Interaction of excitons with an incompressible quantum liquid

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Some general properties of exciton spectra of an interacting two-dimensional electron-hole system have been established. In the quantum limit both the frequencies and the matrix elements of exciton transitions are independent of environment, i.e., of filling factors of electrons and holes, if all the electron-hole interactions are equal in magnitude, $V_{ee} = V_{hh} = |V_{eh}|$. Only when these symmetry restrictions are lifted, do the absorption and emission spectra acquire nontrivial properties reflecting electronic correlations in a system. We consider an exciton against the background of an incompressible quantum liquid for both symmetric and nonsymmetric systems. It is always strongly coupled to the environment. This interaction may be considered as a polaron effect that is due to the dressing of an exciton by magnetorotons, but it shows very specific features because of restrictions imposed by the Pauli exclusion principle. The energy spectrum of a dressed exciton, the electric charge distribution, and some other properties of it are investigated. When the anisotropy parameter reaches its critical value the absolute minimum of the energy spectrum shifts from the point $k = 0$ to a circle, as a result of which abrupt changes in the emission spectrum are expected. A model of the ground state is proposed.

I. INTRODUCTION

The incompressible two-dimensional (2D) electron liquid[1] which underlies the fractional quantum Hall effect[2] is one of the most remarkable objects of the modern solid-state physics. The fundamental property of this liquid is the absence of zero-gap branches in the spectrum of elementary excitations.[1] The lowest branch of this spectrum is usually termed as magnetoroton (MR) and can be described rather well in terms of a single-mode approximation.[3] The excitations lying above this branch cannot be classified, being investigated only by numerical methods, e.g., by using the spherical model.[4] Reviews on different aspects of the problem can be found in Refs. 5 and 6. All the experimental data were based initially on the magnetotransport properties only.

Later on, spectroscopic methods permitting an independent observation of phase transitions in interacting 2D electron systems were also developed.[7–10] They are based on the spectroscopy of the emission arising from the recombination of 2D electrons with holes. The spectra, corresponding to recombination of 2D electrons with free holes and also with holes trapped on acceptors, can be resolved quite well and significantly differ from each other. In the investigation of the extrinsic emission, which is due to radiative trapping of 2D electrons by neutral acceptors, experimentalists made a very important observation[7,8] that the position of the emission band as a function of the filling factor ν exhibits some kind of singularities at the same fractional values of ν at which the fractional quantum Hall effect is also observed. These singularities were described initially in terms of steps, and from the heights of the steps the gaps Δ in the energy spectrum of different liquid phases were estimated.

A considerable difficulty which is inherent in the problem is due to the fact that all the competing energies have the same scale $e^2/\epsilon l(H)$, where ϵ is the dielectric constant, $l(H)$ is the magnetic length, and H is a magnetic field. Therefore, there is no small parameter in the problem, and any analysis of experimental data must be based on a theory which takes consistently into account the interelectron interaction. In the luminescence spectra this interaction should be manifested, in particular, by a considerable intensity of satellites which arise from shake-up processes (or Auger processes, which are in fact the same for the problem under consideration).[11,12] The existence of satellites strongly influences the shape of the emission band and the position of its center of gravity. As applied to the trapping of 2D electrons by neutral impurities there are shake-up processes which reduce the singularity in the ν dependence of the position of the emission band from a step to a cusp down.[13] The latter belongs to the same type as the singularity in the ground-state energy.[14] The analysis of the most recent experimental data performed in terms of the model investigated in Ref. 13 is in reasonable agreement with the theory and resulted in a considerable increase in the values of Δ, especially in the region of not too strong magnetic fields.[15] The quantitative interpretation of the experimental data on the recombination of free holes[9,10] still meets considerable difficulties, and the origin of them is not only in the effect of impurities which makes the problem much more intricate, but first of all in the interelectron interaction which must be consistently taken into account. The fact that prior to the optical transition the impurity center is neutral is of crucial importance for the theory of extrinsic emission,[13] since in its initial state the 2D electronic system is not perturbed by the impurity. The situation is quite different for the recombination of 2D electrons with free holes since holes strongly perturb the

electronic system. As an initial state one may consider an exciton existing against the background of the incompressible Laughlin 2D liquid. Under such conditions the effect of the 2D liquid on the exciton proves to be very strong.

In this paper a magnetoexciton against the background of the incompressible liquid is considered, and some general statements on the spectra of magnetoexcitons valid in the quantum limit are established (for a short summary of data see Ref. 16). There is a considerable difference in the behavior of symmetric systems such as a usual quantum well and of nonsymmetric systems like a skewed well or a double well; it was discussed previously as applied to the interaction of two excitons.[17] The former systems possess a hidden symmetry, due to which the optical spectrum of an exciton is independent of the background against which it appears or disappears, i.e., independent of the number of electrons and holes forming this background. Nevertheless, the independence of frequencies and matrix elements of exciton transitions of the background does not imply the absence of coupling of an exciton to it. On the contrary, this coupling is strong and the effect of it was investigated by us in some detail as applied to an exciton existing against the background of the incompressible quantum liquid, $\nu = \frac{1}{3}$. It actually results in forming a specific polaron due to the dressing of an exciton by MR's. The distinctive feature of the "exciton plus incompressible liquid" system is the existence of restrictions imposed by the Pauli exclusion principle. The polaron effect and these restrictions have an opposite effect on the position of exciton levels and actually compete with each other. For symmetric systems their contributions to the energy cancel out for $\mathbf{k} = \mathbf{0}$ excitons, but the distribution of electron density is strongly perturbed. For nonsymmetric systems the sign of the energy shift is opposite compared to the usual polaron shift. The dispersion law for a dressed exciton in both symmetric and nonsymmetric systems is investigated. In the latter systems the absolute minimum of the exciton energy spectrum shifts from the point $k = 0$ to a circle of radius $k \approx 1/l(H)$ when the asymmetry parameter reaches its critical value, and at this instant the emission spectrum should change abruptly. The remarkable property of dressed excitons is the nearly complete screening of a hole by the electron charge of a medium, and this property persists even for $kl \gg 1$.

II. HAMILTONIAN OF AN ELECTRON-HOLE SYSTEM

We assume in what follows that the widths of confinement layers are small for both electrons and holes and that a magnetic field is strong enough. Therefore all the charge carriers are at the ground Landau level of the lowest subband. There is no band degeneracy, and the level mixing and spin are neglected. Under these conditions the Hamiltonian may be written as

$$H = \tfrac{1}{2}\sum V^{ee}_{1234}\alpha^\dagger_1\alpha^\dagger_2\alpha_3\alpha_4 + \tfrac{1}{2}\sum V^{hh}_{1234}\beta^\dagger_{\bar 4}\beta^\dagger_{\bar 3}\beta_{\bar 2}\beta_{\bar 1} + \tfrac{1}{2}\sum V^{eh}_{1234}\alpha^\dagger_1\beta^\dagger_{\bar 3}\beta_{\bar 2}\alpha_4 + \tfrac{1}{2}\sum V^{eh}_{1234}\beta^\dagger_{\bar 4}\alpha^\dagger_2\alpha_3\beta_{\bar 1}\,. \quad (1)$$

The operators α_f stand for electrons in the conduction band in quantum states f and the operators $\beta_{\bar f}$, for holes in the valence band in the states $\bar f$ which are conjugate to the states f. The matrix elements of the interaction potentials $V_{ee}(r) > 0$, $V_{hh}(r) > 0$ and $V_{eh}(r) < 0$ in the electron-hole system enter the Hamiltonian H, with no restrictions imposed on their relative values. The difference between the values of these potentials may be due to the asymmetric shape of a quantum well (there is some spacing $h \neq 0$ between the confinement planes of electrons and holes), to the difference in the work functions of electrons and holes, to the non-Coulomb part of the interaction, etc. If $V_{ee} = V_{hh} = -V_{eh}$, the system will be termed symmetric.[17] The symmetric model is usually used as a standard one in the theory of 2D systems. It will be shown in what follows that the energy and optical spectra of excitons imbedded in an interacting 2D electron system change drastically when the symmetry condition is violated. Equation (1) includes only the interaction of charge carriers, the terms including the width of the forbidden gap and the Landau quantization energy being omitted.

