Surface waves in a semi-infinite antiferromagnetic-paramagnetic superlattice

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(Received 26 December 1991; revised manuscript received 11 May 1992)

The dispersion relations of surface waves in a semi-infinite magnetic superlattice are obtained with use of magnetostatic theory. Numerical calculations are carried out for an antiferromagnetic-paramagnetic superlattice of the diluted magnetic semiconductor $Cd_{1-x}Mn_xTe$, with the applied magnetic field either parallel or perpendicular to the surface.

I. INTRODUCTION

There has been considerable interest in the study of surface magnetic modes in finite systems. Camley and Cottam[1] have investigated both surface and bulk modes in superlattices of magnetic films separated from one another by a nonmagnetic material, with the applied magnetization perpendicular to the surface. Barnás[2] has studied surface and bulk waves in magnetic superlattices formed by N different layers (with N arbitrary). These collective excitations have been observed experimentally by use of Brillouin light scattering.[3]

Our principal aim here is to study the surface modes of spin waves in a semi-infinite superlattice formed by alternating layers of antiferromagnetic and paramagnetic films, with the applied magnetic field either parallel or perpendicular to the surface of the system. We investigate the possible existence of surface modes of spin waves in superlattices of diluted magnetic semiconductor (DMS) films. Typical examples of DMS's are the ternary systems $A^{\mathrm{II}}_{1-x}\mathrm{Mn}_xB^{\mathrm{VI}}$, for example, $Cd_{1-x}Mn_xTe$ $(0<x<0.75)$, where Cd is replaced by magnetic Mn atoms at random sites.[4,5] The concentration x represents the degree of the replacement, which determines the magnetic properties of the DMS. For $x<0.17$, $Cd_{1-x}Mn_xTe$ is paramagnetic for all tempertures; for $x>0.17$, the coupling is antiferromagnetic between neighboring Mn ions below the critical temperature $T_c(x)$.

II. THEORY

We consider a semi-infinite superlattice (SL) in which antiferromagnetic layers with thickness d_1 (for example, $Cd_{1-x_1}Mn_{x_1}Te$ film with $x_1>0.17$) alternates with thickness d_2 paramagnetic layers (for example, $Cd_{1-x_2}Mn_{x_2}Te$ film with $x_2<0.17$). We define the z axis as being along the growth direction of the superlattice, so that the unit cell is $n=0,1,2,\ldots$ $(z\geq 0)$. The SL periodic distance is $L=d_1+d_2$. Without loss of generality, we assume that the surface belongs to antiferromagnetic layers.

In a semi-infinite superlattice, the periodicity is broken and therefore Bloch's theorem does not hold. In this case the scalar magnetic potential ϕ in a magnetostatic equation will have the form[1,4]

$$\phi_1(x,y,z)=\exp(-hnL)\{A_1\exp[iQ_1(z-nL)]+B_1\exp[-iQ_1(z-nL)]\}\times\exp[i(q_xx+q_yy)]\ ,\quad nL<z<nL+d_1\ ; \tag{1a}$$

$$\phi_2(x,y,z)=\exp(-hnl)\{A_2\exp[iQ_2(z-nL-d_1)]+B_2\exp[-iQ_2(z-nL-d_1)]\}\times\exp[i(q_xx+q_yy)]\ ,\quad nL+d_1<z<(n+1)L\ . \tag{1b}$$

Here the wave vectors $Q_{1,2}$ are given by

$$Q_{1,2}^2=(\mu_{xx}^{(1,2)}q_x^2+\mu_{yy}^{(1,2)}q_y^2)/\mu_{zz}^{(1,2)}\ , \tag{2}$$

and the real part of the decay factor h must be positive so that the amplitudes of these surface excitations decay exponentially as n increases.

