Infrared absorption due to electron–lattice-vibration scattering in AlAs/GaAs quantum wells

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The infrared-absorption coefficient due to interaction of electrons with optical phonons is calculated as a function of frequency for an electron gas in AlAs/GaAs quantum wells. The absorption coefficient is given by the real part of the dynamical conductivity. A finite-temperature force-force correlation-function method is used to represent the Kubo expression for the conductivity. Calculations are made with use of a bulk description for the optical phonons and also by inclusion of the confinement of the phonons for several well widths and several carrier densities. It is found that the effects of the confinement on the phonon spectrum reduce the absorption. The absorption also is found to increase with increasing well width and increasing carrier concentration.

I. INTRODUCTION

During the past decade the optical properties of semiconductor heterostructures have became an important field in view of their applications in improving the performance of optoelectronic devices.[1] It is well known that light absorption in a solid occurs due to scattering either by defects and/or lattice vibrations.[2] In bulk semiconductors inelastic scattering by longitudinal optical phonons is well understood. More recently, relaxation processes involving electrons interacting with polar modes in multiple-quantum-well (QW) structures like those formed by AlAs/GaAs have attracted much attention. In this area much work has been done in understanding dc transport in those heterostructures,[3] as much as cyclotron resonance,[3,4] recombination time for intrasubband and intersubband transitions,[5] exciton linewidths,[5,6] and other properties in these systems.

It is well known that in semiconductor quantum-well structures like GaAs/AlAs the carriers are confined to the quantum well, here the GaAs, as a result of the conduction- and valence-band offsets. The bulk-LO-phonon dispersions of the GaAs and AlAs do not overlap, however, and thus in GaAs/AlAs quantum-well structures there are significant deviations from the bulk phonon spectra.[7–9] It has been found in quantum-well systems that there are bulk confined phonons in the GaAs well region, bulk confined phonons in the AlAs barrier region, and also interface modes. Recently there has been considerable interest in the effects of this phonon confinement on the electron-phonon coupling and scattering rates. Calculations of the intersubband and intrasubband transition rates of electrons in quantum wells due to LO-phonon coupling have been made including these effects of confinement on the phonon spectra.[5,11] In the present work we give a detailed treatment of the optical absorption due to the electron-phonon interaction including the effects of phonon confinement by calculating the full optical absorption of an electron gas in the quantum well as a function of frequency within a Kubo formalism. Screening by the electron gas can be included in the random-phase approximation but we have neglected it in the present work.[10] For the sake of comparison, calculations are also made for a bulk description of the phonons assuming that they exchange momentum with the electron gas only in the plane parallel to the interface. This approach has been taken by Das Sarma *et al.*,[10] but, as we demonstrate in this work, at least from the point of view of absorption and recombination this interaction is clearly unrealistic.

Several approaches have been used to describe the effects of confinement on the phonon spectra and the resulting effects on the electron-phonon interaction.[5] Continuum approaches using the "slab modes" of a free-ion slab[12,13] and the "guided modes" of a model layered structure[11] have been used. These models differ in the boundary conditions applied at the interfaces. A more detailed continuum approach also has been developed recently.[14] Recently a simple microscopic lattice dynamical model for phonons in a quantum well has been developed for a description of the confined phonons, and fits to results for the vibrations provide analytic forms for the electron-phonon interaction matrix elements.[15] We use this description of the electron-phonon coupling in the present study. In a recent work a more detailed lattice-dynamics study tends to substantiate this approach.[16]

The absorption coefficient corresponds to the real part of the dynamic conductivity. A simple way to obtain that quantity is to start from Kubo's equation for finite frequency and write this expression in terms of the force-force correlation function.[2] Forces here refer to the frictional forces acting on the center of mass of the electron system.[17]

In Sec. II, the coupling to the interface and confined

bulk phonons are given, and the frictional force due to scattering by these modes is obtained. In Sec. III we calculate the real part of the dynamic conductivity based on the finite-temperature force-force correlation function method. In Sec. IV we present the numerical results and discuss them.

II. FRICTIONAL FORCES DUE TO SCATTERING BY PHONONS

A. Preliminaries

We study a system composed of an electron gas confined to the GaAs well of an AlAs/GaAs structure. For the sake of simplicity we will take the barriers for electron confinement to be infinite. Then electrons will be well described by the wave functions:

$$\Psi_{n,\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{A}} \exp(i\mathbf{k}\cdot\mathbf{r})\phi_n(z) \,. \tag{2.1}$$

In the above equation the electron state is identified by the subband index n and the wave vector $\mathbf{k} \equiv (k_x, k_y)$ parallel to the AlAs/GaAs interface. We assume a QW of width L. Then the wave function is normalized in a box of volume $v = LA$. The function $\phi_n(z)$ describes the quantized motion in the z direction. The total energy of the state $(n, \mathbf{k})$ is

$$E_n(\mathbf{k}) = E_n + \frac{\hbar^2 k^2}{2m^*} \,, \tag{2.2}$$

with

$$E_n = \frac{\hbar\pi^2 n^2}{L^2} \,. \tag{2.3}$$

The wave function $\phi_n(z)$, since we assume the barrier to be infinite, is given by

$$\phi_n(z) = \frac{2}{\sqrt{L}} \sin\frac{n\pi z}{L}, \quad n = 1, 2, \ldots \,. \tag{2.4}$$

