Quantum-degeneracy effects in the mobility of the electron fluid on the surface of helium

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The dc mobility of two-dimensional electrons on the surface of helium liquid is calculated with the quantum degeneracy of the electron distribution and the effect of multisubband occupation taken into account. The subband structure is obtained from a self-consistent Schrödinger equation with the inclusion of the Hartree and exchange-correlation potentials. It is shown that the effect of higher subband population of electrons is small for low temperatures and the calculated mobility gives better agreement with the experimental results than that one obtained with the Maxwellian electron distribution for moderate electron concentrations. The effect of screening is briefly discussed.

The two-dimensional electron layer deposited over a surface of liquid helium is very interesting for investigating the transport properties of systems of reduced dimensionality. In contrast with low-dimensional systems found in semiconductor microstructures, the electron fluid on helium has been considered to a large extent as a classical system described by a Maxwellian distribution.[1,2] The transport properties have been exhaustively studied in the one-particle approximation by solving the Boltzmann equation.[3–7] Effects of electron correlations have been included in the high-frequency conductivity by screening the electron scattering in the memory-function approach.[8,9] Recently, the displaced-Maxwellian-distribution approximations have been employed for the case where the electron-electron collision frequency is much greater than the electron momentum relaxation frequency and the system is highly correlated.[10] Most of the work has been concerned with the transport problem when only the ground electric subband is occupied by the electrons. However, an instability of the electron layer was reported recently and attributed to collisions between ground-state and excited-state electrons.[11]

There exists some work on the influence of quantum effects on the properties of the two-dimensional electron gas,[12] and the degenerate regime was studied for the case of electrons on helium films.[13] In the present work we include the effect of quantum degeneracy in the electron distribution and the effect of multisubband occupation on the surface mobility. This paper is concerned with the calculation of the dc mobility based on the Boltzmann equation by considering both of the above effects.

We extend the previous theory[6] based on the usual Boltzmann transport equation to the case of more than one occupied subband. In this case the surface mobility is given by

$$\mu=-\frac{e\hbar^2}{m}K\ , \tag{1}$$

where

$$K=\tfrac{1}{2}\sum_n\int\frac{d^2k}{2\pi^2}k^2\left[\frac{\partial f_0}{\partial E(k)}\right]\tau(E)\ . \tag{2}$$

The collision time $\tau(E)$ is due to the scattering of electrons with impurities, the oscillations of the surface (ripplons), and the vapor atoms. These weak interactions are responsible for the dc resistivity. We assume Matthiessen's rule, i.e., $1/\tau=1/\tau_R+1/\tau_G$, since the two scattering mechanisms occur in different regimes. The electron distribution function f_0 is given by the Fermi-Dirac distribution. At low temperatures, intersubband scattering is expected to be sufficiently weak, and can be neglected in such a way that electrons in different subbands can be regarded as independent current carriers. The explicit expressions of the collision times are written as

$$\frac{1}{\tau_R}=\frac{m}{2\pi\hbar^3}\int_0^{2\pi}d\theta|F_n^R(q)|^2(2N_q+1)(1-\cos\theta) \tag{3}$$

and

$$\frac{1}{\tau_G}=\frac{N_G m}{\pi\hbar^3}\int_{-\infty}^{\infty}dp\,|F_n^G(p)|^2\ , \tag{4}$$

where

$$F_n^{R,G}(p)=\int_0^{\infty}|\Psi_n(z)|^2V_p^{R,G}(z)dz\ . \tag{5}$$

In the above expressions $V_q^R(z)$ is the electron-ripplon interaction determined by the theory of Shikin and Monarkha,[3] N_q is the ripplon distribution function $q=2k\sin(\theta/2)$, and $V_p^G(z)=U_G e^{ipz}$, where U_G is the strength of the electron-atom contact pseudopotential related to the cross section of a He atom, and N_G is the density of the He gas.

The subband structure, specified by energy levels E_n and wave functions $\Psi_n(z)$, was obtained from a self-consistent calculation of the Schrödinger equation, where the total potential in the z direction includes the Hartree

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potential, the exchange-correlation in the local-density approximation, the clamping electric field, and the image potential smoothed by a continuously varying dielectric constant between the media.[14] In Fig. 1, we show the energy shifts of the first and second excited subbands relative to the ground state as a function of the electron density. We see that for densities above 10^8 cm^{-2} the energies increase dramatically. For low densities our results reproduce those obtained from calculations using the one-electron approximation.[1,2]

The Fermi energy E_F, which appears in the Fermi-Dirac distribution of Eq. (1), is determined as a function of the electron density through the equation

$$N=\frac{m}{\pi\hbar^2}(k_BT)\sum_n \ln\{1+\exp[E_n(k=0)-E_F]/k_BT\}\ . \tag{6}$$

