Resonances in transmission through an oscillating barrier

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We calculate electron transmission through a point barrier oscillating at frequency ω. Transmission resonances occur before the first emitted sideband channel opens, that is, for electron energies $E \leq \hbar\omega$, and which strongly suggest the formation of quasibound states in the time-dependent potential. We confirm the existence of these quasibound states, obtaining both the binding energy and electron lifetime, by computing the complex energy poles of the transmission amplitude. The resulting transmission resonances are analogous to those found in other multiple quantum channel scattering problems, such as the transmission through a donor impurity in a quasi-one-dimensional wire.

I. INTRODUCTION

Interest in the electron transmission through a potential barrier which oscillates in time began as a possible way to determine the quantum-mechanical "tunneling time" through the barrier.[1–6] More recently, oscillating barriers have been used as a model system[7–15] which permit explicit calculation of inelastic transmission coefficients, in an attempt to simulate electrical conduction in the presence of phonons, light, or applied ac voltages. Transmission through such an oscillating potential is a type of *phase-coherent inelastic scattering*, and does not describe either dissipative transport or the loss of electronic phase coherence in a conductor.

An oscillating barrier functions as a *dynamical trap* for electrons having emitted sideband energies below the continuum.[2] Electrons trapped as evanescent waves near discontinuities or defects appear to be a common feature in multichannel quantum scattering problems. For an electron scattering from a defect in a quasi-one-dimensional wire,[16–24] electrons in evanescent subbands above the Fermi energy accumulate around the static defect, and produce transmission resonances analogous to those created by electrons trapped in evanescent sidebands below the continuum for transmission through the oscillating barrier. Thus transmission through the oscillating barrier, which produces multiple energy transmission channels, is analogous in many respects to transmission through a static defect in a confined geometry, in which the multiple spatial transmission channels consist of the confined electron waveguide modes.

II. OSCILLATING BARRIER VERSUS ELECTRON WAVEGUIDE

In this paper we study the electron transmission through a one-dimensional potential $V(x,t)$ oscillating at frequency ω, similar to that in Ref. 8, where

$$V(x,t) = [V_s + V_d \cos(\omega t)]\delta(x) . \tag{1}$$

A plane wave at energy E incident on this oscillator will produce transmitted and reflected sidebands at energies $E + n\hbar\omega$, where n is the sideband "channel index." The multichannel transmission coefficient T_{mn} (from channel n to channel m) through this potential determines the dc conductance[13] through the Büttiker-Landauer formula[25,26]

$$G = \frac{2e^2}{h} \sum_{mn} T_{mn} = \frac{2e^2}{h} T . \tag{2}$$

Our method to calculate T_{mn} through the oscillating potential is given in Appendix A. For the oscillating barrier only the $n = 0$ sideband is incident on the scattering potential. Therefore $T = \sum_m T_{m0}$ in Eq. (2), where the sum over m runs over all propagating channels. We graph[27] the transmission coefficient T through this oscillator in Fig. 1(a).

A different multichannel scattering problem, transmission through a short-ranged impurity in an electron waveguide, was studied in Refs. 16–22. In Ref. 17 the potential energy is

$$V(x,y) = V_c(y) + \gamma\delta(x)\delta(y - y_i) , \tag{3}$$

where the confinement potential $V_c(y)$ gives rise to a set of normal confinement modes, labeled by a channel index n, at the subband energies E_n. The two-terminal Landauer conductance from Eq. (2) through this scattering potential is shown in Fig. 1(b). (Parameters are the same as Fig. 6 in Ref. 17.) For conduction through an impurity in a waveguide, a new input channel is populated whenever the Fermi energy crosses a confinement subband. Therefore both m and n are summed over all propagating channels to obtain the transmission coefficient in Eq. (2).

The greatest difference between Figs. 1(a) and 1(b) is simply that the conductance in Fig. 1(b) requires a sum over incoming channels which is absent in Fig. 1(a). When only a single channel is incident, transmission through the oscillating potential in Fig. 1(a) and transmission through the donor impurity ($\gamma < 0$) in a waveguide in Fig. 1(b) are nearly identical.[28] That is, both Figs. 1(a) and 1(b) show a sharp "dip" in transmission just before the first new scattering channel opens. Ref-

