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Experimental studies of the localization transition in the quantum Hall regime

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We study experimentally the localization-to-delocalization transition in the transport regime between adjacent integer-quantum-Hall plateaus. We use small Hall bar geometries at millikelvin temperatures so that the phase-breaking length exceeds the sample size. Under these conditions the width ΔB of the transition region scales with the size W of the sample according to $W \propto (\Delta B)^{-\nu}$. We obtain a universal scaling exponent $\nu = 2.3 \pm 0.1$. This result agrees with the predictions of several theoretical approaches to the metal-insulator transition in the integer-quantum-Hall regime. The numerical result agrees with the findings of the trajectory network model ($\nu = 2.5 \pm 0.5$), the percolation picture including quantum tunneling ($\nu = \frac{7}{3}$), and recent numerical studies (e.g., $\nu = 2.34 \pm 0.04$). The temperature exponent of the inelastic-scattering rate can be measured in the same experiment. We obtain results in the range from $p = 2.7 \pm 0.3$ to $p = 3.4 \pm 0.4$, which are considerably larger than commonly assumed values. Small reproducible magnetoresistance fluctuations are observed, which do not substantially influence the scaling behavior. By studying the effect of current heating, it is shown that noise heating does not play a role in the measurements. We discuss the present results in comparison with previous experimental and theoretical investigations.

I. INTRODUCTION

Recent experimental and theoretical work of several groups has clearly shown that the delocalization-to-localization (or metal-insulator) transition in a two-dimensional electron gas (2DEG) in the quantum Hall regime shows scaling behavior.[1–14] Early experimental studies[1–5] have analyzed the temperature (T) dependence of the width of the transition region as a function of the magnetic field (ΔB) or of the energy (ΔE). It was generally found that the width shrinks as the temperature goes to zero according to a power law of the form $\Delta B \propto T^{\kappa}$. The question of whether the exponent κ is a universal quantity or not, however, has been a point of discussion. Wei *et al.*[1] studied the behavior of a specific $In_{0.53}Ga_{0.47}As/InP$ heterostructure, and they found an exponent of $\kappa = 0.42 \pm 0.04$ in the two lowest, spin-split Landau levels (LL's). They proposed that this exponent should be a universal constant. However, the present authors have shown in a systematic study of low-mobility $Al_xGa_{1-x}As/GaAs$ heterostructures[3] that in such systems the exponent κ is *not* universal. Instead, it was demonstrated that this exponent increases with decreasing carrier mobility from values of 0.28 ± 0.06 (in a sample with comparatively high mobility) up to 0.81 ± 0.04 in a low-mobility sample. Other groups have studied Si-MOSFET's (metal-oxide-semiconductor field-effect transistors),[2,4,5] and they also found non-universal results for the exponent κ.

While this exponent could not be computed up to now, the behavior of the localization length of electronic states close to the critical energy E_c or the critical field B_c is considered to be understood.[7–14] It is now generally accepted that the localization length ξ diverges close to the critical point according to

$$\xi \propto |E - E_c|^{-\nu} \propto |B - B_c|^{-\nu}, \quad (1)$$

where ν is the critical exponent.[7–11] Close enough to the critical point the quantities $E - E_c$ and $B - B_c$ are proportional (which is fulfilled in the measurements discussed in this work). Furthermore, various techniques have allowed the computation of the exponent ν, and they strongly suggest a universal value close to $\nu = \frac{7}{3}$, at least for the lowest Landau level.[10–13] The connection between the measured exponent κ and the computed exponent ν is given by the temperature exponent $p/2$ of the inelastic-scattering length (or phase-coherence length) $L_{in} = L_0 T^{-p/2}$ (where L_0 is a constant prefactor). The inelastic-scattering length acts as an effective sample size. The exponents are related via $\kappa = p/(2\nu)$,[8] where diffusive quantum transport is assumed.[15] The exponent p is not known for high magnetic fields.