For simplicity, we consider the slightly simpler case in which only the two lowest subbands are occupied by the electrons due to thermal excitations. In this case, Eq. (6) converts into a second-order algebraic equation for $\exp(E_F/k_BT)$. Figure 2 shows the calculated surface mobility limited by ripplon and gas scattering as a function of the temperature for two different values of the density. We see the effect of the occupation of the two subbands as one increases the temperature for fixed densities. For $N=10^9$ cm^{-2}, the contribution coming from the second subband is negligible for temperatures below 4.5 K, while for $N=10^8$ cm^{-2}, the effect is sizeable for T above 2.8 K. Then, in the temperature range where the electron-ripplon interaction is dominant ($T=0.5-1$ K), the effect of subband occupation can be completely neglected. In Fig. 3, we present a typical plot of the mobility against the density for two different values of temperature. In some sense, the effects of the electron-electron interaction in our calculation are included by the Hartree and exchange-correlation potentials used for determining the subband structure. The Hartree potential forces the electron against the surface, and, by decreasing the mean distance of the electron from the surface, the electron scattering increases. As one expects, the mobility decreases with increasing density. The effect of quantum degeneracy is important, as we can see from Fig. 4, where

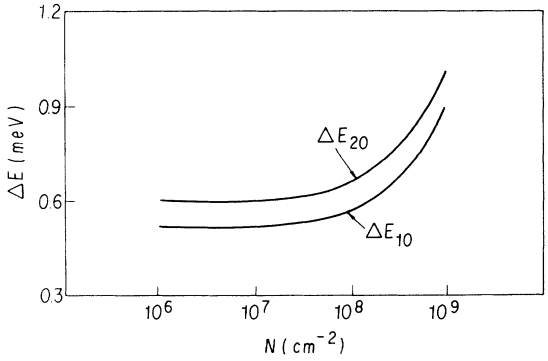

FIG. 1. Energy shifts of the first excited subbands as a function of the surface electron density, calculated from a self-consistent potential that includes the Hartree potential and the exchange-correlation potential.

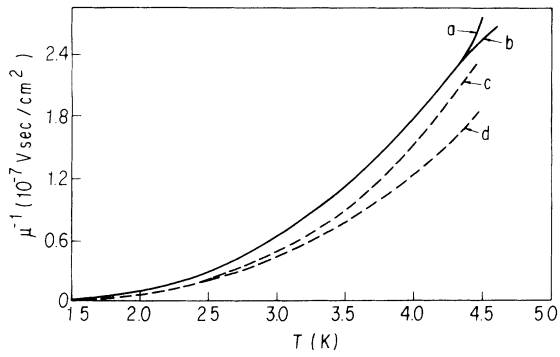

FIG. 2. Calculated inverse mobility as a function of temperature for two densities: $N=10^9$ cm^{-2} (solid line) and $N=10^8$ cm^{-2} (dashed line). Curves a and c are obtained with the two subbands E_0 and E_1 included. Curves b and d are the case in which only the lowest subband is assumed to be occupied.

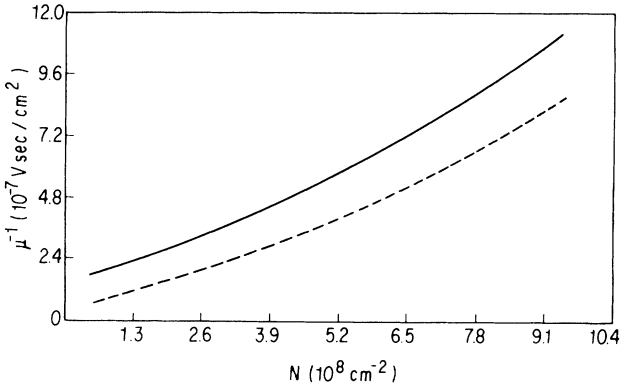

FIG. 3. Inverse mobility as a function of the electron density for two values of temperature: $T=0.8$ K (solid line) and 0.6 K (dashed line).

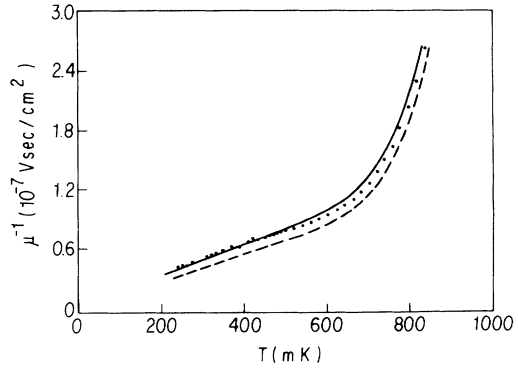

FIG. 4. Inverse mobility as a function of the temperature for a density $N=1.05\times10^8$ cm^{-2}. The solid line represents the results from our calculation, and the dashed line represents the theoretical estimates of the approach of Saitoh and Monarkha. The experimental points are taken from Ref. 15.

our results are compared with experimental results[15,16] and those coming from the calculation using the classical Maxwell distribution for the electrons. We see clearly the decrease of the mobility when this effect is incorporated, and one gets very good agreement with the experimental result for $N \simeq 10^8$ cm^{-2}. At higher densities, the differences between the one-electron theories and experiments become appreciable due to the electron-electron interaction in the plane. The usual procedure is to take into account the screening effect, which can be included in terms of a dielectric function $\epsilon(q)$. The electron-ripplon and the electron-atom potentials that appear in Eq. (5) will be screened out by dividing $F_n^{R,G}$ by the dielectric function. We have performed a calculation along this direction by using the random-phase approximation for the dielectric function. Unfortunately, our result does not agree with the experimental one at high densities. This feature seems to be the same as in the classical case in which the conductivity of the screened electrons is almost identical to the one given by the one-particle approximation in the Boltzmann-equation approach.[9] The role of electron correlations in the transport properties is still an open problem in the electrons-on-helium system.

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