It is convenient to introduce the operators of collective neutral excitations in the conduction and valence bands which will be termed plasmon operators, the 2D momentum $\mathbf{q}$ may be ascribed to them:

$$a(\mathbf{q}) = \left(\frac{2\pi}{A}\right)^{1/2}\sum_p \alpha^\dagger_{p-\frac{q_y}{2}}\alpha_{p+\frac{q_y}{2}}\exp(ipq_x), \quad (2)$$

$$b(\mathbf{q}) = \left(\frac{2\pi}{A}\right)^{1/2}\sum_p \beta^\dagger_{-p-\frac{q_y}{2}}\beta_{-p+\frac{q_y}{2}}\exp(ipq_x). \quad (3)$$

Here the Landau gauge $A_y = x$, $A_x = A_z = 0$ is used, $l(H)$ is chosen as a unit of the length, and A is a normalization area. For these operators the following conditions hold:

$$a^\dagger(\mathbf{q}) = a(-\mathbf{q}), \qquad b^\dagger(\mathbf{q}) = b(-\mathbf{q}). \quad (4)$$

The plasmon operators have correct transformational properties with respect to the operator $\hat T_{\mathbf{r}}$ of magnetic translations

$$\exp(i\hat T_{\mathbf{r}})\, a(\mathbf{q})\, \exp(-i\hat T_{\mathbf{r}}) = a(\mathbf{q})\, \exp(-i\mathbf{q}\cdot\mathbf{r}), \quad (5)$$

and quite analogously for $b(\mathbf{q})$, but they are not virtually the operators of creation (annihilation) of MR excitations. The following commutation rules hold for them:[18]

$$[a(\mathbf{q}), a(\mathbf{q}')] = -[b(\mathbf{q}), b(\mathbf{q}')] = -2i\left(\frac{2\pi}{A}\right)^{1/2}\sin\frac{(\mathbf{q}\times\mathbf{q}')}{2}\,a(\mathbf{q}+\mathbf{q}')\,, \quad (6)$$

$$[a(\mathbf{q}), b(\mathbf{q}')] = 0\,.$$

The origin of the noncommutativity of these operators is a restriction on the electron density imposed by the Pauli exclusion principle, here

$$(\mathbf{q}\times\mathbf{q}') = q_x q_y' - q_y q_x' \quad . \tag{7}$$

It is also convenient to introduce the operators

$$c(\mathbf{q}) = a(\mathbf{q}) - b(\mathbf{q}), \qquad d(\mathbf{q}) = a(\mathbf{q}) + b(\mathbf{q}). \tag{8}$$

$c(\mathbf{q})$ has a simple physical meaning: it is the Fourier component of the total electric charge density in both the conduction and the valence band. When rewritten in terms of these operators the Hamiltonian is

$$H = H_1 + H_2 \quad , \tag{9}$$

$$H_1 = \frac{1}{4\pi}\sum_{\mathbf{q}}\{\tilde{V}_s(q)c(\mathbf{q})c(-\mathbf{q}) + \tilde{V}_a(q)d(\mathbf{q})d(-\mathbf{q}) + \tilde{v}(q)[c(\mathbf{q})d(-\mathbf{q}) + d(\mathbf{q})c(-\mathbf{q})]\} \quad , \tag{10}$$

$$H_2 = -\tfrac{1}{2}\left\{\tilde{V}_e\hat{N}_e + \tilde{V}_h\hat{N}_h\right\} \quad , \tag{11}$$

$$\tilde{V}_e = \int \frac{d\mathbf{q}}{(2\pi)^2}\tilde{V}_{ee}(q), \qquad \tilde{V}_h = \int \frac{d\mathbf{q}}{(2\pi)^2}\tilde{V}_{hh}(q). \tag{12}$$

Here $\hat{N}_e$, $\hat{N}_h$ are the operators of the number of electrons and holes, and the potentials $V_s(q)$, $V_a(q)$, and $v(q)$ are defined as

$$\begin{aligned} V_s &= (V_{ee} + V_{hh} - 2V_{\rm eh})/4 \quad , \\ V_a &= (V_{ee} + V_{hh} + 2V_{\rm eh})/4 \quad , \\ v &= (V_{ee} - V_{hh})/4 \quad . \end{aligned} \tag{13}$$

The tilde sign over all the potentials in (10)–(12) indicates that they are multiplied by a factor $\exp(-q^2/2)$, for the Coulomb potential $V(q) = 2\pi/q$. It follows from (6) and (9)–(11) that $a(\mathbf{q})$ and $b(\mathbf{q})$ do not commute with H at $q \neq 0$. Therefore, if ψ is an eigenfunction of the Hamiltonian H, $a(\mathbf{q})\psi$ is not its eigenfunction. This is in agreement with the above statement that $a(\mathbf{q})$ are not the creation operators of real elementary excitations, they form only a complete basis in the Hilbert space.

For a symmetric system

$$V_s = V_{ee} \quad , \qquad V_a = v = 0 \quad . \tag{14}$$

Equations (9)–(11) for H written in terms of the operators $c(\mathbf{q})$ and $d(\mathbf{q})$ are especially convenient for investigating optical transitions since the commutation rules of the exciton operators with these two operators are rather different (see Sec. III).

The system of the operators $a(\mathbf{q})$ and $b(\mathbf{q})$ is redundant in the Hilbert space in the sense that different excited states which may be obtained from the ground state by the successive operation of several operators $a(\mathbf{q})$ and $b(\mathbf{q})$ are linearly dependent. There exist some simple identities by which this redundancy may by manifested, e.g.,

$$\sum_{\mathbf{q}}\mathbf{q}\,f(\mathbf{q})\,a(\mathbf{q})\,a(-\mathbf{q}) = \sum_{\mathbf{q}}\mathbf{q}\,g(\mathbf{q})\,b(\mathbf{q})\,b(-\mathbf{q}) = 0 \quad ; \tag{15}$$

they hold for arbitrary even functions $f(\mathbf{q}) = f(-\mathbf{q})$, $g(\mathbf{q}) = g(-\mathbf{q})$.