In this situation of possibly conflicting experimental and theoretical results, a clarification from the experimental side is desirable and in fact possible. We have performed a separate measurement of the localization length exponent ν and the scattering length exponent $p/2$.[6] The basis of the method is that at low enough temperature the phase-coherence length is larger than the sample size in small enough samples. The experiments strongly suggest a universal behavior of the exponent ν, both independent of sample and of Landau level. While part of this work has been published previously,[6] in the present study we discuss the separate determination of critical exponents on the basis of additional data. This is done in Secs. II and III. In Sec. IV we discuss the role of "mesoscopic fluctuations" in such measurements. In Sec. V it is shown that eventual experimental problems due to noise heating of the electronic system do not play a role. Finally, in Sec. VI we compare our results with the previous experiments in different semiconductor heterostructures and in Si MOSFET's, and with the results for the localization length exponent ν as obtained from different theoretical approaches. The work is summarized in Sec. VII.

II. THE EXPERIMENTAL TECHNIQUE

For the experiments, we use small Hall bar geometries with widths ranging from 8 to 64 μm. In these geometries, the width ΔB of the localization transition region is found to saturate below a certain characteristic temperature T_c which increases with decreasing sample width. Below the saturation temperature, the width of the transition region scales with the width of the sample. By comparing the results for samples of different size, the exponent ν can be determined. Moreover, by analyzing the characteristic temperatures for the respective sample widths, the temperature exponent $p/2$ of the inelastic-scattering length may be obtained.

The samples under study are low-mobility Al/Ga/As/GaAs heterostructures with intentionally built-in scatterers in the region of the two-dimensional electron gas (for details on the "δ-doping" technique employed, see Ref. 16). The layer sequence of the samples is the following: on a GaAs substrate with a 2-μm GaAs buffer layer, a 23-nm undoped $Al_xGa_{1-x}As$ spacer layer ($x = 0.33$) is grown, followed by a 50-nm Si-doped Al/Ga/As layer and a 10-nm GaAs cap layer. At a distance of 2.2 nm from the Al/Ga/As/GaAs interface an ultrathin sheet of scatterers is introduced into the GaAs which consists of either Si or Be atoms. These impurities act as attractive and repulsive Coulomb scatterers, respectively. The doping level, carrier concentration, and carrier mobility are given in Table I. After the growth process, the samples are etched into structures with four Hall bars with nominal widths of W=8, 16, 32, and 64 μm and a length-to-width ratio of 3. (The actual widths are given in the figures.) The sample layout is such that, e.g., all lateral dimensions of the 64-μm Hall bar are twice as large as those of the 32-μm Hall bar. Each Hall bar is thus characterized by a single parameter only (the width). The sample was mounted in the glass tail of a ^{3}He-^{4}He dilution refrigerator. Samples 1 and 2 were fully photoexcited using a red light-emitting diode (LED), in order to further increase the homogeneity of the sample, while sample 3 was studied in the dark. The electron concentrations and mobilities thus obtained in the samples are given in Table I. The measurements were performed in the temperature range from 25 mK to 1 K. The resistance traces were obtained with a standard low-frequency lock-in technique. It was carefully checked that neither noise nor the measuring current of 0.5 nA led to electron heating. We have eliminated noise heating by carefully shielding all connections to the sample and by using low-pass filters both at room temperature and at low temperature. As a further check, we have simulated the effect of electron heating by studying the effect of current heating (with currents I =1–100 nA) at base temperature. The results are different for different LL's and in different samples, as will be shown explicitly in Sec. V. This is in striking contrast to the universal behavior of the localization length exponent ν to be reported now.

TABLE I. Type of scatterers, density of scatterers n_s, electron concentration n_e, and mobility μ_e of the samples (Asterisks denote after illumination).