The boundary conditions require the continuity of ϕ and b_z, while b_z, by use of Maxwell equations, is given by[2]

$$b_z=-\frac{\partial\phi}{\partial z}-\sum_i 4\pi\chi_{z_i}\frac{\partial\phi}{\partial x_i}\ . \tag{3}$$

When we employ these two boundary conditions at $z=nL$ and $z=nL+d_1$ $(n\neq 0)$, we have

$$A_1+B_1-\exp(hL)\exp(iQ_2d_2)A_2-\exp(hL)\exp(-iQ_2d_2)B_2=0\ , \tag{4a}$$

$$(Q_1\mu_{zz}^{(1)}+a)A_1+(-Q_1\mu_{zz}^{(1)}+a)B_1-(Q_2\mu_{zz}^{(2)}+b)\exp(hL)\exp(iQ_2d_2)A_2-(-Q_2\mu_{zz}^{(2)}+b)\exp(hL)\exp(-iQ_2d_2)B_2=0\ , \tag{4b}$$

$$\exp(iQ_1d_1)A_1+\exp(-iQ_1d_1)B_1-A_2-B_2=0\ , \tag{4c}$$

$$(Q_1\mu_{zz}^{(1)}+a)\exp(iQ_1d_1)A_1+(-Q_1\mu_{zz}^{(1)}+a)\exp(-iQ_1d_1)B_1-(Q_2\mu_{zz}^{(2)}+b)A_2-(-Q_2\mu_{zz}^{(2)}+b)B_2=0\ , \tag{4d}$$

where

$$a=4\pi(\chi_{zx}^{(1)}q_x+\chi_{zy}^{(1)}q_y),\quad b=4\pi(\chi_{zx}^{(2)}q_x+\chi_{zy}^{(2)}q_y)\ ,$$
$$\mu_{ii}^{(1,2)}=1+4\pi\chi_{ii}^{(1,2)}\quad (i=x,y,z)\ . \tag{5}$$

As we have assumed, the surface layer belongs to the antiferromagnetic layers ($Cd_{1-x_1}Mn_{x_1}Te$). When we discuss the surface wave, it is convenient to eliminate A_2 and B_2 in Eq. (4) and to obtain a set of two homogeneous equations only with two unknowns A_1 and B_1.

$$[M+N\exp(hL)]A_1+[M'+N'\exp(hL)]B_1=0\ ,$$
$$[Q+R\exp(-hL)]A_1+[Q'+R'\exp(-hL)]B_1=0\ , \tag{6}$$

where

$$M=Q_1\mu_{zz}^{(1)}+a$$
$$-[(Q_2\mu_{zz}^{(2)}+b)\exp(iQ_2d_2)$$
$$-(-Q_2\mu_{zz}^{(2)}+b)\exp(-iQ_2d_2)]/[2i\sin(Q_2d_2)]\ ,$$
$$N=2Q_2\mu_{zz}^{(2)}\exp(iQ_1d_1)/[2i\sin(Q_2d_2)]\ ,$$
$$M'=M-2Q_1\mu_{zz}^{(1)},\quad N'=N\exp(-2iQ_1d_1)\ ,$$
$$Q=(Q_1\mu_{zz}^{(1)}+a)\exp(iQ_1d_1)+(Q_2\mu_{zz}^{(2)}+b)$$
$$\times\exp[i(Q_1d_1+Q_2d_2)]$$
$$-(-Q_2\mu_{zz}^{(2)}+b)\exp[i(Q_1d_1+Q_2d_2)]/[2i\sin(Q_2d_2)],$$
$$R=Q_2\mu_{zz}^{(2)}i/\sin(Q_2d_2)\ ,$$
$$Q'=[Q-2\exp(iQ_1d_1)Q_1\mu_{zz}^{(1)}]\exp(-2iQ_1d_1)\ ,$$
$$R'=R\ . \tag{7}$$

Now we consider the case of the surface ($z=0$). Employing the wave-vector equation (2) outside the semi-infinite superlattice, we have ϕ of the form

$$\phi(x,y,z)=C\exp(q_\perp z)\exp[i(q_xx+q_yy)]\quad (z<0)\ . \tag{8}$$

Here $q_\perp^2=q_x^2+q_y^2$ with $q_\perp>0$.