III. EXCITON OPERATORS: HIDDEN SYMMETRY

The operator of annihilation of an exciton $\mathcal{A}(\mathbf{q})$ may be obtained from $b(\mathbf{q})$ by the substitution $\beta^{\dagger}_{-p-q_y/2} \to \alpha_{p+q_y/2}$

$$\mathcal{A}(\mathbf{q}) = \left(\frac{2\pi}{A}\right)^{1/2}\sum_{p}\alpha_{p+\frac{q_y}{2}}\beta_{-p+\frac{q_y}{2}}\exp(ipq_x) \quad . \tag{16}$$

It has the same transformational properties as the operators $a(\mathbf{q})$ and $b(\mathbf{q})$ do. The commutation rules have the form

$$[c(\mathbf{q}), \mathcal{A}(\mathbf{k})] = 2i\left(\frac{2\pi}{A}\right)^{1/2}\sin\frac{(\mathbf{k}\times\mathbf{q})}{2}\,\mathcal{A}(\mathbf{q}+\mathbf{k}) \quad , \tag{17}$$

$$[d(\mathbf{q}), \mathcal{A}(\mathbf{k})] = -2i\left(\frac{2\pi}{A}\right)^{1/2}\cos\frac{(\mathbf{k}\times\mathbf{q})}{2}\,\mathcal{A}(\mathbf{q}+\mathbf{k}) \quad . \tag{18}$$

One can use (9)–(11) and (16) to find the energy of an exciton in an empty crystal, i.e., of a bare exciton $\mathcal{A}(\mathbf{k})\,|0\rangle$:[19–21,17]

$$\begin{aligned} E_{\rm ex}(\mathbf{k}) &= \int \frac{d\mathbf{q}}{(2\pi)^2}\tilde{V}_{\rm eh}(q)\cos(\mathbf{k}\times\mathbf{q}) \\ &= -\left(\frac{\pi}{2}\right)^{1/2} I_0(k^2/4)e^{-k^2/4} \quad , \end{aligned} \tag{19}$$

where $I_0(x)$ is the modified Bessel function, and the last expression holds when $V_{\rm eh}(q) = -2\pi/q$. The eigenfunctions of a bare exciton in the configurational representation are

$$\phi^{\rm ex}_{\mathbf{k}}(\mathbf{r},\mathbf{R}) = (2\pi A)^{-1/2}\exp\left[-\frac{\mathbf{r}^2}{4} - \frac{(\mathbf{r}\times\mathbf{k})}{2} - \frac{\mathbf{k}^2}{4} + iX(k_y - y) + ik_yY\right] \quad , \tag{20}$$

where $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$, $\mathbf{R} = (\mathbf{r}_e + \mathbf{r}_h)/2$, and $\mathbf{r}_e$ and $\mathbf{r}_h$ are the coordinates of an electron and a hole.

For symmetric systems an important consequence follows from (10)–(14). Since the operator $\mathcal{A}^\dagger(\mathbf{k}=0)$ commutes with all the operators $c(\mathbf{q})$, for a symmetric system

$$[H_1, \mathcal{A}^\dagger(\mathbf{k}=0)] = 0 \quad . \tag{21a}$$

The operator H_2 does not commute with $\mathcal{A}^\dagger(\mathbf{k})$ since

$$[\hat{N}_e, \mathcal{A}^\dagger(\mathbf{k})] = [\hat{N}_h, \mathcal{A}^\dagger(\mathbf{k})] = \mathcal{A}^\dagger(\mathbf{k}) \quad . \tag{21b}$$

However, it is immaterial since H_2 only shifts the energy of a system by $(\tilde{V}_e + \tilde{V}_h)/2$ per exciton added to it and does not change its wave function. Therefore, if ψ is the wave function for an initial, "small," system, then

$$\Psi = \mathcal{A}^\dagger(\mathbf{k}=0)\,\psi \tag{22}$$

is the exact wave function for a "large" system including one additional exciton, i.e., an additional electron-hole pair. The energies of these states are interrelated by the relation

$$E = \varepsilon - (\tilde{V}_e + \tilde{V}_h) = \varepsilon + E_{\mathrm{ex}}(\mathbf{k}=0) \tag{23}$$

as follows from (11)–(13) and (19), and also from the symmetry condition (14). Thus the energies of the states Ψ and ψ differ only by $E_{\mathrm{ex}}(\mathbf{k}=0)$ which is the energy of a bare exciton, i.e., the existence of an arbitrary electron-hole background, neutral or charged, described by the function ψ does not influence the energy of the exciton with momentum $\mathbf{k}=0$. This statement generalizes the well-known result that the scattering amplitude of two excitons tends to zero when the momenta of both of them tend to zero, $\mathbf{k} \to \mathbf{0}$.[19,22] It is necessary to stress that the zero shift of the $\mathbf{k}=0$ level does not mean the absence of interaction. It will be shown in Secs. IV and VI that the interaction strongly affects the distribution of the electron density.

If the functions ψ and Ψ are written in the configurational representation, the operation of the operator $\mathcal{A}^\dagger(\mathbf{k}=0)$ on ψ results in its multiplication by the wave function of a bare exciton $\phi^{\mathrm{ex}}_{\mathbf{k}}(\mathbf{r}_e|\mathbf{r}_h)$, with $\mathbf{k}=0$, with the subsequent antisymmetrization of the product. The functions Ψ obtained by Eq. (22) will be termed multiplicative. Despite the fact that they make up only a small part of all the quantum states of the large system, they play a very important role in the optical phenomena since the interaction of electrons with light is described by the operators $\mathcal{A}(\mathbf{k}=0)$, $\mathcal{A}^\dagger(\mathbf{k}=0)$. Therefore, only the transitions to multiplicative states, or the transitions from them, are allowed in optical spectra, and the frequencies of optical transitions are equal to the frequency of the exciton transition $E_{\mathrm{ex}}(\mathbf{k}=0)$ in an empty crystal irrespective of ν. Of course, this statement holds only for perfect symmetric systems. The independence of the transition frequency of ν is somewhat like the Kohn theorem.[23] The very important difference is that for intraband transitions this frequency coincides with the cyclotron frequency which is independent of the electron-electron interaction,[23] while for interband transitions it equals the exciton frequency (19) which is shifted by the Coulomb attraction between an electron and a hole.

The conditions (21a) and (21b) reflect the existence of hidden symmetry inherent in symmetric systems. They impose strict restrictions on both the energy and the optical spectra.

The criterion (14) must be fulfilled when a symmetric system is of the general type. It may be softened when holes are absent. In this case the criterion may be found from the condition

$$[\mathcal{A}^\dagger(0), H_1]\ |\mathrm{el}\rangle \propto \mathcal{A}^\dagger(0)\ |\mathrm{el}\rangle \quad , \tag{24}$$

where $|\mathrm{el}\rangle$ is an arbitrary purely electronic state. It has the form

$$V_{ee} + V_{\mathrm{eh}} = 0 \quad , \tag{25}$$

as might be expected. If it is satisfied, then $\mathcal{A}^\dagger(0)\,|\mathrm{el}\rangle$ is the eigenfunction for a large system provided the $|\mathrm{el}\rangle$ is the eigenfunction for a small system.

The above results hold for an exciton with $\mathbf{k}=0$ in a symmetric system. Any generalization of them involves considerable difficulties. Some of them have the same nature as in the spin-wave problem and are associated with the kinematic interaction which was discussed in detail by Dyson.[24] Its origin in the problem under consideration lies in the existence of the Fermi limit $n_0 = 1/2\pi$ for the electron density. This restriction is imposed by the Pauli exclusion principle. An additional and very complicated problem arises from the nonconservation of the number of MR's, which leads to a polaron effect. Under these conditions the kinematical interaction proves to be highly significant as distinct from the spin-wave scattering problem,[24] since it limits the electron density induced around an exciton dressed by MR's. Since the problem is highly involved and hardly permits an analytical treatment, and also that it includes several different aspects which must be clarified, we widely use in what follows the results of numerical computations performed for an exciton existing against the background of the incompressible liquid, $\nu = \frac{1}{3}$. They are combined with the results based on symmetry arguments.

IV. SPHERICAL GEOMETRY

This approach was developed by Haldane in Ref. 4 where a hierarchical scheme for incompressible quantum liquids was proposed. In this geometry a magnetic monopole resides at the center of a sphere, a magnetic flux through it, expressed in units of the flux quantum $\phi_0 = hc/e$, is an integer $2S$, and the radius of the sphere equals $R = S^{1/2}$ in units of $l(H)$. The spherical geometry is the most adequate in spectroscopic applications, since the continuous rotation group and the associated selection rules are retained. The computational technique was developed in Ref. 25 and described in detail in Ref. 26 (for reviews see Refs. 5 and 6).

