Effect of nonlocality and anisotropy on Gaussian fluctuations around the flux-lattice state

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The theory of Gaussian fluctuations around the mean-field flux-lattice state, which was developed within (Ginzburg-Landau) GL theory on the basis of Landau-level expansion of the order parameter, is applied to deriving the elastic free energy for arbitrary field orientations in a uniaxially anisotropic superconductor. It is found that the results obtained for nonlocal elastic terms coincide with those resulting from London limits, indicating the validity of the present approach in GL theory. Furthermore we show that, by examining the Gaussian fluctuation corrections to thermodynamic quantities in strongly type-II (three-dimensional) superconductors, theories formulated in infinite-κ limits are valid in fluctuation-dominated regimes of mixed states. Recent experimental results for clean samples of high-T_c oxides are discussed from the viewpoint of fluctuation theory.

I. INTRODUCTION

In previous papers,[1-3] Gaussian (harmonic) fluctuations around the Abrikosov flux-lattice state were studied in Ginzburg-Landau (GL) theory, as is consistent with mean-field theory, on the basis of Landau-level (LL) expansion of the order parameter in order to see[3] how the elastic modes are derived from within fluctuations in the order parameter. It was shown in Ref. 3 (called I below) that the two (transverse and longitudinal) elastic modes correspond to a lowest ($n=0$) and a next ($n=1$) LL modes, respectively, and that the superconducting long-range order in the flux lattice is absent. Furthermore, these results were found to be justified by the analysis in the London limit. However, considerations were limited in I to the isotropic system and simplest $B\|\hat{Z}$ case for uniaxially anisotropic systems, where B is the flux density and $\hat{Z}$ is the anisotropy axis of the crystal.

In the present paper, the approach in I is applied to examining two features characteristic of the cuprate high-T_c superconductors, i.e., uniaxial crystal anisotropy and strong nonlocality in the elastic response. For simplicity, we neglect the layer structure throughout this paper. First, we derive in Sec. II the elastic free energy for arbitrary field configuration in an uniaxially anisotropic system. As far as we know, the full expression of the elastic free energy in such a general case has not been derived according to GL theory in the literature. We find that the resulting nonlocal terms, as expected, coincide with those calculated in the London limit, indicating the validity of the procedures in the GL theory of I. Next, in order to see the implication of the nonlocality and anisotropy, we examine the Lindemann criterion[4,5] of flux-lattice melting for an arbitrary field configuration and point out the validity of the corresponding result in the infinite-κ limit[1,2] (κ is the GL parameter) where the fluctuations in the gauge field are entirely neglected. This result is further emphasized in Sec. III by calculating Gaussian contributions to thermodynamic properties of order parameter fluctuations (including the massive mode neglected in I) around the flux-lattice state.[6] We find again that, in the range where the theory is justified, the contributions of the gauge-field fluctuation can safely be neglected and show that the character of order-parameter fluctuations in the three-dimensional (3D) case is, consistent with the study[7] from the high-temperature side, one-dimensional-like.[6,7] This is due to the fact that there are no characteristic scales of superconducting fluctuations in directions perpendicular to the applied field other than the vortex spacing (i.e., the magnetic length) and thus suggests the absence[3,7] of a true superconducting transition in the region above H_{c1}. In Sec. IV implications of these results are discussed in relation to experimental results in high-T_c superconductors, and the Appendix is devoted to a brief explanation of the calculations in Sec. II.

II. ELASTIC MODES FOR ARBITRARY FIELD CONFIGURATIONS

In this section we derive the elastic free energy of the flux-lattice state for an arbitrary field orientation with respect to a uniaxially anisotropic crystal. We focus on the anisotropic 3D GL model

$$G=\int d^3r\left\{a\left[\xi_0^2\sum_{i,j}\left[i\partial_i-\frac{2\pi}{\phi_0}A_i\right]\psi^*M_{ij}^{-1}\left[-i\partial_j-\frac{2\pi}{\phi_0}A_j\right]\psi-\left[1-\frac{T}{T_c}\right]|\psi|^2\right]+\frac{b}{2}|\psi|^4+\frac{1}{8\pi}(\operatorname{curl}\mathbf{A})^2\right\}, \tag{2.1}$$

where $M_{ij}^{-1}=\delta_{i,j}+(\eta-1)\delta_{iZ}\delta_{jZ}$ ($i,j=X,Y,Z$; $\eta\le 1$) is an inverse-mass tensor, a,b are positive constants, T_c the zero-field transition temperature, ξ_0 the coherence length along the easy (X-Y) plane, and ϕ_0 the flux quantum. For simplicity, we neglect the discrete layer structure throughout this paper. The configuration given in Fig. 1 of Ref. 8(a) is considered. The vortex frame (x-y-z) is defined by a rotation θ ($0\le\theta\le 90°$) of the crystal frame (X-Y-Z) about the Y

($=y$) axis. That is,

$$x=X\cos\theta-Z\sin\theta,\quad y=Y,\quad z=Z\cos\theta+X\sin\theta\ , \tag{2.2}$$

and the flux density is expressed by $\mathbf{B}=B(\hat{\mathbf{Z}}\cos\theta+\hat{\mathbf{X}}\sin\theta)=B\hat{\mathbf{z}}$.

