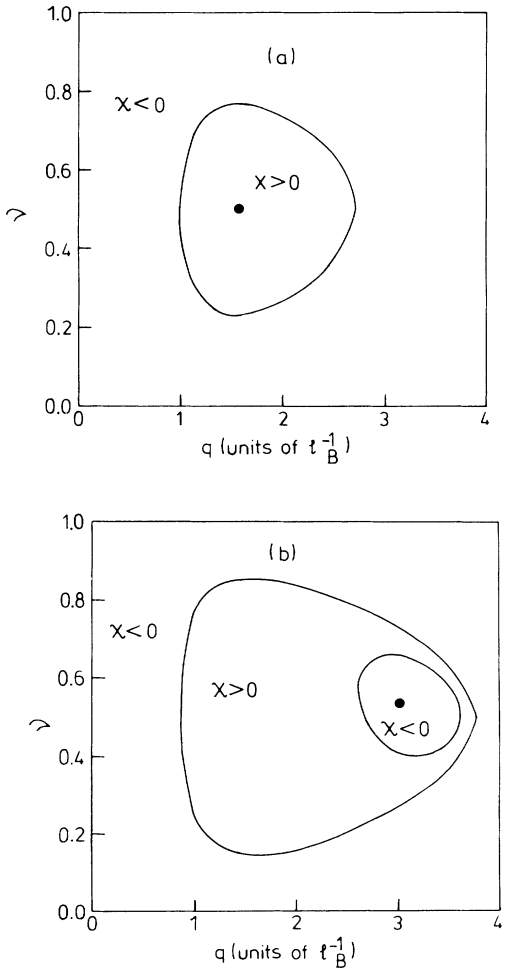

FIG. 1. Sign diagram of the static dielectric susceptibility: (a) for $T=0.10$ (units of $\hbar\omega_c$); when $T\to 0.139$ the enclosed region converges to the marked point; (b) for $T=0.070$; the smaller closed area reduces to the indicated point when $T\to 0.0754$.

$\chi<0$) is reached so that the system becomes prepared in the state corresponding to a point inside the bounded stability region.

The dielectric susceptibility is presented in Fig. 2 as a function of ν for three wave vectors of distinct regions of Fig. 1(b): the external region [Fig. 2(a)], the symmetrical (with respect to $\nu=\frac{1}{2}$) region [Fig. 2(b)], and the asymmetrical one [Fig. 2(c)]. In the first case [Fig. 2(a)], as well as for any q if $T>T_c$, the susceptibility (2.25) is not qualitatively different, but only larger, with respect to the RPA susceptibility. As discussed above, the situation is completely changed in Figs. 2(b) and 2(c). The dielectric susceptibility has changes of sign from $-\infty$ to $+\infty$ as well as through zero. As the temperature is lowered, the poles approach 0 and 1, respectively—and the zeros too, but more slowly—with the negative susceptibility between them becoming larger.

III. DIELECTRIC FUNCTION

The static dielectric function can be found combining Eq. (2.25) and the first half of Eq. (2.8).

In the low-temperature limit

$$\epsilon(q)=\frac{1-\frac{\sqrt{2/\pi}}{q}e^{-q^2/4}[I_0(q^2/4)]^{-1}}{1+\frac{2\pi}{q}\nu\chi_1^{(\mathrm{RPA})}(q)+(2/\pi^3)^{1/2}\nu e^{-q^2/4}q\zeta(q)[I_0(q^2/4)]^{-1}}, \tag{3.1}$$