Also, by use of the continuity of ϕ and b_z at $z=0$, we get

$$(q_\perp-iQ_1\mu_{zz}^{(1)}-ia)A_1+(q_\perp+iQ_1\mu_{zz}^{(1)}-ia)B_1=0\ . \tag{9}$$

Reference 1 has pointed out that, under magnetostatic theory, there are no surface modes in the perfect semi-infinite ferromagnetic or antiferromagnetic-nonmagnetic superlattice structure in which the amplitudes of surface waves increase exponentially from the surface to the interior. But in the DMS superlattice under study, we cannot obtain this conclusion directly from the above Eqs. (6), (7), and (9), and a later numerical calculation shows that the conclusion is not true in our case.

Equations (6) and (9) are our main results. Combining these two equations, we can obtain two relations without the unknown parameters A_1 and B_1:

$$\exp(-hL)=-(N-N'Y)/(M-M'Y)\ ,$$
$$\exp(-hL)=-(Q-Q'Y)/(R-R'Y) \tag{10}$$

with $\mathrm{Re}(h)>0$ and

$$Y=[q_\perp-i(Q_i\mu_{zz}^{(1)}+a)]/[q_\perp+i(Q_1\mu_{zz}^{(1)}-a)]\ .$$

Obviously, the dispersion relation of surface modes can be given by

$$(N-N'Y)/(M-M'Y)=(Q-Q'Y)/(R-R'Y)\ , \tag{11}$$

with $\mathrm{Re}(h)>0$.

Thus we can investigate the surface waves of ferromagnetic, antiferromagnetic, or paramagnetic superlattices. In the next section, we will discuss an antiferromagnetic-paramagnetic DMS superlattice.

III. RESULTS

The $Cd_{0.30}Mn_{0.70}Te$-$Cd_{0.89}Mn_{0.11}Te$ superlattice is an example of an antiferromagnetic-paramagnetic superlattice at liquid-helium temperatures.[4] The experimental parameters are known as follows: the sublattice saturation magnetization $M_0=0.2$ kG ($M_0^{(2)}=0.07$ kG for the paramagnetic phase), the exchange field $H_E=200$ kG, the anisotropy field $H_A=30$ kG, and the applied magnetic field $H_0=60$ kG. In the following we will present numerical applications to this superlattice in the case of the applied fields either parallel or perpendicular to the surface,

(i) The applied magnetic field $\mathbf{H}_0$ is parallel to the surface (perpendicular to the z axis). We take $\mathbf{H}_0$ to be along x axis so that we have

$$\mu_{yy}^{(1,2)}=\mu_{zz}^{(1,2)}=\mu_{1,2},\quad \mu_{xx}^{(1,2)}=1,\quad \chi_{zx}^{(1,2)}=0,\quad \chi_{zy}^{(1,2)}=\chi_{1,2}\ ,$$

and

$$Q_{1,2}^2=-q_y^2-\mu_{1,2}^{-1}q_x^2\ , \tag{12}$$

where

$$\mu_1=(\Omega^2-y_+^2)(\Omega^2-y_-^2)/(\Omega^2-x_+^2)(\Omega^2-x_-^2)\ ,$$
$$\mu_2=(\Omega^2-x_2^2)/(\Omega^2-x_1^2)\ ,$$
$$4\pi\chi_1=i\Omega x_A^2(x_+-x_-)/(\Omega^2-x_+^2)(\Omega^2-x_-^2)\ ,$$
$$4\pi\chi_2=i\Omega(x_2^2-x_1^2)/(x_1(\Omega^2-x_1^2)\ ,$$

with

$$x_\pm=\pm|\gamma|H_0+|\nu|(H_A^2+2H_AH_E)^{1/2}\ ,$$
$$x_A=|\gamma|(8\pi M_0H_A')\ ,$$
$$y_\pm=\tfrac{1}{2}(x_A^2+x_+^2+x_-^2)$$
$$\pm\tfrac{1}{2}[(x_A^4+2x_A^2(x_+-x_-)^2+(x_+-x_-)^2]^{1/2}\ , \tag{13}$$
$$x_1=|\gamma|(H_0-4\pi M_0^{(2)})\ ,$$
$$x_2=|\gamma|[H_0(H_0-4\pi M_0^{(2)})]^{1/2}\ .$$