In the spherical geometry quantum states are classified by eigenvalues $L(L+1)$ and L_z of the square of the

angular momentum $\hat{L}^2$ and its projection, and all the levels are $2L+1$ fold degenerate. The spin of the particles, electrons and holes, equals S. The eigenfunctions of electrons are polynomials

$$e_m(\omega) = \left\{ \frac{2S+1}{4\pi} \binom{2S}{S+m} \right\}^{1/2} u^{S+m} v^{S-m} \quad , \tag{26}$$

where $\binom{2S}{S+m}$ are the binomial coefficients, $-S \le m \le S$, u and v are components of a spinor

$$u = \cos(\vartheta/2) e^{i\varphi/2}, \quad v = \sin(\vartheta/2) e^{-i\varphi/2} \quad , \tag{27}$$

ϑ and φ are spherical coordinates, and ω is a unit vector on the sphere. The eigenfunctions of holes are obtained from e_m by the time inversion

$$g_{-m} = (-1)^{S-m} e_m^* \quad . \tag{28}$$

The connection between the spherical and plane geometries is established by the relation $L = kR = kS^{1/2}$ which relates the momentum k and the angular momentum L.

The eigenfunction of an exciton with quantum numbers LM is

$$\phi^{\rm ex}_{LM}(\omega_e|\,\omega_h) = \sum_{m_1 m_2} \langle Sm_1\ Sm_2|LM\rangle\, g_{m_1}(\omega_h)\, e_{m_2}(\omega_e), \tag{29}$$

where $\langle Sm_1\ Sm_2|LM\rangle$ are the Clebsch-Gordan coefficients. If one introduces the annihilation operators α_m and $(\alpha_v)_m$ for the states e_m in the conduction and valence band, respectively, and the hole operators $\beta_m = (\alpha_v)^\dagger_{-m}$, then the annihilation operator of an exciton is

$$\mathcal{A}_{LM} = \sum_{m_1 m_2} (-1)^{m_1} \langle Sm_1\ Sm_2|LM\rangle\, \beta_{m_1} \alpha_{m_2} \quad . \tag{30}$$

By analogy, plasmon operators for the conduction and valence bands may be introduced

$$a_{LM} = \sum_{m_1 m_2} (-1)^{m_1} \langle Sm_1\ Sm_2|LM\rangle\, \alpha^\dagger_{-m_1} \alpha_{m_2} \quad , \tag{31}$$

$$b_{LM} = \sum_{m_1 m_2} (-1)^{m_1} \langle Sm_1\ Sm_2|LM\rangle\, \beta^\dagger_{-m_2} \beta_{m_1} \quad , \tag{32}$$

$a_{LM} = (-1)^M a^\dagger_{L\,-M}$. If one introduces the operators $c_{LM} = a_{LM} - b_{LM}$ and $d_{LM} = a_{LM} + b_{LM}$ similarly to (8), then the Hamiltonian takes the form analogous to (9)–(12). It is important that only the c_{LM} operators enter into H_1 for symmetric systems, quite analogously to (10). One can easily check that the following commutation rules hold:

$$[c_{LM}\,,\mathcal{A}_{L'M'}] = \sum_{L''} F\,(LM\ L'M'|\ L''\ M+M')\,\mathcal{A}_{L''M+M'}, \tag{33}$$

$$[a_{LM}\,,a_{L'M'}] = \sum_{L''} F\,(LM\ L'M'|\ L''\ M+M')\,a_{L''M+M'}, \tag{34}$$

where

$$\begin{aligned} F\,(LM\ L'M'|\ L''\ M+M') &= (-1)^{1+S+M+M'}[1-(-1)^{L+L'+L''}] \\ &\times (2L+1)^{1/2}(2L'+1)^{1/2}(2L''+1)^{1/2} \\ &\times \begin{pmatrix} L & L' & L'' \\ M & M' & -(M+M') \end{pmatrix} \begin{Bmatrix} L & L' & L'' \\ S & S & S \end{Bmatrix} \quad . \end{aligned} \tag{35}$$

Here the parenthesis denotes the $3j$ Wigner coefficient and the brace the $6j$ Racah coefficient.

These equations are important since they enable one to express in terms of the spherical model the analogs of Eq. (21) and of the statement about the absence of a level shift for $\mathbf{k} = 0$ excitons. It follows from (35) that the square bracket becomes zero if $L = 0$ or $L' = 0$. Therefore, the interaction does not shift the energy levels of the $L = 0$ excitons in a symmetric system.

The coefficients (35) go to zero also when $L = L'$, $M = M'$. In fact, if $2L+L''$ is an even integer the square bracket equals zero, and if it is an odd integer the $3j$ coefficient equals zero since it acquires a factor $(-1)^{2L+L''}$ with the permutation of two identical columns.

We have made most of the computations for a system consisting of 5 electrons and 1 hole ($5e+1h$) for $2S = \nu^{-1}(N-1) = 9$, i.e., for $\nu = \frac{1}{3}$ in the final state where there are $N = 4$ electrons and no holes. In order to check computer programs, numerous results for multielectron systems obtained by different authors have been reproduced.

In Fig. 1 the energy spectrum of a symmetric system

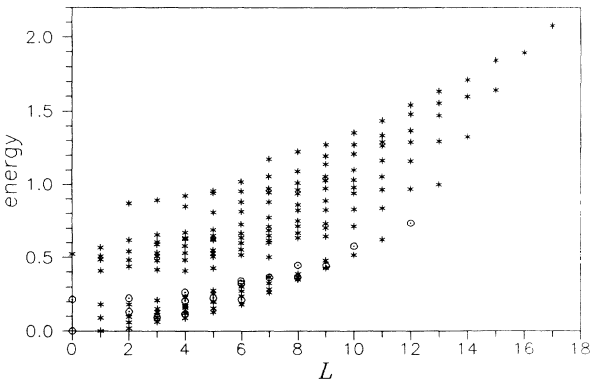

FIG. 1. The energy spectrum of a symmetric system $5e+1h$. The multiplicative states are shown by circles, the other states by asterisks.

is shown for all values of L. It consists of two parts separated by a gap having a width of about 0.2 (in units of $e^2/\epsilon l$). The gap opens since the pseudopotential $V_0 < 0$ (in the sense of Ref. 25), which is the largest in magnitude among all the pseudopotentials, corresponds to the electron-hole attraction. The gap separates the states, in which this strong bond is not broken, from those in which it is broken. It has a scale of $E_{\rm ex}(\mathbf{k}=0)$ which is determined by V_0. This statement is supported by the dependence of the gap width on the system parameters. It increases when $|V_0|$ rises as compared to its value for a Coulomb potential, and rapidly decreases when the spacing h between the confinement planes of electrons and holes increases; it is altogether absent at $h \approx 0.5$ (see also Sec. VI). When $|V_0|$ is large enough the arrangement of levels under the gap is nearly independent of $|V_0|$. In Fig. 1 multiplicative states are marked. All of them belong to the lower part of the spectrum and are located under the gap.