Sample No.	Scatterers	n_s 10^{14} m^{-2}	n_e 10^{15} m^{-2}	μ_e m^2/Vs
1	Si	0.3	4.85*	15.5
2	Be	2.0	4.0*	5.8
3	Si	2.0	3.3	3.4

III. EXPERIMENTAL RESULTS

In Fig. 1(a) we present experimental results for the measured longitudinal resistance R_{xx} in LL $N = 1 \uparrow$ (the arrow stands for the spin direction), i.e., for the region of filling factors between 2 and 3 for sample 1, normalized to the maximum value as a function of magnetic field B close to the critical magnetic field B_c. The curves are given for the four widths, at a temperature of 25 mK. The half-width ΔB of the R_{xx} peak increases with decreasing width. The reproducible fluctuations visible in the smallest structure will be discussed below. In Fig. 1(b) we show the Hall resistance R_{xy} in the same LL. Clearly the maximum slope of the Hall resistance as a function of magnetic field decreases with decreasing width. In this low-temperature region, the transport coefficients are independent of temperature, as can be seen from Fig. 2. In Fig. 2 we show the half-width of R_{xx} in LL $N = 1 \uparrow$ for sample 1 as a function of temperature for the four different widths. At low temperatures, the respective half-width saturates for a given sample width. The saturation

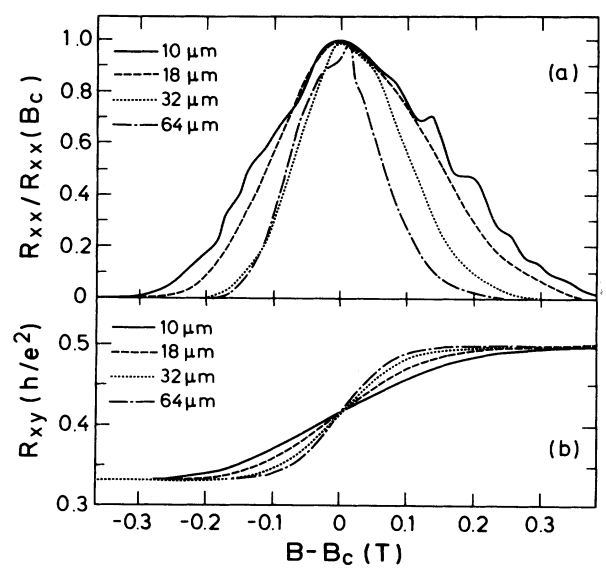

FIG. 1. Transport data for sample 1 in Landau level $N = 1 \uparrow$ at a temperature of $T = 25$ mK. (a) Normalized longitudinal resistance R_{xx} as a function of magnetic field B close to the critical field B_c. (b) The Hall resistance R_{xy} in units of h/e^2 as a function of $B - B_c$.

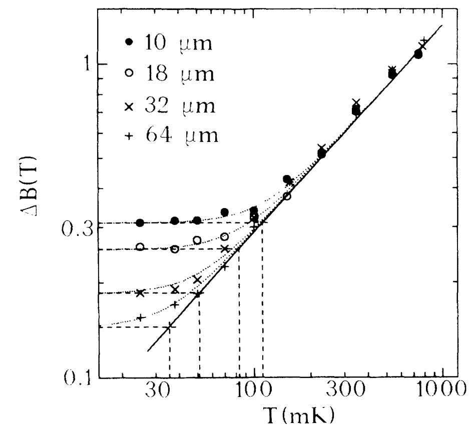

FIG. 2. The half-width ΔB as a function of temperature for sample 1 in Landau level $N = 1 \uparrow$ for the four sample widths. Horizontal dashed lines: extrapolation to zero temperature. The vertical dashed lines correspond to the characteristic temperatures T_c. Full line: high-temperature fit. The dotted fit curves are discussed in the text.

values are indicated by the horizontal dashed lines. The saturation starts at lower temperatures with increasing sample size. The corresponding characteristic temperatures T_c are indicated by the vertical dashed lines. At high temperatures, the measured values practically coincide, independent of width. The full line is a fit to the high-temperature data and corresponds to a temperature exponent of $\kappa = 0.72 \pm 0.05$. In Fig. 3, data are shown for the maximum slope $(\partial\rho_{xy}/\partial B)^{\max}$ in sample 1 in LL $N = 2 \uparrow$. These data show a saturation behavior which is similar to that in Fig. 2; the temperature exponent of the high-temperature behavior is $\kappa = 0.67 \pm 0.06$. We note here that at these low temperatures the influence of the thermal broadening of the Fermi function no longer plays a role.

