PHYSICAL REVIEW B VOLUME 46, NUMBER 24 15 DECEMBER 1992-II

## Rapid Communications

*Rapid Communications are intended for the accelerated publication of important new results and are therefore given priority treatment both in the editorial office and in production. A Rapid Communication in* **Physical Review B** *should be no longer than four printed pages and must be accompanied by an abstract. Page proofs are sent to authors.*

# Dot and antidot edge magnetoplasmons for a two-dimensional electron gas in a ring geometry

C. R. Proetto*
*International Centre for Theoretical Physics, 34100 Trieste, Italy*
(Received 27 August 1992)

The dynamical response of a classical two-dimensional electron gas confined in a ring geometry under a perpendicular magnetic field is analyzed. Within the hydrodynamical approach and in the strong magnetic-field limit, a new set of antidot edge magnetoplasmons is obtained, corresponding to density oscillations circulating along the inner boundary of the ring and whose frequency *increases* with magnetic field. The associated self-induced distribution of densities and currents are presented, together with an analysis of the size dependence of these perimeter waves.

A two-dimensional electron gas (2DEG) with confining boundaries in a perpendicular magnetic field exhibits low-frequency plasma oscillations associated with the edge of the sample. These edge magnetoplasmons (EMP's) in the 2DEG both on the surface of liquid helium[1-3] and in etched semiconductor heterojunctions[4-8] have received much attention in recent years.

The theoretical studies to date[6,9-11] have beeen restricted to simply connected geometries (dots); we propose here that in more complicated geometries, like a ring, a new EMP will arise, due to the propagation of density fluctuations on the inner boundary of the sample.

Our model is a classical 2DEG confined in a ring of inner radius $a$ and outer radius $b$. The ring contains a static positive and uniform background with density per unit area $e\,n_0\,(e>0)$ and a compressible electron fluid with areal charge density $-e(n_0+n)$, where $n$ is the self-induced density ($n \ll n_0$). The system is under a perpendicular magnetic field $B$ along the $z$ direction and surrounded by dielectric material with dielectric constant $\epsilon_1$ for $z>0$ and $\epsilon_2$ for $z<0$. In order to get a closer resemblance to the actual experimental setup of electrons on the surface of liquid helium, we also include two grounded metallic electrodes above ($z=h$) and below ($z=-h$) the 2DEG.

In this paper, we adopt the hydrodynamic approach[12] to study the magnetoplasma excitations of such electron fluid confined to a ring; the same theoretical framework has been used previously to study the EMP of dots,[9,11] strips,[10] and quite recently, the magnetoplasma excitations of parabolic quantum wells.[13] The starting points of this frictionless, unretarded, and linearized hydrodynamic model are the equations of continuity and the Euler equation,

$$\frac{\partial n}{\partial t}+\boldsymbol{\nabla}\cdot(n_0\mathbf{v})=0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{s^2}{n_0}\boldsymbol{\nabla}n-\frac{e}{m}\boldsymbol{\nabla}\phi-\omega_c\hat{\mathbf{z}}\times\mathbf{v}=0, \tag{2}$$

where $\phi$ and $\mathbf{v}$ are the self-induced potential and velocity, respectively, $\omega_c=eB/mc$ the cyclotron frequency, and $s$ is an effective wave speed that arises from the compressibility of the fluid. The symbol $\boldsymbol{\nabla}$ is the two-dimensional operator $(\partial/\partial\rho)\hat{\rho}+(\partial/\rho\partial\theta)\hat{\theta}$, with $\rho$ and $\theta$ the radial and polar coordinates, respectively.

For $z\neq 0$, the self-induced potential satisfies Laplace's equation, while at $z=0$ it obeys the usual boundary conditions that $\phi$ be continuous and that its normal derivative should have the following discontinuity:

$$\epsilon_1\left.\frac{\partial\phi}{\partial z}\right|_{0^+}-\epsilon_2\left.\frac{\partial\phi}{\partial z}\right|_{0^-}=4\pi en\Theta(\rho-a)\Theta(b-\rho), \tag{3}$$

where $\Theta$ is the Heaviside step function.