Let us introduce the function

$$\rho_{\mathbf{k}}(\mathbf{r},\mathbf{r}_h) = \int |\Psi_{\mathbf{k}}(\mathbf{r}_1,\ldots,\mathbf{r}_{N+1}|\,\mathbf{r}_h)|^2 \times \sum_{i=1}^{N+1} \delta(\mathbf{r}_i-\mathbf{r})d\mathbf{r}_1\cdots d\mathbf{r}_{N+1} \quad , \qquad (36)$$

where the summation is performed over all the electrons. It describes the distribution of the electron density for a fixed position of the hole $\mathbf{r}_h$. In the spherical geometry $\mathbf{r}\to\omega(\vartheta,\varphi)$, $\delta(\mathbf{r})\to\delta(\omega)/S$, $\mathbf{k}\to(L,M)$ and the Fermi limit is $n_0(S)=(2S+1)/4\pi S$. In Fig. 2 this function is displayed for a symmetric system for $L=0$, $\vartheta_h=0$, i.e., when the hole resides at the North Pole. Three curves shown in it correspond to the three states of the $5e+1h$ system with $L=0$, and their energies may be found in Fig. 1. For comparison the same function is shown for a bare exciton, in the plane limit it equals $\exp(-r^2/2)$. It is seen that even for multiplicative states 1 and 2 the density distribution is quite different from that for a bare exciton. Therefore, the zero level shift of the $\mathbf{k}=0$ level does not imply the absence of interaction, as has been mentioned above (Sec. III). The necessity of a strong redistribution of the electron density becomes obvious if one finds its value, for a bare exciton, at the point where a hole resides. It equals

$$\rho_{\rm ex}(k) = \int |\phi^{\rm ex}_{\mathbf{k}}(\mathbf{r}=0\,,\mathbf{R})|^2\,d\mathbf{R} = \frac{1}{2\pi}\exp(-k^2/2) \quad , \qquad (37)$$

as follows from (20). In the plane geometry $\rho_{\rm ex}(k) = n_0 = 1/2\pi$ for $k=0$, and in the spherical geometry $\rho^{\rm ex}_{LM} = n_0(S)$ for $L=0$, i.e., in both cases it is equal to the Fermi limit. A minimum in the electron density is seen on curve 1, then the density saturates. The existence of oscillations is typical of the screening of electric charge by the incompressible liquid.[27,28,3] Oscillations on curves 2 and 3 corresponding to the excited states of the liquid are much stronger, and saturation is only expected at large distances exceeding the diameter of a sphere.

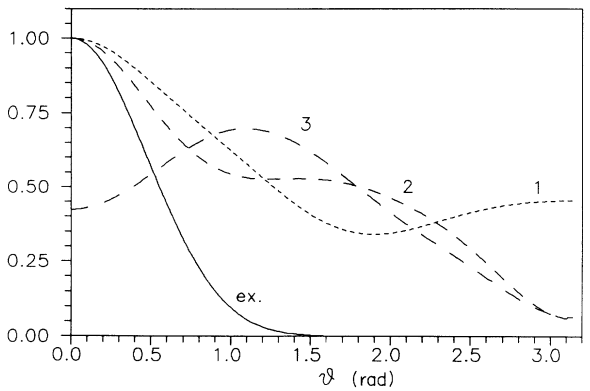

FIG. 2. Angular distribution of the electron density. The function $\rho_{LM}(\vartheta,\vartheta_h=0)$ [Eq. (36)] is shown, i.e., the hole resides at the North Pole. The density is normalized to $n_0(S)$. Curves 1, 2, and 3 correspond to the three states with $L=0$ (Fig. 1), their energies $E_1 < E_2 < E_3$. The curve ex. represents a bare exciton.

V. DISPERSION LAW OF A DRESSED EXCITON

The set of minimum values of the energy for all values of L considered as a function of $L=\sqrt{S}k$ represents the dispersion law $E^*_{\rm ex}(k)$ of an exciton against the background of the liquid, at least for small values of k. This dispersion law, constructed for a symmetric system by using the data of Fig. 1, is shown in Fig. 3(a). For comparison the dispersion law of a bare exciton $E_{\rm ex}(L)$ and that of MR's in the absence of excitons $\omega_{\rm mr}(L)$ are shown.

Conspicuous is a strong suppression of the exciton dispersion, i.e., a strong and rather specific polaron effect. As distinct from the usual pattern, in virtue of Eqs. (21) the level $k=0$ is not shifted, but the dispersion for a dressed exciton turns out to be much weaker than for a bare one. It is striking that the $E^*_{\rm ex}(L)$ values for $L=0$ and $L=1$ practically coincide, their difference being about 10^{-4}. It is at least an order of magnitude less than it follows from the square law interpolation between the points $L=0$ and 2. This coincidence is not accidental. With changes in the number of particles ($N=3$ or 5), and also in the interaction potential [the $(r^2+\alpha^2)^{-1/2}$ law with different values of α instead of the Coulomb law] the coincidence of the energies of these two states persists. However, it disappears as soon as the symmetry condition (25) is violated and simultaneously a level shift at $k=0$ appears [Fig. 3(b)]. These data prompt one to conjecture that for symmetric systems the dispersion law of a dressed exciton does not include k-square terms.

The exciton states rapidly change its nature with increasing L. Let us consider a sequence of normalized trial functions $(1-\nu)^{-1/2}\mathcal{A}^{\dagger}_{LM}|{\rm L}\rangle$, where $|{\rm L}\rangle$ is the eigenfunction of a Laughlin liquid. The function with $L=0$ is exact, but the scalar products of trial and exact functions with given L,M rapidly decrease when L increases. They equal 0.91 for $L=1$, 0.74 for $L=2$, and 0.58 for $L=3$ (all the data for $N=4$ and $\nu=\frac{1}{3}$). At

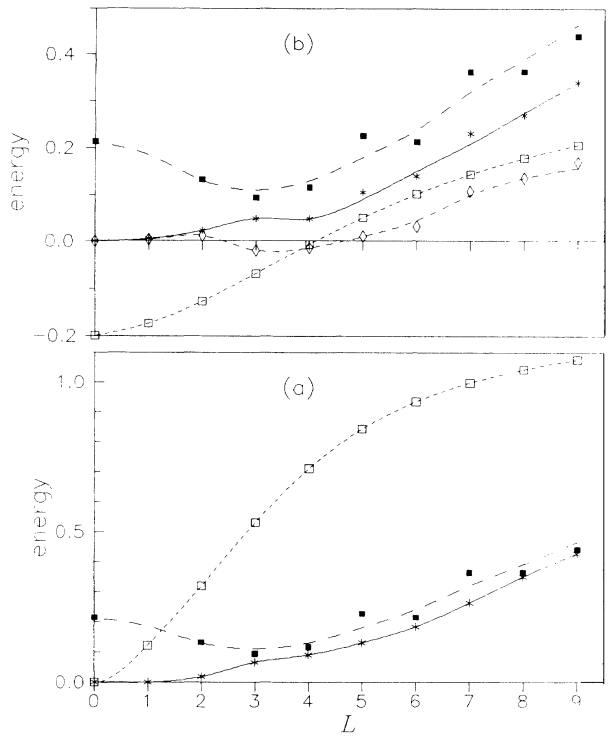

FIG. 3. Energy spectrum of excitons and magnetorotons. □: a bare exciton; ∗: a dressed exciton; ■: magnetorotons. The curves are only guides for the eye. (a) Symmetric system; (b) nonsymmetric system, $h = 0.5$, in the latter case the MR energy is plotted from the bottom of the spectrum of a dressed exciton. By diamonds ◇ the dispersion law of a dressed exciton is shown for $h = 1.0$.