In the whole temperature range the temperature dependence of the half-width ΔB is described by the dotted fit curves displayed in Fig. 2. These fits are based on the following (simplified) analysis. We assume the validity of Matthiesen's rule in the form

$$\Gamma_{\text{tot}} = \Gamma_{\text{in}} + \Gamma_{\text{edge}}. \quad (2)$$

Γ_{in} is the rate of inelastic-scattering processes and Γ_{edge} is the rate of scattering processes at the edge of the sample. For both individual scattering rates diffusive transport is assumed, which means

$$\Gamma_{\text{in}} = \frac{D}{L_{\text{in}}^2} = \frac{D}{L_0^2}T^p \quad \text{and} \quad \Gamma_{\text{edge}} = \frac{D}{W^2}. \quad (3)$$

D is the diffusion coefficient, L_{in} is the inelastic-scattering length, and W is the width of the sample. At high temperatures we have $\Gamma_{\text{tot}} = \Gamma_{\text{in}} = (D/L_0^2)T^p$, and thus

$$\Delta B = bT^{\kappa} = b(T^p)^{1/(2\nu)} = b\left(\frac{L_0^2}{D}\Gamma_{\text{tot}}\right)^{1/(2\nu)}. \quad (4)$$

By inserting Eqs. (2) and (3) into Eq. (4) one obtains the fit equation for the half-width ΔB at arbitrary temperature T and sample width W,

$$\Delta B = b\left(T^p + \frac{L_0^2}{W^2}\right)^{1/(2\nu)}. \quad (5)$$

It is easily verified that this equation yields the correct behavior at both low and high temperatures. For constructing the fits we have used the exponent ν and the saturation half-widths, as obtained from the low-temperature results, and the prefactor b together with the exponent p, as inferred from the high-temperature behavior. Using these given values, the half-width ΔB in the transition regime can be obtained. The fit equation for the maximum slope in Fig. 3 can be derived in a similar manner. As can be seen from Figs. 2 and 3 the fits provide a good representation of the measured data.

The saturation of the transport data at temperatures smaller than 100 mK is a clear indication that at these temperatures the inelastic-scattering length is larger than the sample width [correspondingly, in Eq. (5) the second term dominates the first]. Therefore the magnetic-field range of the extended states, and thus ΔB and $(\partial\rho_{xy}/\partial B)^{\max}$, is only determined by the sample width. Electron states with a localization length larger (smaller) than the width do (do not) contribute to a finite R_{xx}. This size dependence can be used to obtain the critical exponent ν of the localization length $\xi \propto [(\partial\rho_{xy}/\partial B)^{\max}]^{\nu}$. In Fig. 4 we show the low-temperature saturation values of $(\partial\rho_{xy}/\partial B)^{\max}$ in samples 1 and 2 for the four widths on a double logarithmic plot. Within experimental error these values lie on a straight line with a slope corresponding to the value of the exponent ν. As can be seen from the figure, the slopes for different Landau levels in both samples coincide within experimental error. The overall result for the exponent is $\nu = 2.3 \pm 0.1$ (given in more detail in Ref. 6). This can be *directly* compared with theoretical results, as will be done in Sec. VI.

Using the data of Figs. 2 and 3 it is also possible to obtain the temperature exponent $p/2$ of the inelastic-scattering length $L_{\text{in}} \propto T^{-p/2}$. We extrapolate the low-

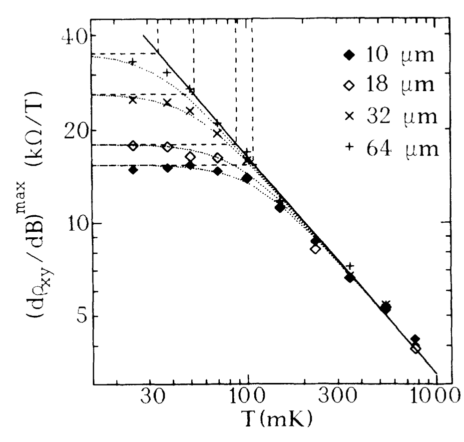

FIG. 3. Same as Fig. 2, for the maximum slope $(\partial\rho_{xy}/\partial B)^{\max}$ in sample 1 in Landau level $N = 2 \uparrow$.