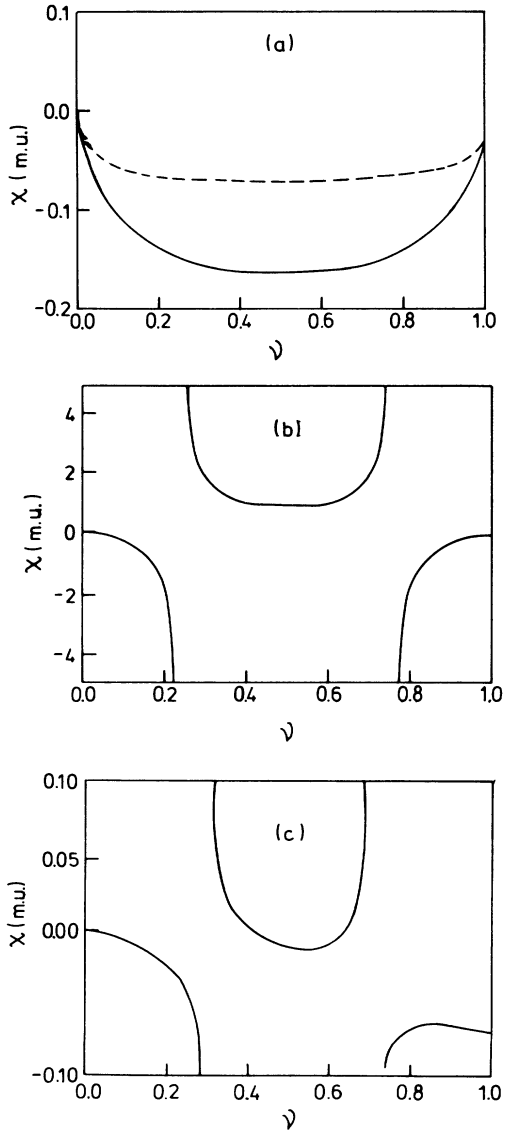

FIG. 2. Static dielectric susceptibility, in magnetic units (m.u.), vs the filling factor, for three wave vectors corresponding to all distinct regions of Fig. 1 ($T=0.07$): (a) $q=0.5$, the dashed line describing the RPA; (b) $q=1.0$; (c) $q=3.2$.

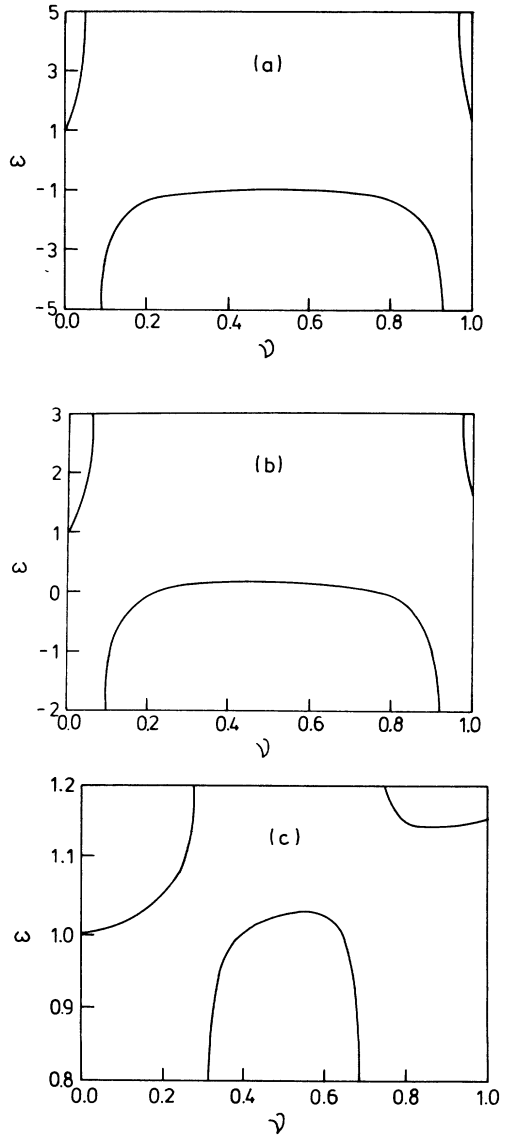

FIG. 3. The static dielectric function vs the filling factor for the same three wave vectors of Fig. 2, i.e., $q=0.5$ (a), $q=1.0$ (b), and $q=3.2$ (c), for $T=0.070$.

the low-temperature singularity of the RPA being now removed, the screening becoming perfect only in the long-wavelength limit.