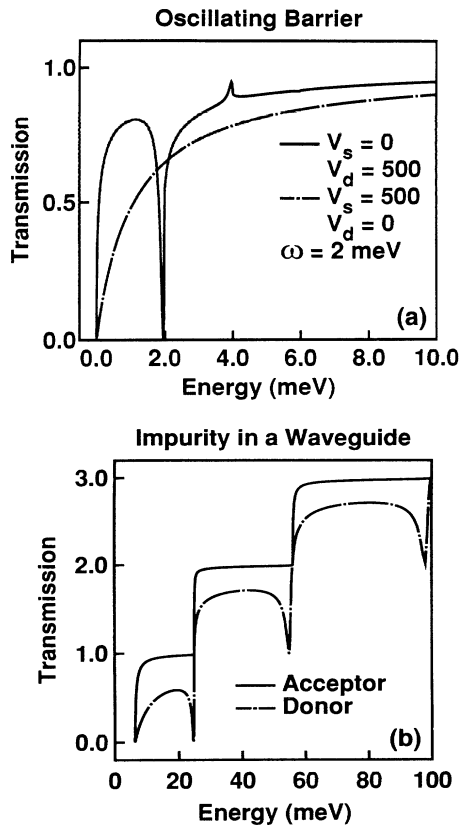

FIG. 1. (a) Transmission through an oscillating potential compared to (b) transmission through a donor impurity in an electron waveguide. The sharp drop in transmission before (a) the first sideband and (b) the first subband channel opening indicate the formation of quasibound states in the scattering potential.

erence 17 argues the drops in transmission in Fig. 1(b) correspond to "quasidonor levels" forming in the impurity. A somewhat similar[29] type of "quasibound state" produces the sharp drop in transmission through the oscillating potential in Fig. 1(a).

III. QUASIBOUND STATES IN A TIME-DEPENDENT POTENTIAL

Linear systems theory[30] gives the prescription for finding the natural frequencies (or eigenenergies) of any linear system. In particular, if a transfer function $t_{mn}(E)$ relates the incoming wave $\psi_n^{\rm inc}$ to the scattered wave $\psi_m^{\rm trans}$ by

$$\psi_m^{\rm trans}(E) = t_{mn}(E)\psi_n^{\rm inc}(E)\ , \tag{4}$$

then the natural energies are found by locating the poles of $t_{mn}(E)$, namely

$$\frac{1}{t_{mn}(E_R + iE_I)} = 0\ . \tag{5}$$

The poles[31] of $t_{mn}(E)$ will occur at a complex energy $E = E_R + iE_I$. At this complex energy, a scattered wave can be produced even if no incident wave is present. Equation (5) is equivalent to locating either the poles of the S matrix[32] or poles of the Green function.

The energies E_R and E_I have a simple physical interpretation. The full time-dependent wave function $\psi(x,t)$ can be written

$$\psi(x,t) = \psi(x)e^{Et/i\hbar} = \psi(x)e^{E_R t/i\hbar}e^{E_I t/\hbar}\ . \tag{6}$$

Thus, E_R gives the binding energy of the state and $|E_I|/\hbar$ its decay rate. If $E_I = 0$ the state is "bound," while $E_I \neq 0$ describes a "quasibound state." We must have $E_I < 0$ for the linear system to be stable.

Figure 2(a) shows the complex energy poles and zeros of the transmission coefficient $T_{00}(E)$ through the oscillating potential [from Fig. 1(a)], while Fig. 2(b) does the same thing for $T_{11}(E)$ through the donor impurity in an electron waveguide [from Fig. 1(b)]. (There may be more transmission zeros off the real energy axis in higher subbands/sidebands which we have not located.) The close proximity of the pole and zero pair in Fig. 2 is consistent with the sharp dip in transmission from Fig. 1. Designing a linear system with a zero on the real energy axis, and an adjacent pole slightly off the real energy axis, produces just such a "narrowband reject" filter.[33]

Bound states in the oscillating potential are very leaky, since the matrix elements for emission and absorption of oscillator quanta are equal. The corresponding pole in transmission through the oscillator moves rapidly off the real energy axis before it moves much below the new channel threshold, producing a broader "dip" in transmission through the oscillator than for the donor impu-

