Coulomb quantum kinetics in a dense electron gas

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The semiclassical Boltzmann equation for a dense electron gas is generalized to a quantum kinetic equation beyond the approximation of isolated collisions. The resulting quantum kinetic equation for the Wigner function contains memory effects, which are determined by the retarded and advanced nonequilibrium Green's functions of the scattered electrons and the screened Coulomb potential. A closed set of equations for the distribution and the spectral functions is given which is exact within the generalized Kadanoff-Baym ansatz and the random-phase approximation. Simplifying approximations are given which result in a quantum kinetic equation with memory kernels similar to those obtained for the electron-phonon scattering. In the limit of completed collisions, the quantum kinetic equation reduces to a Boltzmann equation in which the energy conservation is smeared out due to the finite time interval and due to collision broadening.

I. INTRODUCTION

Femtosecond pump and probe spectroscopy with semiconductors[1-4] allows one to investigate the regime of ultrafast relaxation kinetics in an electron-hole plasma governed by Coulomb and electron-hole interactions. Monte Carlo simulations[5-9] and direct numerical integrations[10-13] of the semiclassical Boltzmann equation have been used to describe the time development of the nonequilibrium electron distributions. Closely related studies (see, e.g., Refs. 14 and 15) of the nonequilibrium electron kinetics are required to describe the transport in semiconductor microstructures. The large electric fields in these small devices cause large deviations from equilibrium distributions.

It is obvious that on a very short time scale the Boltzmann picture of individual, successive collisions breaks down and has to be replaced by a quantum kinetic theory (see, e.g., Refs. 15–20). The quantum kinetic equations can be derived in the framework of density matrices (see, e.g., Refs. 15–17), or of nonequilibrium Green's functions (see, e.g., Refs. 18–20).

The quantum kinetics is characterized by the absence of a strict energy conservation and by the presence of memory effects. In particular, for the electron–LO-phonon interaction[15-20] the quantum kinetic equation for the electron distribution is firmly established and already well studied. Zimmermann has shown recently,[17] by direct numerical integration of the fully retarded kinetic equation, that the electron distribution shows initially relaxation oscillations with the frequency of a LO phonon before it enters the regular relaxation regime. The transition from the quantum-beat regime to the relaxation regime illustrates beautifully the transition from coherent to irreversible kinetics. It is important to note that quantum beats are only present in a quantum kinetic equation with memory; they are lost if the retardation is neglected.

The quantum kinetics of a dense electron gas with Coulomb kinetics is less developed and understood, even though several studies have been devoted to some aspects of this problem, particularly for transport in strong fields (see, e.g., Refs. 21–23). In the present paper we will derive a closed quantum kinetic description of a dense, spatially homogeneous electron gas with Coulomb interaction in the absence of any field. The idea is to study for this rather basic system the initial quantum kinetic regime, starting from a given initial nonequilibrium distribution (e.g., generated by a femtosecond laser pulse). We assume that a separation between a "macroscopic" time scale and a "microscopic" time scale is not possible. Kadanoff and Baym[24] have shown that the assumption of such a separation allows a quantum-mechanical derivation of the semiclassical Boltzmann equation also for a Coulomb system with screening. The macroscopic coordinate is introduced as the central time $T=(t_1+t_2)/2$ of a two-time single-particle nonequilibrium function $G(t_1,t_2)$; it describes the slow change of the distribution function. The microscopic time $t=t_1-t_2$ describes spectral properties, i.e., the fast oscillations and decay of the microscopic correlations. Therefore one changes with a Fourier transformation from t to ω. Products of two two-time functions imply in the Green's-function theory a time integral over common inner time variable. In (T,ω) space, these products are simple if the variations of all functions with the macroscopic variable are sufficiently slow. We will avoid such assumptions and stay for the whole derivation strictly in the original two-time representation. We will use a nonequilibrium Green's-function theory[25] rather than a density-matrix theory because the former is particularly well suited to describe consistently the kinetic and the spectral properties. It will be shown that the kinetic equation couples to the spectral functions of the electron and of the screened Coulomb potential. The Green's-function theory allows one to derive, on the same level of approximations, equations for the spectral functions. However, it is in principle a matter of individual preference whether nonequilibrium Green's-function or density-matrix theory is used.