Generally such structures are grown in such a way that impurities in the wide gap region provide carriers in the two-dimensional channel. We assume that those carriers are electrons provided by donor impurities (e.g., Si) in the AlAs. Normally samples are grown in which the carrier density N_e varies in the order of $10^{10} - 10^{12}$ cm^{-2}. In that case it is reasonable to take the so-called quantum limit, in which the Fermi level lies between the first and the second subbands. In this limit,

$$E_F = \frac{\pi\hbar^2}{m^*} N_e \,. \tag{2.5}$$

In what follows we assume, consistent with the fact that the electron system is confined in the z direction, that phonons exchange momentum with the carriers only in the x-y plane. Furthermore, we also impose a quasi-two-dimensional character to the phonon system in such a way that its wave vector is represented by $\mathbf{q} \equiv (q_x, q_y)$. The potential imposed by the lattice vibration to an electron at position $\mathbf{r}$ is

$$V_{\rm ph}(\mathbf{r}) = \sum_{\mathbf{q},\nu} f_\nu(x, y, z; \mathbf{q}) A^\nu_{\mathbf{q}} \,, \tag{2.6}$$

where $A^\nu_{\mathbf{q}} = a^\nu_{\mathbf{q}} + a^{\nu\dagger}_{-\mathbf{q}}$ is the well-known phonon operator for a branch ν. The above expression is valid both for confined and interface modes. To treat the interaction with bulk-LO modes we will need a special treatment, which we will comment on later. Futhermore, we will assume

$$f_\nu(x, y, z; \mathbf{q}) = e^{i\mathbf{q}\cdot\rho} F_\nu(\mathbf{q}, z) \,, \tag{2.7}$$

with $\rho \equiv (x, y)$. Then the electron-phonon potential operator becomes

$$\hat{V} = \int d^3r\, \hat{\Psi}^\dagger(\mathbf{r}) V_{\rm ph}(\mathbf{r}) \hat{\Psi}(\mathbf{r}) \tag{2.8}$$

or

$$\hat{V} = \sum_{n_1,n_2} \sum_{\mathbf{k},\mathbf{q},\nu} \left(\int dz\, \phi^*_{n_1}(z) F_\nu(\mathbf{q}, z) \phi_{n_2}(z) \right) \times A^\nu_{\mathbf{q}} c^\dagger_{n_1,\mathbf{k}+\mathbf{q}} c_{n_2,\mathbf{k}} \,, \tag{2.9}$$

where $c^\dagger_{n,\mathbf{k}}$ is the creation operator for an electron in a state $(n, \mathbf{k})$. As we will show later, we will need the expression in large parentheses for the transition $1 \to l$ for the electron states. Then we define

$$\Phi^{1l;\nu}_{e\text{-ph}}(q) = \int dz\, \phi^*_l(z) F_\nu(q, z) \phi_1(z) \,. \tag{2.10}$$

This is now calculated for the three kinds of interactions: scattering by bulk-LO phonons, confined phonons, and interface phonons.

B. The special case of bulk-LO phonons

In this case the lattice vibrations carry three-dimensional momentum $\mathbf{Q} \equiv (\mathbf{q}, q_z)$. The polar interaction is given by

$$V_{\rm LO}(\mathbf{r}) = \sum_{\mathbf{Q}} \frac{\lambda e^{i\mathbf{q}\cdot\mathbf{r}}}{v^{1/2} Q} A^{\rm LO}_{\mathbf{Q}} \,, \tag{2.11}$$

with

$$\lambda = [2\pi e^2 \hbar w_{\rm LO1}(1/\epsilon_{\infty 1} - 1/\epsilon_{01})]^{1/2} \,. \tag{2.12}$$

From now on we will use the subscript 1 for GaAs and 2 for AlAs. Then, using Eq. (2.9),

$$\hat{V}_{\rm LO} = \sum_{n_1,n_2} \sum_{\mathbf{k},\mathbf{Q}} \frac{\lambda}{v^{1/2}} \frac{1}{Q} \left(\int dz\, e^{iq_z z} \phi_{n_1}(z) \phi_{n_2}(z) \right) \times A^{\rm LO}_{\mathbf{Q}} c^\dagger_{n_1,\mathbf{k}+\mathbf{q}} c_{n_2,\mathbf{k}} \,. \tag{2.13}$$

For the time being we define

$$\Phi^{1l;{\rm LO}}_{e\text{-ph}}(\mathbf{q}) = \frac{\lambda}{v^{1/2}} \sum_{q_z} \frac{F_{1l}(q_z)}{(q^2 + q_z^2)} \,, \tag{2.14}$$

with

$$F_{1l}(q_z) = \int dz\, e^{iq_z z} \phi_l(z) \phi_1(z) \,. \tag{2.15}$$

We must pay attention to the fact that the definition Eq. (2.14) cannot be used directly in the potential given by Eq. (2.9). However, it will be very useful as we will show later. With a little algebra we obtain

$$F_{11}(q_z) = \frac{8\pi^2}{q_z L} \sin\frac{q_z L}{2} \frac{1}{(q^2L^2 - 4\pi^2)} , \tag{2.16}$$

$$F_{12}(q_z) = 16\pi^2 q_z L \cos\frac{q_z L}{2} \frac{1}{(q_z^2L^2 - 9\pi^2)(q_z^2L^2 - \pi^2)} . \tag{2.17}$$