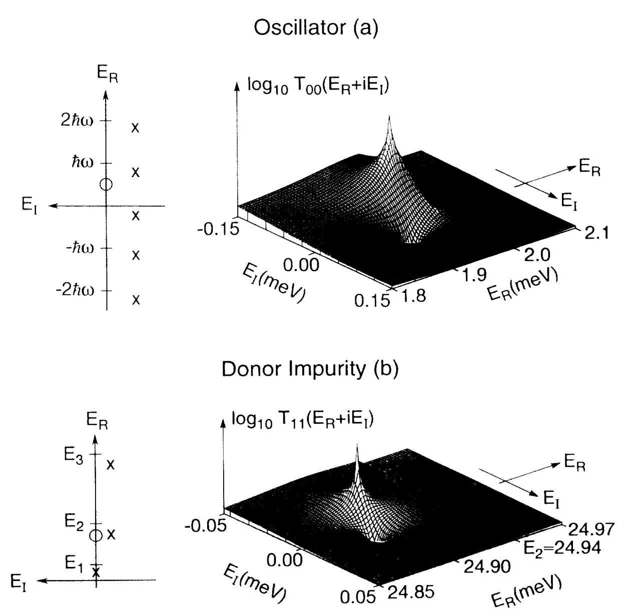

FIG. 2. Transmission coefficient (a) $T_{00}(E)$ through the oscillator of Figs. 1(a) and 1(b) $T_{11}(E)$ through the donor impurity in a waveguide of Fig. 1(b) as a function of complex energy. Pole locations (crosses) define the quasibound states. The adjacent pole and zero near (a) $E = \hbar\omega$ and (b) $E = E_2$ produce the sharp resonances in the transmission coefficient in Fig. 1.

rity in a waveguide. In contrast, the quasidonor levels in a waveguide can have very long lifetimes, and can produce sharp transmission resonances even well below a new subband channel opening.

Poles of the transmission coefficient have a simple physical interpretation as the natural eigenstates of the scattering potential. Unfortunately, transmission zeros have no such simple physical interpretation. However, transmission zeros and complex energy poles of the transmission coefficient always seem to appear together[23,24] in the multichannel scattering from an attractive potential. For example, the transmission coefficient through an an acceptor impurity (repulsive potential having $\gamma \geq 0$) in an electron waveguide has no poles and no transmission resonances. Further, if a strong enough static, repulsive, potential barrier is combined with an oscillating barrier, the resulting transmission coefficient has no poles or transmission resonances. Poles seem to appear in the multichannel transmission coefficient only when attractive potentials are present, and appear with a corresponding transmission zero.

IV. TRANSMISSION RESONANCES

Transmission through an oscillating potential nicely illustrates the large differences between attractive and repulsive potentials in multichannel quantum scattering problems. Although we no longer present the structure of complex poles and zeros corresponding to each transmission coefficient shown in this section, these quasibound states and transmission zeros are still present near any transmission resonances.

A. Weak oscillators

When V_d is small, so that the oscillator is weak, Fig. 3 shows that the transmission T drops abruptly when a new sideband transmission channel opens at $E = \hbar\omega$. When an attractive static barrier is combined with the weak oscillating barrier, there is an interesting "Fano" resonance[34–37] behavior in T well before the first emitted sideband can propagate, shown also in Fig. 3. Fano resonances have been observed in atomic systems,[35] and may also be present in both quasi-one-dimensional wires[18] and semiconductor superlattices.[36,37] The spacing from the Fano resonance to the first sideband channel opening is just the binding energy in the (static) attractive potential well.[11] Appendix B shows that these two types of resonances exhaust the possibilities for weak oscillating barriers.

B. Strong oscillators

As the oscillating barrier strength increases, the transmission zero [originally at the first emitted sideband channel opening ($E = \hbar\omega$) in Fig. 1(a)] has moved halfway to the edge of the continuum in Fig. 4. As the oscillating barrier strength further increases, this zero drops below the continuum edge, and a second zero forms near $E = \hbar\omega$ (not shown). As V_d continually increases, this pattern appears to repeat indefinitely. The movement of these transmission zeros to lower energies with increasing strength of the oscillating barrier is due to an increased binding energy, analogous to Fig. 7 of Ref. 17. Transmission through the oscillating barrier is generally larger than through a static barrier of the same strength. This is reasonable, since the average barrier strength is smaller for the oscillating potential.

Figure 5 shows the transmission coefficient when a strong, static, repulsive potential is combined with the oscillator. Only small "threshold" features are present in the transmission coefficient.[8] Adding the strong, repulsive barrier has destroyed the transmission resonances. However, making the oscillator even stronger recovers the transmission resonances. A pronounced drop in T develops at the first sideband emission threshold in Fig. 5, showing that a quasibound state again forms in the time-dependent potential. Since a quasibound state cannot gradually form with increasing strength of an attractive potential (it is either present or not), this result interpolates sensibly between the purely dynamical barrier (where the quasibound state exists) and the static repulsive barrier (where no quasibound state exists).

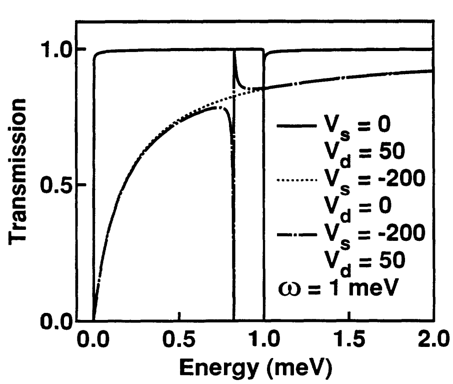

FIG. 3. Combining the oscillating barrier with an attractive static potential well ($V_s = -200$, $V_d = 50$) produces a "Fano" transmission resonance when the first emitted sideband becomes bound in the quantum well.