$L \approx 3$, which is actually the threshold for the MR emission, $E^*_{\rm ex}(k) \approx \omega_{\rm mr}(k)$, a considerable change in the shape of the curve $E^*_{\rm ex}(k)$ occurs. Such a point may be the termination point of the exciton spectrum. The specific feature of the problem under consideration is that the matrix element of the exciton scattering depends not only on momentum transfer as is usually the case in the polaron problem. It depends also on the momentum of the final state, and this dependence is critical near the threshold since the exciton with $k = 0$ does not interact with MR's. Usually under the conditions of a weak coupling such suppression of the interaction promotes the appearance of a termination point, without any singularity or with a weak singularity in it.[29] However, there exists another competing possibility, namely the appearance of bound states of an exciton with a MR;[29] the weak exciton dispersion near $k = 0$ favors it. Since $E^*_{\rm ex}(L) < \omega_{\rm mr}(L)$ for all values of L and for $L \geq 3$ both curves are nearly parallel, it is plausible that the latter possibility is realized, then the spacing between the curves has the meaning of binding energy. In such states nearly all the momentum is transferred by MR's, the momentum of an exciton being small.

In order to check the validity of this model of bound states, it is convenient to introduce in the Hilbert space of a large system the basis set of functions obtained as the direct product of the eigenfunctions $\phi^{\rm ex}_{L'M'}$ of a bare exciton and of the eigenfunctions $\psi_{L''M''}$ for a small system. From them one can construct antisymmetric functions $f^{L'}_{LM}$ belonging to the total angular momentum L and including the bare exciton in different states having angular momentum L'. The problem is that the functions $f^{L'}_{LM}$ with different values of L' are not orthogonal. We orthogonalized this set by the usual iterative procedure starting from small L' values. Calculations show that the projections of the exact exciton states Ψ_{LM} onto these new $f^{L'}_{LM}$ functions with $L' = 1$ and 2 provide the dominant contribution to the normalization integral for $L \geq 4$. For instance, this contribution equals 0.71 for $L = 4$, 0.76 for $L = 5$, 0.81 for $L = 6$, and 0.90 for $L = 7$. The contributions coming from $L' = 1$ and 2 have comparable magnitudes. The state $L' = 0$ makes no contribution since it is not coupled to MR's. These data support the proposed model of bound states. We believe that all the above-mentioned general conclusions are correct despite the fact that the accuracy of the dispersion laws of excitons and MR's is low in the high-L region because of a small number of particles in the system.

The existence of bound states "exciton plus MR" is also suggested by the analysis of the energy spectrum at small angular momenta $L \leq 3$, where stable dressed excitons definitely exist. The minimum MR frequency (roton minimum) is $\omega_{\rm mr} \approx 0.1$ [Fig. 3(a)]. However, the first excitation energy for the states with $L = 2$ is approximately twice as small (Fig. 1). It is possible only if a bound state of a MR with an exciton is formed, it may be considered as a local MR mode. Such states are well known for different strongly coupled systems, e.g., 3D and 1D polarons and solitons.[30,31]

In Fig. 3(b) a typical energy spectrum of an asymmetric system is shown. In computations the spacing between an electron and a hole was chosen as $(R^2|\omega_e - \omega_h|^2 + h^2)^{1/2}$, hence $V_{ee} > |V_{\rm eh}|$. It is seen that the level $L = 0$ is shifted, but the sign of the shift is opposite to that of the usual polaron shift. The reason for this can be understood if one takes it into account that two competing effects contribute to the shift. One contribution comes from the polaron dressing of an exciton by MR's. The other originates from the Pauli exclusion principle, since an exciton appears against the background of the liquid, the electrons of which fill a considerable part of the phase space. The latter contribution is somewhat allied to the Burstein-Moss shift. It has been mentioned at the end of Sec. IV that the density of an electron cloud in a bare exciton is high at $\mathbf{r}_e = \mathbf{r}_h$ and it coincides with n_0. Therefore restrictions imposed by the Pauli exclusion principle are very stringent. In symmetric systems the shifts which are due to the polaron effect and to the Pauli exclusion principle cancel out exactly at $k = 0$. In nonsymmetric systems the latter contribution predominates and determines the sign of the level shift at $k = 0$.

The energy values for $L = 0$ and $L = 1$ differ markedly, which is indicative of the appearance of a k-square term in the dispersion law. The binding energies of the states with $L \geq 4$ are considerably larger than for $h = 0$, which may be attributed to the fact that the projections of the functions Ψ_{LM} onto the functions $f^{L'=0}_{LM}$ make substantial contributions to the normalization integrals. At

$h \approx 0.5$ an inflection point on the curve $E^*_{ex}(L)$ arises and for higher values of h an extra minimum exists. At $h = h_{cr} \approx 0.85$ this minimum becomes the absolute one, i.e., the bottom of the exciton band shifts from the point $k = 0$ to a circle. In this region of h values the simplest model of a dressed exciton is a neutral entity consisting of a hole screened by two quasielectrons and a third quasielectron orbiting around this complex. The distribution of the screening charge at the level $L = 3$, when described in terms of Fig. 2, changes strongly for $h > 0.5$. The charge density $\rho_L(\vartheta, \vartheta_h = 0)$ averaged over all values of M [cf. Eq. (36)] drops at $\vartheta = 0$, the charge distribution becomes wider, and the total amount of the screening charge accumulated in the Northern Hemisphere is close to $\frac{2}{3}$ for $h \approx 0.85$. Unfortunately, the size of the sphere was too small to specify the distribution of the third quasielectron. The shift of the band minimum has a considerable effect on the emission spectra (Sec. VII). The instability of the $\mathbf{k} = 0$ state closely resembles the appearance of a charge-density wave in a double quantum well when h reaches some critical value which is nearly the same as h_{cr} for an exciton (see Refs. 32–34 and the references therein).

VI. SCREENING OF A HOLE CHARGE

Figures 2 and 3(a) make it clear how drastically the properties of excitons are changed by the joint effect of the polaron dressing and the Pauli exclusion principle. In this section we shall define and investigate one more function describing the distribution of electrons, namely, the electron density at the point where a hole resides. Let us define the function:

$$\Phi_{\mathbf{k}}(\mathbf{r}_h) = \int |\Psi_{\mathbf{k}}(\mathbf{r}_1, \cdots, \mathbf{r}_{N+1} | \mathbf{r}_h)|^2 \times \sum_{i=1}^{N+1} \delta(\mathbf{r}_i - \mathbf{r}_h) d\mathbf{r}_1 \cdots d\mathbf{r}_{N+1} \quad . \tag{38}$$

For it the following inequality holds:

$$\rho(k) = \int \Phi_{\mathbf{k}}(\mathbf{r}_h) d\mathbf{r}_h \leq n_0 = \frac{1}{2\pi} \quad , \tag{39}$$

which reflects the existence of the Fermi limit n_0. The hole density is

$$n_{\mathbf{k}}(\mathbf{r}_h) = \int |\Psi_{\mathbf{k}}(\mathbf{r}_1, \ldots, \mathbf{r}_{N+1} | \mathbf{r}_h)|^2 d\mathbf{r}_1 \cdots d\mathbf{r}_{N+1}, \tag{40}$$

and the reduced electron density on a hole is

$$\rho_{\mathbf{k}}(\mathbf{r}_h) = \Phi_{\mathbf{k}}(\mathbf{r}_h)/n_{\mathbf{k}}(\mathbf{r}_h) \quad . \tag{41}$$

It characterizes the screening degree of the hole charge. For a homogeneous system the $\Psi_{\mathbf{k}}$, $n_{\mathbf{k}}$, and $\rho_{\mathbf{k}}$ only depend on k and are independent of $\mathbf{r}_h$. For a bare exciton the $\rho_{\mathbf{k}}$ coincides with $\rho_{ex}(k)$, Eq. (37).