Our derivation relies strongly on the generalized Kadanoff-Baym ansatz (GKBA) (Ref. 18) between the kinetic two-point function and the one-point distribution function. Lipavský and co-workers[18] showed that this GKBA takes properly into account the causal time evolution of the two-point function and gave a systematic derivation of this relation, which allows one, at least in principle, to check for a given system the quality of the GKBA. All these features are not present in the original Kadanoff-Baym ansatz, which was an intuitive generalization of an equilibrium relation.

In Sec. II we evaluate all self-energies within the random-phase approximation (RPA). Using the GKBA one can express the scattering rates by retarded electron distribution functions. The retardation is determined by the retarded electron Green's function and by the propagator of the screened Coulomb potential. The particlelike potential propagator can be expressed exactly in terms of a convolution of the particlelike polarization function and the retarded and advanced screened Coulomb potential. The particlelike polarization contains the distribution of the electrons in the initial and final states of the scattering partner at still earlier times. Within RPA and the GKBA the equations of the retarded electron Green's function and the retarded screened Coulomb potential can be evaluated and form, together with the quantum kinetic equation, a closed set of integral equations.

In Sec. III we describe simplifying approximations for the spectral functions. With these approximations one obtains a quantum kinetic equation with one simple retardation of all distribution functions of the initial and final states of the two scattering particles. The retardation is determined by the collision broadening of all four involved particle states but also dispersively by phase cancellation effects which are obtained from the integration over the transferred momentum and the momentum of the scattering partner. This simplified Coulomb quantum kinetic equation is suitable for numerical integration. If one disregards, in a further approximation, the retardation of the distribution, one gets a Boltzmann-like equation in which the energy conservation is broadened by the finite evolution time and by the collision broadening.

II. DERIVATION OF A CLOSED QUANTUM KINETIC DESCRIPTION

The two-point particle Green's function $G_k^<(t_1,t_2)=i\langle a_k^\dagger(t_2)a_k(t_1)\rangle$ forms the basis of the kinetic theory. For the Keldysh nonequilibrium Green's functions the notation of DuBois[25] is used. For simplicity a spatially homogeneous system is assumed in order to concentrate fully on the time arguments. a_k is the annihilation operator of an electron in momentum state k (the vector notation is suppressed). Obviously the equal-time limit of the particle function $G_k^<(t,t)=if_k(t)$ yields the particle distribution function f_k. The equation of motion for the particle propagator in the equal-time limit reads[18,19,26]

$$\frac{\partial}{\partial t}f_k(t)=-\int_{t_0}^{t}dt'[\,\Sigma_k^<(t,t')G_k^>(t',t)+G_k^>(t,t')\Sigma_k^<(t',t)-\Sigma_k^>(t,t')G_k^<(t',t)-G_k^<(t,t')\Sigma_k^>(t',t)\,]\,. \tag{1}$$

Here $\Sigma_q^<(t_1,t_2)$ is the particle self-energy. $G_k^>(t_1,t_2)=-i\langle a_k(t_1)a_k^\dagger(t_2)\rangle$ is the hole propagator and $\Sigma_k^>(t_1,t_2)$ its self-energy. The first two terms on the right-hand side of Eq. (1) give the scattering rate into state k, while the last two terms describe the scattering rate out of state k. As shown in Eq. (1) these scattering rates are convolution time integrals between the self-energies and the particle propagators. The kinetic equation (1) connects thus a one-time distribution function to two-time particle and hole propagators and is thus not closed. Kadanoff and Baym[24] (KB) suggested intuitively how the two-time propagators should be connected approximately with the one-time distribution function. The KB ansatz yields indeed the semiclassical Boltzmann equation, but fails to give any retardation effects. Lipavský and co-workers[18] developed a systematic theory for this connection. In lowest order their result, called GKBA, is a slight but very important generalization of the old KB ansatz which expresses correctly the causality of the time development of the two-time propagators using its equal-time value as an initial value:

$$-iG_k^{\gtrless}(t_1,t_2)=G_k^r(t_1,t_2)G_k^{\gtrless}(t_2,t_2)-G_k^{\gtrless}(t_1,t_2)G_k^a(t_1,t_2)=\mp i[\,G_k^r(t_1,t_2)f_k^{\gtrless}(t_2)-f_k^{\gtrless}(t_1)G_k^a(t_1,t_2)\,]\,. \tag{2}$$