Before considering GL theory, it is instructive to see how the nonlocal (i.e., tilt and compressional) elastic terms for the $\theta\neq0$ case is derived in the London limit where $|\psi|$ is uniform [$|\psi|^2=(\phi_0/2\pi\xi_0\lambda_L)^2/8\pi a$, where λ_L is the London penetration depth]. One can easily calculate them along lines similar to that given in Appendix D of I. Defining displacement fields from the fluctuation part of the topological condition and integrating out the gauge-field fluctuations, we can express the result for the nonlocal terms of the elastic free energy in the form

$$\delta G_L=\frac{1}{8\pi}\left[\frac{\phi_0}{2\pi\lambda_L}\right]^2\int\frac{d^3q}{(2\pi)^3}\left[|\mathbf{v}_\perp(q)|^2+\eta|v_Z(q)|^2-\frac{1}{d_L}\left[|\mathbf{v}_\perp(q)|^2+\eta|v_Z(q)|^2+\eta^{-1}\lambda_L^2|\mathbf{q}_\perp\cdot\mathbf{v}_\perp(q)+\eta q_Z v_Z(q)|^2+\frac{(\eta^{-1}-1)}{1+\lambda_L^2q^2}\lambda_L^2|\mathbf{q}\times\mathbf{v}(q)|_Z^2\right]\right] \tag{2.3}$$

$$=\frac{1}{8\pi}\left[\frac{\phi_0}{2\pi}\right]^2\int\frac{d^3q}{(2\pi)^3}\frac{1}{d_L}\left[|\delta\boldsymbol{\omega}_\perp(q)|^2+\frac{1+\eta^{-1}\lambda_L^2q^2}{1+\lambda_L^2q^2}|\delta\omega_Z(q)|^2\right]\ , \tag{2.4}$$

where $d_L=1+\lambda_L^2(\eta^{-1}q_\perp^2+q_Z^2)$ and the vector $\mathbf{P}_\perp$ denotes the projection of $\mathbf{P}$ into the X-Y plane. As in Appendix D of I, we neglect terms of no interest here with nonzero reciprocal-lattice vectors [this case was denoted in Ref. 9 as the flux (vortex) liquid limit]. Then the calculation used in obtaining the v field is the same as in $\theta=0$ (i.e., $B\parallel\hat{\mathbf{Z}}$) case.[3] The deviation of the vorticity $\delta\boldsymbol{\omega}$ can be written in the form

$$\delta\boldsymbol{\omega}=\text{curl }\mathbf{v}=\frac{2\pi}{\phi_0}B(\partial_z\mathbf{s}-\hat{\mathbf{z}}\,\text{div}s)\ ,\qquad \mathbf{v}=\nabla\chi-\frac{2\pi}{\phi_0}(\mathbf{B}\times\mathbf{s})\ . \tag{2.5}$$

Here χ is the longitudinal fluctuation of the order-parameter phase and $\mathbf{s}$ the displacement field. Using $\mathbf{q}\cdot\delta\boldsymbol{\omega}(q)=0$, one can verify that the expression (2.4) is just the general expression in the London limit representing interactions among the vortices in an uniaxially anisotropic material [compare, for instance, with Eq. (2) in Ref. 9]. Using (2.5), the nonlocal terms of the elastic energy (2.4) can be written in a more explicit form, which we will give in deriving the corresponding result to (2.4) according to the GL theory. Recently, Sardella[10] also derived expression (2.4) with (2.5) in the London limit. Note that (2.4) is a gauge-invariant result (i.e., independent of the χ field). This fact will be useful later in calculating the elastic free energy within the GL framework. As shown in I, in spite of this gauge invariance, the superconducting long-range order is absent in the 3D flux lattice because of the fact that, for instance, in the London gauge the gradient of phase $\nabla\chi$ is identified with the displacement field through $\text{div}\mathbf{v}=0$, which is just the longitudinal component of the Maxwell equation.[3,11]

We note that, reflecting the diagonalized form of the inverse-mass tensor in (2.1), the expression (2.4) is "diagonalized," irrespective of θ, in the crystal frame. As will be seen below in (2.7), however, when expressed in terms of the displacement field, the elastic free energy includes a cross term between the tilt angle $\partial_z s_x$ and the compression (or dilation) $-\text{div}s$ except the $\theta=0$ and 90° ($B\perp\hat{\mathbf{Z}}$) cases, since the symmetry axis for the displacements is not a crystal axis but the vortex axis $\mathbf{B}(\parallel\hat{\mathbf{z}})$. This feature implies the following in the strongly nonlocal ($\lambda_L q\gg1$) and anisotropic ($\eta\ll1$) limit: According to (2.4), the fluctuation

$$\delta\omega_Z=-2\pi\phi_0^{-1}B(\sin\theta\,\partial_z s_x+\cos\theta\,\text{div}s)$$

in this case is relatively suppressed, indicating that, as expected, the vortex cores in $\theta\neq0$ tend to become parallel to the X-Y plane (when $\theta=0$, this merely means that the compression $-\text{div}s$ is unimportant compared to the tilt deformation). We note that the local distribution of flux density is uniform and parallel to the z axis since, in the nonlocal limit, magnetic-field fluctuations do not follow the order-parameter fluctuations $\delta\boldsymbol{\omega}$. By contrast, in the local ($\lambda_L q\ll1$) limit, such a cross term vanishes, and the elastic energy has the usual isotropic form since the tilt and compressional terms in this limit are dominated by the isotropic magnetic energy.

In a recent paper, Blatter, Geshkenbein, and Larkin[12] tried to derive the elastic moduli for an arbitrary field configuration (in the London limit) by invoking the nonlocal limit (we note that the nonlocal limit is not equivalent to the case of vanishing electric charge, namely, to the infinite-κ limit in this paper). The tilt modulus they obtained for the deformation $\partial_z s_x$ does not agree with the corresponding one in (2.4) [or (2.7)], although other moduli in Ref. 12 are consistent with our results. It suggests that the scaling rule argued in Ref. 12 is not correct in general [see, however, Eq. (2.15) below].