It is convenient and reasonable to discuss the case in which q_x or q_y vanishes.[4] Here we restrict ourselves only to the case $q_y=0$, which gives

$$Q_{1,2}^2=-\mu_{1,2}^{-1}q_x^2\ . \tag{14}$$

Figure 1 shows the frequency of surface waves in which μ_1 and μ_2 are positive ($Q_{1,2}=-i\mu_{1,2}^{-1/2}q_x$) as a function of q_x for several selected d_1 (with fixed $d_2=5.0$)

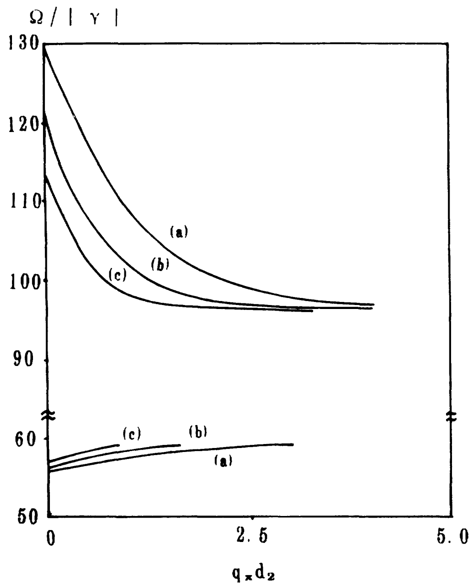

FIG. 1. Dispersion relations for surface waves in the perfect ($\Delta=0$) $Cd_{0.30}Mn_{0.70}Te$-$Cd_{0.80}Mn_{0.11}Te$ (AF-P) superlattice in the case of the applied magnetic field $\mathbf{H}_0$ is parallel to the surface (along the x axis). The different curves correspond to (with fixed $d_2=5.0$) (a) $d_1=3.0$, (b) $d_1=5.0$, and (c) $d_1=8.0$.

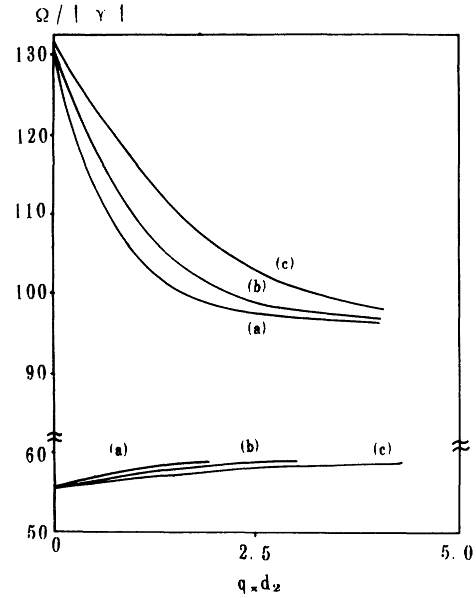

FIG. 2. Dispersion relations for surface waves in the altered surface $Cd_{0.30}Mn_{0.70}Te$-$Cd_{0.89}Mn_{0.11}Te$ (AF-P) superlattice in the case of the applied magnetic field $\mathbf{H}_0$ is parallel to the surface (along the x axis). The different curves correspond to (with fixed $d_1=3.0$, $d_2=5.0$) (a) $\Delta=1.0$, (b) $\Delta=0.0$, and (c) $\Delta=-1.0$.

in the perfect DMS-DMS superlattice. In our antiferromagnetic-paramagnetic (AF-P) superlattice, it is true that there exist two frequency regions ($53.578<\Omega/|\gamma|<59.12$, $59.56<\Omega/|\gamma|<173.578$) of surface waves.[3,4] They have a different tendency when the wave vector q_x increases: The upper-band frequencies decrease sharply and the lower-band frequencies increase smoothly as q_x is increased. And, similarly, we find that, as d_1 is increased, the lower-band frequencies also increase, while the upper-band frequencies decrease. If we note the allowed region of q_x, we find the region decreases as d_1 is increased.

