Scattering of electrons from fractons and magnons in dilute two-dimensional antiferromagnets: Temperature-dependent resistivity in high-T_c cuprates

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Based on the *s-d* exchange model, we investigate the temperature-dependent resistivity arising from the scattering of electrons off fractons and magnons in dilute two-dimensional Heisenberg antiferromagnets. The result shows that the existence of fractons will lead to a linear temperature dependence of the resistivity over a wide temperature range and its slope $d\rho/dT$ is nearly a constant, which is consistent with experimental results in high-T_c oxides, while the magnon scattering will contribute a resistivity varying as $T^{3/5}$ almost over the whole temperature region.

I. INTRODUCTION

The nearly linear temperature dependence of the normal-state resistivity in a wide temperature range is one of the extraordinary properties of copper oxide superconductors.[1] Quite a few types of transport mechanisms have been proposed to account for this unusual behavior,[2-8] but none of them to date has been widely accepted; thus, the understanding of normal state transport properties in the cuprates remains of great interest.

The common feature of all high-temperature oxide superconductors found so far is the existence of planes formed by the copper and oxygen atoms (CuO_2). The linear $\rho(T)$ found in cuprate superconductors has led to the general belief that this is a universal feature of the CuO_2 planes. As it is well known, the localized spins presented on the Cu^{2+} ions in Cu-O planes exhibit dynamic Heisenberg antiferromagnetic (AF) intralayer order throughout the insulating as well as the metallic composition regimes. For undoped parent compounds, which are known to be AF insulators, it is found that the spin-wave theory gives a good description of the magnetic dynamics.[9,10] Hole doping strongly reduces the effective intralayer Cu-Cu superexchange coupling constant J_{ij}, and destroys quickly the long-range AF order.[11,12] Nevertheless, neutron scattering[13,14] and NMR data[15] suggest that there are still short-range AF fluctuations in the metallic regime. In so far as the magnetic properties are concerned, we may assume that the effect of adding holes to the Cu-O planes be accounted for by randomly breaking some of the superexchange couplings between two adjacent copper *d* electrons. This picture can be modeled by using of the bond percolation networks in a dilute two-dimensional antiferromagnet. Percolation networks appear to be homogeneous at length scales L longer than the percolation correlation length ξ_p, and exhibit fractal characteristic at shorter length scales $L < \xi_p$.[16,17] The excitations on a fractal lattice are termed fractons.[16] Thus, there are two types of magnetic excitations in this dilute two-dimensional antiferromagnet, namely magnons and fractons (magnonlike). We have calculated the interaction between the conduction electrons and fractons (vibration excitations on a fractal network), and the result accounts well for the resistivity minimum phenomena observed in weak-scattering metallic glasses at low temperatures. It is demonstrated[18-20] that fractons play an essential role in many stages of physics for topologically disordered systems.

Much effort has been devoted[8] to investigating the effect of the interaction between electrons and phonons on the temperature-dependent resistivity in normal state of high-T_c cuprates. The experimental behaviors could be qualitatively explained based on this scattering mechanism. However, it is still not clear whether the scattering between the charged carriers and the magnetic excitations, which is believed to exist in the high-T_c copper oxides,[21,22] may yield the desired temperature dependence of the normal-state resistivity. In this paper, we carry out a calculation of the resistivity in a dilute two-dimensional antiferromagnet. Our approach to the problem is based on the assumption that the carriers (mobile electrons) scatter from magnetic excitations (fractons and magnons) which are external to the current-carrier system, and their interaction be in essence *s-d* exchange coupling.[23]

The paper is organized in the following way. In Sec. II, we will calculate the resistivities arising from the scatterings of extended electrons off fracton and magnon excitations in a dilute two-dimensional antiferromagnet. We do this by deriving an expression for the self-energy of electrons. The random breaking of superexchange coupling is treated in the effective-medium approximation. We will present some numerical solutions to the problems in Sec. III. This includes the temperature dependences of resistivity and its slope $d\rho/dT$, for both fracton and magnon scatterings. The final section, IV, will give a summary of this paper.