Here $G_k^<(t,t)=if_k^<(t)=if_k(t)$ and $G_k^>(t)=-if_k^>(t)=-i[1-f_k(t)]$, where $f_k(t)$ is the Wigner distribution function (here for a spatially homogeneous system). $G_k^r(t_1,t_2)$ and $G_k^a(t_1,t_2)$ are the retarded and advanced Green's functions, respectively. For $t_1>t_2$ only the term with the retarded Green's function contributes. This function describes the time development of $G_k^{\gtrless}(t_1,t_2)$ away from its diagonal value at the earlier time t_2. For $t_1<t_2$ the time development is governed by the advanced Green's function. For $t_1=t_2$ the relation (2) is obviously exact. From (2) one recovers the KB ansatz by the following rough approximation:

$$G_k^{\gtrless}(t_1,t_2)\simeq\pm f_k^{\gtrless}((t_1+t_2)/2)\times[G_k^r(t_1,t_2)-G_k^a(t_1,t_2)]=\mp if_k^{\gtrless}((t_1+t_2)/2)A_k(t_1,t_2)\,,$$

where $A_k(t_1,t_2)$ is the spectral function. While the KB ansatz also has the correct equal-time limit, its midpoint approximation for the distribution functions violates causality. Recently, a further interesting derivation of the GKBA has been given.[27] In the following the GKBA of Eq. (2) will be used to close the kinetic equation (1).

For Coulomb scattering the self-energies $\Sigma_k^{\gtrless}(t_1,t_2)$ are given in RPA by

$$\Sigma_k^{\gtrless}(t_1,t_2)=i\sum_q G_{k-q}^{\gtrless}(t_1,t_2)V_{sq}^{\gtrless}(t_1,t_2)\ . \tag{3}$$

The particlelike screened Coulomb potential $V_{sq}^{<}(t_1,t_2)$ will first be expressed in terms of the particlelike polarization function $L_q^{<}$. In obvious matrix notation we write (the explicit momentum and time arguments and integrals are suppressed)

$$V_s^{\eta_1\eta_2}=V\delta_{\eta_1\eta_2}+VL^{\eta_1\eta_3}V_s^{\eta_3\eta_2} \tag{4}$$

and particularly

$$V_s^{+-}=V(L^{++}V_s^{+-}+L^{+-}V_s^{--}) \tag{5}$$

or

$$V_s^{<}=V(L^tV_s^{<}-L^{<}\bar{V}_s^t)\ , \tag{6}$$

where L^t and $\bar{V}^t$ are time- and antitime-ordered functions. They can be expressed in terms of particlelike, retarded, and advanced functions: $L^t=L^r+L^{<}$ and $\bar{V}_s^t=V_s^{<}-V_s^a$. Equation (6) becomes

$$V_s^{<}=VL^rV_s^{<}+VL^{<}V_s^a\ . \tag{7}$$

The retarded potential obeys the equations

$$V_s^r=V+VL^rV_s^r \tag{8a}$$

and

$$V_s^r=V+V_s^rL^rV\ . \tag{8b}$$

Putting the Coulomb potential V from Eq. (8a) into the last term of the right-hand side of Eq. (7), one gets

$$V_s^{<}=L^r(VV_s^{<}-VV_s^rL^{<}V_s^a)+V_s^rL^{<}V_s^a\ . \tag{9}$$

By using Eq. (7) again one gets

$$V_s^{<}=L^r(VV_s^{<}-V_s^rV_s^{<}+VV_s^rL^rV_s^{<})+V_s^rL^{<}V_s^a\ . \tag{10}$$

For the third term on the right-hand side we use Eq. (8b) and get a complete compensation between the resulting first four terms. (Note that V is a scalar, not a matrix.) The simple final result is

$$V_{sq}^{\gtrless}(t_1,t_2)=\int_{t_0}^{t_1}dt_3\int_{t_0}^{t_2}dt_4V_{sq}^r(t_1,t_3)L_q^{\gtrless}(t_3,t_4)V_{sq}^a(t_4,t_2)\ . \tag{11}$$

This result (see also Ref. 28) means that one can express exactly the two-time particlelike potential in terms of a convolution of the retarded potential, the particlelike polarization $L_q^{<}$, and the advanced potential. This result is a generalization of the corresponding equilibrium result given, e.g., by Kadanoff and Baym.[24]