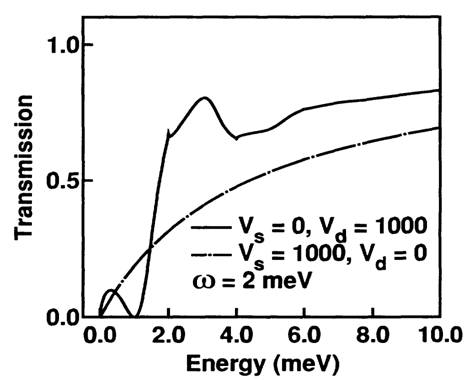

FIG. 4. A strong oscillator ($V_s = 0$, $V_d = 1000$) pulls the transmission zero lower in energy as the quasibound states in the oscillator move lower in energy. Eventually, for strong enough oscillators, the resonance is pulled below the band edge and new resonances appear at $E = \hbar\omega$ (not shown).

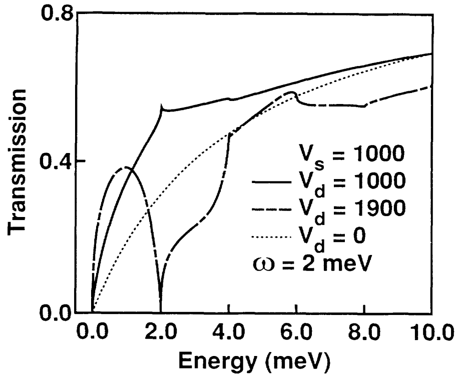

FIG. 5. Adding a strong repulsive potential barrier to the oscillator destroys the quasibound states ($V_s = 1000$, $V_d = 1000$). Bound states are recovered ($V_s = 1000$, $V_d = 1900$) by increasing the oscillating barrier strength.

If the oscillator frequency is now varied (at a fixed Fermi energy and oscillator strength), the resulting transmission resonances are shown in Fig. 6. Resonances both with the edge of the conduction band (i.e., $\hbar\omega \simeq 1$ meV) and with our proposed quasibound states (i.e., $\hbar\omega \simeq 2$ meV) are visible. Other "multiphonon" transmission resonances at frequencies $\hbar\omega < 1$ meV are also present. However, for frequencies $\hbar\omega > 2$ meV, there are no further transmission resonances. Too large of an oscillation frequency evidently causes the first emitted sideband to lie lower in energy than a bound state, so that no transmission resonances occur above a certain frequency. Figure 6 resembles the Shubnikov–de Haas oscillations of the conductivity σ_{xx} versus magnetic field observed in two-dimensional electron gases and in metals. The sideband separation $\hbar\omega$ for transmission through the oscillating barrier corresponds to the Landau-level separation $\hbar\omega_c$, where the cyclotron frequency ω_c is proportional to the magnetic field. The conductivity σ_{xx} corresponds to the transmission coefficient.

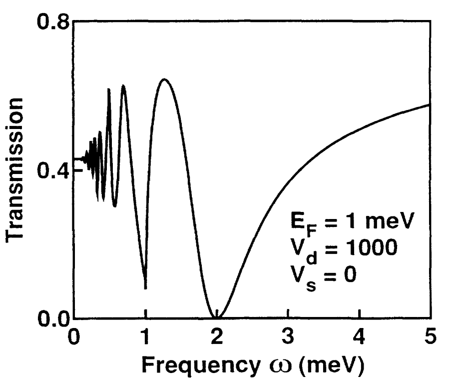

FIG. 6. Total transmission T vs oscillating barrier frequency. The graph is reminiscent of the Shubnikov–de Haas oscillations in the conductivity σ_{xx} vs magnetic field for a two-dimensional electron gas.

V. CONCLUSION

The energy dependence of the transmitted flux through an oscillating potential illustrates the strong differences between repulsive versus attractive potentials in multichannel scattering problems. As a consequence of leaky bound states formed in the partially attractive scattering potential, transmission resonances occur at many different energies, not simply near the propagation threshold of a new scattering channel. By locating the complex energy poles of the transmission amplitude, we establish both the existence and nature of these bound states in an oscillating potential.

Transmission resonances occur when the incident electron energy is a multiple of the oscillator frequency from the edge of the continuum or from a bound state formed in the oscillator. Combining the oscillating barrier with a static, attractive, potential well produces "Fano" transmission resonances, while combining the oscillator with a strong enough static, repulsive, potential barrier eliminates the transmission resonances. Poles in the transmission amplitude appear together with these transmission resonances.