II. THEORETICAL CALCULATION

We proceed with the Hamiltonian of the s-d exchange model,

$$H=\sum_{k\mu}\varepsilon_k C^{\dagger}_{k\mu}C_{k\mu}+H_d - \frac{I}{N}\sum_{l\alpha}\sum_{kk'}\sum_{\mu\mu'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_{l\alpha}}\mathbf{S}_{l\alpha}\cdot\boldsymbol{\sigma}_{\mu\mu'}C^{\dagger}_{k\mu}C_{k'\mu'} , \quad (1)$$

where $C^{\dagger}_{k\mu}$ and $C_{k\mu}$ are the creation and annihilation operators for electrons, $S_{i\alpha}$ is the localized spin operator of a copper d electron on site $R_{i\alpha}$ in the α sublattice, the indices α take the value of 1 for the spin-up sublattice and 2 for the spin-down sublattice, σ is the Pauli matrice, I is the s-d exchange parameter, and N is the number of Cu sites.

The Heisenberg Hamiltonian H_d is given by

$$H_d=\sum_{\langle ij\rangle} J_{ij}\mathbf{S}_{i\alpha}\cdot\mathbf{S}_{j\bar{\alpha}} . \quad (2)$$

Note that if $\alpha=1$, then $\bar{\alpha}=2$, and vice versa.

It is believed[24] that doping creates holes on the O^- sites in the CuO_2 planes, since the superexchange interaction between the two Cu ions is mediated via the O ions; the introducing of hole on the O^- sites will have a drastic effect on destroying the AF bonds. We may assume that the effective exchange-coupling constant J_{ij} obeys the probability density

$$P(J_{ij})=p\delta(J_{ij}-J)+(1-p)\delta(J_{ij}) \quad (3)$$

and the concentration of broken bonds is $1-p$. J is the exchange-coupling constant in the undoping regime.

Defining the one-particle retarded Green's function for electrons

$$G(\mathbf{k},E)=\langle\langle C_{k\uparrow}|C^{\dagger}_{k\uparrow}\rangle\rangle_E=[E-\varepsilon_k-\Sigma(\mathbf{k},E)]^{-1} . \quad (4)$$

For the second order in I, we obtain the self-energy $\Sigma(\mathbf{k},E)$ for electrons

$$\Sigma(\mathbf{k},E)=-IS+\frac{I^2}{N^2}\sum_{ll'}\sum_{k'k''}\sum_{\mu\mu'}\sum_{\alpha\alpha'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_{l\alpha}}e^{i(\mathbf{k}-\mathbf{k}'')\cdot\mathbf{R}_{l'\alpha'}}\langle\langle \mathbf{S}_{l\alpha}\cdot\boldsymbol{\sigma}_{\uparrow\mu'}C_{k'\mu'}|\mathbf{S}_{l'\alpha'}\cdot\boldsymbol{\sigma}_{\mu\uparrow}C^{\dagger}_{k''\mu}\rangle\rangle_E . \quad (5)$$

For the Heisenberg antiferromagnetic Hamiltonian H_d, we introduce the Dyson-Maleev transformation

$$S^-_{i\alpha}=a_i^{+} , \quad S^{\dagger}_{i\alpha}=(2S-a_i^{+}a_i)a_i , \quad S^Z_{i\alpha}=S-a_i^{+}a_i , \quad (6a)$$

when $\alpha=1$ (spin-up sublattice), and

$$S^-_{j\alpha}=-b_j , \quad S^+_{j\alpha}=-b_j^{+}(2S-b_j^{+}b_j) , \quad S^Z_{j\alpha}=b_j^{+}b_j-S , \quad (6b)$$

when $\alpha=2$ (spin-down sublattice). a_i, b_j are the Bose operators.

Then the Hamiltonian (2) is transformed into[25]

$$H_d=-\tfrac{1}{2}JZS^2Np+\sum_{\langle ij\rangle}J_{ij}[S(a_i^{\dagger}a_i+b_j^{+}b_j-a_i^{+}b_j^{+}-a_ib_j) +\tfrac{1}{2}a_i^{+}(b_j^{+}-a_i)^2b_j] \quad (7)$$

where Z is the number of the nearest neighbors.