In RPA the polarization bubble is given by

$$L_q^{\eta_1\eta_2}(t_1,t_2)=-i\eta_1\sum_{k'}G_{k'+q}^{\eta_1\eta_2}(t_1,t_2)G_{k'}^{\eta_2\eta_1}(t_2,t_1)\ . \tag{12}$$

From Eq. (12) one gets

$$L_q^{\gtrless}(t_1,t_2)=-i\sum_{k'}G_{k'+q}^{\gtrless}(t_1,t_2)G_{k'}^{\lessgtr}(t_2,t_1)\ . \tag{13}$$

With these results the first term of the quantum kinetic equation reads

$$-\int_{t_0}^{t}dt'\Sigma_k^{<}(t,t')G_k^{>}(t',t)=\sum_{q,k'}\int_{t_0}^{t}dt'\int_{t_0}^{t}dt_3\int_{t_0}^{t'}dt_4V_{sq}^r(t,t_3)G_{k'+q}^{<}(t_3,t_4)G_{k'}^{>}(t_4,t_3)V_{sq}^a(t_4,t')G_{k-q}^{<}(t,t')G_k^{>}(t',t)\ . \tag{14}$$

In order to be able to use the GKBA, one has to establish a definite order between the times t_3 and t_4. By splitting the t_3 integral one gets

$$\begin{aligned}&\sum_{q,k'}\int_{t_0}^{t}dt'G_{k-q}^r(t,t')G_k^a(t',t)f_{k-q}(t')[1-f_k(t')]\int_{t_0}^{t'}dt_4V_{sq}^a(t_4,t')\\&\times\left[\int_{t_0}^{t_4}dt_3V_{sq}^r(t,t_3)G_{k'+q}^a(t_3,t_4)G_{k'}^r(t_4,t_3)f_{k'+q}(t_3)[1-f_{k'}(t_3)]\right.\\&\left.+\int_{t_4}^{t}dt_3V_{sq}^r(t,t_3)G_{k'+q}^r(t_3,t_4)G_{k'}^a(t_4,t_3)f_{k'+q}(t_4)[1-f_{k'}(t_4)]\right]\ .\end{aligned} \tag{15}$$

The second term of Eq. (1) which belongs to the rate-in is obtained from Eq. (15) by interchanging in all spectral functions the time arguments and by replacing all retarded functions by advanced ones and vice versa. By these means the second term becomes just the complex conjugate of the first one. The total resulting kinetic equation which one gets from Eq. (1) is

$$\frac{\partial}{\partial t}f_k(t)=\sum_{q,k'}\int_{t_0}^{t}dt'G^r_{k-q}(t,t')G^a_k(t',t)f_{k-q}(t')[1-f_k(t')]$$
$$\times\int_{t_0}^{t'}dt_4V^a_{sq}(t_4,t')\left[\int_{t_0}^{t_4}dt_3V^r_{sq}(t,t_3)G^a_{k'+q}(t_3,t_4)G^r_{k'}(t_4,t_3)f_{k'+q}(t_3)[1-f_{k'}(t_3)]\right.$$
$$\left.+\int_{t_4}^{t}dt_3V^r_{sq}(t,t_3)G^r_{k'+q}(t_3,t_4)G^a_{k'}(t_4,t_3)f_{k'+q}(t_4)[1-f_{k'}(t_4)]\right]+\{\text{c.c}\}$$
$$-\sum_{q,k'}\int_{t_0}^{t}dt'G^r_{k-q}(t,t')G^a_k(t',t)[1-f_{k-q}(t')]f_k(t')$$
$$\times\int_{t_0}^{t'}dt_4V^a_{sq}(t_4,t')\left[\int_{t_0}^{t_4}dt_3V^r_{sq}(t,t_3)G^a_{k'+q}(t_3,t_4)G^r_{k'}(t_4,t_3)[1-f_{k'+q}(t_3)]f_{k'}(t_3)\right.$$
$$\left.+\int_{t_4}^{t}dt_3V^r_{sq}(t,t_3)G^r_{k'+q}(t_3,t_4)G^a_{k'}(t_4,t_3)[1-f_{k'+q}(t_4)]f_{k'}(t_4)\right]+\{\text{c.c}\}\ . \tag{16}$$

Equation (16) is the central quantum kinetic equation. The collision rates contain the distribution functions of the scattered particles before and after the collision as required by the Pauli principle. However, the initial- and final-state occupation probabilities of the considered electron which is scattered from $k\leftrightarrows k-q$ enter at the retarded time $t'<t$, and those of the scattering partner enter at still earlier times t_3,t_4. These memory effects are the trademark of the quantum kinetic regime. Due to the extra retardation introduced by screening, they are particularly involved in a dense Coulomb system.