C. Electrons interacting with confined phonons

In this case we use the model for phonon confinement:[15,18]

$$V_{\rm ph}(\mathbf{r}) = \frac{\lambda}{v^{1/2}} \sum_{\mathbf{q}} \sum_{n,\alpha} e^{i\mathbf{q}\cdot\rho} t_n(q) u_{n\alpha}(z) A_{\mathbf{q}}^{n,\alpha} , \tag{2.18}$$

with $n = 2, 3, \ldots$ and $\alpha = +$ or $-$. Note that as discussed in Ref. 15 the bulk confined mode given by $n = 1$ is not present as it corresponds to the interface mode. Then, Eq. (2.9) applies straightforwardly, and

$$F_{n,\alpha}(q,z) = \frac{\lambda}{v^{1/2}} t_n(q) u_{n,\alpha}(q) , \tag{2.19}$$

with

$$t_n(q) = 1/\sqrt{2I_n}, \tag{2.20}$$

$$I_n = \frac{1}{L}\int dz \left[q^2\phi_n^2 + \left(\frac{d\phi_n}{dz}\right)^2\right], \tag{2.21}$$

$$\phi_{n+} = \sin\frac{\mu_n \pi z}{L} + \frac{C_n z}{L}, \quad n = 3, 5, 7, \ldots, \tag{2.22}$$

$$\phi_{n-} = \cos\frac{\mu_n \pi z}{L} - (-1)^{n/2}, \quad n = 2, 4, 6, \ldots, \tag{2.23}$$

$$C_n = -2\sin\frac{\mu_n \pi}{2} . \tag{2.24}$$

Then,

$$\Phi^{n_1 n_2; n\alpha}(q) = \frac{\lambda}{v^{1/2}} t_n(q) G_{n_1 n_2}^{n\alpha} , \tag{2.25}$$

where

$$G_{n_1 n_2}^{n\alpha} = \int dz \phi_{n_1}(z) u_{n\alpha}(z) \phi_{n_2}(z) , \tag{2.26}$$

which, for the intrasubband transition $1 \to 1$ and the intersubband transition $1 \to 2$, are

$$G_{2n}^{11} = \tfrac{3}{2}\delta_{n1} - (-1)^n(1 - \delta_{n,1}) , \tag{2.27}$$

$$G_{2n+1}^{12} = \frac{16 C_{2n+1}}{9\pi^2} - 16\pi^2(\mu_{2n+1}\pi)\cos\frac{\mu_{2n+1}\pi}{2} \times \frac{1}{[(\mu_{2n+1}\pi)^2 - \pi^2][(\mu_{2n+1}\pi)^2 - 9\pi^2]} . \tag{2.28}$$

The values of μ_l are those given by the roots of

$$\tan\left(\frac{\mu_l \pi}{2}\right) = \frac{\mu_l \pi}{2} . \tag{2.29}$$

For the intrasubband transition the only nonzero contributions are $\alpha = +$ and $n = 2, 4, 6, \ldots$. For the intersubband transition they are $(1 \to 2)$, $\alpha = -$, and $n = 3, 5, 7, \ldots$.

D. Electrons interacting with interface modes

In this case we have to consider the two branches of each of the symmetric and antisymmetric modes for which[19]

$$V_{\pm}^{s}(\mathbf{r}) = \sum_{\mathbf{q}} e^{i\mathbf{q}\cdot\rho} e^{-qL/2} \frac{e^{qz} + e^{-qz}}{\sqrt{1+\gamma}} v_s^{\pm} A_{\pm}^{s}(q), \tag{2.30}$$

$$V_{\pm}^{a}(\mathbf{r}) = \sum_{\mathbf{q}} e^{i\mathbf{q}\cdot\rho} e^{-qL/2} \frac{e^{qz} - e^{-qz}}{\sqrt{1-\gamma}} v_a^{\pm} A_{\pm}^{a}(q) , \tag{2.31}$$

where

$$\gamma = e^{-qL} , \tag{2.32}$$

$$v_{s,a}^{\pm} = \left(\frac{2\pi^2 |f_{\pm}^{s,a}|}{\hbar\omega_{\pm}^{s,a} A q}\right)^{1/2} , \tag{2.33}$$

$$|f_{\pm}^{s,a}| = \frac{[\hbar^2(\omega_{\pm}^{s,a})^2 - T_1^2][\hbar^2(\omega_{\pm}^{s,a})^2 - T_2^2]}{\hbar^2[(\omega_{+}^{s,a})^2 - (\omega_{-}^{s,a})^2](\epsilon_1^{s,a} + \epsilon_2^{s,a})} , \tag{2.34}$$

$$\epsilon_{1,2}^{s} = \epsilon_{\infty\,1,2}(1 - \gamma) , \tag{2.35}$$

$$\epsilon_{1,2}^{a} = \epsilon_{\infty\,1,2}(1 + \gamma) , \tag{2.36}$$

and $\omega_{\pm}^{s,a}$ are the dispersion curves for the respective modes. In Eq. (2.34), $T_1 = \hbar\omega_{\rm TO1}$ and $T_2 = \hbar\omega_{\rm TO2}$.