Transmission through an oscillating potential barrier shares many common features with the scattering from a donor impurity in an electron waveguide. The pole and zero structure of the transmission amplitude in both problems is similar. A finite sized donor impurity in an electron waveguide can even produce transmission resonances with the characteristic "Fano" line shape.[18] The important feature which these two different scattering potentials have in common is that they both trap electrons in cutoff channels near the defect.

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APPENDIX A: SCATTERING FROM AN OSCILLATING POINT BARRIER

To obtain the transmission coefficient through the scattering potential $V(x,t)$ from Eq. (1), we solve the one-dimensional time-dependent Schrödinger equation following Eqs. (3.8)–(3.17) of Ref. 5. This is equivalent to inverting a matrix equation for the scattered wave amplitudes A_n,

$$-2ik_n\delta_{n,0} = (\Gamma_s - 2ik_n)\,A_n + \Gamma_d(A_{n+1} + A_{n-1})\;, \quad \text{(A1)}$$

where $\Gamma_s = 2mV_s/\hbar^2$ and $\Gamma_d = mV_d/\hbar^2$. To obtain the wave vector k_n we take the positive square root of

$$E_n = E + n\hbar\omega = \frac{\hbar^2k^2}{2m} + n\hbar\omega = \frac{\hbar^2k_n^2}{2m}\;, \quad \text{(A2)}$$

so that

$$k_n = +k_0 \left(1 + \frac{n\hbar\omega}{E_0}\right)^{1/2} , \tag{A3}$$

with $n = 0, \pm1, \pm2, \ldots$. Choosing a large enough matrix in Eq. (A1) to diagonalize fulfills current conservation, ensures proper "normalization of the wave function," and encompasses the "feedback between elastic and inelastic scattering" emphasized in Ref. 8.

Since n in Eq. (A2) can be negative, we will have imaginary k_n's. To be consistent with Eqs. (3.12) and (3.13) of Ref. 5 we require that $k_n = i\kappa_n$, where $\kappa_n > 0$ for the evanescent sidebands. These imaginary wave vectors represent evanescent states trapped around the oscillating barrier. If the current incident on the oscillating barrier is turned off, electrons trapped in these evanescent states will eventually leak away from the oscillator. The wave-function transmission amplitude t_{mn} and transmission coefficient T_{mn} are obtained from

$$t_{m0} = A_m, \qquad T_{mn} = \frac{k_m}{k_n} t^*_{mn} t_{mn} . \tag{A4}$$

Equation (A1) is obtained from the scattering solutions of the time-dependent Schrödinger equation[5] with the potential in Eq. (1). The appropriate scattering wave function for $x < 0$ is the sum of the incident wave plus reflected waves in all possible sidebands

$$\psi_L(x,t) = e^{ik_0 x} e^{-iE_0 t/\hbar} + \sum_n B_n e^{-ik_n x} e^{-iE_n t/\hbar}, \tag{A5}$$

while the transmitted wave for $x > 0$ is also a sum over all possible sidebands

$$\psi_R(x,t) = \sum_n A_n e^{ik_n x} e^{-iE_n t/\hbar}. \tag{A6}$$

Enforcing continuity of the wave function for all time t, together with the derivative jump condition at $x = 0$ obtained by integrating the time-dependent Schrödinger equation across the δ function in Eq. (1), directly yields Eq. (A1).

APPENDIX B: WEAK OSCILLATORS

If the oscillator strength Γ_d is weak, only the lowest-order sidebands are significant. We therefore regard the $n = \pm1$ sidebands as small compared to the incident wave ($|A_{\pm1}|^2 \ll 1$), and the second-order sidebands as being approximately zero ($|A_{\pm1}|^2 \gg |A_{\pm2}|^2 \simeq 0$). Hence,

$$\begin{pmatrix} 0 \\ -2ik_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \Gamma_s - 2ik_{-1} & \Gamma_d & 0 \\ \Gamma_d & \Gamma_s - 2ik_0 & \Gamma_d \\ 0 & \Gamma_d & \Gamma_s - 2ik_1 \end{pmatrix} \times \begin{pmatrix} A_{-1} \\ A_0 \\ A_1 \end{pmatrix} . \tag{B1}$$

Direct diagonalization of Eq. (B1) gives

$$\begin{pmatrix} A_{-1} \\ A_0 \\ A_1 \end{pmatrix} = \begin{pmatrix} \dfrac{-\Gamma_d}{\Gamma_s - 2ik_{-1}} \\ 1 \\ \dfrac{-\Gamma_d}{\Gamma_s - 2ik_1} \end{pmatrix} \times \frac{(-2ik_0)(\Gamma_s - 2ik_{-1})(\Gamma_s - 2ik_1)}{D} , \tag{B2}$$

where the determinant D is

$$D = (\Gamma_s - 2ik_0)(\Gamma_s - 2ik_{-1})(\Gamma_s - 2ik_1) - 2\Gamma_d^2 [\Gamma_s - i(k_1 + k_{-1})] . \tag{B3}$$

We have numerically verified that Eq. (B1) well describes the transmission coefficient shown in Fig. 3.