In order to close this quantum kinetic description, one has to calculate the spectral (i.e., retarded and advanced) functions of the screened Coulomb potential and of the electron propagator. The RPA retarded screened Coulomb potential obeys Eq. (8a) which is explicitly

$$V^r_{sq}(t_1,t_2)=V_q\delta(t_1-t_2)$$
$$+V_q\int_{t_2}^{t_1}dt_3L^r_q(t_1,t_3)V^r_{sq}(t_3,t_2)\ . \tag{17}$$

The retarded polarization L^r_q is given by

$$L^r_q(t_1,t_2)=L^{++}_q(t_1,t_2)+L^{+-}_q(t_1,t_2)$$
$$=L^t_q(t_1,t_2)-L^<_q(t_1,t_2)\ . \tag{18}$$

Using the identity

$$G^t_k(t_1,t_2)=\Theta(t_1-t_2)G^>_k(t_2,t_1)$$
$$+\Theta(t_2-t_1)G^<_k(t_2,t_1) \tag{19}$$

one gets

$$L^r_q(t_1,t_2)=\Theta(t_1-t_2)[L^>_q(t_1,t_2)-L^<_q(t_1,t_2)]\ , \tag{20}$$

which reduces with the GKBA to

$$L^r_q(t_1,t_2)=-i\sum_{k'}G^r_{k'+q}(t_1,t_2)G^a_{k'}(t_2,t_1)$$
$$\times[f_{k'}(t_2)-f_{k'+q}(t_2)]\ . \tag{21}$$

From the polarization function (23) one gets in lowest order with the free-particle Green's function

$$G^r_{k+q}(t_1,t_2)=-i\Theta(t_1-t_2)e^{-ie_{k+q}(t_1-t_2)}\ ,$$
$$G^a_k(t_2,t_1)=i\Theta(t_1-t_2)e^{ie_k(t_1-t_2)} \tag{22}$$

and with $t=t_1-t_2$ the result

$$L^r_q(t_1,t_2)=-i\Theta(t)\sum_k e^{i(e_k-e_{k+q})t}[f_k(t_2)-f_{k+q}(t_2)]\ . \tag{23}$$

In equilibrium one gets with a Fourier transform with respect to t the well-known Lindhard formula

$$L^r_q(\omega)=\sum_k\frac{f_k-f_{k+q}}{\omega+i\delta+e_k-e_{k+q}}\ . \tag{24}$$

Finally, one has to address the problem of calculating the retarded electron Green's function from its Dyson equation which in the integral form is given by

$$G^r_k(t_1,t_2)=G^{r0}_k(t_1,t_2)$$
$$+\int_0^{t_1}dt_3\int_0^{t_2}dt_4G^{r0}_k(t_1,t_3)$$
$$\times\Sigma^r_k(t_3,t_4)G^r_k(t_4,t_2)\ , \tag{25}$$

with the retarded RPA Coulomb self-energy $\Sigma^r_k=\Sigma^t_k-\Sigma^<_k$. Using Eq. (19) and a corresponding representation of the screened Coulomb potential

$$V^t_{sq}(t_1,t_2)=V_q\delta(t_1-t_2)+\Theta(t_1-t_2)V^<_{sq}(t_1,t_2)$$
$$+\Theta(t_2-t_1)V^>_{sq}(t_1,t_2) \tag{26}$$

and

$$\lim_{t_2\to t_1}G^t_k(t_1,t_2)=G^<_k(t_1,t_1)\ , \tag{27}$$

one finds for the retarded self-energy the form

$$\Sigma_k^r(t_1,t_2)=\Sigma_k^x(t_1)\delta(t_1-t_2)+\Theta(t_1-t_2)[\Sigma_k^>(t_1,t_2)-\Sigma_k^<(t_1,t_2)]\ , \tag{28}$$

where $\Sigma_k^x(t_1)$ is the instantaneous exchange self-energy

$$\Sigma_k^x(t_1)=i\sum_q G_{k-q}^<(t_1,t_1)V_q=-\sum_q f_{k-q}(t_1)V_q\ . \tag{29}$$