Now we turn to the derivation of the elastic free energy in GL theory. We apply the GL approach in I to the $\theta\neq0$ case. This approach is based upon the statement that, if the elastic free energy is derived from the GL free energy expressed in terms of the order-parameter field, the displacement fields have to be identified with fluctuation amplitudes of modes of the order parameter. First, we find it convenient to first perform the (volumepreserving) scale transformation

$$x \to \tilde{x} = \gamma^{-1} x , \quad y \to \tilde{y} = \gamma y ,$$
$$z = \tilde{z} , \quad \gamma^4 = \cos^2\theta + \eta \sin^2\theta . \tag{2.6}$$

In this new frame, we can expand the order parameter, as in I, in terms of the orthogonal and complete Landau-level basis functions with the triangular lattice symmetry introduced by Eilenberger.[1] The completeness of these functions is easily shown by noting that they are nothing but magnetic Bloch states[13] (see the Appendix). Since we wish to focus on large-κ systems, where κ is the GL parameter, relatively small $O(\kappa^{-2})$ corrections arising from local magnetic-field contributions rapidly varying on the scale of vortex spacing can be neglected as in I. Again, this approximation corresponds to the flux-liquid limit in Ref. 9. Since this approximation does not affect shear-energy terms independent of gauge-field fluctuations (see Refs. 3 and 17) and the present approach does not need a similar continuum approximation for the order-parameter field[4] at all, we can derive all elastic moduli on the same footing. As usual, the mean-field solution will be assumed to belong to the lowest ($n=0$) LL. Consistent with this, we can restrict ourselves to the fluctuations within the lowest and next ($n=1$) LL.[3] Then deriving the elastic free energy for $\theta \neq 0$, as will be sketched in the Appendix, is tedious but straightforward. As a result, we obtain the elastic free energy

$$\delta G^{\theta}_{\rm GL} = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[\frac{B^2}{4\pi} \frac{1}{d_{H\theta}} \left\{ q_z^2 |s_{yq}|^2 + |\sin\theta\, \mathbf{q}\cdot\mathbf{s}_q - \cos\theta\, q_z s_{xq}|^2 + \frac{1+\eta^{-1}\lambda_{H\theta}^2 q^2}{1+\lambda_{H\theta}^2 q^2} |\cos\theta\, \mathbf{q}\cdot\mathbf{s}_q + \sin\theta\, q_z s_{xq}|^2 \right\} \right.$$
$$\left. + C^{\theta}_{66} [(\tilde{q}_x^2 + \tilde{q}_y^2) |\tilde{\mathbf{s}}_q|^2 - |\mathbf{q}\cdot\mathbf{s}_q|^2] + \frac{B^2}{4\pi} \gamma'^4 \frac{r_B^2}{2\lambda_{H\theta}^2} q_z^2 \left\{ \frac{1}{d_{H\theta}} |\tilde{\mathbf{s}}^T_q|^2 + |\tilde{\mathbf{s}}^L_q|^2 \right\} \right] , \tag{2.7}$$

where $\gamma'^4 = 1 + \eta - \gamma^4$, $r_B^{-2} = 2\pi B / \phi_0$, and the new displacement field

$$\tilde{\mathbf{s}}_q = \gamma^{-1} s_{xq} \hat{\mathbf{x}} + \gamma s_{yq} \hat{\mathbf{y}} \tag{2.8}$$

satisfies $\tilde{\mathbf{q}}\cdot\tilde{\mathbf{s}}^T_q = (\tilde{\mathbf{q}} \times \tilde{\mathbf{s}}^L_q)_z = 0$. The penetration depth $\lambda_{H\theta}$ and the upper critical field $H^{\theta}_{c2}(T)$ in the $\theta \neq 0$ case are expressed in terms of the coherence length $\xi(T) \equiv \xi_0 / \sqrt{1 - T/T_c}$ by

$$\lambda_{H\theta}^2 = \beta_A \kappa^2 \xi^2(T) / [1 - B / H^{\theta}_{c2}(T)] ,$$
$$H^{\theta}_{c2}(T) = \phi_0 / 2\pi\gamma^2 \xi^2(T) = H_{c2}(T)/\gamma^2 ,$$

where $\beta_A = 1.16$ and the shear modulus C^{θ}_{66} and denominator $d_{H\theta}$ in (2.7) become

$$C^{\theta}_{66} = \frac{0.476}{16\pi(\beta_A \kappa)^2} H^2_{c2}(T) \left[1 - \frac{B}{H^{\theta}_{c2}(T)} \right]^2$$
$$\simeq \frac{\phi_0 H_{c2}(T)}{64\pi^2 \lambda_{H\theta}^2} \left[1 - \frac{B}{H^{\theta}_{c2}(T)} \right] , \tag{2.9}$$
$$d_{H\theta} = 1 + \lambda_{H\theta}^2 (\eta^{-1} q_\perp^2 + q_Z^2) .$$

The expression $C^{\theta}_{66} (\tilde{q}_x^2 + \tilde{q}_y^2) |\tilde{\mathbf{s}}_q|^2$ of the shear term can be understood by rewriting it in another form:

$$\overline{C}^{\theta}_{66} \left[\gamma^6 q_x^2 |s_{yq}|^2 + \gamma^{-2} q_y^2 |s_{xq}|^2 + \gamma^2 \sum_{i=x,y} q_i^2 |s_{iq}|^2 \right] , \tag{2.10}$$

with

$$\overline{C}^{\theta}_{66} \simeq \frac{\phi_0}{64\pi^2 \lambda_L^2} H^{\theta}_{c2}(T) ,$$

where the factor $1 - B/H^{\theta}_{c2}(T)$ in (2.9) due to the shift of T_c in $B \neq 0$ was replaced by 1 and $\lambda_{H\theta}$ by λ_L, according to the usual definition of the London limit. Then the shear moduli for the first and second terms of (2.10) coincide with $C^{(e)}_{66}$ and $C^{(h)}_{66}$ calculated in Ref. 8(b) in the London limit, respectively, if the factor $H^{\theta}_{c2}(T)$ in $\overline{C}^{\theta}_{66}$ is replaced by B. Of course, it is essential to use the Eilenberger basis with triangular lattice symmetry in order to derive these shear energy terms. As noted by Ikeda, Ohmi, and Tsuneto,[2] if the corresponding basis with square lattice symmetry is used, as expected, one obtains results indicating a shear instability.