Finally, Eqs. (1)–(3) will be suplemented by the usual hard-wall boundary conditions of many hydrodynamic treatments of finite systems. These boundary conditions require that there be no normal component of velocity at the boundaries of the electron fluid; from (2), the mathematical expression of this condition is

$$\left(\omega\frac{\partial}{\partial\rho}-\frac{i\omega_c}{\rho}\frac{\partial}{\partial\theta}\right)\left.\left(s^2\frac{n}{n_0}-\frac{e}{m}\phi\right)\right|_{\rho=a,b}=0. \tag{4}$$

While hard-wall boundary conditions should apply for filled wells with vertical sides, as is the case for electrons on the surface of liquid helium, its applicability to the case of electrons in etched semiconductor heterojunctions is not obvious.

Rotational invariance around the $z$ axis and translational invariance in time implies that all the unknown quantities have an angular and time dependence of the form $e^{i(l\theta-\omega t)}$, where $l$ is an integer and $\omega$ the frequency of the normal mode. In the fully screened limit $h\to 0$, the self-induced density and potential are related by a local relation and the previous set of basic equations reduces to the following two equations:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}-\frac{l^2}{x^2}+\frac{\omega^2-\omega_c^2}{\Omega_0^2+s^2/a^2}\right)n(x)=0, \quad (5)$$

$$\left(\omega\frac{\partial}{\partial x}+\frac{\omega_c l}{x}\right)n(x)\bigg|_{x=1,b/a}=0, \quad (6)$$

where (5) is the result of some manipulation with (1), (2), and (3), (6) is the boundary condition (4), $x=\rho/a$, and $\Omega_0^2=4\pi e^2 n_0 h/ma^2(\epsilon_1+\epsilon_2)=c_p^2/a^2$ , where $c_p$ is a screened plasmon velocity. At this stage the problem has been reduced to the solution of an ordinary second-order differential equation (5) with the mixed boundary condition (6); a similar procedure was used in Ref. 2 to analyze the EMP of a disk. Considering the trivial role played by the parameter $s$ in Eq. (5) (it just gives a renormalization of $\Omega_0$) in what follows we will take $s=0$.

Taking into account that the origin is excluded from the region where solutions of (5) should be obtained, the more general solution when $\omega>\omega_c$ is given by

$$n(x)=AJ_l(\lambda x)+BY_l(\lambda x), \quad (7)$$

where $A$ and $B$ are coefficients, $J_l$ and $Y_l$ are the Bessel functions,[14] and $\lambda$ a parameter that should be chosen so as to satisfy the boundary conditions. Replacing (7) in (5) we obtain the dispersion relation

$$\omega^2=\omega_c^2+\Omega_0^2\lambda^2, \quad (8)$$

which is similar to that for an infinite 2DEG, but with the two-dimensional bulk plasma frequency replaced by $\lambda\Omega_0$, which is magnetic-field and ring size dependent.

Substitution of (7) in (6) yields the equation which determines the eigenfrequencies $\omega$,

$$\left[\lambda J_{l-1}(\lambda)+l\left(\frac{\omega_c}{\omega}-1\right)J_l(\lambda)\right]\left[\frac{\lambda b}{a}Y_{l-1}\left(\frac{\lambda b}{a}\right)+l\left(\frac{\omega_c}{\omega}-1\right)Y_l\left(\frac{\lambda b}{a}\right)\right]$$
$$-\left[\lambda Y_{l-1}(\lambda)+l\left(\frac{\omega_c}{\omega}-1\right)Y_l(\lambda)\right]\left[\frac{\lambda b}{a}J_{l-1}\left(\frac{\lambda b}{a}\right)+l\left(\frac{\omega_c}{\omega}-1\right)J_l\left(\frac{\lambda b}{a}\right)\right]=0. \quad (9)$$