The expression for the self-energy equation (5) contains two Green's functions,

$$\langle\langle S^-_{l\alpha}C_{k'\downarrow}|\mathbf{S}_{l'\alpha'}\cdot\sigma_{\mu\uparrow}C^{\dagger}_{k''\mu}\rangle\rangle_E\equiv G^{\alpha\alpha'}_{ll'}(E)$$

and

$$\langle\langle S^Z_{l\alpha}C_{k'\uparrow}|\mathbf{S}_{l'\alpha'}\cdot\sigma_{\mu\uparrow}C^{\dagger}_{k''\mu}\rangle\rangle_E\equiv P^{\alpha\alpha'}_{ll'}(E) .$$

their equations of motion can be calculated as follows:

$$(E-\varepsilon_{k'})G^{\alpha\alpha'}_{ll'}(E)=\langle S^-_{l\alpha}S^+_{l'\alpha'}\delta_{k'k''}+2S^Z_{l\alpha}C^{\dagger}_{k''\downarrow}C_{k'\downarrow}\delta_{ll'}\delta_{\alpha\alpha'}+C_{k'\downarrow}C^{\dagger}_{k''\uparrow}S^-_{l\alpha}\delta_{ll'}\delta_{\alpha\alpha'}\rangle + \frac{(-1)^{\bar{\alpha}}}{2}\sum_j J_{lj}[f(\mathbf{r}_{jj})-g(\mathbf{r}_{lj})-2S][G^{\alpha\alpha'}_{ll'}(E)+G^{\bar{\alpha}\alpha'}_{jl'}(E)] \quad (8)$$

and

$$(E-\varepsilon_{k'})P^{\alpha\alpha'}_{ll'}(E)=\langle S^Z_{l\alpha}S^Z_{l'\alpha'}\delta_{k'k''}-C^{\dagger}_{k''\downarrow}C_{k'\uparrow}S^+_{l\alpha}\delta_{ll'}\delta_{\alpha\alpha'}\rangle+\frac{(-1)^{\alpha}}{2}\sum_j J_{lj}g(\mathbf{r}_{lj})[P^{\alpha\alpha'}_{ll'}(E)+P^{\bar{\alpha}\alpha'}_{jl'}(E)] . \quad (9)$$

In order to obtain Eqs. (8) and (9), we have made approximations

$$b_j^{+}b_j^{+}b_j\approx\langle b_j^{+}b_j\rangle b_j^{+} ,$$

$$2b_j^{+}a_lb_j\approx\langle b_j^{+}b_j\rangle a_l+\langle a_lb_j\rangle b_j^{+} .$$

Note that we have[25]

$$\langle a_l^{+}b_j\rangle=\langle a_lb_j^{+}\rangle=\langle a_la_{l'}\rangle=\langle b_j^{+}b_{j'}^{+}\rangle=\langle b_jb_{j'}\rangle=0 , \quad (10)$$

and in Eqs. (8) and (9), we have set

$$f(\mathbf{r}_{lj})=\langle a_l^{+}a_j\rangle=\langle b_l^{+}b_j\rangle , \qquad g(\mathbf{r}_{lj})=\langle a_lb_j\rangle=\langle a_l^{+}b_j^{+}\rangle . \quad (11)$$

As the Boson operators a_i, b_j are defined for two sublattices, respectively, we should form two sets of Boson operators α_q, β_q for spin-wave excitations by combining them with Bogoliubov transformation,

$$a_l = \sqrt{2/N} \sum_q (\cosh\theta_q \alpha_q + \sinh\theta_q \beta_{-q}^{+}) e^{i\mathbf{q}\cdot\mathbf{r}_l} ,$$
$$b_j^{+} = \sqrt{2/N} \sum_q (\sinh\theta_q \alpha_q + \cosh\theta_q \beta_{-q}^{+}) e^{i\mathbf{q}\cdot\mathbf{r}_j} . \tag{12}$$

Using this, we then have

$$f(\mathbf{r}_{lj}) = \frac{2}{N} \sum_q \cosh(2\theta_q) \exp(-i\mathbf{q}\cdot\mathbf{r}_{lj})(n_q + \tfrac{1}{2}) - \tfrac{1}{2}\delta_{lj} ,$$
$$g(\mathbf{r}_{lj}) = \frac{2}{N} \sum_q \sinh(2\theta_q) \exp(-i\mathbf{q}\cdot\mathbf{r}_{lj})(n_q + \tfrac{1}{2}) , \tag{13}$$

and

$$\cosh(2\theta_q) = \Delta q , \quad \sinh(2\theta_q) = \eta\gamma_q\Delta_q ,$$
$$\Delta_q = \frac{1}{(1-\eta^2\gamma_q^2)^{1/2}} , \quad \gamma_q = \frac{1}{Z} \sum_\delta e^{i\mathbf{q}\cdot\delta} . \tag{14}$$

The δ's here are vectors from the site to its nearest neighbors and n_q is the Bose-Einstein distribution function of the magnetic excitations (fracton and magnon) with frequency ω_q. From the numerical calculation of Ref. 25, we have exactly enough $\eta \approx 1$.