The relations between the displacement fields s_q and the fluctuation amplitudes of the order parameter, when expressed in the new frame (2.6), are the same as in $\theta = 0$ case of Ref. 3 (see the Appendix). One finds that the nonlocal terms in (2.7) precisely coincide with those in (2.4) in the London limit except for the usual difference in the definition of the penetration depth. This should be expected when taking account of the following fact. The expressions of the nonlocal terms, namely, of the interaction potential between the flux lines are, as already understood in Ref. 3, primarily determined through the minimal coupling between the order parameter and gauge fields by the gradient terms for these fields in the original free energy. Obviously, they are of the same form both in the London limit and GL theory. When $\theta \neq 0$, the tilt moduli [coefficients of $(\partial_z \mathbf{s})^2$ terms] in GL theory depend on the tilt directions even in the local limit ($\lambda q \ll 1$) since the order-parameter rigidity appearing in the last line (so-called "magnetization" terms) of (2.7) is different in the x and y directions from each other. In Ref. 14 a computation of the compression modulus was performed according to the GL approach of Ref. 4. In contrast to our result shown above, however, its expression in Ref. 14 is inconsistent with that in the London limit and thus not justified.

It is easy to introduce a pinning potential term in this Landau-level formalism. Consider the following random-potential term taking care of local variations of the transition temperature:

$$\delta G_r \equiv \int d^3r \, V(r) (\psi_0^* \delta\psi + \text{c.c.}) , \tag{2.11}$$

where V is a random potential. Using (A7) and (A10) in the Appendix and (A1) of Ref. 3 and expressing the deviation $2\,\mathrm{Re}\psi_0^*\delta\psi$ of squared order parameter in terms of displacement fields, (2.11) can be, to leading order, rewritten in the form

$$\delta G_r \simeq -\int d^3r\, V(r)(s\cdot\partial)|\psi_0|^2 \,, \tag{2.12}$$

which is just the well-known[15] phenomenological form. Of course, this form of the pinning potential does not change in the infinite-κ limit.

Next, we will comment on the Lindemann criterion for the (if any) melting line of the flux lattice within anisotropic 3D GL theory [the Lindemann criterion is merely an estimation of the melting line and thus is insufficient to understand a possible mechanism of the melting transition. Furthermore, competition between the layer structure and vortex spacing in strong fields could change, for $B\perp\hat{Z}$, the equilibrium lattice structure and thus the shear modulus. Therefore it might be questionable in general to apply directly the result given below to estimating the melting (softening) positions in real systems with strong anisotropy. See, however, below and Sec. IV]. In order to derive the expression of the Lindemann criterion for an arbitrary field configuration, it is convenient to use the elastic free energy in the infinite-κ limit[2,3] where there are no gauge-field fluctuations:

$$\delta G^{\infty} = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left\{ \left[\frac{\phi_0}{2\pi}\right]^2 \left[\frac{\gamma'^4}{4\pi\lambda_{H\theta}^2} q_z^2 + B^{-2}C_{66}^{\theta}(\tilde{q}_x^2+\tilde{q}_y^2)^2\right]|\chi_q|^2 + \frac{B^2}{4\pi\lambda_{H\theta}^2}\left[\left[\gamma^2+\frac{\gamma'^4}{2}r_B^2 q_z^2\right]|\tilde{s}_q^L|^2 + \frac{(1-\eta)\sin 2\theta}{2\gamma} r_B^2 (iq_z\chi_q \tilde{s}_{yq}^{L*} + \mathrm{c.c.})\right]\right\} , \tag{2.13}$$

with

$$\tilde{s}_q^T \simeq i r_B^2 (\tilde{\mathbf{q}}\times\hat{z})\chi_q \,, \tag{2.14}$$

where χ_q is the fluctuation of the order-parameter phase defined in (2.5) [(2.13) is easily obtained by following procedures in the Appendix and neglecting contributions of gauge-field fluctuations]. For instance, one can see that the result of the Lindemann criterion for $\theta=0$ in Ref. 4 is nicely obtained in terms of (2.13) and (2.14), indicating that low-q_x (and q_y) fluctuations are irrelevant[5] in the fluctuation-dominated regime of the flux (vortex) lattice. In general, thermal softening of the flux lattice in a strongly type-II material will be dominated by the shear mode with no accompanying gauge-field fluctuations. Therefore we define below the Lindemann criterion according to the mean-square average of the transverse displacement (2.14). Actually, the longitudinal elastic mode in (2.13) is massive, since, as shown in Ref. 3, this mode belongs to the higher ($n=1$) LL. It reflects the fact that the flux lattice in the infinite-κ limit is incompressible in the limit of vanishing wave number. Furthermore, we note that, if one tries to derive (2.13) on the basis of the London limit, the $q_z^2|\tilde{s}_q^L|^2$ term in the second line of (2.13) is lost [see (D11) of Ref. 3].

When $\theta\neq 0, 90°$, $\tilde{s}^T$ and $\tilde{s}_y^L$ are intrinsically coupled to each other. Therefore, in order to obtain the effective free energy for $\tilde{s}^T$, $\tilde{s}_y^L$ corresponding to one component of the complex fluctuation amplitude a_1 defined in the Appendix, has to be integrated out. Consequently, we find

$$\delta G_{\mathrm{eff}}^{\infty} \simeq \frac{1}{2}\int\frac{d^3q}{(2\pi)^3}\left[\frac{\eta\gamma^{-4}B^2}{4\pi\lambda_{H\theta}^2(\tilde{q}_x^2+\tilde{q}_y^2)} q_z^2 + C_{66}^{\theta}(\tilde{q}_x^2+\tilde{q}_y^2)\right]|\tilde{s}_q^T|^2 \,, \tag{2.15}$$

where higher-order terms in $q_z^2 r_B^2$ were neglected.[16] We note that (2.15) is also obtained by integrating out $\mathbf{q}\cdot\mathbf{s}$ in (2.7) and assuming $\tilde{q}_x, \tilde{q}_y \gg q_z$. This expression indicates that the scaling rule[12] is effectively satisfied within the lowest ($n=0$) LL, to which the shear mode $\tilde{s}^T$ as well as the mean-field solution belongs. When $\theta\neq 0$, it is natural to calculate the mean-square displacement in the new frame (2.6) where the mean-field solution is, as usual, the isotropic triangular lattice. Following Refs. 4 and 5, we will use the expression $C_{66}^L \simeq \phi_0 B/64\pi^2\lambda_L^2$ of the shear modulus in the London limit[17] and in the isotropic case [see the context following (2.10)]. We obtain the following (scaled) mean-square displacement:[4,5]