Figure 1 shows that there also exist true surface modes in perfect DMS superlattices. This result is con-

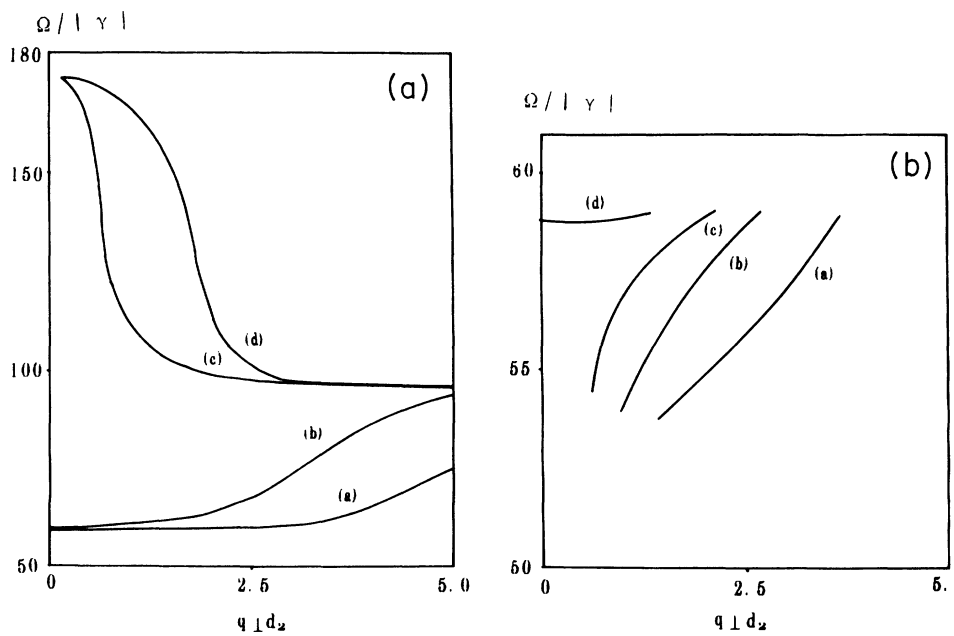

FIG. 3. Same as Fig. 1 except in the case where the applied magnetic field $\mathbf{H}_0$ is perpendicular to the surface (with fixed $d_2=5.0$, $\Delta=0.0$): (a) $d_1=3.0$, (b) $d_1=4.0$, (c) $d_1=5.0$, and (d) $d_1=8.0$.

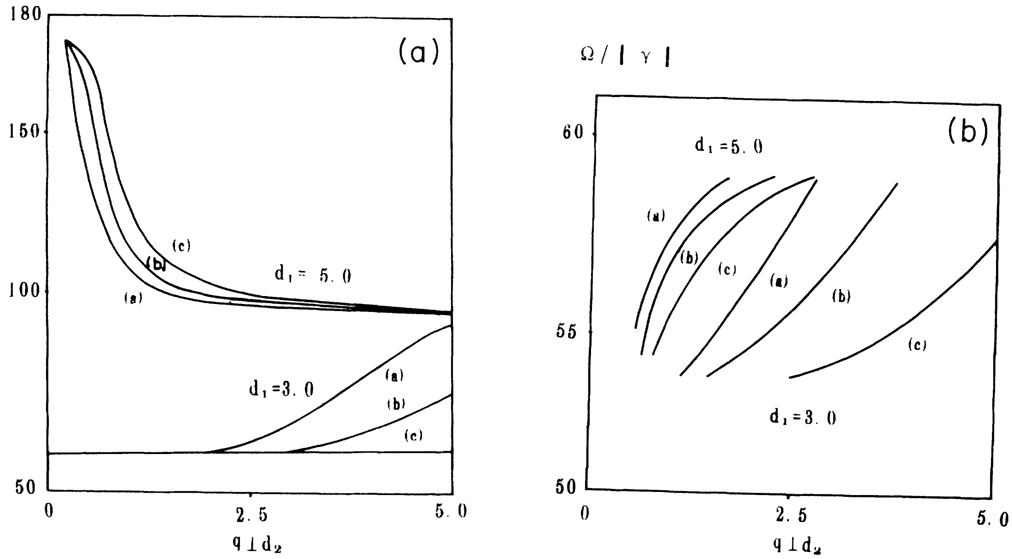

FIG. 4. Same as Fig. 2 except in the case where the applied magnetic field $\mathbf{H}_0$ is perpendicular to the surface (with fixed $d_1 = 3.0$ and 5.0, $d_2 = 5.0$): (a) $\Delta = 1.0$, (b) $\Delta = 0.0$, and (c) $\Delta = -1.0$.