Equations (8) and (9) are in the site representation, and their Fourier transforms are not simple since the presence of the holes has introduced the random exchange-coupling constant J_{ij} for the adjacent Cu spins. We treat them in the effective-medium approximation[26] in which J_{ij} is replaced by a uniform coupling constant $\bar{J}(E)$. $\bar{J}(E)$ is chosen in such a way that the scattering produced by one bond, for which the original coupling constant J is maintained, is zero on the average.

According to Ref. 26, the $\bar{J}(E)$ is determined by

$$\left\langle \frac{\bar{J}(E) - J_{ij}}{J_{ij}[1-\epsilon G_0(\epsilon)] + [Z/2 - 1 + \epsilon G_0(\epsilon)]\bar{J}(E)} \right\rangle = 0 . \tag{15}$$

For a square lattice $G_0(\epsilon)$ is given by

$$G_0(\epsilon) = \int_0^\infty \exp[(-2+\epsilon)x][I_0(x)]^2 dx , \tag{16}$$

where $\epsilon = E/\bar{J}(E)$ and $I_0(x)$ is the modified Bessel function of order 0.

For Y-Ba-Cu-O and La-Sr-Cu-O systems, we expect $\bar{J}(E)/E_F < 1$,[27,28] then from Eqs. (3), (15), and (16), we obtain approximately

$$\bar{J}(E) = J[p + (J/E)(p-0.5)^2] \tag{17}$$

for the bond percolation network on a two-dimensional lattice.

Taking the Fourier transformation of Eq. (8), we have

$$(E - \varepsilon_{k'}) G^{\alpha\alpha'}(\mathbf{q}, E) = A^{\alpha\alpha'}(\mathbf{q}) + \frac{(-1)^\alpha}{2} Z\bar{J} \times [F_1 G^{\alpha\alpha'}(\mathbf{q}, E) + G_1 G^{\bar{\alpha}\alpha'}(\mathbf{q}, E)] , \tag{18}$$

where

$$F_1 = \frac{2}{N} \sum_{q_1} [(1-\gamma_{q_1}^2)\Delta_{q_1}(n_{q_1} + \tfrac{1}{2})] - \frac{4S+1}{2} ,$$
$$G_1 = \frac{2}{N} \sum_{q_1} [(\gamma_{q_1} - \gamma_{q+q_1}\gamma_{q_1})\Delta_{q_1}(n_{q_1} + \tfrac{1}{2})] - \frac{4S+1}{2}\gamma_q , \tag{19}$$

and

$$A^{11}(\mathbf{q}) = 2S\{2\Delta_q(n_q + \tfrac{1}{2}) - \tfrac{1}{2} + f_{k'}[1 - f(0)/S]\}\delta_{k'k''} ,$$
$$A^{22}(\mathbf{q}) = 2S\{2\Delta_q(n_q + \tfrac{1}{2}) + \tfrac{1}{2} + f_{k'}[f(0)/S - 1]\}\delta_{k'k''} , \tag{20}$$
$$A^{12}(\mathbf{q}) = A^{21}(\mathbf{q}) = -4S\gamma_q\Delta_q(n_q + \tfrac{1}{2})\delta_{k'k''} .$$

Here $f_{k'}$ is the Fermi-Dirac distribution function of electrons with wave vectors k'.