Then in the present case,

$$\Phi^{1ls\pm}(q) = \frac{e^{-qL/2}}{\sqrt{1+\gamma}} v_s^{\pm} \int dz\, \phi_l(z)(e^{qz} + e^{-qz})\phi_1(z) , \tag{2.37}$$

$$\Phi^{1la\pm}(q) = \frac{e^{-qL/2}}{\sqrt{1-\gamma}} v_a^{\pm} \int dz\, \phi_l(z)(e^{qz} + e^{-qz})\phi_1(z) . \tag{2.38}$$

We call the integrals in Eqs. (2.37) and (2.38) $G_{s,a}^{1l}$. They can be easily evaluated and result in

$$G_a^{11} = G_s^{12} = 0 , \tag{2.39}$$

$$G_s^{11} = \frac{8\pi^2}{qL} \frac{e^{qL/2}(1 - e^{-qL})}{q^2L^2 + 4\pi^2} , \tag{2.40}$$

$$G_a^{12} = \frac{16\pi^2 qL}{(q^2L^2 + \pi^2)} \frac{e^{qL/2}(1 + e^{-qL})}{q^2L^2 + 9\pi^2} . \tag{2.41}$$

We see, therefore, that only the symmetric modes contribute to the intrasubband transition, and only the anti-symmetric modes contribute to the intersubband transition.[20]

E. Frictional forces and the Kubo equation

The Kubo expression for the conductivity is more commonly expressed in terms of the current-current correla-

tion function which in the finite-temperature formalism is

$$\pi_{\mu\mu}(i\omega) = -\frac{1}{v}\int_0^\beta d\tau e^{i\omega\tau}\langle T_\tau j_\mu(\tau) j_\mu(0)\rangle \,, \tag{2.42}$$

where j_μ is the μ component of the current operator, which can itself be expressed in terms of the electron creation and destruction operators corresponding to the states given by the wave vector $\mathbf{k}$ in the x-y plane and subband index n:

$$j_\alpha = \sum_{n_1,n_2}\sum_{\mathbf{k}_1,\mathbf{k}_2} p_\alpha^{n_1\mathbf{k}_1;n_2\mathbf{k}_2} c^\dagger_{n_1\mathbf{k}_1} c_{n_2,\mathbf{k}_2} \,. \tag{2.43}$$

The matrix element is

$$p_\alpha^{n_1\mathbf{k}_1;n_2\mathbf{k}_2} = \frac{e\hbar}{2mi}\int d^2\rho \int dz\{\psi^*_{n_1\mathbf{k}_1}(\mathbf{r})\nabla_\alpha\psi_{n_2\mathbf{k}_2}(\mathbf{r}) - \psi_{n_2\mathbf{k}_2}(\mathbf{r})\nabla_\alpha\psi^*_{n_1\mathbf{k}_1}(\mathbf{r})\} \,, \tag{2.44}$$

which becomes a simple expression in the x direction:

$$j_x = \frac{e\hbar}{m}\sum_{n,\mathbf{k}} k_x c^\dagger_{n\mathbf{k}} c_{n\mathbf{k}} \,. \tag{2.45}$$

A simple derivation[2] brings the current-current correlation function into a force-force correlation function which is much more convenient in the present case in studying the dynamic conductivity. Force here means the frictional force acting on the center of mass of the electron system due to lattice imperfections and vibrations. In the case of scattering by phonons,

$$\pi(i\omega) = -\frac{e^2\hbar^2}{vm^2(i\omega)^2}\sum_{\mathbf{q},\nu} q_x^2 \sum_{n_1,n_2,\mathbf{k}}\sum_{n_1',n_2',\mathbf{k}'}\int_0^\beta d\tau(e^{i\omega\tau}-1)\langle T_\tau c^\dagger_{n_1\mathbf{k}+\mathbf{q}}(\tau)c_{n_2\mathbf{k}}(\tau)c^\dagger_{n_1'\mathbf{k}'-\mathbf{q}}(0)c_{n_2'\mathbf{k}'}(0)\rangle$$
$$\times\langle T_\tau\hat\Phi^{n_1n_2\nu}(\mathbf{q},\tau)\hat\Phi^{n_1'n_2'\nu}(-\mathbf{q},0)\rangle \,, \tag{2.46}$$

where we made use of the notation

$$\hat\Phi^{n_1n_2\nu}(\mathbf{q},\tau) = \Phi^{n_1n_2\nu}(\mathbf{q})A_\nu(\mathbf{q},\tau) \,. \tag{2.47}$$