trary to the conclusion of Ref. 1. The reason is that we here discuss the antiferromagnetic-paramagnetic DMS/superlattice, in which we have $Q_2 \neq q_\perp$ and different boundary condition, instead of ferromagnetic or antiferromagnetic-nonmagnetic superlattices used in Ref. 1. But here we also adopt the idea in Ref. 1, changing the thickness of surface layer d_1 to $d_1 + \Delta$, to investigate the influence of the altered surface on surface waves. This case, only this time with Y replaced by $Y' = Y\exp(-2iQ_1\Delta)$ in Eq. (10), has been plotted in Fig. 2. We find that the frequencies increase as Δ is decreased and the allowed region of q_x changes little in the upper band, while in the lower band, the frequencies decrease and the allowed region of q_x increases as Δ is decreased.

(ii) The applied magnetic field $\mathbf{H}_0$ is perpendicular to the surface (along the z axis). In this case we have

$$\mu_{zz}^{(1,2)} = 1, \quad \mu_{xx}^{(1,2)} = \mu_{yy}^{(1,2)} = \mu_{1,2} , \\ \chi_{zx}^{(1,2)} = \chi_{zy}^{(1,2)} = 0 , \quad Q_{1,2}^2 = -\mu_{1,2} q_\perp^2 , \tag{15}$$

where μ_1 and μ_2 are given in Eq. (13).

The case where both μ_1 and μ_2 are positive ($Q_{1,2} = -i\mu_{1,2}^{1/2} q_\perp$) is shown in Fig. 3 in the perfect DMS superlattice. There also exist two frequency regions ($53.578 < \Omega/|\gamma| < 59.12$, $59.56 < \Omega/|\gamma| < 173.578$), which has been plotted in Figs. 3(a) and 3(b), respectively. We find that, in the upper band, the surface waves are very sensitive to d_1/d_2 (with fixed $d_2 = 5.0$): The frequencies increase in $d_1/d_2 < 1.0$ and decrease in $d_1/d_2 \geq 1.0$ as $q_\perp$ is increased [see Fig. 3(a)]. This phenomenon persists in DMS superlattices with the surface altered (see Fig. 4). And the upper-band frequencies increase in $d_1/d_2 < 1.0$, while in $d_1/d_2 \geq 1.0$ the frequencies decrease, as Δ is increased.

In summary, we have investigated the surface modes in a semi-infinite diluted magnetic semiconductor superlattice with the applied magnetic field either parallel or perpendicular to the surface. We find that, even in perfect superlattices, there exist surface waves in our antiferromagnetic-paramagnetic DMS superlattice, in which the conclusion of Ref. 1 is not suitable. The high-frequency surface modes (or optical branches) are also found.[6,7] We also find interesting features of surface waves in an applied magnetic field parallel or perpendicular to the surface. Comparing these two cases, we obtain, very reasonable one, that the frequencies decrease more abruptly in the perpendicular case. We hope our results can be confirmed experimentally.

ACKNOWLEDGMENT

This work was partly supported by the National Natural Science Foundation of China.

[1]R. E. Camley and M. G. Cottam, Phys. Rev. B **35**, 189 (1987).
[2]J. Barnás, J. Phys. C **21**, 1021 (1988); **21**, 4097 (1988); J. Phys. Condens. Matter **2**, 7173 (1990).
[3]I. K. Schuller and M. Grimsditch, J. Appl. Phys. **55**, 2491 (1985).
[4]M. Villeret, S. Rodriquez, and E. Kartheuser, Phys. Rev. B **39**, 2583 (1989); Appl. Phys. Lett. **53**, 447 (1988).
[5]J. Furdyna and N. Samarth, J. Appl. Phys. **61**, 3526 (1988).
[6]A. Babcenco and M. G. Cottam, J. Phys. C **14**, 5347 (1981).
[7]A. Mauger and D. L. Mills, Phys. Rev. B **28**, 6553 (1983).