It is instructive, using the spherical geometry, to investigate the angular dependence of the functions $\rho_{LM}(\omega_h)$ defined similarly to $\rho_{\mathbf{k}}$, for different quantum states L, M. It turns out that for a symmetric system the functions $\rho_{LM}(\omega_h)$ are quite different for two groups of states, those lying under the gap and above it (Sec. IV). For all the states belonging to the former group $\rho_{LM}(\vartheta)/n_0(S) \approx 1$, their difference does not exceed $\approx 1\%$ for all values of ϑ. On the contrary, for the states belonging to the latter group the difference $1 - \rho_{LM}/n_0(S)$ is large and strongly depends on ϑ. For instance, in Figs. 4(a) and 4(b) are shown the data for the states with $M = 0$; for them $\rho_{L0}(\pi - \vartheta) = \rho_{L0}(\vartheta)$. Curves 1 and 3 of Fig. 4(a) are typical of all the states lying under the gap. For them $\rho_{LM} \approx n_0(S)$; it only weakly depends on ϑ and differs drastically from the analogous curve for a bare exciton having the same quantum numbers L and M. Curves 4 and 5 are typical of the states lying above the gap: $\rho_{LM}(\vartheta)$ considerably deviates from $n_0(S)$ and strongly depends on ϑ. This behavior remains unchanged up to $L = 12$, i.e., the largest value of L for which the gap exists [Fig. 4(b)]. Therefore, for symmetric systems the screening is practically complete for all the states lying under the gap, for all values of L, M and also of ϑ. However, the situation changes rapidly for nonsymmetric systems when h increases. For instance, curve 3 in

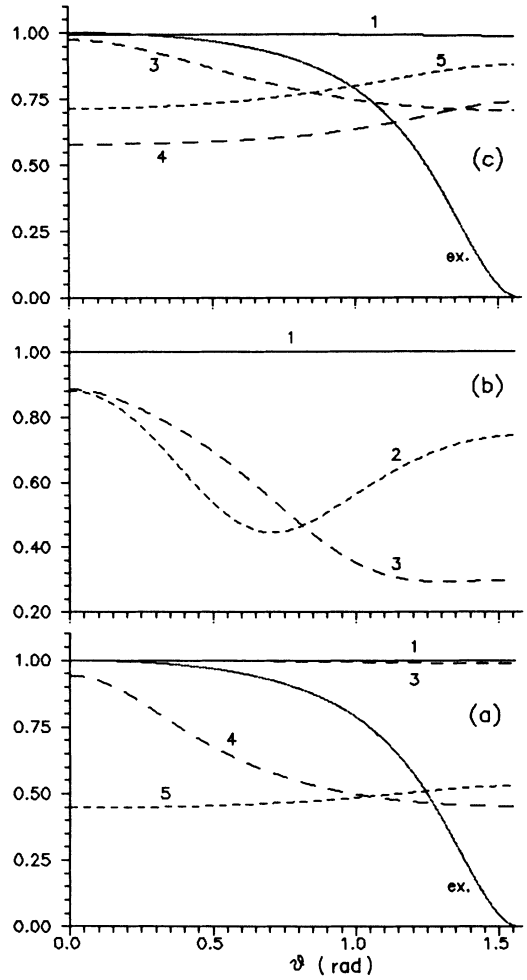

FIG. 4. Screening of the hole charge. The function $\rho_{LM}(\vartheta)/n_0(S)$ [Eq. (41)] for a system with $N = 4$. Numbers by the curves are sequential numbers of the levels with given values of L, $M = 0$; ex.: a bare exciton with the same values of L, $M = 0$. (a) $h = 0$, $L = 1$; (b) $h = 0$, $L = 12$; (c) $h = 0.5$, $L = 1$.

Fig. 4(c) strongly deviates from $n_0(S)$; it has an absolutely different shape than curve 3 in Fig. 4(a).

From calculations performed in the spherical geometry one can find the quantities

$$\rho_L = \sum_M \Phi_{LM}(\omega_h) / \sum_M n_{LM}(\omega_h) = 4\pi \sum_M \Phi_{LM}(\omega_h). \tag{42}$$

The latter sum over M is independent of ω_h since the sum of the δ functions entering (38) is a spherical tensor of zero rank. Of course, $\rho_L \to \rho_k$ in the limit $S \to \infty$, $k = L/R$. For a bare exciton expression (42) may be easily calculated. In this case the wave function (29) equals $\phi^{\rm ex}_{LM}(\omega|\omega) \propto Y_{LM}(\vartheta, \varphi)$ at coincident values of the arguments, $\omega_e = \omega_h = \omega$, where Y_{LM} is a spherical function. This expression determines the angular dependence of $\Phi_{LM}(\omega)$, and the ρ_L is

$$\rho_{\rm ex}(L) = n_0(S) \frac{(2S+1)!\,(2S)!}{(2S+L+1)!\,(2S-L)!}, \tag{43}$$

in the limit $S \to \infty$ it becomes $\rho_{\rm ex}(k)$ [Eq. (37)]. Calculations performed for a symmetric system with $N = 4$ show that for the states lying under the gap the scale of deviations of ρ_L from $n_0(S)$ does not exceed 1%. The equation $\rho_L = n_0(S)$ is exact for all the multiplicative states as straightforward but somewhat cumbersome calculations show.

For the system under consideration any modeling of quantum states is much more complicated than for the usual polaron problem since in the latter one electrons and phonons are independent particles, while in the former one both excitons and MR's are formed by electrons belonging to the same Fermi sea. It is worth mentioning in this connection that a strong screening of a hole in a symmetrical system, which has been established above, is in agreement with the model of a bound state of a MR with a slow exciton (Sec. IV), since the screening in the exciton is strong only when k is small [Eq. (37)].

VII. OPTICAL SPECTRA

Due to hidden symmetry, Eq. (21), inherent in symmetric systems, only multiplicative states are exhibited in optical spectra, and the energies of these states at $\mathbf{k} = 0$ depend solely on the energy of a bare exciton [cf. Eq. (23)]. Therefore, of much more interest are the spectra at $h \neq 0$. For calculating the intensities of optical transitions the equation

$$I_{LM} \propto \left| \int \Psi^*_{LM}(\mathbf{r}_1, \ldots, \mathbf{r}_N, \mathbf{r}_h | \mathbf{r}_h) \psi_{LM}(\mathbf{r}_1, \ldots, \mathbf{r}_N)\, d\mathbf{r}_1 \cdots d\mathbf{r}_N d\mathbf{r}_h \right|^2 \tag{44}$$

was used, which provides a correct description of the 2D properties, but does not include the dependence of the probability of vertical tunneling on L, M.

In Fig. 5 the basic parameters of the emission spectrum are shown as a function of h. It is seen that the spectrum changes greatly at $h = h_{\rm cr} \approx 0.85$. This phenomenon is caused by a change in the energy spectrum described in Sec. V. When the absolute minimum of the exciton spectrum $E^*_{\rm ex}(k)$ is at the point $k = 0$, the optical transition from this state to the zero-roton final state is allowed and the band corresponding to it predominates in the emission spectrum. Yet for $h > h_{\rm cr}$ the zero-roton transition is forbidden and the shake-up transition to a single-roton state predominates. Therefore, at $h = h_{\rm cr}$ the position of the emission band $\overline{\omega}$ (its first moment) changes abruptly, and its intensity drops by an order of magnitude. Its width γ, the square root of the second moment, also changes. Because of a small number of particles in the final state, $N = 4$, the accuracy of numerical results is not high, but the existence of discontinuities at $h = h_{\rm cr}$ is the general property of emission spectra. On the contrary, the absorption spectra do not show any singularities at this point, and the zero-roton transition to the $k = 0$ state predominates in them. An analogous behavior may also be expected in some extrinsic spectra as a function of the distance h from the impurity center to the confinement layer.[12]

The dependence of the position of the emission band on S, i.e., on ν^{-1}, is shown in Fig. 6. It shows an ir-