1. Transmission resonances

The interesting factor from Eq. (B2) for transmission resonances (zeros) in the lowest sideband is

$$\Gamma_s + 2\kappa_{-1} = 0 , \tag{B4}$$

where $k_{-1} = i\kappa_{-1}$. Equation (B4) can be satisfied for $0 \le E \le \hbar\omega$ only if the static barrier is attractive ($V_s \le 0$). Equation (B4) predicts a transmission zero at the "Fano" resonance condition

$$E - \hbar\omega = -\frac{mV_s^2}{2\hbar^2} , \tag{B5}$$

that is, where the electron in the first emitted sideband aligns with the bound state in the attractive potential well.[38] For a purely oscillating potential, Eq. (B4) shows the resonance condition is $E = \hbar\omega$, at the first sideband opening.

2. Poles of the transmission amplitude

We find the poles of the transmission amplitude t_{00} through a weak oscillator by setting $D = 0$ in Eq. (B3). For a pure oscillator ($V_s = 0$) operating at high frequency, we find its "quasibound" states at the complex energy

$$E_R \simeq n\hbar\omega, \qquad E_I \simeq -\frac{1}{8\hbar\omega}\left[\frac{mV_d^2}{2\hbar^2}\right]^2 , \tag{B6}$$

so that lower frequencies or stronger oscillators increase the escape rate. Setting $D = 0$ from Eq. (B3) gives only $E_R = 0$, but since the determinant of any 3×3 submatrix of Eq. (A1) would give the same result for E_I with E_R offset by $\hbar\omega$, we obtain Eq. (B6) in general. Equation (B6) correctly predicts the imaginary part of the pole in Fig. 2(a), but fails to give the small, yet finite, binding energy of this pole.

If the high-frequency, weak oscillator is combined with a large, static, attractive, potential well ($V_s < 0$), we again set $D = 0$ in Eq. (B3) and obtain

$$E_R \simeq n\hbar\omega - \frac{\hbar^2}{2m}\left[\frac{mV_s}{\hbar^2} - \frac{1}{4}\sqrt{\frac{\hbar^2}{2m(\hbar\omega)}}\left(\frac{mV_d}{2\hbar^2}\right)^2\right]^2 \equiv n\hbar\omega - E_B \tag{B7}$$

and

$$E_I \simeq -\frac{\hbar^2}{4m}\left(\frac{mV_d}{\hbar^2}\right)^2 \sqrt{\frac{E_B}{\hbar\omega}} \,. \tag{B8}$$

Both the binding energy and leakage rate increase with the oscillator strength V_d. Reference 11 studied how the "Fano" transmission resonance through this potential shifts lower in energy and broadens "due to interactions" as the oscillating barrier is made stronger. The movement of these complex energy poles, i.e., the formation of quasibound states in the oscillator, is consistent with this trend. If the weak oscillator is combined with a large, static, repulsive, potential barrier, we do not find any complex energy poles of the transmission amplitude, as expected.

APPENDIX C: FORMATION OF QUASIBOUND STATES

1. Donor impurity in an electron waveguide

The transmission coefficients through the potential in Eq. (3) can be calculated analytically for a simple two-subband model of a waveguide.[17] The exact current transmission amplitude $\tilde{t}_{11}$ in this model, where $T_{ij} = \tilde{t}_{ij}\tilde{t}^*_{ij}$, is

$$\tilde{t}_{11} = \frac{1 + (mV_{22}/\hbar^2\kappa_2)}{1 + (mV_{22}/\hbar^2\kappa_2) + i\,(mV_{11}/\hbar^2 k_1)} \,. \tag{C1}$$

E_1 and E_2 are the first and second confinement subband energies, the wave vector k_1 is determined from $E - E_1 = \hbar^2 k_1^2/2m$, and the evanescent wave vector κ_2 is found from $E_2 - E = \hbar^2\kappa_2^2/2m$. The coupling constants are $V_{22} = \gamma|\chi_2(y_i)|^2$ and $V_{11} = \gamma|\chi_1(y_i)|^2$, where $\chi_1(y_i)$ and $\chi_2(y_i)$ are the normal mode wave functions evaluated at the lateral impurity position y_i. We locate the poles of the transmission amplitude by setting $0 = 1/\tilde{t}_{11}$.