In the regime $\omega<\omega_c$, according to Eq. (8), $\lambda^2<0$, which means that we should replace $\lambda$ by $i\lambda$ in (9); using the properties of the Bessel functions of imaginary arguments[14] we obtain in the regime $\omega<\omega_c$ the eigenvalue equation

$$\left[\lambda I_{l-1}(\lambda)+l\left(\frac{\omega_c}{\omega}-1\right)I_l(\lambda)\right]\left[-\frac{\lambda b}{a}K_{l-1}\left(\frac{\lambda b}{a}\right)+l\left(\frac{\omega_c}{\omega}-1\right)K_l\left(\frac{\lambda b}{a}\right)\right]$$
$$-\left[-\lambda K_{l-1}(\lambda)+l\left(\frac{\omega_c}{\omega}-1\right)K_l(\lambda)\right]\left[\frac{\lambda b}{a}I_{l-1}\left(\frac{\lambda b}{a}\right)+l\left(\frac{\omega_c}{\omega}-1\right)I_l\left(\frac{\lambda b}{a}\right)\right]=0, \quad (10)$$

where $I_l$ and $K_l$ are now the modified Bessel functions. Equations (9) and (10) are the central result of our paper; before proceeding with its numerical solution, we will obtain some analytical results from them.

A first important point is that Eqs. (9) and (10) are not invariant under the change $l$ by $-l$, *if* $\omega_c\neq 0$; this means that while the solutions corresponding to the normal modes with $l$ and $-l$ are degenerate if $\omega_c=0$, as soon as the magnetic field is turned on, a splitting arises between both modes. In real samples, however, any deviation from the cylindrical symmetry will give a coupling between the $l$ and $-l$ modes, and consequently a splitting even at $\omega_c=0$.

A second interesting question is how many EMP's have a ring in the strong magnetic field regime $\omega_c/\Omega_0\gg 1$ for a given $l$? The answer can be easily found from Eq. (10) (by definition, the existence of EMP implies $\omega<\omega_c$) by making use of the asymtotic expansions of the modified Bessel functions,[14] with the result that we found *two* EMP's,

$$l>0:\quad \omega_+(\omega_c/\Omega_0\gg 1)\rightarrow l\Omega_0+O(1/\omega_c), \quad (11)$$

$$l<0:\quad \omega_-(\omega_c/\Omega_0\gg 1)\rightarrow -l\frac{a}{b}\Omega_0+O(1/\omega_c), \quad (12)$$

which, remembering that $\Omega_0=c_p/a$, could be rewritten in the transparent way $\omega_+\sim lc_p/a$, $\omega_-\sim -lc_p/b$, which shows that the positive $l$ mode $\omega_+$ (the antidot EMP) corresponds to a density perturbation that circulates around the inner edge, in the counterclockwise sense, with the screened plasmon velocity $c_p$, while the negative $l$ mode $\omega_-$ (the dot EMP), corresponds to a density perturbation circulating along the outer boundary, in the clockwise sense. This last mode is the one discussed extensively in the literature both experimentally and theoretically, and its main feature is that its frequency decreases as the magnetic field increases, reaching asymptotically the limiting value (12). No anomalous solution of (10) exist for $l=0$, which is consistent with the fact that for these radial modes, the angular component of the velocity is zero when $\omega_c=0$ [see Eq. (15) below].

Turning now to the numerical results, we display in Fig. 1 the antidot EMP frequencies $\omega_+$ (dashed lines) and the dot EMP frequencies $\omega_-$ (full lines) for several values of $l$; the dotted line corresponds to the cyclotron frequency $\omega_c$ and $b/a=2$. It should be pointed out that for a given $l$, Eqs. (9) and (10) have an infinite number of solutions, corresponding to different values of the radial quantum number; of all these solutions, only the one corresponding to the lowest radial quantum number (no nodes in the density in the radial direction) gives rise to the EMP, and these are the only ones plotted in Fig. 1.