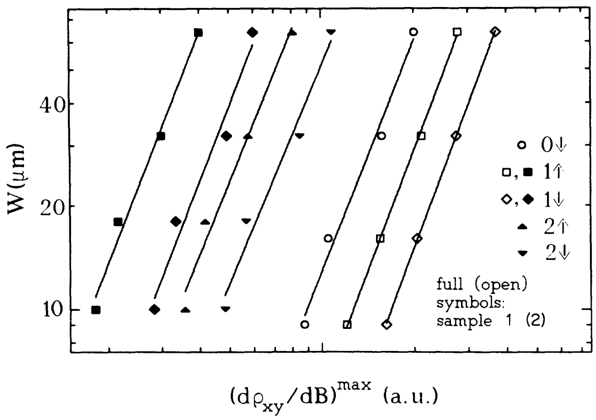

FIG. 4. The sample width W as a function of the saturation maximum slope $(\partial\rho_{xy}/\partial B)^{\max}$ for samples 1 and 2 in the three lowest Landau levels. Straight lines: fits. The curves are offset for clarity. The absolute values for the 64-μm-wide sample 1 are (in kΩ/T) LL 1 $\uparrow$: 32, 1 $\downarrow$: 36, 2 $\uparrow$: 35, 2 $\downarrow$: 32.5, and in sample 2 we obtain LL 0 $\downarrow$: 27, 1 $\uparrow$: 52, 1 $\downarrow$: 37.

temperature value of ΔB and $(\partial\rho_{xy}/\partial B)^{\max}$ for a given width to higher temperatures (horizontal dashed line) and the high-temperature fit (full line) to low temperatures. These lines intersect at a temperature T_c. We argue that at the temperature T_c, the inelastic-scattering length is approximately equal to the sample width, so that an absolute value for the inelastic-scattering length is also obtained. In this way, four values of L_{in} are determined for four temperatures. From $W = L_{\text{in}} \propto T_c^{-p/2}$ we obtain values between $p = 2.7$ and 3.4. Furthermore, an evaluation of the measured results for the exponent κ obtained in Ref. 3 is possible. By using the equation $\kappa = p/(2\nu)$ and the present universal result of $\nu = 2.3 \pm 0.1$ we arrive at a range of $p = 1.3$ to 3.7 for the numerical value of the inelastic-scattering rate exponent. Apparently, the exponent p depends on the concentration and distribution of the scatterers close to the 2DEG. The results for p in zero magnetic field [$p = 1$ ("dirty metal limit"[17]) and $p = 2$ (obtained from Fermi liquid theory)] thus appear to be generally inapplicable in high magnetic fields.

IV. THE ROLE OF FLUCTUATIONS

In samples where the phase-coherence length is larger than the sample size, naturally the question arises regarding the influence of quantum interference phenomena leading to reproducible magnetoresistance fluctuations (or "mesoscopic fluctuations"). In fact, the magnetoresistance traces of the structures in Fig. 1 show small fluctuations. Their amplitude is less than 10% of the maximum R_{xx} value of the transition for the smallest structure, and is smaller in the larger samples. We also find that the oscillations are stronger in the spin-up LL than in the spin-down LL.

The amplitude of the fluctuation effects can be compared with results reported by Simmons *et al.*[18] in 1 – 2-μm-wide Hall geometries fabricated from high-mobility Al/Ga/As/GaAs heterostructures at temperatures of 25 mK. In Fig. 1 of Ref. 18, quasiperiodic fluctuations in the vicinity of the R_{xx} minima are shown. It can be estimated that the strength of the fluctuations in the transition region between adjacent Hall plateaus is roughly 25% of the R_{xx} maximum. In comparison to our results one finds that the amplitude of the fluctuations is stronger in smaller samples. This corresponds qualitatively to the results of several theoretical investigations which shall now be shortly discussed. Jain and Kivelson[19] analyzed resonant tunneling between edge states on opposite edges of a narrow sample via a bound state in the center of the sample. Among other findings, their results showed that the fluctuations are more pronounced at lower temperatures and in narrower samples, which corresponds to our experimental results. Comparable investigations were performed by Büttiker.[20] He found that the (negative) fluctuations become stronger with a decreasing number of occupied edge states. In numerical studies by Lee, McLennan, and Datta[21] which confirm the conclusions of Ref. 19, the authors also obtain "anomalous" resistance maxima which are small compared to the R_{xx} maxima.