For long wavelengths, but at finite temperatures,

$$\epsilon(q)=1+\frac{1}{q}\frac{\nu(1-\nu)}{T-\sqrt{\pi/2}\nu(1-\nu)} . \tag{3.2}$$

The electron-hole symmetry is present in this limit because the terms influenced by the kinetic-energy operator in the Hamiltonian (which does not have this symmetry) vanish. They are $\chi_1(q)$ and the component of the exchange term denoted by $\Phi(k)$ in Eq. (A4) which depends on the perturbed-oscillator wave functions (2.22).

Equation (3.2) can be obtained directly from Eq. (1.2). Assuming no external field the Hartree-Fock energy of the lowest Landau level [Eqs. (2.10 and (2.11)] is given only by the exchange term with ψ_k replaced by f_k.[20] Using Eq. (A1), one obtains (see also Ref. 18) $\varepsilon=-\sqrt{\pi/2}\nu$, which combined with the Fermi function $\mathcal{F}((\varepsilon-\mu)/T)=\nu=2\pi\langle\rho\rangle$ yields an equation for the thermodynamic density of states (TDOS), $D_T=\partial\langle\rho\rangle/\partial\mu$, wherefrom Eq. (1.2) leads to Eq. (3.2).

It can be seen from Eq. (3.2) that, if $T<T^*\approx 0.313$ (in units of $\hbar\omega_c$), the dielectric constant may become negative for some filling factors around $\frac{1}{2}$, provided that the wavelength is large enough. This fact, even if unusual, is not surprising and is specific to strongly correlated systems.[6] In Système International (SI) units $T^*\approx 26$ K for a magnetic field of induction $B=12$ T and an effective mass $\approx 0.2m$ as in the silicon samples used in Refs. 14 and 15.

In those papers negative TDOS, previously predicted,[16] are reported for temperatures below $T^*\approx 4$ K. The negative values appear at filling factors close to integers, unlike Eq. (3.2) in which $\nu=\frac{1}{2}$ is the critical one. The reason for this difference can be better understood in terms of $D_T^{-1}=D_{TD}^{-1}+D_{TC}^{-1}$ since the contributions of the disorder and of the Coulomb interaction can be separated. The disorder, inherent in the real case, yields $D_{TD}^{-1}>0$ (Ref. 14) and consequently T^* is lowered. On the other hand, in the low-temperature limit, the electron liquid becomes more strongly correlated and the Coulomb interaction needs to be more accurately taken into account, as has also been pointed out in Refs. 14 and 15.

In Fig. 3 the dielectric function is plotted versus the filling factor for the three wave vectors chosen in Fig. 2, the temperature being $T=0.070$. For $q=0.5$, even if the susceptibility is not qualitatively different from results of the RPA, the dielectric function changes sign discontinuously (from $+\infty$ to $-\infty$), according to inequalities (1.1) [Fig. 3(a)]. For $q=1$, these inequalities are violeted, ϵ crosses continuously the horizontal axis in the forbidden interval corresponding to unstable states [Fig. 3(b)]. Inside the "island" of stability, e.g., for $q=3.2$, the dielectric function is continuous and again positive [Fig. 3(c)].

IV. CONCLUSIONS

The static dielectric susceptibility of the electrons partially occupying the lowest Landau level has been obtained in an approximation leading to qualitatively different behavior for low temperatures from RPA results.

While in the RPA, the dielectric susceptibility is negative definite, under the proposed extended approximation, for some wave vectors, it is positive inside the region of the (T,ν) plane bounded by

$$T(\nu)=0.557\nu(1-\nu) , \tag{4.1}$$

and diverges on both sides of this boundary. Therefore Eq. (4.1) gives the critical temperature for a spontaneous transition to a CDW state. This is the second-order transition studied in Refs. 1 and 3.

For $T<0.0754$ (in units of $\hbar\omega_c$), an "island" of filling factors and wave vectors appears inside the instability region, in which the dielectric susceptibility becomes again negative, this time continuously. Consequently a small harmonic external potential, like those produced by a periodic microstructured gate obtained by microlithographic techniques,[12] could induce stable modulations of the particle density having different wavelengths than those of the spontaneous CDW. If the temperature rises or if the filling factor (or the wave vector) is modified, as the system leaves the "island" it will automatically adopt the most favorable density modulation, regardless of the form of the small external potential. This transition from continuously modulable states to spontaneous CDW's seems to be higher than second order.