$$\langle \tilde{s}^{T^2}\rangle = \frac{\lambda_{H\theta}kT}{B^2 r_B^2}\,\frac{\gamma B}{(4\pi\eta C_{66}^L)^{1/2}} \,, \tag{2.16}$$

where k is the Boltzmann constant. When calculating (2.16), as in Ref. 4, we chose $\sqrt{2}/r_B$ as the upper cutoff $q_{\max}$ of the scaled transverse wave number by assuming a circular Brillouin zone.[4] The value $q_{\max}=\sqrt{2}/r_B$ means that the Landau-level degeneracy is correctly taken into account:

$$\sum_{q_\perp} = \frac{S}{2\pi r_B^2} \,, \tag{2.17}$$

where S is the system area in directions perpendicular to B. Using (2.16), the Lindemann criterion gives the following estimate of the (if any) melting temperature $T_m(B)$:

$$1-\frac{T_m}{T_c} \simeq \frac{(\kappa\xi_0)^2\gamma}{10\Lambda_{T_c}c_L^2}\left[\frac{2\pi}{\phi_0\eta}B\right]^{1/2} \,, \tag{2.18}$$

where $\Lambda_T = \phi_0^2/16\pi^2 kT$ and c_L is the Lindemann number.[4] This expression is essentially the same as those given in Refs. 12 and 18. Clearly, the derivation given above indicates that the melting (softening) of the flux lattice is controlled by the shear mode, namely, by the *fluctuations* in the lowest LL with no gauge-field fluctuations.

III. GAUSSIAN-FLUCTUATION CORRECTIONS TO THERMODYNAMIC PROPERTIES

As pointed out in Sec. II, contributions of gauge-field fluctuations are negligible in the elasticity near the (if any) melting line in systems with the strong nonlocality. We show below that the same result is found by calculating Gaussian contributions of order-parameter fluctuations around the flux-lattice solution to thermodynamic quantities. As already understood by the methods from high[7,19] and low[3] temperatures, no true superconducting phase transitions are expected in the mixed state of three-dimensional type-II superconductors above H_{c1}. Even in such a case, it is useful to construct the mean-field solution by assuming broken gauge symmetry and to investigate[6] the thermodynamic fluctuation effects around it.

For simplicity, we consider the $\theta=0$ case studied in I and, as in Sec. II, restrict ourselves to the $n=0$ and 1 LL fluctuations. According to I, it allows us to regard $\varepsilon/2h$, as well as κ^{-1}, as a small parameter, where $h=B/H_{c2}(0)$ and $\varepsilon=1-h-T/T_c$. For the present purpose, we have only to calculate the mean-square order parameter $\langle|\psi|^2\rangle$ (or the entropy reduction), which is expressed in the form

$$f=\frac{b}{a}\langle|\psi|^2\rangle=\beta_A^{-1}\varepsilon+\sum_{n=0,1}\sum_{\sigma=\pm}f_n^\sigma\ ,$$
$$f_n^\sigma=-\frac{k}{2\Delta C}\frac{1}{\Omega}\sum_q\frac{\partial}{\partial\varepsilon}\ln E_n^\sigma\ , \tag{3.1}$$

where ΔC is the specific-heat jump at T_c, Ω the sample volume, and the constants a and b are defined in (2.1). In obtaining (3.1) we neglected the massive gauge-field fluctuations, defined in I as $\delta A'$, which do not follow the order-parameter fluctuations. [Their contribution, which can lead to a change of the Landau potential[20] in the region $|\varepsilon|\sim(k/4\pi\Delta C\xi_0^3)^2\kappa^{-6}$, is obviously unimportant in strongly type-II superconductors. Especially, it is ineffective in the finite-field case, because the order-parameter fluctuations in this case cannot reach[19] a true critical (or massless) regime and thus should not be affected by the infinite-ranged gauge-field fluctuations when approaching from above T_c.] The eigenvalues E_n^σ of the $n=0$ and 1 fluctuation modes for the anisotropic and $\theta=0$ cases are easily obtained following procedures in I (see the Appendix) and are given by

$$E_0^+=2\varepsilon+\eta\bar q_z^2-\frac{\bar q_\perp^2}{1+\kappa_T^2\bar q^2}\ ,$$
$$E_0^-=C\frac{\varepsilon}{h}\bar q_\perp^2+\frac{h\kappa_T^2}{1+\kappa_T^2(\bar q_z^2+\eta^{-1}\bar q_\perp^2)}\left[1+\frac{\eta}{2h\kappa_T^2}\right]\bar q_z^2\ ,$$
$$E_1^+=\frac{2h\kappa_T^2}{1+\kappa_T^2\bar q^2}\bar q^2+\eta\bar q_z^2+\frac{3}{1+\kappa_T^2(\bar q_z^2+\eta^{-1}\bar q_\perp^2)}\bar q_\perp^2\ ,$$
$$E_1^-=\frac{2}{1+\kappa_T^2(\bar q_z^2+\eta^{-1}\bar q_\perp^2)}(h\kappa_T^2\bar q_z^2+\tfrac{1}{2}\bar q_\perp^2)+\eta\bar q_z^2\ , \tag{3.2}$$

where $\bar q=q\xi_0$, $C=0.12\beta_A^{-1}$, and $\kappa_T^{-2}=\kappa^{-2}\varepsilon\beta_A^{-1}$. E_0^+ is the spectrum of the massive (amplitude) mode which was not taken into account in I, and E_0^- and E_1^+ correspond to the elastic modes [see (5.2) in I]. When obtaining E_1^+, we used $|\mathbf{q}\cdot\mathbf{s}_q|^2=q_\perp^2|s_q^L|^2$. In the first terms of $E_0^\pm$, we neglected corrections[1] proportional to ε and with higher power in $\bar q_\perp^2/h$. Inclusion of them does not change our principal results (see below). The $q_\perp$ integral is done in just the same way as in the Lindemann criterion (see Sec. II) and as in the corresponding calculation in the infinite-κ limit.[6] Then it is straightforward to carry out the q integrals in f_n^σ. For simplicity, a term independent of ε is omitted in the f_0^- result given below since it is absorbed into a negligible renormalization of T_c and does not contribute at all to quantities of practical interest such as the specific heat $C_{\rm sp}=\Delta C\,\partial f/\partial\varepsilon$. For $f_0^\pm$, we obtain