As a summary of these considerations we conclude that reproducible magnetoresistance fluctuations are observed in our experiment, as may be expected in a situation where the phase-coherence length is larger than the sample size. Their amplitude agrees with estimates obtained from theoretical considerations as well as from experimental studies. Nevertheless, in the samples and at the temperatures used, the strength of the fluctuations is so small that the scaling behavior is not substantially influenced.

V. ELECTRON HEATING

In principle, a saturation of a temperature-dependent quantity such as the half-width ΔB could originate from an electronic temperature increased in comparison to the bath temperature. This experimental problem shall now be discussed in some detail. For the interpretation of our results it is essential that the electronic temperature in the samples is not significantly larger than the measured bath temperature. Such a problem might be most important in the smallest samples. Assuming that between any pair of contacts the same high-frequency noise current is flowing, this would lead to the largest electronic temperature in the smallest sample because here the current *density* is largest. However, we are convinced that this is *not* the case for several reasons.

First, we have again compared our data with the results by Simmons *et al.*[18] In a sample of 1 – 2-μm width they reported that a current of 0.3 nA did not lead to electron heating at a bath temperature of 25 mK. This corresponds to an even larger current *density* than the 0.5 nA we used in our samples with widths down to 9 μm, so that we conclude that the electronic temperature will not be significantly enhanced in our measurements either.

Second, in sample 3 we did not observe saturation

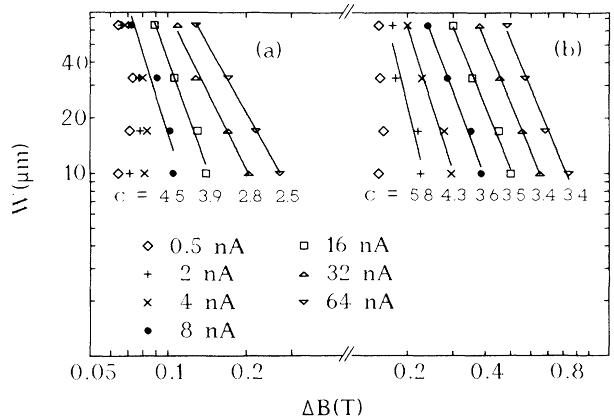

FIG. 5. Sample width W as a function of the half-width ΔB in R_{xx} for currents from 0.5 to 64 nA in sample 3 on a double-logarithmic plot, corresponding to $W \propto (\Delta B)^{-c}$. Regression lines with numerical value of the slope. Lattice temperature: 25 mK. (a) Landau level $N = 1 \downarrow$, (b) $N = 0 \downarrow$.

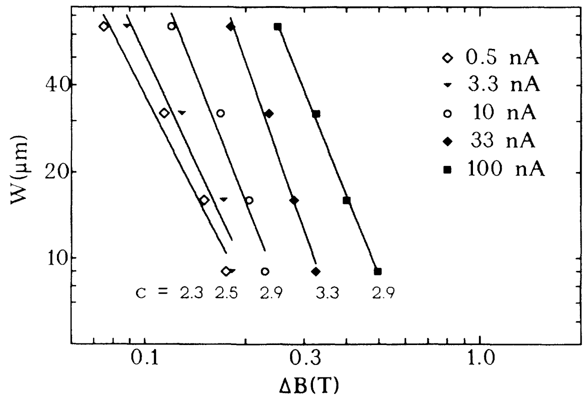

FIG. 6. Sample width W as a function of ΔB for currents from 0.5 to 100 nA in Landau level $N = 1 \downarrow$ for sample 2. Lattice temperature: 25 mK.

due to the localization transition (although the analysis is complicated here by larger sample inhomogeneities). We have to assume that in this sample the inelastic-scattering length L_{in} at a given temperature is smaller than in the other samples. Quantitatively, we conclude that $L_{\text{in}}(70 \text{ mK}) < 10\ \mu\text{m}$. On the other hand, it would be difficult to understand why in this sample noise heating should be less effective. We note that an identical experimental setup was used, when compared to the measurements in the other two samples.

