Hopfield models and spin-density waves in metallic spin glasses

M. B. Weissman
Department of Physics, University of Illinois at Urbana–Champaign, 1110 West Green Street, Urbana, Illinois 61801

P. G. Wolynes
School of Chemical Sciences, University of Illinois at Urbana–Champaign, 505 South Matthews, Urbana, Illinois 61801

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Some parallels are found between the spin-interaction Hamiltonian produced by incipient spin-density waves and the multicomponent Mattis models which give Hopfield associative memories. The approximate criterion found by Feigelman and Ioffe for the crossover from spin glass to "spoiled helical" order is seen to follow easily from this analogy. The analogy allows a rather trivial extension of the Feigelman-Ioffe criterion to cases, such as $Cu_{1-x}Mn_x$, in which there are multiple but not infinitely multiple spin-density wavelike components together with short-range magnetic order.

The order parameter or parameters of spin-glass (SG) phases of real materials remain highly elusive.[1,2] The closest thing to a snapshot of such an order parameter is provided by neutron-scattering results on $Cu_{1-x}Mn_x$ and $Ag_{1-x}Mn_x$, which show short-range (about 4 nm) spin-density-wave (SDW) order.[3,4] Werner,[4] following a very early proposal by Overhauser,[5] has interpreted these results to mean that "the prototype spin glass is not a spin glass," but rather just an SDW antiferromagnet broken into many small domains along 12 possible SDW **q** vectors. Mydosh[3] has argued for the importance of local ferromagnetic correlations, promoted by atomic-short-range order (ASRO), in breaking up the postulated simple antiferromagnetism and producing macroscopic behavior quite distinct from simple SDW's. On the other hand, finite-size effect measurements[6] and mesoscopic fluctuation measurements[7] show that the thermodynamic and dynamic correlation lengths are much longer than the range of the SDW-like order, so that it is not appropriate to simply view these spin glasses as SDW's decorated with local moments. Therefore, it is worth considering what sort of order should form in these prototypical SG materials, given what the neutron-scattering results suggest about the Hamiltonian.

Feigelman and Ioffe[8–10] have systematically investigated the behavior of spin Hamiltonians based on SDW-like terms in the magnetic susceptibility, $\chi(q)$. They found a phase diagram including both SG phases and ones with what they dub "spoiled helical" order, which resembles the Overhauser phase except with some important extra degrees of freedom due to the finite range of the interaction.

In this paper we exploit the similarity of Hamiltonians with SDW-like sinusoidal coupling of randomly placed spins to variants of the Mattis model[11] in order to make a highly simplified schematic theory. We use the Mattis analogy to show that the nature of the frozen phase in these relatively realistic spin-glass models can be understood in terms of the behavior of Hopfield associative memories. Within this highly simplified picture, it becomes easy to predict approximately the effects of having multiple, but still discrete, SDW **q**'s and of having other competing short-range magnetic order. Both of these effects are important in $Cu_{1-x}Mn_x$, but hard to treat in the full Feigelman-Ioffe approach. In passing, we try to provide an accessible cartoon version of the Feigelman-Ioffe approach and try to remind the reader of the distinction between Mattis models and antiferromagnets.

We may start by considering a dilute solution of magnetic atoms in a host that has a sharp peak in $\chi(q)$ for only one direction of q. ($Y_{1-x}Gd_x$ and related materials provide approximate examples.[3,12]) The (infinite range) interaction between spins via the inducted polarization of the conduction electrons is then of the very simple form

$$\begin{aligned} H_{ij} &= -JS_i\cdot S_j \cos[\mathbf{q}\cdot(\mathbf{r}_i - r_j)] \\ &= -JS_i\cdot S_j\{\cos(\mathbf{q}\cdot\mathbf{r}_i)\cos(\mathbf{q}\cdot\mathbf{r}_j) \\ &\qquad + \sin(\mathbf{q}\cdot\mathbf{r}_i)\sin(\mathbf{q}\cdot\mathbf{r}_j)\} . \end{aligned} \quad (1)$$