With the GKBA one gets

$$\Sigma_k^r(t_1,t_2)=-\sum_q f_{k-q}(t_2)V_q\delta(t_1-t_2)-i\Theta(t_1-t_2)\sum_q G_{k-q}^r(t_1,t_2)\{[1-f_{k-q}(t_2)]V_{sq}^<(t_1,t_2)+f_{k-q}(t_2)V_{sq}^>(t_1,t_2)\}\ . \tag{30}$$

In equilibrium the retarded self-energy Eq. (30) reduces to the usual RPA self-energy.[29]

Now we have derived a closed system of equations which is, within RPA and the GKBA, exact: The quantum transport equation (16) for the electron distribution, Eqs. (17) and (21) for the retarded potential, and Eqs. (25) and (30) for the retarded electron Green's function. The particlelike potentials in Eq. (30) can be eliminated with Eq. (11). The advanced functions do not have to be calculated separately, they can be obtained from the retarded functions by the relations $G_k^a(t_1,t_2)=[G_k^r(t_2,t_1)]^*$ and $V_{sq}^a(t_1,t_2)=[V_{sq}^r(t_2,t_1)]^*$.

These equations have to be solved self-consistently, which seems to be quite involved even with the use of supercomputers. Simplifying approximations are necessary to proceed with numerical analysis.

III. SIMPLIFYING APPROXIMATIONS

In transport problems with electron-phonon scattering damped free-particle approximations, of the form used to derive Eq. (23), have been used[18,19] for the electron spectral functions

$$\begin{aligned}G_k^r(t_1,t_2)&=-i\Theta(t_1-t_2)e^{(-ie_k-\gamma_k)(t_1-t_2)}\ ,\\ G_k^a(t_1,t_2)&=i\Theta(t_2-t_1)e^{(-ie_k+\gamma_k)(t_1-t_2)}\ ,\end{aligned} \tag{31}$$

where γ_k has to be calculated from the imaginary part of the retarded electron self-energy. But normally a self-consistent treatment of γ_k is omitted and some reasonable collision broadening is used. A mean energy renormalization by the real part of the retarded self-energy can be included in the single-particle energy e_k. The retarded potential also needs some extra simplifying approximations, because its integral equation is still too difficult even if one uses Eq. (31) for the polarization function. A large simplification is obtained if one uses a nonequilibrium version of the plasmon pole approximation.[30] In the time representation we get

$$V_{sq}^r(t_1,t_2)=V_q\delta(t_1-t_2)-V_q\Theta(t_1-t_2)\frac{\omega_{\rm pl}^2(t_2)}{\omega_q(t_2)}\sin[\omega_q(t_2)(t_1-t_2)]\ , \tag{32}$$

with

$$\begin{aligned}\omega_{\rm pl}^2(t_2)&=\frac{4\pi e^2 n(t_2)}{\epsilon_0 m}\ ,\\ \omega_q^2(t_2)&=\omega_{\rm pl}^2(t_2)\left[1+\frac{q^2}{\kappa^2(t_2)}\right]+Cq^4\ ,\end{aligned} \tag{33}$$

where the time-dependent plasma frequency $\omega_{\rm pl}(t_2)$ and the frequency of the effective plasmon pole $\omega_q(t_2)$ enter in Eq. (32) at the earlier time t_2 (and not at the central time T_{12}) as the polarization (23) shows. If one studies the femtosecond relaxation by Coulomb scattering the total number of electrons $n(t)=n$ is constant and therefore the plasma frequency is also constant. The inverse screening length κ can be obtained by integrating the long-wavelength limit of (23) over the microscopic time $t=t_1-t_2$. In three dimensions (3D), one finds

$$\kappa_{\rm 3D}^2(t_2)=\frac{4\pi e^2}{\epsilon_0}\int^\infty \frac{de}{e}\rho(e)f(e,t_2)\ , \tag{34}$$

assuming that the nonequilibrium distribution is isotropic, i.e., depends only on the energy e; $\rho^{\rm 3D}(e)$ is a parabolic density of states.