Expanding the square roots in κ_2 and k_1, which is valid if the real part of the pole (E_R) is not too near a subband edge and the imaginary part of the pole (E_I) is not too large, one finds the "donor level" pole at a purely real energy

$$E_R \simeq E_1 - \frac{mV_{11}^2}{2\hbar^2}, \qquad E_I = 0, \tag{C2}$$

and the "quasidonor level" pole at the complex energy

$$E_R \simeq E_2 - \frac{mV_{22}^2}{2\hbar^2} \equiv E_2 - E_B,$$

$$E_I \simeq -\frac{mV_{12}^2}{2\hbar^2}\left[\frac{4E_B}{E_2 - E_1}\right]^{1/2}, \tag{C3}$$

where $V_{12}^2 = V_{11}V_{22}$. The imaginary part (E_I) of the "quasidonor level" pole is proportional to the coupling strength V_{12}^2 between the lowest two subbands as expected. The expression for E_I from Eq. (C3) contains the ratio of the two state densities, $N_1(E_2 - E_B)/N_2(E_2 - E_B) \simeq \sqrt{E_B}/\sqrt{E_2 - E_1}$, greatly suppressing the leakage rate out of the quasidonor level.

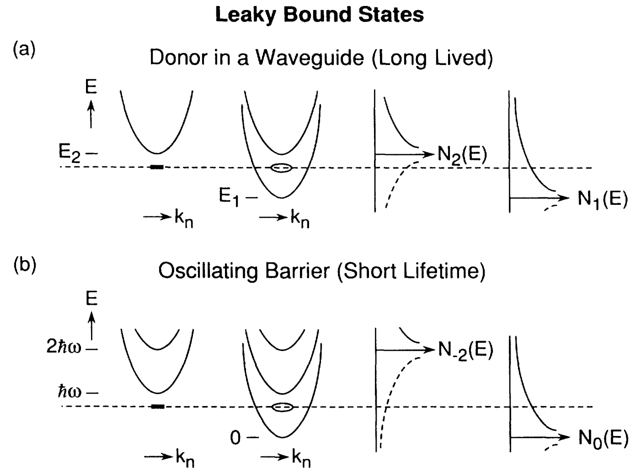

FIG. 7. The quasibound state for (a) the donor impurity in a waveguide is long lived, since the density of states in the decay channel N_1 (solid) is smaller than the evanescent density of states N_2 (dashed) responsible for the binding. The opposite situation holds for (b) the oscillating potential, making the quasibound state very leaky.

2. Oscillating barrier

Equation (B6) shows that an electron never really binds to the oscillator if the oscillator is weak. Instead, the pole moves out along the imaginary energy axis as the oscillator strength V_d increases, producing a very leaky quasibound state. Stronger oscillators do eventually produce the small but finite binding energy seen in Fig. 2(a). We can partially understand this because the electron couples with equal strength to adjacent sidebands, one of which is evanescent and the other propagating. But at the bound-state energy, occurring just slightly below the new channel threshold at $E \le \hbar\omega$, the density of states in the decay channel (propagating states) is actually larger than the density of states in the binding channel (evanescent states from the second emitted sideband channel responsible for the binding). As a result, quasibound states in the oscillator are very leaky. The opposite situation holds for the donor impurity in a waveguide, where the density of states of the decay channel is small compared to the number of available states in the binding channel, and the quasibound states are therefore long lived. This is shown schematically in Fig. 7.

APPENDIX D: SIDEBAND ASYMMETRY (TUNNELING TIME)

Transmission through a weak oscillator is of interest in the study of tunneling times.[1–6] Reference 1 defines a function which measures the relative strength of the transmitted sideband currents

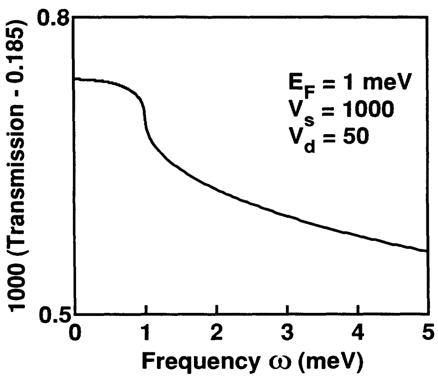

FIG. 8. For weak oscillators, the "crossover frequency" for tunneling through a point barrier occurs when the first emitted sideband aligns with the band edge, and does not depend on the properties of the barrier.

$$F(\omega) = \frac{k_1|A_1|^2 - k_{-1}|A_{-1}|^2}{k_1|A_1|^2 + k_{-1}|A_{-1}|^2} = \frac{T_{10} - T_{-10}}{T_{10} + T_{-10}}, \quad \text{(D1)}$$

which is given by $F(\omega) = \tanh(\omega\tau_{BL})$ for a rectangular barrier subject to a small modulation at frequency ω. If we plot $F(\omega)$ versus ω, there will be a "crossover frequency" at $\omega = 1/\tau_{BL}$. For a point barrier, one can easily show $\tau_{BL} = 0$. That is, it takes "no time" to tunnel through the point barrier. $F(\omega)$ found in Ref. 1 holds provided $E_F \gg \hbar/\tau_{BL}$.