Let us examine one of the two terms [e.g., $\cos(\mathbf{q}\cdot\mathbf{r}_i)\cos(\mathbf{q}\cdot\mathbf{r}_j)$] in the right-hand side of Eq. (1). Since the probability density function for $\cos(x)$ peaks at ± 1, we may approximate this term in H by $-J\varepsilon_i\varepsilon_j\mathbf{S}_i\cdot\mathbf{S}_j$, where the ε_i are randomly assigned the values ± 1. If there were only one such term in the Hamiltonian, we would be dealing with a problem that has already been solved, since this is simply the Mattis model. It is a model for ferromagnetic order of the fictitious spins $\mathbf{S}_i' \equiv \varepsilon_i\mathbf{S}_i$. Taking into account the actual distribution of absolute values of $\cos(x)$ would give a finite density of single-spin excitations at low energies but otherwise make no essential difference.

At this point, it is helpful to remember that even the simple one-parameter Mattis model should not be thought of as either antiferromagnetic or as ferromagnetic, since the transformation required to obtain either such model is equally random. To belabor this familiar point a bit in the context of SDW-like interactions, if the randomly placed spins happened to come out uniformly spaced one half of an interaction wavelength apart in some region, their frozen state would look perfectly antiferromagnetic. If, however, they happened to be spaced one wavelength apart, the frozen state would look perfectly ferromagnetic. The identification[3,4] of the randomly placed spins, coupled sinusoidally, with an antiferromagnet is thus inappropriate. There is no need to invoke the presence of some nonrandom local ferromagnetic coupling[3] (unlikely to be important in more dilute spin glasses) to explain the nonantiferromagnetic behavior of dilute materials with SDW-like coupling, since even the most idealized case would be Mattis-like, not antiferromagnetic. (We refer here to the behavior of the randomly placed local moments. The conduction electrons can show true SDW antiferromagnetism,[8–10] but the conduction-electron contribution to the magnetic susceptibility is small relative to that of the local moments in realistic cases.)

A more important difference than the inclusion of some weakly coupled spins between Eq. (1) and the Mattis model occurs when we include both the cos and sin terms. While the expectation of their product is zero, these terms are not independent, since they cannot both simultaneously have large absolute value. If we make the approximation that for any spin either $\sin(\mathbf{q}\cdot\mathbf{r}_i)$ or $\cos(\mathbf{q}\cdot\mathbf{r}_i)$ is zero while the other is ± 1, we obtain a simple picture of two parallel Mattis systems, again having no special thermodynamics. If we make the opposite approximation, treating the two terms as independent, we have a Hamiltonian which is the sum of two independent Mattis models, whose properties we shall discuss below.[13]

In either case, the approximation breaks the true statistical translational invariance of the Hamiltonian. We thus lose information on fluctuations of the phase of the sinusoidal order, which is crucial to understanding the correlation length in the ordered phase for finite-range forces,[9] but not essential to getting some feel for some qualitative features of the low-temperature phase. It is this finite correlation length which can give a spoiled helical phase rather than a simple Overhauser phase. Feigelman and Ioffe already noted that the finite-range version of this single-**q** Hamiltonian produces a regime of Mattis-like behavior, although with a range of metastable states provided by the lack of rigidity of the phase of the magnetization.[9] (As Feigelman has reminded us, for purely X-Y or Heisenberg spins, the combination of rotational and translation invariance allows the phase of the helix to decouple from the relative spin order, giving only one Mattis component, not two.)

In either approximation, we find that ratio of the number of independent Mattis components of the Hamiltonian seen by each spin to the number of spins within range of that Mattis component is just twice the inverse of the number of spins (or equal for pure X-Y or Heisenberg spins). The approximate value of this ratio will be important as we draw the analogy to associative memories.