$$f_0^+\simeq-\frac{1}{2}g_3\frac{1}{\sqrt{2\varepsilon}}\left[1-\frac{3}{2}\frac{1}{h\kappa_T^2}\ln\frac{\eta h}{\varepsilon}\right]\ , \tag{3.3}$$

$$f_0^-\simeq-\frac{1}{4}g_3\left[\frac{C}{\varepsilon}\right]^{1/2}\left[1-\frac{2C}{h}\varepsilon-\frac{\eta}{4h\kappa_T^2}\right]\ , \tag{3.4}$$

where g_3 is the coupling constant defined in Ref. 7,

$$g_3=\frac{k}{\Delta C}\frac{B}{\phi_0\xi_0}\eta^{-1/2}\ . \tag{3.5}$$

The leading (first) terms of $f_0^\pm$ *precisely* coincide with results[6] found in the infinite-κ limit. They, as in the Lindemann criterion, result from the $\bar q_\perp\simeq\sqrt{2h}$ region, meaning that there are no characteristic length scales other than the vortex spacing of order-parameter fluctuations in directions perpendicular to B. Since the longitudinal gauge-field fluctuation contributes[3] to the transverse elastic mode due to the broken gauge symmetry assumed in mean-field theory, an additional κ-independent (the second) term appears in f_0^-. Together with still smaller $O(\kappa^{-2})$ corrections in $f_0^\pm$, however, it can be neglected compared to the first term of f_0^- since ε/h is being assumed from the outset to be small. Taking account of the corrections to $E_0^\pm$ mentioned above merely leads to negligible modifications of the numerical factors of each term in (3.4) and of the first term in (3.3) [it does not change the last term in (3.3) at all since this term arises from $\bar q_\perp\simeq0$ region]. In the same way, we can calculate the contributions $f_1^\pm$ of $n=1$ modes and obtain $f_1^+\simeq-g_3\sqrt{\varepsilon\eta}/4h\kappa^3$, $f_1^-\simeq-g_3\sqrt{\eta}/8\kappa\sqrt{\varepsilon}$, which are again negligible in the large-κ ($\sim10^2$) materials of interest. The result that all contributions arising from gauge-field fluctuations are higher order in ε/h is nontrivial and the main conclusion in this section. Therefore we can say that the contributions of gauge-field fluctuations are negligible in the fluctuation region below (and thus above) the melting line. Furthermore, as the temperature dependence of the first terms in $f_0^{\pm}$ shows, the order-parameter fluctuations in 3D systems can be regarded to be one-dimensional-like,[7] resulting in the scaling behavior[21] $\varepsilon \sim B^{2/3}$, and, as usual, are dominated by the massive mode. It indicates that the contributions of elastic modes in a system with no disorder are negligible in the specific heat and the in-plane conductivity showing the flux flow. These results justify the statements on low-temperature behavior obtained according to the theory of Ref. 7, where the first terms in (3.3) and (3.4) were used in order to check the validity of the theory.

IV. REMARKS

We explicitly showed in preceding sections that gauge-field fluctuations can be neglected in describing Gaussian fluctuations around the flux-lattice state of strongly type-II ($\kappa \gg 1$) superconductors and that the theories constructed in the infinite-κ limit are valid in such a case. This is due to the fact that, as far as $\lambda \gg r_B$, the vortex spacing r_B is the only characteristic length of order-parameter fluctuations in directions perpendicular to the vortex axis. Thus this conclusion should be valid even in the region (near H_{c2}) where no rigid flux lattice appears. In fact, transport and thermodynamic phenomena in such region of cuprate high-T_c superconductors have been discussed according to infinite-κ fluctuation theory[7] approaching from higher temperature. The theory has been confirmed on the quantitative level[11] by recent measurements[21] in the flux-flow and fluctuation regions for clean crystal samples in a field parallel to the c axis ($B \parallel c$). Furthermore, the independence[22] of the resistivity in Bi-Sr-Ca-Cu-O on the in-plane field component is easily explained according to the theory of Ref. 7 if the strongly anisotropic limit is taken.[23] Actually, in order to extend the theory[7] to an arbitrary field configuration (except $B \perp c$) in this limit, as expected, one has only to replace the field strength in the formulas of Ref. 7 by the out-of-plane component of the field.

Recently, it has been pointed out in Ref. 18 that a Lindemann criterion in anisotropic 3D theory, which is essentially the same as (2.18) of the present paper, explains well the angular (and field) dependence of observed dissipation peaks, suggestive of the flux-lattice melting in untwinned single-crystal Y-Ba-Cu-O. Below, we would like to give some comments closely related to this significant experimental result.

In studying the elastic response for $\theta \equiv 90° - \theta' \neq 0$ in Sec. II, we neglected the discrete layer structure and assumed an anisotropic 3D model. This approximation seems to be valid if a few conditions are satisfied. We will explain below this by taking 90 K Y-Ba-Cu-O as an example. According to Ref. 24, the onset angle θ'^* of the intrinsic pinning in this material is extremely small ($\lesssim 0.5°$). Thus the Lindemann criterion given in Sec. II is expected to be useful at least when $\theta' > \theta'^*$. Furthermore, when the strength of the applied field is not so strong, the resistivity curves for $\theta' \simeq 0$ calculated in the limit of vanishing layer spacing according to the theory of Ref. 7 are consistent with experimental data in the flux-flow and fluctuation regions of Refs. 25 where the intrinsic pinning has not been observed possibly because of a small misalignment. We give in Fig. 1 typical theoretical curves (one can easily extend the theoretical calculations in Ref. 7 to the present situation,[23] although it is necessary to use a fact commented in Ref. 16 of the paper). The magnitude of the deviation among the two curves, as already commented in Sec. 5 of Ref. 7, roughly agrees with the experimental data[25] (in passing, we find that the deviation tends to vanish with increasing the anisotropy η^{-1}). It suggests that, consistent with the result in Ref. 18, the fluctuation effects in 90 K Y-Ba-Cu-O in the moderate angle and field ranges can be understood according to the anisotropic 3D model.