The solution to Eq. (18) can be found easily,

$$\begin{bmatrix} G^{11}(\mathbf{q},E) & G^{12}(\mathbf{q},E) \\ G^{21}(\mathbf{q},E) & G^{22}(\mathbf{q},E) \end{bmatrix} = \frac{1}{D_1(\mathbf{q},E)} \begin{bmatrix} (E - \varepsilon_{k'} + \frac{1}{2}Z\bar{J}F_1)A^{11} + \frac{1}{2}Z\bar{J}G_1A^{21} & (E - \varepsilon_{k'} + \frac{1}{2}Z\bar{J}F_1)A^{12} + \frac{1}{2}Z\bar{J}G_1A^{22} \\ (E - \varepsilon_{k'} - \frac{1}{2}Z\bar{J}F_1)A^{21} - \frac{1}{2}Z\bar{J}G_1A^{11} & (E - \varepsilon_{k'} - \frac{1}{2}Z\bar{J}F_1)A^{22} - \frac{1}{2}Z\bar{J}G_1A^{12} \end{bmatrix} \tag{21}$$

with

$$D_1(\mathbf{q},E) = (E - \varepsilon_{k'})^2 - \tfrac{1}{4}Z^2\bar{J}^2(F_1^2 - G_1^2) . \tag{22}$$

In the same way, we obtain

$$\begin{bmatrix} P^{11}(\mathbf{q},E) & P^{12}(\mathbf{q},E) \\ P^{21}(\mathbf{q},E) & P^{22}(\mathbf{q},E) \end{bmatrix} = \frac{B(\mathbf{q})}{D_2(\mathbf{q},E)} \begin{bmatrix} (E - \varepsilon_{k'}) - \frac{1}{2}Z\bar{J}(F_2 + G_2) & -(E - \varepsilon_{k'}) + \frac{1}{2}Z\bar{J}(F_2 + G_2) \\ -(E - \varepsilon_{k'}) - \frac{1}{2}Z\bar{J}(F_2 + G_2) & (E - \varepsilon_{k'}) + \frac{1}{2}Z\bar{J}(F_2 + G_2) \end{bmatrix} , \tag{23}$$

with

$$F_2=\frac{2}{N}\sum_{q_1}\gamma_{q_1}^2\Delta_{q_1}(n_{q_1}+\tfrac{1}{2}),\quad B(\mathbf{q})=NS\delta(\mathbf{q})[S-2f(0)]\delta_{k'k''},$$
$$G_2=\frac{2}{N}\sum_{q_1}\gamma_{q_1}\gamma_{q+q_1}\Delta_{q_1}(n_{q_1}+\tfrac{1}{2}),\quad D_2(\mathbf{q},E)=(E-\varepsilon_{k'})^2-\tfrac{1}{4}Z^2\bar{J}^2(F_2^2-G_2^2). \tag{24}$$

Substituting Eqs. (21) and (23) into Eq. (5), we obtain the following result for the self-energy of electrons:

$$\Sigma(\mathbf{k},E)=-IS+\frac{2SI^2}{N}\sum_q\frac{4(E-\varepsilon_{k-q})(1-\gamma_q)\Delta_q(n_q+\frac{1}{2})+\frac{1}{2}Z\bar{J}(F_1^2-G_1^2)[2f_{k-q}(1-f(0)/S)-1]}{(E-\varepsilon_{k-q})^2-\frac{1}{4}Z^2\bar{J}^2(F_1^2-G_1^2)}. \tag{25}$$

Note that the contribution to the self-energy of electrons from Green's function $P_{ll'}^{\alpha\alpha'}(E)$ is zero because the effects of its four components cancel out, as one can see from Eq. (23).

The relaxation rate τ_k^{-1} of electrons with momentum k is given by

$$\tau_k^{-1}=-2\,\mathrm{Im}\Sigma(\mathbf{k},E+i\delta). \tag{26}$$

Thus we obtain the relaxation rate

$$\frac{1}{\tau_k}=\frac{4\pi SI^2}{N}\sum_q(\{2(1-\gamma_q)\Delta_q(n_q+\tfrac{1}{2})-[f_{k-q}(1-f(0)/S)-\tfrac{1}{2}]\sqrt{(F_1-G_1)/(F_1+G_1)}\}\delta(E-\varepsilon_{k-q}-\tfrac{1}{2}Z\bar{J}\sqrt{F_1^2-G_1^2})$$
$$+\{2(1-\gamma_q)\Delta_q(n_q+\tfrac{1}{2})+[f_{k-q}(1-f(0)/S)-\tfrac{1}{2}]\sqrt{(F_1-G_1)/(F_1+G_1)}\}$$
$$\times\delta(E-\varepsilon_{k-q}+\tfrac{1}{2}Z\bar{J}\sqrt{F_1^2-G_1^2})). \tag{27}$$

The resistivity is given by the Drude formula

$$\rho=m^*/ne^2\tau_F, \tag{28}$$

where m^* is the effective electron mass and n the concentration of the conduction electrons.