If we grant that the behavior of a single-**q** dilute random system could resemble that of a ferromagnet (or antiferromagnet) with a random Mattis transformation, consistent with experiment,[12] we have a clue as to the behavior of a multi-**q** system. Each **q** vector produces its own coupling between the spins. The correlations between these couplings are small for spins more than a few lattice spacings apart. Thus we should consider the behavior of spins whose Hamiltonian is the sum of a number of independent Mattis Hamiltonians.

Such multicomponent Mattis Hamiltonians are nothing new.[2,13–15] For a finite number M of component Hamiltonians, they are known as the Hopfield associative memory model. Such models have a transition to a SG state when M becomes too large compared with the number of interacting spins, N_c.[14,15] In the limit as M goes to infinity one gets exactly the Sherrington-Kirkpatrick SG Hamiltonian, whose fascinating properties have been much studied.[16] Perhaps the neutron-scattering results on $Cu_{1-x}Mn_x$ are not so far from SG theory as one might first guess.

When $M/N_c \equiv \alpha < 0.05$ the lowest-energy states are close to being ground states of individual Mattis components of the Hamiltonian. (That's why associative memories work.)[2,14,15] For larger α a SG ground state is found, along with the Mattis-like states. Above $\alpha \approx 0.1$ the system behaves, in mean-field theory, essentially like a Sherrington-Kirkpatrick SG.[2,14,15]

Thus the analogy to associative memories suggests a simplified test for what type of behavior will be exhibited by randomly placed spins interacting sinusoidally. If α, the total number of Mattis-like components of the Hamiltonian per number of spins, is greater than about 0.05, complicated SG order is to be expected even locally. That would not by itself tell us the nature of the global phase, but it might undermine some of the arguments supporting a simple scaling picture of short-range Ising spin glasses and lead to the interesting hierarchical superparamagnet proposed by Feigelman and Ioffe.[17] For $\alpha < 0.15$, some local Mattis-like (helical) order could remain. If, on the other hand $\alpha < 0.05$, we would expect that locally the order would become sinusoidal, and that the Feigelman-Ioffe spoiled helical phase would be found.

We may check whether our simplified approach of treating the multiple-**q** interaction as the sum of Mattis terms agrees with the Feigelman-Ioffe treatment for a simple case. When $\chi(q)$ is isotropic, but sharply peaked within a range κ of q, one has, in real space, an interaction of the form $J[\sin(qr)/qr]e^{-\kappa r}$.[8] While this interaction is reminiscent of the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction,[18] it is more tractable mathematically. For *Cu*Mn and *Ag*Mn the extended tail is also realistic, in that multilayer experiments[19] show longer-range coupling than expected from RKKY interactions.

The isotropic SDW-like Hamiltonian then couples spins within a volume κ^{-3}. Within this volume $\chi(\mathbf{q}_1)$ and $\chi(\mathbf{q}_2)$ produce essentially identical contributions to the Hamiltonian for $|\mathbf{q}_1-\mathbf{q}_2|<\kappa$, but produce nearly uncorrelated contributions for $|\mathbf{q}_1-\mathbf{q}_2|>\kappa$. Then the effective number of independent **q** vectors is about q^2/κ^2, and the effective number of independent Mattis contributions is about twice as large. So long as this number is greater than about c/κ^3, where c is the number density of spins per volume, SG-like behavior will appear within each coherence volume.

Thus, by consideration of the similarity of the single-**q** Hamiltonian to two Mattis components and by considering the known behavior of multicomponent Mattis models, i.e., Hopfield associative memories, we arrive at a crude criterion for spin-glass-like behavior of spins with finite-range isotropic SDW-like coupling, namely,

$$q^2\kappa/c > 1 ,$$

where we have not tried to keep dimensionless prefactors. It is very reassuring that the more formal and thorough treatment of Feigelman and Ioffe, completely independent of the associative memory analogy, gives essentially the same criterion, although with a specified numerical coefficient: $q^2\kappa/4\pi c > 1$.