At present, it is theoretically unclear whether[26,27] the melting can be a true phase transition, in other words, whether the long-ranged translational order is possible in the mixed state. Even if a true melting (or freezing) transition is possible, as suggested also in Ref. 26, no remarkable feature is expected to appear in thermodynamic quantities near the melting point, since much of the fluctuation entropy has to be spent when crossing over from the fluctuation regime near T_c to the flux-flow regime where no rigid flux lattice is still built (note that the fundamental model describing the thermodynamics of mixed state is the GL free energy, but not the elastic free energy). This seems to be consistent with the experimental data for cuprate high-T_c superconductors and with the result found in Sec. III. In relation to this, it will be interesting to examine experimentally how the dissipation peaks reported in Refs. 18 and 28 are reflected in the corresponding resistivity curves in the configurations $B \perp I$ and especially $B \parallel I$. A direct comparison between the melting curve in Ref. 28 and the in-plane resistivity data for an untwinned crystal in Ref. 21 suggests that, in contrast to the viewpoint in other works,[29] the melting (or softening) of the 3D flux lattice with weak disorder occurs in the region where Ohmic resistivity is finite. The corresponding resistivity data in $B \parallel I$, closely related to the absence of the phase coherence,[3] may be useful in understanding the meaning of the dissipation peaks.[27]

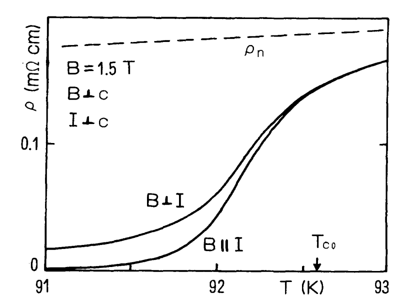

FIG. 1. Theoretical resistivity curves in the configuration $B_{\perp}c$ of 90 K Y-Ba-Cu-O. They (solid curves) were obtained within the anisotropic 3D GL model according to the theory of Ref. 7. The parameters used for calculations are, except the $B=0$ transition temperature T_{c0} (arrow) and the extrapolated normal resistivity (dashed line), the same as in Fig. 3(a) of Ref. 7. I denotes the direction of applied current. In obtaining them, a result (Ref. 16) quoted in Ref. 7 was used.

Note added in proof: (a) Recently, we learned of an experimental work by H. Safar, P. L. Gammel, D. J. Bishop, J. P. Rice, and D. M. Ginsberg [Phys. Rev. Lett. **69**, 824 (1992)], who argued the presence of a 3D first-order melting transition where the sample disorder starts to affect the resistivity. In contrast to transport, thermodynamic quantities are not expected to be sensitive to the melting transition even in real samples. A recent 2D Monte Carlo simulation by Y. Kato and N. Nagaosa (unpublished) shows that a first-order melting transition is present but extremely weak. (b) Quite recently, Z. Hao and J. R. Clem [Phys. Rev. B **46**, 5853 (1992)] argued that the field and angular dependence of fluctuations characteristic of the 3D GL model is always obtained, as in the $\mathbf{B}\|\hat{\mathbf{Z}}$ case, within the lowest LL. As shown in the present paper and Ref. 23, this is not correct.

APPENDIX

Here we will comment on the details of calculation in GL theory which were not given in the text. As mentioned in Sec. II, we represent the order parameter in the new frame (2.6) and work in the gauge

$$A_{\rm ext}=-B(\hat{\mathbf{X}}\cos\theta-\hat{\mathbf{Z}}\sin\theta)y=-By\hat{\mathbf{x}}\ .$$

For brevity, we set $r_B=1$ below. The Eilenberger basis function[1] in this frame is given by

$$\phi_n(\tilde{r}|\tilde{r}_0)=\left[\frac{k}{2^n n! S\pi^{1/2}}\right]^{1/2} D_+^n \times \sum_{p=\text{integer}} C_p \exp[i(kp+\tilde{q}_x)\tilde{x}-ikp\tilde{q}_y -\tfrac{1}{2}(y+kp+\tilde{q}_x)^2]\ , \quad (\text{A1})$$

where $\tilde{q}=\tilde{\mathbf{r}}_0\times\hat{\mathbf{z}}$, $D_+=-i\partial_{\tilde{x}}+\tilde{y}-\partial_{\tilde{y}}$ is the raising operator, S the (normalized) system area in x-y plane, and k^2 and C_p are $\pi\sqrt{3}(2\pi)$ and $\exp[i(\pi/2)p^2]$ (1.0) in the triangular (square) lattice, respectively. The function (A1) satisfies the orthonormalization

$$\int d^2\tilde{r}\,\phi_n(\tilde{r}|\tilde{r}_0)\phi_m^*(\tilde{r}|\tilde{r}_0')=\delta_{n,m}\delta_{\tilde{r}_0,\tilde{r}_0'}\ , \quad (\text{A2})$$

which is different from that used in I. The basis (A1) is nothing but the magnetic Bloch state, which, for example, in the triangular lattice satisfies the magnetic translation operation[13]

$$T\left[\frac{2\pi}{k}\hat{\mathbf{x}}\right]\phi_n(\tilde{r}|\tilde{r}_0)=e^{i2\pi\tilde{q}_x/k}\phi_n(\tilde{r}|\tilde{r}_0)\ ,$$
$$T\left[\frac{\pi}{k}\hat{\mathbf{x}}+k\hat{\mathbf{y}}\right]\phi_n(\tilde{r}|\tilde{r}_0)=e^{i(\pi\tilde{q}_x/k+k\tilde{q}_y)}\phi_n(\tilde{r}|\tilde{r}_0)\ , \quad (\text{A3})$$

where

$$T(a\hat{\mathbf{x}}+b\hat{\mathbf{y}})=\exp[a\partial_{\tilde{x}}+b(\partial_{\tilde{y}}+i\tilde{x})]\ . \quad (\text{A4})$$