Assuming a zeroth-order approximation for the phonon part of $\pi(i\omega)$,

$$\pi(i\omega) = -\frac{e^2\hbar^2}{Lm^2(i\omega)^2}\sum_{\mathbf{q},\nu} q_x^2 \sum_{n_1,n_2}\sum_{n_1',n_2'}\Phi^{n_1n_2\nu}(\mathbf{q})\Phi^{n_1'n_2'\nu}(-\mathbf{q})\int_0^\beta d\tau(e^{i\omega\tau}-1)\Xi^{n_1n_2;n_1'n_2'}(\mathbf{q},\tau)\mathcal{D}_\nu^{(0)}(\mathbf{q},\tau) \,, \tag{2.48}$$

where we have introduced

$$\Xi^{n_1n_2;n_1'n_2'}(\mathbf{q},\tau) = -\frac{1}{A}\sum_{\mathbf{k},\mathbf{k}'}\langle T_\tau c^\dagger_{n_1\mathbf{k}+\mathbf{q}}(\tau)c_{n_2\mathbf{k}}(\tau)c^\dagger_{n_1'\mathbf{k}'-\mathbf{q}}(0)c_{n_2'\mathbf{k}'}(0)\rangle \,. \tag{2.49}$$

$\mathcal{D}_\nu^0(\mathbf{q},\tau)$ is Matsubara's expression for the bare phonon propagator.

At this point we make the approximation

$$\Xi^{n_1n_2;n_1'n_2'}(\mathbf{q},\tau) \approx P^{(1)}_{n_1n_2}(\mathbf{q}\tau)\delta_{n_1n_1'}\delta_{n_2n_2'} \,, \tag{2.50}$$

which consists in assuming only a single polarization diagram expressed in the frequency space in terms of the electron propagators as

$$P^{(1)}_{n_1n_2}(\mathbf{q},i\omega) = \frac{2}{A}\sum_{\mathbf{p}}\frac{1}{\beta}\sum_{i,p}\mathcal{G}^{(0)}_{n_1}(\mathbf{p},ip)\mathcal{G}^{(0)}_{n_2}(\mathbf{p}+\mathbf{q},ip+i\omega) \,. \tag{2.51}$$

In the following we will evaluate the absorption for zero temperature. This is appropriate for temperature well below room temperature because in the case $k_BT \ll \hbar\omega_{\mathrm{LO}}$ and $k_BT \ll E_{F,1}$, the electron Fermi energies correspond to the densities studied here. On this low-temperature limit the absorption is dominated by phonon emission processes. Results for the absorption at high temperature can be obtained straightforwardly from the results of the formalism given here.

III. DYNAMIC CONDUCTIVITY

After a long but straightforward calculation we obtain an expression for the real part of the dynamic conductivity[2] taking into account the absorption by the lattice polarization (LO-like phonons) at low temperatures:

$$\mathrm{Re}[\sigma(\omega)] = \frac{e^2mE_F^2}{2\pi^2\hbar^4\omega^3}\sum_{l\nu}\left(\int_{x_1}^{x_4} dx\, x^{1/2}U^{l\nu}(x)[1-g^{l\nu}(x)]^{1/2}\int_{x_2}^{x_3} dx\, x^{1/2}U^{l\nu}(x)[1-g^{l\nu}(x)-a^{l\nu}(x)]^{1/2}\right) \,, \tag{3.1}$$

where we have taken the Fermi level to tie between the first and the second subbands,

$$a^{l\nu} = \frac{\hbar\omega - \hbar\omega_\nu(x)}{E_F} , \tag{3.2}$$

$$g^{l\nu} = \frac{[x - e^{l\nu}(x)]^2}{4x} , \tag{3.3}$$

and

$$e^{l\nu}(x) = \frac{\hbar\omega - \hbar\omega_\nu(x) - \Delta_{l1}}{E_F} . \tag{3.4}$$

We have used a dimensionless variable $x = (q/k_F)^2$ and the integrals in Eq. (3.1) are performed in the range where the functions under square roots are positive. The function $U^{l\nu}(x)$ in Eq. (3.1) describes the electron interaction with the several phonon modes, and the index ν spans over the four branches of the interface phonons $(s, a, +, -)$ and the two possible cases for the confined modes. We have also explored, for the sake of comparison, the results we would obtain if the scattering were governed by bulk dispersionless LO phonons. Therefore, Eq. (3.1) is general.

In the case of bulk-LO phonons,

$$U^{1l;\mathrm{LO}}(x) = 64\pi^3 l^2 \lambda^2 \int_0^\infty dy\, y^2 \frac{\left\{ \begin{matrix} \sin^2 y/2 \\ \cos^2 y/2 \end{matrix} \right\}}{(x^2+y^2)[y^2-(l+1)^2\pi^2]^2[y^2-(l-1)^2\pi^2]^2} , \tag{3.5}$$

where the upper function in the numerator corresponds to the intraband ($l = 1$) transition, and the lower function corresponds to the intersubband transition to the second subband ($l = 2$).