III. NUMERICAL STUDY

Up to this point, we have not used the specific magnetic scattering mechanism. As noted above, we assume that there are two kinds of magnetic excitations (fractons and magnons) in CuO_2 planes. It has been predicted[16] and confirmed by experiments[20] that in a percolation network, propagating phonons or magnons at long wavelengths and low energies should cross over (at the crossover frequency ω_c) to localized fracton excitations at higher energies and short length scales where the network has a fractal geometry. The magnon density of states is given by

$$N(\omega)\sim\omega^{d-1}\quad(\omega<\omega_c), \tag{29}$$

where d is the Euclidean dimensionality, $\omega_c\sim V_s/\xi_p$, and V_s is the velocity of the spin wave.

For frequencies above ω_c, the density of states of fractons is given by

$$N_F(\omega)\sim\omega^{d_f-1}\quad(\omega>\omega_c). \tag{30}$$

Here, d_f is the fracton dimension. For a two-dimensional percolation network, d_f is expected to equal 1.33.[16]

The excitation energy of a magnon has been calculated as[25]

$$\hbar\omega=2\sqrt{2}\bar{J}(E)m_1qa_0\quad(\omega<\omega_c), \tag{31}$$

with $m_1=S+0.078974$ and a_0 the distance between two adjacent copper atoms. By contrast, following a scaling analysis of Rammal and Toulouse,[29] we know that the excitation frequency ω of the fracton varies with wave vector q as

$$\omega=\omega_c(q\xi_p)^{D/d_f}=\omega_F(qa_0)^{D/d_f}\quad(\omega>\omega_c). \tag{32}$$

D is the fractal dimensionality and equals 1.9 for $d=2$,[16] and ω_F is the fracton cutoff frequency defined in terms of the crossover frequency ω_c by $\omega_F=\omega_c(\xi_p/a_0)^{D/d_f}$.

As a result, the sum over q in Eqs. (13), (19), (25), and (27) can be transformed into an integral according to

$$\frac{1}{N}\sum_q=\frac{a_0^2}{2\pi^2}\int_0^{q_{\max}}q\,dq\int_0^{\pi}d\theta \tag{33}$$

for the magnon excitations, and

$$\frac{1}{N}\sum_q=\frac{2K_{D-1}a_0^D}{(2\pi)^D}\int_{1/\xi_p}^{1/a}q^{D-1}dq\int_0^{\pi}(\sin\theta)^{D-2}d\theta \tag{34}$$

for the fracton excitations, respectively, where $K_D=\pi^{D/2}/\Gamma(D/2)$. The percolation correlation length ξ_p depends upon the concentration p for bond occupancy, it is expected[30] that

$$\xi_p=a_0|p-p_c|^{-4/3}. \tag{35}$$

For a quantum percolation problem, p_c has been found to be 0.76 for the bond percolation on a square lattice.[31]

We are now able to calculate the resistivity as a function of temperature using Eqs. (27) and (28). For simplicity, we limit our evaluation to Y-Ba-Cu-O systems; the results for La-Sr-Cu-O systems are qualitatively similar to them. The parameters concerned are given as follows: $k_F=0.5$ Å^{-1},[27] $J=0.12$ eV,[28] $a_0=3.8$ Å,[32] $m^*=5$ m,[33]