Third, as already mentioned, we have simulated the effect of noise heating by studying the effect of current heating on the transport properties at a bath temperature of 25 mK. We have used the same low-frequency current at larger amplitudes in the range from $I = 1$ to 100 nA. In Fig. 5 we show the width W of sample 3 as a function of the half-width ΔB of the R_{xx} peak for currents in the range from 0.5 to 64 nA in the LL's $N = 0 \downarrow$ and $N = 1 \downarrow$. In the present sample 3, at low currents, the half-widths are approximately equal because saturation does not occur as described above. (The results are not identical because the influence of inhomogeneities is different in the individual Hall bars.) At higher currents, larger half-widths ΔB are found. The respective ΔB is larger in samples with smaller width because there the current density and thus the electronic temperature are larger. Figure 6 shows similar data for sample 2 in LL $N = 1 \downarrow$. Here, at $I = 0.5$ nA the localization-induced saturation behavior is observed. Within a certain error, one obtains power laws of the form $W \propto (\Delta B)^{-c}$ from Figs. 5 and 6. However, *the exponent c in general depends on the current.* In sample 3 we obtain a value of $c \approx 3.5$ in LL $N = 0 \downarrow$ for larger currents, while in LL $N = 1 \downarrow$ the exponent c decreases with increasing current, and no constant value for c is found as in LL $N = 1 \downarrow$. In sample 2, finally, the exponent c varies unsystematically with the current. In conclusion, it is found that current heating leads to a sample- as well as Landau-level-dependent behavior, which should also be the case for noise heating. This is in striking contrast to the universal behavior of the saturation due to the localization-to-delocalization transition which was reported above. All these facts strongly indicate that noise heating does not play a role in our measurements.

VI. DISCUSSION

Various authors have suggested universality for the critical exponent ν of the localization length, independent of Landau level and the details of the random potential.[7,9–11] Trugman[7] provides the result $\nu = \frac{4}{3}$ of the classical percolation picture, while in the work of Pruisken[9] only universality as such is suggested and no numerical value of ν is obtained. On the basis of the trajectory network model, Chalker and Coddington[10] derive the value $\nu = 2.5 \pm 0.5$. Mil'nikov and Sokolov[11] also use the percolation picture, but they include quantum tunneling and thus arrive at $\nu = \frac{7}{3}$. The two latter results correspond to the case of a potential with long-range fluctuations. On the other hand, the numerical studies performed to date correspond to the situation where the correlation length d of the random potential is either zero or of the order of the magnetic length $l_c = \sqrt{\hbar/(eB)}$. For the lowest LL, various numerical studies obtain results close to $\nu = \frac{7}{3}$. To be specific, Aoki and Ando[8] calculate $\nu \approx 2$, Huckestein and Kramer[12] find $\nu = 2.34 \pm 0.04$, and Mieck[13] determines $\nu = 2.3 \pm 0.08$. While in Refs. 8–13 only bulk Landau states are considered, in a recent work by Ando[14] both edge states and bulk Landau states are included in the numerical study. His result of $\nu = 2.2 \pm 0.1$ is very close to those of the other numerical studies. There have also been attempts to numerically study higher LL's, but for the first LL $N = 1$ these studies have not yet been able to conclusively obtain scaling.[8,13] This discrepancy may be resolved if larger system sizes are used than those in the cited studies.

Within experimental error, our values obtained for the three lowest (spin-split) LL's $N = 0$, $N = 1$, and $N = 2$ coincide. Furthermore, this coincidence is found in two samples with different mobilities. This fact strongly supports the universality statement. In addition, our result agrees with the numerical value obtained by the various theoretical approaches mentioned above. This excellent agreement between the completely different approaches allows the conclusion that the localization length diverges as expressed by the power law of Eq. (1), with a universal exponent $\nu \approx 2.3$.