For the contribution of the confined modes we have

$$U^{1l;\mathrm{conf}}(x) = \lambda^2 \left\{ \delta_{1,l} \sum_{n=1}^{\infty} |G_{2n}^{11}|^2 \frac{1}{2(a_{2n}x^2 + b_{2n})} \delta_{2,l} \sum_{n=1}^{\infty} |G_{2n+1}^{12}|^2 \frac{1}{2(a_{2n+1}x^2 + b_{2n+1})} \right\} , \tag{3.6}$$

where the functions G are defined in Eqs. (2.26) and (2.29):

$$a_{2n} = \tfrac{3}{2} + \frac{\sin \mu_{2n}\pi}{2\mu_{2n}\pi} - 4(-1)^n \frac{\sin(\mu_{2n}\pi/2)}{\mu_{2n}\pi} , \tag{3.7}$$

$$b_{2n} = \frac{\mu_{2n}\pi}{2}(\mu_{2n}\pi - \sin \mu_{2n}\pi) , \tag{3.8}$$

$$a_{2n+1} = \frac{1}{2\mu_{2n+1}\pi} + \tfrac{1}{12} \sin^2 \frac{\mu_{2n+1}\pi}{2} + \frac{2}{(\mu_{2n+1}\pi)^2} \sin^2 \frac{\mu_{2n+1}\pi}{2} - \frac{1}{\mu_{2n+1}\pi} \sin \mu_{2n+1}\pi , \tag{3.9}$$

$$b_{2n+1} = \frac{\mu_{2n+1}\pi}{2} \sin \mu_{2n+1}\pi + \frac{(\mu_{2n+1}\pi)^2}{2} . \tag{3.10}$$

Finally, for the scattering by interface modes, we have

$$U^{1l;s\pm}(x) = \lambda^2 \frac{e^{-x}}{1+e^{-x}} \frac{F_\pm^s}{x\hbar\omega_\pm^s} [G_s^{11}(x)]^2 \delta_{1,l} , \tag{3.11}$$

$$U^{1l;a\pm}(x) = \lambda^2 \frac{e^{-x}}{1-e^{-x}} \frac{F_\pm^a}{x\hbar\omega_\pm^a} [G_a^{12}(x)]^2 \delta_{2,l} , \tag{3.12}$$

respectively, for the interaction with the symmetric (s) and antisymmetric (a) modes with the functions G defined in Eqs. (2.39)–(2.41). In order to express the previous equations explicitly in terms of λ^2 as it appears in both bulk and confined cases, we have defined the function $F_\pm^{s,a}$ as

$$F_\pm^{s,a} = \frac{f_\pm^{s,a}}{\hbar\omega_{\mathrm{LO}1}(1/\epsilon_{\infty 1} - 1/\epsilon_{01})} . \tag{3.13}$$

IV. RESULTS

In the present work we have calculated the infrared absorption due to electron–LO-phonon coupling as a function of frequency for a quasi-two-dimensional electron gas confined to a GaAs quantum well. A force-force correlation approach to the Kubo expression for the dynamic conductivity has been used. We have treated the effects of confinement on the phonon system using results based on a recent lattice-dynamics study, and for comparison we also have calculated the absorption using a bulk phonon description of the phonons.

In Fig. 1 we show the absorption coefficient per unit volume in the quantum well resulting from a bulk description of the phonons and that including the effects of the phonon confinement. In the case of confined phonons

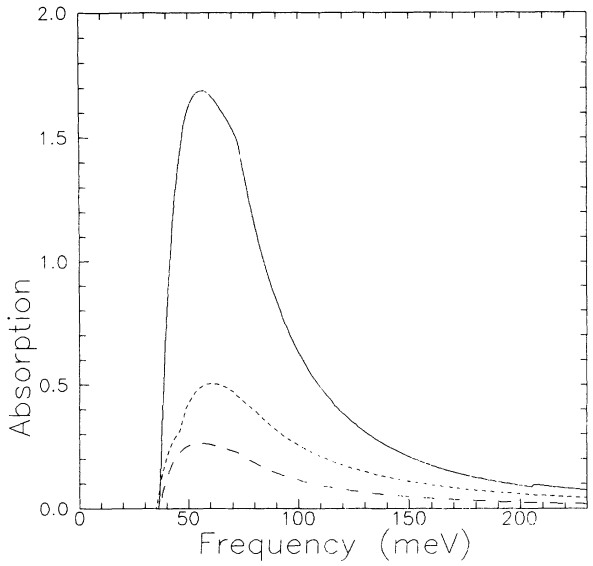

FIG. 1. Real part of the dynamic conductivity (absorption coefficient per unit volume inside the QW) in units of $\frac{e^2m}{2\pi^2\hbar} \times 10^3$ as a function of energy in meV, for a QW of width L=100 Å and carrier density $N_e = 1.0 \times 10^{12}$ cm^{-2}. The solid line represents absorption mediated by bulk-LO phonons, and the small bump feature above 200 meV corresponds to the $1 \rightarrow 2$ transition. The dashed curves correspond to the interface (higher curve) and confined (lower curve) modes.