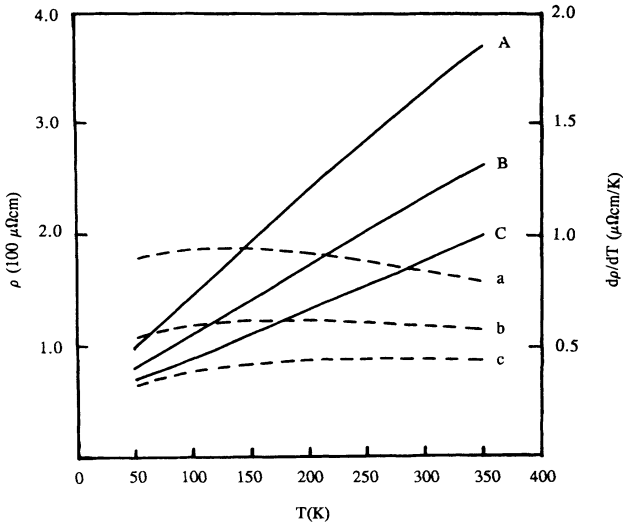

FIG. 1. Temperature dependences of the resistivity (solid curve) and its slope (dashed curve) from the fracton scattering with $I=0.144$ eV. Curves $A(a)$, $B(b)$, $C(c)$ correspond to $p=0.77, 0.78, 0.80$, respectively.

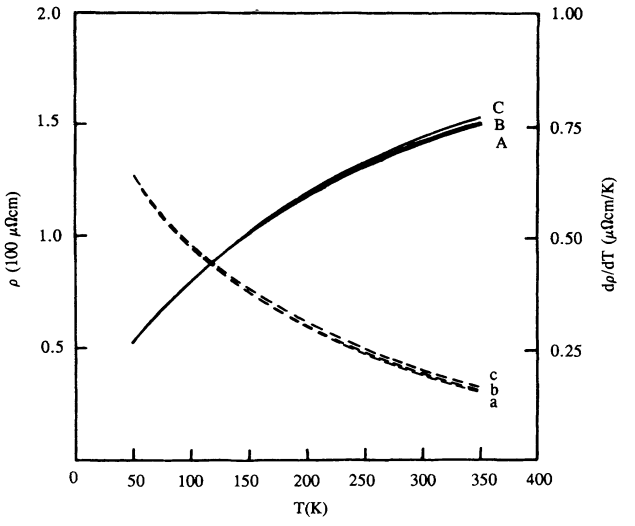

FIG. 2. Temperature dependences of the resistivity (solid curve) and its slope (dashed curve) from the magnon scattering with $I=0.144$ eV. Curves $A(a)$, $B(b)$, $C(c)$ correspond to $p=0.77, 0.78, 0.80$, respectively.

$n=5\times10^{21}$ cm^{-3}.[34] The results for $\rho(T)$ and $d\rho/dT$ from fracton and magnon scatterings are shown in Figs. 1 and 2, respectively. The s-d exchange parameter I is chosen to fit the experimentally measured $d\rho/dT$. It was found that[35] the slopes $d\rho/dT$ fall in a narrow range near 0.6 $\mu\Omega$ cm K^{-1}. So, we obtain $I=0.144$ eV for Y-Ba-Cu-O from Eq. (28) by using the calculation results of Eq. (27) at $p=0.78$. The striking feature of the results is that $\rho(T)$ curves from fractons scattering appear to be linear and the slope $d\rho/dT$ is nearly a constant over most of temperature ranges, which seems to be consistent with the experimental measurements.[1,35] But, the contribution to the resistivity from magnon scattering deviates largely from the T-linear law. Taking the logarithmic coordinate in the ρ-T curve, we find a scaling law $\rho\sim T^{3/5}$. Furthermore, this scaling law holds at different concentrations p for bond occupances near p_c, as one can see from Fig. 2. Experimentally, the T_c maximum occurs at a hole density of about 0.22 holes per Cu-O unit in the CuO_2 planes.[36] In view of Fig. 1, we find that in our calculation the best value of the concentration of broken bonds, which leads to a linear T dependence of resistivity, is $1-p=0.22$. It is also compatible with the experimental results.

IV. SUMMARY

In this paper, we have performed a calculation of temperature-dependent resistivity arising from the scatterings of electrons off fracton and magnon excitations in a dilute two-dimensional antiferromagnet based on the s-d exchange model. The results indicate that the resistivity from fracton scattering is nearly linear to temperature over a wide temperature range, and the slopes $d\rho/dT$ in the ρ-T curve support this result. The contribution to the resistivity from magnon scattering has a relation $\rho\sim T^{3/5}$.

In conclusion, as far as the magnetic excitations are concerned, the scattering from fractons seems to predict well the temperature-dependent resistivity in normal state of high-T_c copper oxides.

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