On the basis of this result, the findings of previous experimental studies[2–5] can be understood. In addition to the present work, where ν and p have been directly determined, the results in different realizations of 2DEG can all be interpreted as manifestations of scaling behavior with a universal localization-length exponent. In large samples of Al/Ga/As/GaAs heterostructures, a power law of the form $\Delta B \propto T^{\kappa}$ was found. In a previous work we have shown that in low-mobility samples κ increases with decreasing mobility,[3] and that in samples with comparatively high mobility a value close to $\kappa = \frac{p}{2\nu} = \frac{2}{2(7/3)} = \frac{3}{7} \approx 0.43$ is found. Here the result $p = 2$ of Fermi liquid theory appears to be a good approximation. This finding is consistent with the work by Dolgopolov *et al.*,[4] who found $\kappa = 0.40$ ($\kappa = 0.37$) in heterostructures with mobilities $\mu_e = 37$ m^2/Vs ($\mu_e = 27$ m^2/Vs).

In In/Ga/As/InP heterostructures the result $\kappa = 0.42$ (Ref. 1) can be interpreted as a combination of the localization-length exponent $\nu = \frac{7}{3}$ and the scattering rate exponent $p = 2$. It would, however, be interesting to verify this assumption by a similar experiment as described in the present work for Al/Ga/As/GaAs heterostructures.

The situation in Si-MOSFET's at present appears to be less well understood. Various studies performed by different authors[2,4,5] lead to largely different results for the temperature exponent κ. In a sample with peak mobility $\mu_e^p = 1.4$ m^2/Vs Wakabayashi, Yamane, and Kawaji[2] found $\kappa = 0.25$ in the filling-factor region n =3–4 and $\kappa = 0.15$ for n =5–6. Dolgopolov *et al.*[4] observed an exponent $\kappa = 0.90$ in three different samples with peak mobilities from $\mu_e^p = 2.1$ m^2/Vs to $\mu_e^p = 2.9$ m^2/Vs for the filling-factor regions n =2–3 and 3–4. D'Iorio, Pudalov, and Semenchinsky[5] studied two samples with peak mobilities $\mu_e^p = 1.87$ m^2/Vs and $\mu_e^p = 4.3$ m^2/Vs. They found that κ did not depend upon mobility and observed the same exponent $\kappa = 0.62$ in the filling-factor regions n =1–2, 2–3, and 3–4. On the other hand, the results were largely filling factor dependent in the LL $N = 1$, varying between $\kappa = 0.20$ and 0.43. Summarizing, we note in particular the large differences in the results for the lowest LL when comparing the results of the different groups, so that there seems to be a substantial sample dependence. However, it has to be emphasized that all these exponents were derived from measurements in the temperature range from 0.2 to 2 K. At lower temperatures the transport data saturate, which may be due to inhomogeneities of the carrier concentration. At temperatures higher than 0.2 K, the system still might not be close enough to the critical point. This problem is eventually enhanced due to the fact that in addition to the spin splitting there is also a valley splitting in silicon — in contrast to the situation in GaAs. Eventually, the valley splitting is not sufficiently well resolved at the temperatures and in the samples used in the studies mentioned above. This may cause complications for the observability of scaling behavior. Nevertheless, in principle all these results could be ascribed to variations of the scattering rate exponent p with a universal localization-length exponent $\nu = \frac{7}{3}$. It appears, however, highly desirable to perform a separate determination of the different exponents in order to verify a general scaling law.

VII. SUMMARY

In conclusion, we have reported the *direct* measurement of the localization-length exponent ν in the integer-quantum-Hall regime. A universal behavior of ν in the three lowest Landau levels is obtained, with a value of ν=2.3±0.1. This value agrees with the result from the percolation picture including quantum tunneling and the result of recent numerical studies. The inelastic-scattering rate exponent p at high magnetic fields can be measured in the same experiment. This exponent is determined by the specific form of the random potential. Reproducible magnetoresistance fluctuations are present in the low-temperature measurements, but their amplitude is small compared to the strength of the usual R_{xx} peaks between quantum Hall plateaus.

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