In fact, constructing the Bloch condition on the basis of (A3), one finds $\tilde{q}$ to be defined, for instance, in the region $0<\tilde{q}_x\le k$ and $0<\tilde{q}_y\le 2\pi/k$. This condition satisfies the relation (2.17). Then, by using the Poisson sum formula, one readily obtains

$$\sum_{\tilde{q}}\phi_0(\tilde{r}|\tilde{r}_0)\phi_0^*(\tilde{r}'|\tilde{r}_0) =\frac{1}{2\pi}\exp\left[-\frac{1}{4}|\tilde{r}-\tilde{r}'|^2-\frac{i}{2}(\tilde{x}-\tilde{x}')(\tilde{y}+\tilde{y}')\right]\ . \quad (\text{A5})$$

The right-hand side of (A5) is just the projection operator into the lowest Landau level $P_0(\tilde{r},\tilde{r}')$, which satisfies the Bargmann identity[30] expressed in the present gauge:

$$\phi_{0L}(\tilde{r})=\int d^2\tilde{r}'\,P_0(\tilde{r},\tilde{r}')\phi_{0L}(\tilde{r}')\ , \quad (\text{A6})$$

where $\phi_{0L}(\tilde{r})$ is an arbitrary function in the lowest LL in this gauge. Therefore, the completeness of the basis (A1) is found in the same way as in the usual orbital representation, which is used in the translationally invariant situation.[7,19]

Now we explain the derivation of the elastic free energy. According to I,[3] the order parameter ψ is expanded in terms of the basis (A1):

$$\psi=\psi_0+\delta\psi =\sum_{m=0}\alpha_{6m}\phi_{6m}(\tilde{r}|0)+\Omega^{-1/2}\sum_n\sum_{q\neq 0}a_{nq}\phi_n(\tilde{r}|\tilde{r}_0)e^{iq_z z}\ . \quad (\text{A7})$$

As mentioned in Sec. II, it is permitted to calculate the mean-field solution in $\kappa^{-1}=0$, and thus the isotropic triangular lattice is trivially obtained in the frame (2.6). Following the procedures in I, we substitute (A7) into (2.1) and integrate out the gauge-field fluctuations. As a result, the fluctuation free energy δG harmonic in $\delta\psi$ is written as

$$\delta G=\int d^3\tilde{r}\,a\xi_0^2\left[\sum_{j=X,Y}|\Pi_j\delta\psi|^2+\eta|\Pi_Z\delta\psi|^2\right]$$
$$-\frac{1}{4}\int\frac{d^3q}{(2\pi)^3}\frac{a\xi_0^2}{d_{H\theta}\alpha_0^2}\left[|\delta j_\perp(q)|^2+\eta|\delta j_Z(q)|^2+\eta^{-1}\lambda_{H\theta}^2|q_\perp\cdot\delta j_\perp(q)+\eta q_Z\delta j_Z(q)|^2+\frac{(\eta^{-1}-1)}{1+\lambda_{H\theta}^2q^2}\lambda_{H\theta}^2|[q\times\delta j(q)]_Z|^2\right]$$
$$+\int d^3\tilde{r}\left[-a(1-T/T_c)|\delta\psi|^2+2b|\psi_0|^2|\delta\psi|^2+\frac{b}{2}(\psi_0^{*2}\delta\psi^2+\text{c.c.})\right]\ , \quad (\text{A8})$$

where $\delta j_i(q)$ is the Fourier transform of

$$\delta j_i = \delta\psi^* \Pi_i \psi_0 + \psi_0^* \Pi_i \delta\psi + \text{c.c.} \; ,$$

and

$$\Pi_X = -i\partial_X + y\cos\theta \; , \quad \Pi_Y = -i\partial_Y \; , \quad \Pi_Z = -i\partial_Z - y\sin\theta \; . \tag{A9}$$

As understood in I, we have only to take account of $n=0$ and 1 modes in order to derive the usual elastic free energy. The second line in (A8) corresponds to the contribution arising from the massive gauge-field fluctuations. The last three terms (without any gauge-invariant derivative) resulting from the original $|\psi|^p$ $(p=2,4)$ terms are necessary to making elastic modes massless,[3] and the shear term arises from the last two terms of (A8). Hereafter, we do not consider the last line of (A8) and focus on the remaining terms including the (gauge-invariant) derivatives. One can see a similarity between these terms and (2.3). In order to derive the tilt and compressional elastic terms from (A8), it is convenient to make use of the fact that the elastic free energy is a gauge-invariant result, i.e., independent of the presence of χ field in (2.5). Accordingly, as noted in Appendix D of I, we can set $\nabla_i\chi = 2\pi(B\times \mathbf{s}^T)_i/\phi_0$ $(i=x,y)$ in (2.5) and substitute the resulting $\mathbf{v}$ field into (2.3). Next, we compare the expression thus obtained in the London limit with the Fourier representation[3] of (A8) without the last line. Then we find that, except the additional magnetization terms, both of them are equivalent to each other if the fluctuation amplitudes a_{0q} and a_{1q} satisfy

$$\alpha_0 \tilde{s}_q^T = -\tilde{r}_0 a_{0q} \; , \quad a_{1q} = \frac{\alpha_0}{\sqrt{2} r_B}(\tilde{s}_{yq}^L - i\tilde{s}_{xq}^L) \; , \quad \tilde{s}_{-q}^{k*} = \tilde{s}_q^k \;\; (k=L \text{ or } T) \tag{A10}$$

[see (2.8)]. However, this is a natural extension of the definition of the displacement fields in $\theta=0$ case of I to $\theta\neq 0$ case, since the order-parameter field in $\theta\neq 0$ recovers its isotropy in the new frame (2.6). As a result, we obtain (2.7) as the GL result. Its nonlocal-elastic terms coincide with those in (2.4) except for the difference in the definition of the penetration depth.

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