the contributions from the bulk confined phonons and the interface phonons are shown separately. The onset of the absorption occurs at a frequency slightly below the bulk-LO-phonon frequency as a result of the dispersion of the interface phonons. The intrasubband transition is found to be larger than the intersubband contribution. The onset of the latter shows a small feature at an energy corresponding to 200 meV for the case of L=100 Å shown here. The most interesting feature shown here is that the optical absorption in the confined phonon description is less than that in the bulk phonon description at all frequencies. This is a result of the selection rules governing transitions involving confined phonons. We note in addition that the interface phonon contribution is larger than that from the bulk confined phonons at all frequencies. The suppression of the scattering by phonon confinement and the dominance of the interface contribution seen here are consistent with the behavior seen in studies of other quantities such as the intersubband and intrasubband electron relaxation rates.[5]

In Fig. 2 we show the absorption per unit area for several well widths. The contributions from the bulk confined and the interface modes are shown separately. Note that the magnitude of the interface contributions are approximately twice that of the bulk confined modes. We see here that the scattering increases as the well width increases. This is consistent with the scattering rate being larger in the bulk than in the quantum well. For wider wells more-confined phonon modes make significant contributions to the scattering.

In Fig. 3 the absorption is given for two carrier densities. Again the interface and confined bulk phonon contributions are shown separately. It is seen here that the scattering rate increases for increasing carrier densities. For larger carrier densities there are more final states to which to scatter, or, in more physical terms, there are more electrons through which to dissipate energy to the final-state phonons.

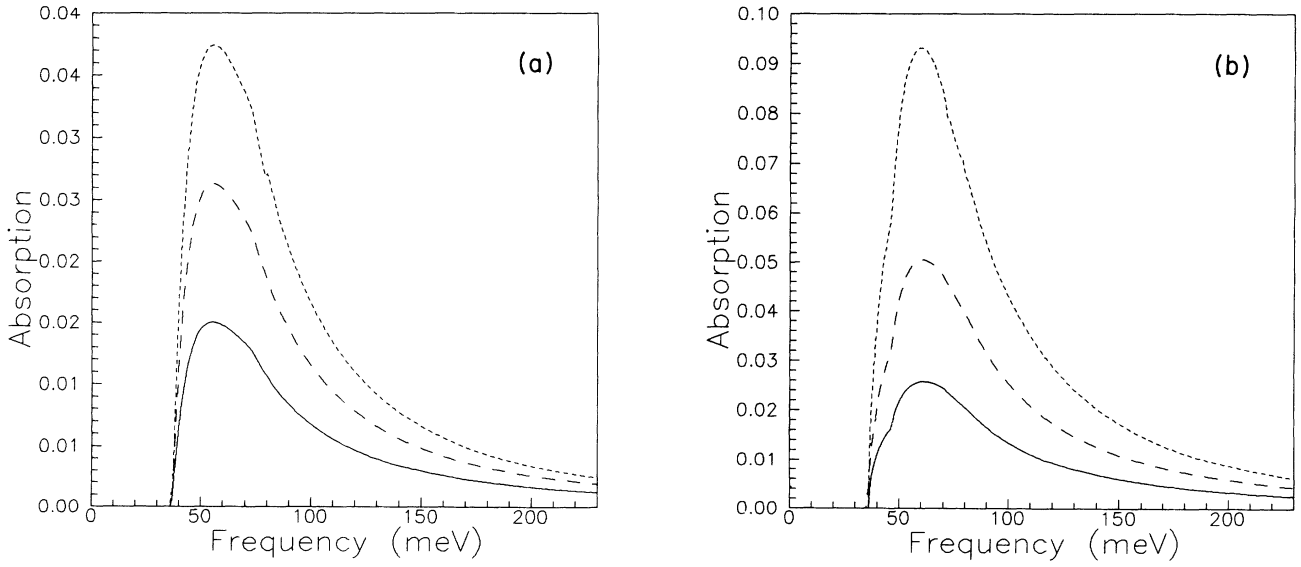

FIG. 2. Absorption (per unit area of the interface in the QW) in units of $\frac{e^2m}{2\pi^2\hbar} \times L$ for the confined (a) and interface (b) modes for $N_e = 1 \times 10^{12}$ cm^{-2}. The three curves, from below to above, correspond to L=50, 100, and 200 Å.