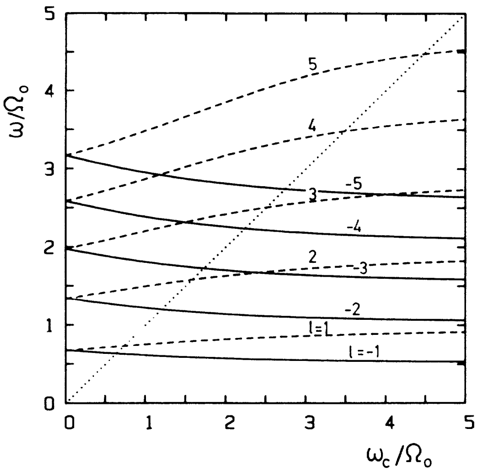


FIG. 1. Edge magnetoplasmon frequencies as function of magnetic field for a ring with $b/a = 2$. The full line ($l < 0$) corresponds to the dot EMP, the dashed line ($l > 0$) to the antidot EMP, and the dotted line is the cyclotron frequency.

As mentioned above, the degeneracy between the $\pm l$ modes is lifted by the magnetic field, and we obtain a dot EMP, whose frequency decreases with magnetic field, and an antidot EMP, whose frequency increases with magnetic field, going asymptotically to the limiting value (11). To the best of our knowledge, no theoretical study has been done previously on this antidot EMP. As expected, for a given value of $|l|$ and $\omega_c$, in the regime $\omega_c/\Omega_0 \gg 1$ where the EMP are well established, the dot EMP is closer to its limiting value $-al\Omega_0/b$ than the antidot EMP to its corresponding value $l\Omega_0$; this is related to the fact that for a given value of $|l|$, finite size effects are more important for the dot EMP than for the antidot EMP.

To clarify this point we have repeated the calculation for the ratio $b/a = 5$; the results are shown in Fig. 2 and there is seen clearly that the dot EMP crosses the line $\omega = \omega_c$ (which gives a rough estimate of the threshold magnetic field at which the EMP begins to develop) much before the antidot EMP (compare, for instance, the $|l| = 5$ case). It is worth noting that for $l \gg 1$, the threshold magnetic field at which the antidot EMP begins to develop becomes independent of $b$ and is given by $\sqrt{l(l-1)}\Omega_0$ [this result could be obtained from Eqs. (9) or (10) in the limit $\omega \to \omega_c$].

We display in Fig. 3 the radial dependence of the self-induced densities for the $|l| = 1$ dot (full line) and antidot (dashed line) EMP for different values of the magnetic field; according to Eq. (7) (generalized to the case $\omega < \omega_c$) the normalized density is given by

$$\frac{n(x)}{n(x_0)} = \frac{I_1(\lambda x) + (B/A)K_1(\lambda x)}{I_1(\lambda x_0) + (B/A)K_1(\lambda x_0)}, \tag{13}$$

where $x_0 = 1$ or $b/a$, depending on which of the EMP's one is considering, $\lambda$ is obtained from the eigenvalue equation (10) at the corresponding magnetic field, and $B/A$ is determined from the boundary condition (6). Note that in the present fully screened limit, $n(x)/n(x_0) = \phi(x)/\phi(x_0)$.

Figure 3 clearly shows that (i) the density associated with the EMP is strongly localized near the corresponding boundary and, (ii) this localization increases as function of magnetic field. For example, if $x \to 1$ and $\omega_c/\Omega_0 \gg 1$, one can obtain from (13) the following asymptotic behavior of the charge density associated with the antidot EMP:

$$\frac{n(x)}{n(1)} \sim \frac{e^{\lambda(1-x)}}{\sqrt{x}} \tag{14}$$

which gives an exponential localization at the inner edge of the ring (within a scale of $\lambda \sim \omega_c/\Omega_0$). An analogous behavior is obtained in the case $x \to b/a$ and $\omega_c/\Omega_0 \gg 1$, as it was obtained previously for the dot geometry.[2]