In 2D, one finds correspondingly

$$\kappa_{\rm 2D}^2(t_2)=\frac{2e^2 m}{\epsilon_0}f(0,t_2)\ . \tag{35}$$

In relaxation experiments in which the originally created electrons are not too energetic the inverse screening length κ will also depend only weakly on time. With the potential (32) the quantum kinetic equation describes processes in which at an early time an electron is scattered $k'\leftrightarrows k'+q$ under emission (absorption) of a plasmon, which propagates in the system for some time before it is absorbed (emitted) by the scattering of the other electron $k\leftrightarrows k-q$ at time t' giving rise to a change of the distribution at time t.

A still simpler approximation is obtained if one uses a statically screened (nonretarded) potential

$$V_{sq}^r(t_1,t_2)=V_{sq}(t_2)\delta(t_1-t_2)\ , \tag{36}$$

where V_{sq} is in the simplest approximation

$$V_{sq}(t)=V_q\frac{q^2}{q^2+[\kappa(t)]^2}\ . \tag{37}$$

The resulting quantum kinetic equation for a statically screened potential is with $t_0=0$:

$$\frac{\partial f_k(t)}{\partial t} = -2\sum_{q,k'}\int_0^t d\sigma\, V_{sq}(t)V_{sq}(t-\sigma)\cos((e_k+e_{k'}-e_{k-q}-e_{k'+q})\sigma)e^{-\Gamma_{kk'q}\sigma}$$
$$\times\{f_k(t-\sigma)f_{k'}(t-\sigma)[1-f_{k-q}(t-\sigma)][1-f_{k'+q}(t-\sigma)]$$
$$-[1-f_k(t-\sigma)][1-f_{k'}(t-\sigma)]f_{k-q}(t-\sigma)f_{k'+q}(t-\sigma)\}\ , \tag{38}$$

where $\Gamma_{kk'q}=\gamma_k+\gamma_{k'}+\gamma_{k-q}+\gamma_{k'+q}$ is the sum of all collision damping coefficients. Now all four distribution functions enter the collision rate at the same retarded time $t-\sigma$, while the statically screened potential enters at the times t and $t-\sigma$. The non-Markovian memory terms extend over a time interval which is given by the sum of the four damping coefficients. These coefficients are determined in turn by the imaginary parts of the retarded Coulomb self-energy. In a complete theory this collision broadening should be included self-consistently. An urgent problem to investigate is in which sense the energy in this system is conserved. But even with $\Gamma_{kk'q}=0$, the memory would have only a finite duration because the integrations over q and k' cause a cancellation due to interferences in the cosine function.

A direct, numerical integration of this nonlinear, retarded quantum kinetic equation (38) seems to be difficult but not impossible, as the recent work of Zimmermann[17] for an electron system interacting with LO phonons shows.

A further simplification is possible in the limit of completed collisions, where all retardations of the distribution and of the screened potential are ignored. In this limit one gets with the much simpler equation

$$\frac{\partial f_k(t)}{\partial t} = -\sum_{q,k'} V_{sq}(t)V_{sq}(t)D(e_k+e_{k'}-e_{k-q}-e_{k'+q})\{f_k(t)f_{k'}(t)[1-f_{k-q}(t)][1-f_{k'+q}(t)]$$
$$-[1-f_k(t)][1-f_{k'}(t)]f_{k-q}(t)f_{k-q}(t)f_{k'+q}(t)\}\ , \tag{39}$$

where $D(\omega)$ is a broadened δ function

$$D(\omega)=\frac{2}{\omega^2+\Gamma^2}[\omega\sin(\omega t)e^{-\Gamma t}-\Gamma\cos(\omega t)e^{-\Gamma t}+\Gamma]\ . \tag{40}$$

The only difference of the quantum kinetic equation in the completed-collision limit to a semiclassical Boltzmann equation is that the energy conserving δ function is replaced by a broadening function D, which takes into account the finite lifetime of a particle state in a plasma and the finite evolution time of the system. Reggiani, Lugli and Jauho[20] have already shown for electron-phonon scattering that the quantum kinetic equation in the completed-collision approximation can be treated again by an extension of the Monte Carlo simulation method.

For detailed application to ultrafast optical semiconductor spectroscopy, one has to generalize this theory to the multiband case. This extension of the theory will be given in a separate publication.

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