However, if the barrier itself plays no role in limiting the tunneling time, the only other energy scale left in the problem is the Fermi energy. We indeed find that the Fermi energy is the crossover frequency for transmission through the point barrier of Eq. (1). That is, a resonance of the incident electron with the edge of the continuum makes $F(\omega)$ "cross over." This is in contrast to the conclusion of Refs. 4 and 5 for transmission through this same potential as discussed below.

Figure 8 shows the transmission coefficient versus oscillation frequency when V_d is small. We see first that the change in T with frequency is small compared to T itself, confirming that the oscillating barrier is weak. Second, there is a qualitative change in the frequency dependence of T when $E = \hbar\omega$. Therefore, $E = \hbar\omega$ is the crossover frequency for the point barrier. Figure 8 is for an opaque barrier which has $E \ll m(V_s/\hbar)^2$, but we have checked that the frequency dependence of T also changes abruptly at $E = \hbar\omega$ if the barrier is transparent. Again, the "crossover frequency" in this problem occurs when the transmitted electrons interact with the band edge, rather than because transmission is limited by the static barrier itself.

We can also construct an analytical proof which suggests the crossover frequency is the Fermi energy. Starting from Eqs. (D1) and (B2), we expand the wave vectors $k_{\pm 1}$ to lowest order in the small parameter $\hbar\omega/E$, such that $k_{\pm 1} \simeq k_0(1 \pm \hbar\omega/2E)$, to obtain

$$F(\omega) = \left(\frac{\hbar\omega}{2E}\right) \frac{\left[\frac{1}{2}m\left(\frac{V_s}{\hbar}\right)^2 - E\right]}{\left[\frac{1}{2}m\left(\frac{V_s}{\hbar}\right)^2 + E\right] - \left(\frac{\hbar\omega}{2E}\right)(\hbar\omega)}. \quad \text{(D2)}$$

On the other hand, if we neglect the difference between k_1 and k_{-1}, which one might think is valid if $E \gg \hbar\omega$, we recover the result (3.25) of Stovneng and Hauge[5] directly from Eqs. (D1) and (B2). Reference 5 obtains their (3.25) using the calculus of residues, but simply diagonalizing a 3×3 matrix gives the same result. We see that Ref. 5 calculates the ratio of probability densities instead of the ratio of transmitted currents $F(\omega)$. Equation (D2) gives $F(\omega) = \hbar\omega/2E$ for opaque barriers and $F(\omega) = -\hbar\omega/2E$ for transparent barriers. This supports our numerical result that the crossover frequency in this problem is the Fermi energy.

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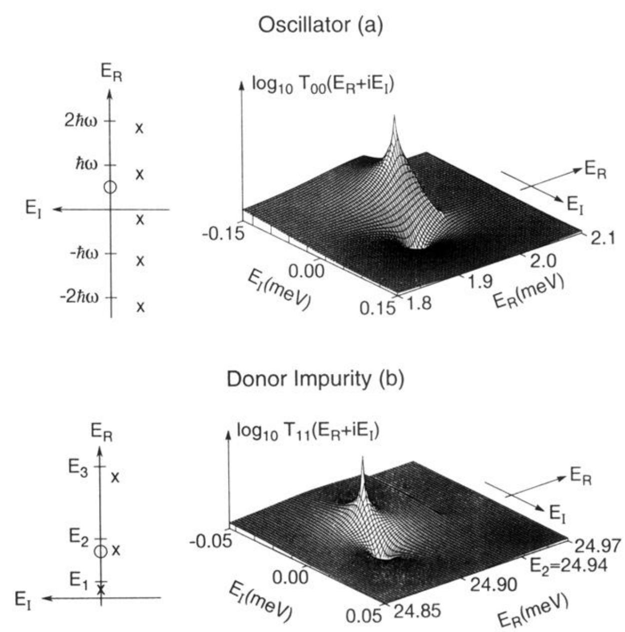

FIG. 2. Transmission coefficient (a) $T_{00}(E)$ through the oscillator of Figs. 1(a) and 1(b) $T_{11}(E)$ through the donor impurity in a waveguide of Fig. 1(b) as a function of complex energy. Pole locations (crosses) define the quasibound states. The adjacent pole and zero near (a) $E = \hbar\omega$ and (b) $E = E_2$ produce the sharp resonances in the transmission coefficient in Fig. 1.