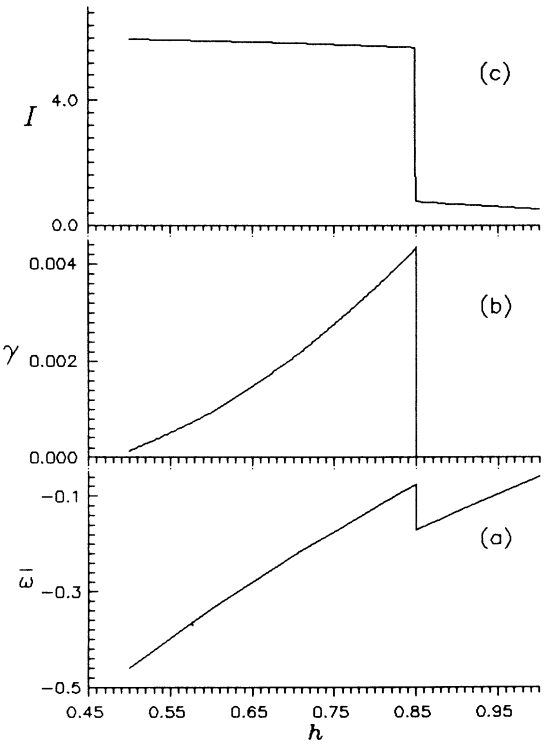

FIG. 5. Basic parameters of the exciton emission band as functions of h. (a) The position of the band; (b) the bandwidth; (c) the intensity.

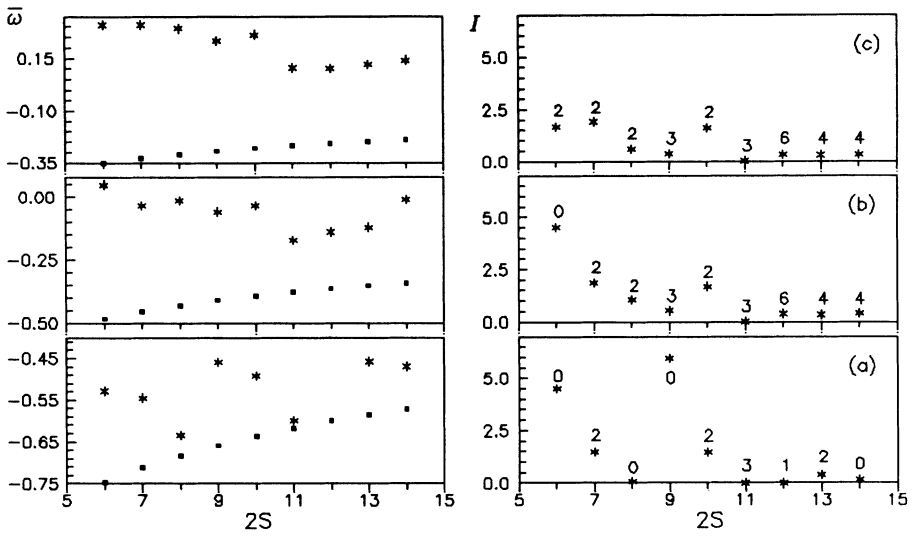

FIG. 6. Exciton emission spectrum vs the inverse filling factor. The left column: the transition frequency; the right column: the transition intensity. Numbers by the asterisks: L values for the corresponding states. Dressed excitons: asterisks; bare excitons: squares. (a) $h = 0.5$; (b) $h = 1.0$; (c) $h = 1.5$.

regular behavior for $9 \leq S \leq 12$. At the edges of this interval the incompressible liquid is formed either in the final ($N = 4$) or in the initial ($N = 5$) state. In the same region the intensity of the emission band also shows a strong dependence on S, which originates from the S dependence of the quantum number L at which the absolute minimum of $E^*_{ex}(L)$ is reached. The underlying physical mechanism is a strong ν dependence of the screening of a hole in the vicinity of $\nu = \frac{1}{3}$, and for gaining a deeper insight into it a systematic study of the bound states of quasielectrons and quasiholes with an exciton is needed.

VIII. CONCLUSION

The results on the spectroscopy of 2D magnetoexcitons obtained in this paper are related to two problems. The first group of results is based on symmetry arguments and is of quite general nature. It is shown that for perfect systems satisfying the following three conditions: (i) they are symmetric, i.e., all Coulomb interactions are equal in magnitude, $V_{ee} = V_{hh} = |V_{eh}|$; (ii) the subband and cyclotron quantizations in them are so strong that the conditions of the quantum limit are fulfilled; and (iii) all charge carriers are at the ground quantum level, both the frequencies and the matrix elements of all allowed exciton transitions are universal, i.e., they are independent of the electron and hole filling factors, the distribution function of elementary excitations, etc. The hidden symmetry inherent in such systems and explicitly expressed by the commutation rules (21) rules out the influence of the interaction of $\mathbf{k} = \mathbf{0}$ excitons with environment on their optical spectra. This statement somewhat resembles the Kohn theorem,[23] but the latter holds for monopolar systems and for intraband, and not interband, transitions. Therefore all nontrivial manifestations of electron interactions may be observed in optical spectra if the hidden symmetry of the system is violated by lowering its geometrical symmetry (skewed or double quantum wells), by taking into account the level mixing, etc. Some phenomena caused by this hidden symmetry have been discussed previously as applied to various specific problems, e.g., the zero scattering amplitude of two slow excitons[19,22,35] and coincidence of the energy of excitons and deexcitons.[36] They have also been observed in some experiments.[37,38] The criterion (21) enables one to establish in what cases electron correlations will, or will not, be exhibited in exciton spectra.

In connection with the criterion (21) for intrinsic systems, it is worth mentioning that in the theory[13] of the trapping of electrons by neutral centers the cusp strength is zero when $h = 0$, i.e., when a center resides in the confinement plane, and the experimental measurement of the gaps by cusp strengths[15] is based on the investigation of the h dependence of the emission spectrum. This means that the necessary condition for the manifestation of electron correlations in optical spectra, which is common to a wide class of intrinsic and extrinsic systems, is symmetry breaking. In such systems a consistent description of the 3D recombination dynamics may turn out to be of importance since the probability of vertical tunneling strongly depends on the particle energy. Hence, if a tunneling path is sufficiently long shake-up processes may be suppressed.

The second problem involves the coupling of an exciton to the electron-hole environment, which is investigated by computational methods as applied to a Laughlin liquid with $\nu = \frac{1}{3}$. This coupling may be treated as the polaron effect associated with the dressing of the exciton by MR's. However, since MR's are elementary excitations of an electron system the restrictions imposed by the Pauli exclusion principle are also of importance. In symmetric systems both contributions cancel out exactly at $\mathbf{k} = 0$, and in nonsymmetric ones the total shift has the sign opposite to that of the usual polaron shift. The dis-

tribution of the electron density in the dressed exciton is strongly changed compared to the bare exciton, and the screening of a hole is highly enhanced. The dispersion of the exciton is suppressed and in its energy spectrum there are MR bound states. When the anisotropy parameter reaches its critical value the absolute minimum of the energy spectrum shifts from the point $k = 0$ to a circle, and this shift should be accompanied by an abrupt change in the emission spectrum. Significant changes in the emission spectra at critical values of certain parameters, the splitting of emission bands and changes in their intensities have been repeatedly observed by different experimental groups, but their possible connection with the mechanism considered here calls for a special investigation.

After this work was completed two recent related papers came to our attention. Dzyubenko and Lozovik[39] have discovered the commutation rule equivalent to (21), and MacDonald and Rezayi[40] proposed the approach to a hidden symmetry in an electron-hole system in terms of mapping between it and a two-component electron system.

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