The obvious question for $Cu_{1-x}Mn_x$ and $Ag_{1-x}Mn_x$ is where, in the spectrum of behavior from isotropic to single **q** and from high c to low c, these materials belong. Neutron scatterings indicates that the magnetic order is not isotropic, but has SDW-like structure peaks at 12 different symmetry-related **q**'s. We have seen that each **q** corresponds to two Mattis components. Thus we are concerned with the case $M \approx 24$, and not with the case described above in which M depends on κ. The relevant criterion to find spin-glass behavior within a single interaction volume becomes roughly

$$24 > 0.05cV_c ,$$

where $V_c \approx 1/\kappa^3$ is the volume range of the SDW-like terms in the Hamiltonian.

The empirical neutron-scattering coherence volume in $Cu_{1-x}Mn_x$ contains about 4000 atoms, in the concentration range studied, $0.05 < x < 0.25$.[3,4] It is unclear from the neutron-scattering results that this coherence volume of the ordered phase represents the range of the Hamiltonian, determined by the width in the peak in $\chi(\mathbf{q})$. However, above T_G the inelastic neutron scattering at the relevant **q**'s shows even shorter coherence lengths.[3]

More important, very recent data on interlayer coupling across various metals show that the apparent interaction range across Cu or Ag is about 4 nm.[19] This is a true measure of the form of the Hamiltonian, not of the properties of the ordered state. Furthermore, it is very weakly dependent on impurities. Thus it is safe to assume $cV_c \approx 4000x$ for $Cu_{1-x}Mn_x$ even at lower x than examined by neutron scattering. The set of SDW **q** vectors, which show very little change (other than changing length along with the Fermi wave vector) for $0.05 < x < 0.25$, continues to be about constant.

Then so long as $24 > 4000x \cdot 0.05$, we would expect that SG behavior, not some approximation of a simple Mattis state, would be found, since the density of Mattis terms is higher than the spin density can support in an associative memory. This condition holds for $x < 0.1$. The agreement of this estimate with the Feigelman-Ioffe result for the same materials is fortuitous, since we have explicitly taken into account the strong anisotropy of $\chi(\mathbf{q})$, which favors the Mattis-like spoiled helical phase, but used much less precise arguments about the appropriate dimensionless prefactors.

At higher x, the same analysis would predict that the Feigelman-Ioffe spoiled helical phase would occur. However, at higher x the role of local spin correlations (which happen to be mostly ferromagnetic) becomes important.[3] Over the range $0.05 < x < 0.25$, the SDW-like coherence length is only about five or six times this short-range-order coherence length.[3] Thus within an SDW coherence volume there are only about 125–216 independent units of locally coherent spins, which is below the minimum (480) required to give a Mattis-like ground state with $M = 24$, but right around the minimum to give some stable Mattis-like states. It is not surprising then to find some sinusoidal short-range order in this concentration regime, but with no evidence of the helical states' playing an important role in the thermodynamics.

Thus we find that the actual Hamiltonian produced by the multi-**q** interaction suggested by the neutron scattering in $Cu_{1-x}Mn_x$ would not be one whose solutions would resemble those of a simple Mattis model (much less an ordinary homogeneous SDW antiferromagnet), but rather would be in the regime which has been traditionally considered to be a SG, at least for low x. As Mydosh suggested,[3] the short-range correlations in $Cu_{1-x}Mn_x$, produced by the RKKY interaction and perhaps enhanced by ASRO, can play an important role. However, that role is not to produce ferromagnetic coupling needed to compete with an otherwise antiferromagnetic coupling (which is not, in fact, present), but to reduce the number of effectively independent spins in a multicomponent Mattis model, helping to prevent the system from settling into a Mattis-like state at high values of x.

The SDW-like coupling in Cu and Ag alloys[3,4,19] may also help explain the apparently hierarchical dynamics found in these materials in mesoscopic fluctuations experiments[7] and nonlinear perturbation experiments.[20] Since such coupling produces a Hamiltonian with more, not less, resemblance to the Sherrington-Kirkpatrick Hamiltonian than would be found for RKKY and other couplings, it is possible that nonhierarchical behavior could be found in other SG's.

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