In the approximation described, the static dielectric function does not diverge as $T\to 0$, like in the RPA. The dielectric constant, as well as the electronic thermodynamic density of states, can take on negative values for $T<0.313$. This means the long-wavelength components of the total (external plus induced) and the external potentials can have opposite signs, which is specific only to strongly correlated states.[6] In terms of the dielectric function, the CDW instabilities are generated by $0\le\epsilon\le 1$.

ACKNOWLEDGMENTS

I am grateful to Professor B. Ciobanu and to Dr. A. B. Fazakas, Dr. P. Gartner, and Dr. G. A. Mezincescu for helpful discussions.

APPENDIX: EXCHANGE-TERM CONTRIBUTION

Fourier transforming the Coulomb potential the exchange term in (2.11) may be expressed in the form

$$T_k^X=-\frac{1}{2\pi}\sum_{k'}\mathcal{F}_{k'}\int\frac{d\mathbf{Q}}{Q}|\langle\psi_k|e^{i\mathbf{Q}\cdot\mathbf{r}}|\psi_{k'}\rangle|^2 . \tag{A1}$$

Approximating in the matrix elements ψ_k with φ_k, Eqs. (2.22) and (2.23), a straightforward calculation gives

$$T_k^X=-\frac{1}{\pi}\int dk'\mathcal{F}_{k'}\frac{(\omega\omega')^{1/2}}{\omega+\omega'}\exp\left[-\frac{\omega\omega'(\xi-\xi')^2}{\omega+\omega'}+\frac{(k-k')^2}{2(\omega+\omega')}\right]K_0\left[\frac{(k-k')^2}{2(\omega+\omega')}\right] , \tag{A2}$$

K_0 being a modified Bessel function.[23] In (A2) the unknown Hartree-Fock energies (2.10) are contained in the Fermi function $\mathcal{F}_k$, but we are interested only in the derivative of (A2) with respect to V for $V=0$ [see Eqs. (2.10)–(2.12)]. The derivatives of ω and ξ may be easily evaluated from Eqs. (2.23), and

$$\left.\frac{\partial \mathcal{F}_{k'}}{\partial V}\right|_{V=0} = -\frac{\nu(1-\nu)}{T}\left.\frac{\partial \varepsilon_{k'}}{\partial V}\right|_{V=0} \tag{A3}$$

[see Eq. (2.16)] can be expressed with the yet-to-be-determined $\eta(q)$ from Eq. (2.15). We get

$$\left.\frac{\partial T_k^X}{\partial V}\right|_{V=0} = \left[\frac{\pi}{2}\right]^{1/2}\frac{\nu(1-\nu)}{T}\eta(q)e^{-q^2/4}I_0\left[\frac{q^2}{4}\right]\cos(qk)+\frac{\nu}{\pi}\Phi(k)\ , \tag{A4}$$

where another modified Bessel function[23] is used,

$$I_0(z)=\sum_{n=0}^{\infty}\frac{(z/2)^{2n}}{(n!)^2}\ ,$$

and

$$\begin{aligned}\Phi(k)&=\int dq\, q^2 e^{-q^2/4}\zeta(q)\cos(qk)\ ,\\ \zeta(q)&=\left[\frac{\pi}{2}\right]^{3/2}\left\{{}_1F_1\left[\tfrac{3}{2};2;-\frac{q^2}{2}\right]-\frac{3}{16}\left[{}_2F_2\left[\tfrac{3}{2},\tfrac{3}{2};\tfrac{1}{2},2;-\frac{q^2}{2}\right]-{}_1F_1\left[\tfrac{5}{2};2;-\frac{q^2}{2}\right]\right]\right\}\end{aligned} \tag{A5}$$

${}_1F_1$ and ${}_2F_2$ being hypergeometric functions.[23]

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