The characterization of the dynamical response is completed in Fig. 4, which gives the normalized $|l| = 1$ angular current [defined as $j_\theta(x) = -en_0v_\theta(x)$, with $v_\theta(x)$ the angular velocity] as function of the radial coordinate,

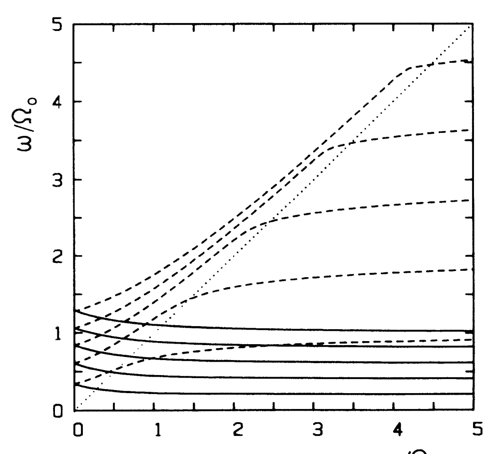


FIG. 2. Same as Fig. 1, but for a ring with $b/a = 5$.

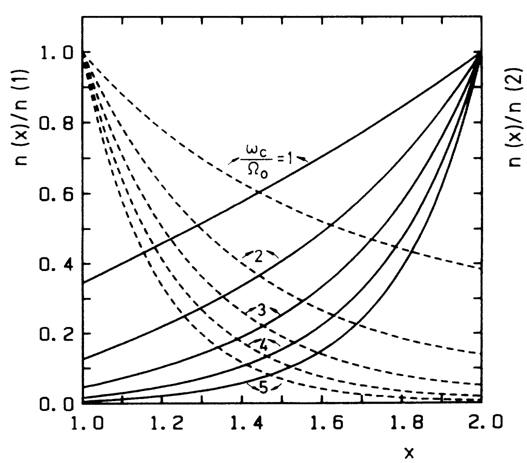


FIG. 3. Self-induced density distributions associated with the dot EMP (full line) and antidot EMP (dashed line), for several values of magnetic field; $|l| = 1$ and $b/a = 2$.

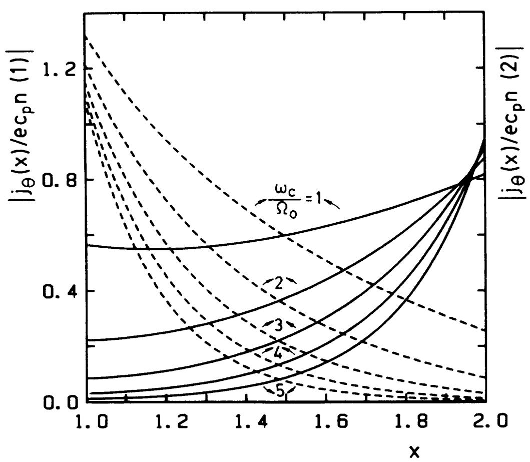

FIG. 4. Self-induced angular current distributions associated with the dot EMP (full line) and antidot EMP (dashed line), for several values of magnetic field; $|l| = 1$ and $b/a = 2$.

for several values of the magnetic field. From Eq. (2), we obtain

$$\frac{j_\theta(x)}{ec_p n(x_0)} = \frac{\Omega_0^2}{\omega^2 - \omega_c^2}\left(\frac{\omega_c}{\Omega_0}\frac{\partial}{\partial x} + \frac{l\omega}{\Omega_0}\frac{1}{x}\right)\frac{n(x)}{n(x_0)} \qquad (15)$$

with $n(x)/n(x_0)$ as given by (15). The behavior of the angular currents is similar to the charge density behavior: with increasing magnetic field they tend to be concentrated on the corresponding edge, within a scale given by $\lambda$. For strong enough magnetic field, the angular current of the antidot (dot) EMP tends asymptotically towards the limiting value $ec_p n(1)$ $[ec_p n(2)]$ from above (below).