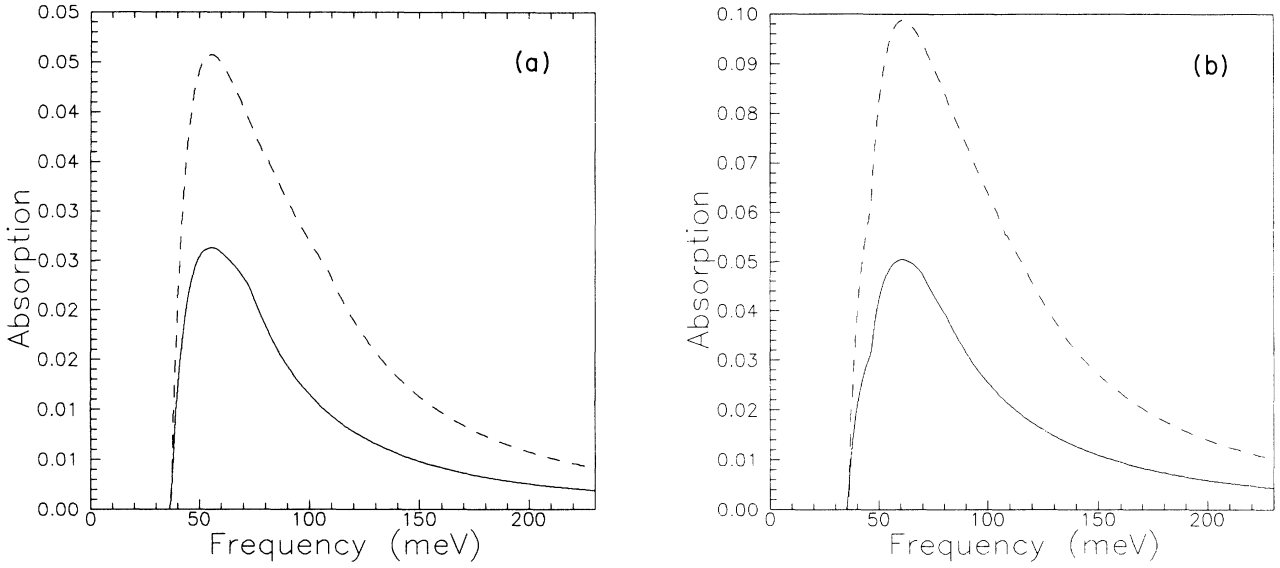

FIG. 3. Absorption curves (in the same units as in Fig. 2) for confined (a) and interface (b) modes for L=100 Å. Dashed curve is for $N_e = 2 \times 10^{12}$ cm^{-2}, and solid curve for $N_e = 1 \times 10^{12}$ cm^{-2}.

In Fig. 4 the total absorption, which is the sum of the contributions from the interface phonons and the bulk confined phonons, is given for the three well widths. The same increase in absorption with increasing well width as given in Fig. 2 is seen here. The total absorption for two carrier densities is shown in Fig. 5 where it is again seen that the absorption increases with increasing carrier density.

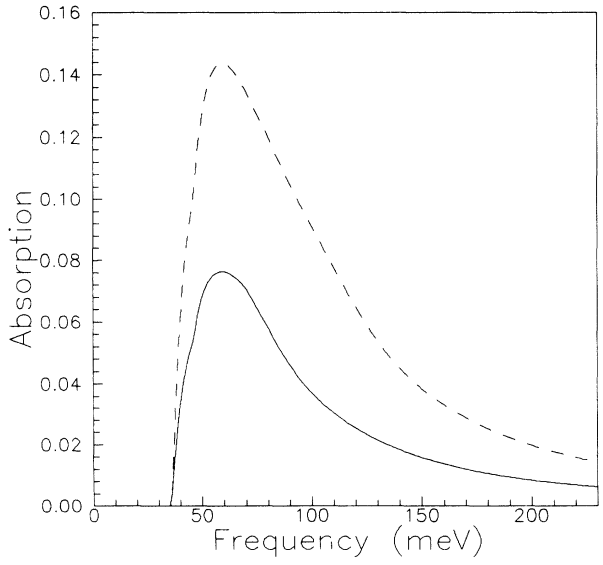

FIG. 4. Absorption curve due to lattice vibration in a quantum well (interface and confined modes) with $N_e = 1.0 \times 10^{12}$ cm^{-2}. From below to above, L=50, 100, and 200 Å. Same units as in Fig. 2.

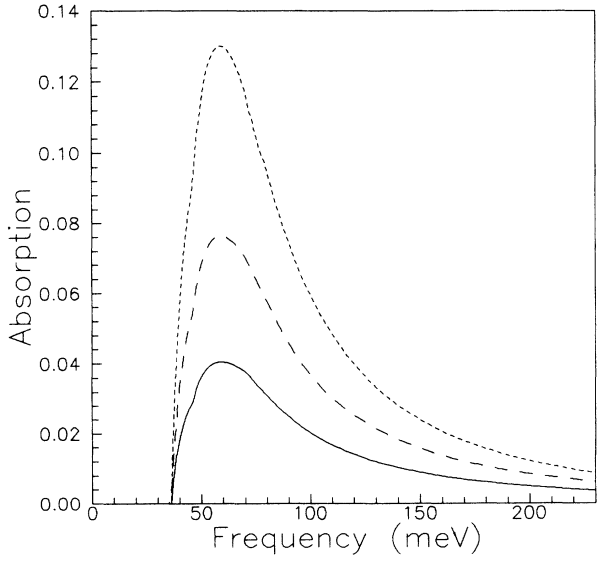

FIG. 5. Absorption curves (total, as above) for a quantum-well width L=100 Å. Dashed line represents $N_e = 2 \times 10^{12}$ cm^{-2}, solid line $N_e = 1 \times 10^{12}$ cm^{-2}.

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[18]Recently it has been pointed out (Ref. 20) that the phonon modes given in Ref. 15 are not fully orthogonalized. In our earlier work (Ref. 5) we checked that the modes of low index and relatively small wave vector are to a good approximation orthogonal and that the modes of higher index give a relatively small contribution to electron scattering rates. This is verified by a comparison of the numerical results for the scattering rates given in Refs. 5 and 20. For these reasons in the interest of simplicity we do not orthogonalize the phonon modes in the present work.

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