Turning now towards the question of the experimental situation, very recently Kern *et al.*[15] have studied the collective excitations of a periodic array of antidots by far-infrared spectroscopy. They observed essentially a high- and a low-frequency resonance; the low-frequency branch starts approximately from $\omega_c$ at small $B$ and then increases in frequency with magnetic field, as opossite to the dot EMP, which decreases in frequency with magnetic field. The mode (at least for strong magnetic field) was associated with an EMP which circulates around the antidot.

While it is tempting to associate this behavior with the similar results obtained for the $l > 0$ modes of Figs. 1 and 2, a comparison between theoretical and experimental results is difficult as the antidot structures have been prepared by etching arrays of holes in a 2DEG while the calculations presented in this work should give a reasonable description of electrons on the surface of liquid helium; this is implicit in our assumption on the presence of grounded electrodes and the hard-wall boundary condition.

In summary, using the hydrodynamic approach to describe the dynamic response of a classical 2DEG confined by a ring, we have obtained *two* edge magnetoplasmons. One of them is associated to a charge density which circulates around the outer perimeter of the ring and whose frequency *decreases* when the magnetic field increases, while the second one (the antidot EMP) represents a charge density which circulates along the inner boundary of the ring, with a frequency which is an *increasing* function of the magnetic field. We hope that the present results will stimulate the search for the experimental detection of this new EMP for electrons on the surface of liquid helium, where the first clear results were obtained for the related dot geometry.

The author is grateful to Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, and M. Tomak for a careful reading of the manuscript.

*Permanent address: (8400) S. C. de Bariloche, Río Negro, Argentina.

[1] D. B. Mast, A. J. Dahm, and A. L. Fetter, Phys. Rev. Lett. **54**, 1706 (1985).

[2] D. C. Glatti, E. Y. Andrei, G. Deville, J. Poitrenaud, and F. I. B. Williams, Phys. Rev. Lett. **54**, 1710 (1985).

[3] P. J. M. Peters, M. J. Lea, A. M. L. Janssen, A. O. Stone, W. P. N. M. Jacobs, P. Fozooni, and R. W. van der Heijden, Phys. Rev. Lett. **67**, 2199 (1991).

[4] S. J. Allen, H. L. Stormer, and J. C. M. Hwang, Phys. Rev. B **28**, 4875 (1983).

[5] T. Demel, D. Heitmann, P. Grambow, and K. Ploog, Phys. Rev. Lett. **64**, 788 (1990).

[6] M. Wassermeir, J. P. Oshinowo, J. P. Kotthaus, A. M. Mac Donald, C. T. Foxon, and J. J. Harris, Phys. Rev. B **41**, 10 287 (1990).

[7] I. Grodnensky, D. Heitmann, and K. von Klitzing, Phys. Rev. Lett. **67**, 1019 (1991).

[8] R. C. Ashoori, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. West, Phys. Rev. B **45**, 3894 (1992).

[9] A. L. Fetter, Phys. Rev. B **32**, 7676 (1985); **33**, 5221 (1986); **33**, 3717 (1986).

[10] V. Cataudella and G. Iadonisi, Phys. Rev. B **35**, 7443 (1987).

[11] V. A. Volkov and S. A. Mikhailov, Zh. Eksp. Teor. Fiz. **94**, 217 (1988) [Sov. Phys. JETP **67**, 1639 (1988)].

[12] S. Lundqvist, in *Theory of the Inhomogeneous Electron Gas*, edited by S. Lundqvist and N. H. March (Plenum, New York, 1983).

[13] J. Dempsey and B. I. Halperin, Phys. Rev. B **45**, 1719 (1992).

[14] F. W. J. Olver, in *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965), p. 355.

[15] K. Kern, D. Heitmann, P. Grambow, Y. H. Zhang, and K. Ploog, Phys. Rev. Lett. **66**, 1618 (1991).