Adiabatic criterion for fermion bound states in the core of a moving vortex

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We derive a criterion which the velocity of a vortex line must satisfy in order that the fermion bound states follow the core adiabatically. The criterion appears strongly violated in vortex tunneling in high-T_c superconductors, which raises issues regarding vortex mass and viscosity in the theory of quantum creep.

The quasiparticle excitation spectrum in a pure superconductor, containing a vortex, has been calculated by Caroli, DeGennes, and Matricon.[1] By solving the Bogoliubov equations[2] for the gap function of a *static* vortex, these authors obtained the lowest energy bound states with a finite spacing of order Δ_∞^2/E_F, where Δ_∞ is the gap in a homogeneous superconductor and E_F is the Fermi energy. For ordinary superconductors with $\Delta_\infty^2/E_F \ll k_B T$, these low lying excitations form a quasicontinuum with a local density of states resembling that of a *normal* cylinder of radius equal to the correlation length. This concept of a normal core has been the basis of several calculations that involve properties of a *moving* vortex. Among these investigations there is the well-known model of vortex viscosity, proposed by Bardeen and Stephen.[3] In this model, dissipation occurs by ordinary resistive processes in a core, whose excitation spectrum is inferred from a calculation made for a *static* vortex.[1] Another example, where the concept of normal core is applied to a moving vortex, is the recent calculation of inertial mass of a vortex in a deformable superconductor.[4] Since the core of the vortex is in a normal state it behaves as a source of elastic inhomogeneity (due to the difference between the specific volumes of the normal and superconducting states). The resulting lattice strain surrounding the flux line will change as the latter moves. The kinetic energy of the ions oscillating under the influence of this motion provides a source of inertial mass, which can reach a significant value for high-T_c superconductors.[4] Both of the above examples point to the need of investigating the excitation spectrum of a moving vortex.

In this paper we attempt to answer the question: Under what conditions do the core bound states follow the moving line adiabatically? This leads us to investigate the adiabatic approximation of the time dependent Bogoliubov equations[2] by an extension of the method of Born and Fock.[5,6] The motion of the vortex line produces time dependent fluctuations of local gap function, which may induce *mixing* of the core bound states (nonadiabatic electron transitions). If the condition of adiabaticity is satisfied, these transitions can be neglected and the Bogoliubov equation can be solved, at each instant of time, for quasistationary bound states in the core of an *instantaneously* displaced line. We confine ourselves to layered superconductors, characterized by a large effective mass in the direction perpendicular to the layers. It turns out that, in this case, the quasiparticle amplitude follows closely the static vortex configuration. The central result of this calculation is an *adiabatic criterion* for the average transverse velocity of a vortex line. This criterion can be put in a form, allowing an interpretation analogous to the well-known Born-Oppenheimer approximation for an electron-nuclear system.[7] We discuss the application of this adiabatic criterion to the problem of quantum tunneling of a vortex. For the case of high-T_c superconductor, we find that the transit time of vortex, as it tunnels between pinning centers, can be so *short* that the adiabatic criterion is grossly *violated*. We discuss the consequences of this result for the determination of vortex parameters used in theory of quantum vortex creep.

The properties of the lowest fermionlike excitations in a clean superconductor with a space and time dependent pair potential $\Delta(\mathbf{r},t)$ can be obtained from the time dependent Bogoliubov equation[2]

$$i\hbar\frac{\partial\hat{\psi}}{\partial t}=\hat{\Omega}(t)\hat{\psi}(t)\ , \tag{1}$$

where $\hat{\Omega}(t)$ is defined as

$$\hat{\Omega}(t)=\begin{vmatrix} H_0 & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -H_0^* \end{vmatrix}\ . \tag{2}$$

In the presence of a magnetic field, oriented along the z axis, the operator H_0 is taken in the form

$$H_0=\frac{1}{2m_\perp}\left[\left(p_x-\frac{e}{c}A_x\right)^2+\left(p_y-\frac{e}{c}A_y\right)^2\right]+\frac{p_z^2}{2m_z}-E_F\ . \tag{3}$$

For layered compounds, the effective mass along the z axis m_z is much larger than the in-plane effective mass $m_\perp$. In the limit $m_z\to\infty$, the Fermi surface is perfectly cylindrical, and the solution of the quasistationary Bogoliubov equations is much simplified, owing to the fact that H_0 is a two-dimensional operator. The adiabatic approximation consists in replacing the nonstationary Eq. (1) by the quasistationary equation

$$\hat{\Omega}(t)\begin{bmatrix}u_n(t)\\ v_n(t)\end{bmatrix}=E_n\begin{bmatrix}u_n(t)\\ v_n(t)\end{bmatrix}, \quad (4)$$

where u_n and v_n are the quasiparticle amplitudes.[6] To derive the criterion for the validity of the adiabatic approximation we take the following ansatz for $\hat{\psi}(t)$

$$\hat{\psi}(t)=\sum_n a_n(t)\begin{bmatrix}u_n(t)\\ v_n(t)\end{bmatrix}\exp\left[-\frac{i}{\hbar}\int_0^t E_n(t')dt'\right]. \quad (5)$$

Inserting Eq. (5) into Eq. (1) and applying the steps outlined in Ref. 6, we obtain with the use of Eq. (4)

$$\dot{a}_k=-\sum_n{}' a_n\frac{\langle k|\dot{\hat{\Omega}}|n\rangle}{E_n-E_k}\exp\left[-i\int_0^t\omega_{nk}(t')dt'\right], \quad (6)$$

where

$$\langle k|\dot{\hat{\Omega}}|n\rangle=\int(u_k^*,v_k^*)\dot{\hat{\Omega}}\begin{bmatrix}u_n(t)\\ v_n(t)\end{bmatrix}d^3r. \quad (7)$$

The prime on the summation in Eq. (6) indicates that the term $n=k$ is to be omitted.

For $k=n$, the coefficient $\langle n|\dot{n}\rangle$ is related to the geometric phase,[8] which is responsible for the Aharonov-Bohm effect.[9] An explicit calculation, based on the approximate wave function of Ref. 1 [see Eq. (14)], shows that $\langle n|\dot{n}\rangle=0$. The absence of the geometric phase is presumably due to the neglect of the magnetic vector potential in the calculation of the fermion wave function.[1]

We now assume that, at $t=0$, the quasiparticle is in the state $|m\rangle$. To find the amplitude a_k for $k\neq m$, we put $a_n=\delta_{nm}$ and assume that the slowly varying quantities E_n, E_k and $\langle k|\dot{\hat{\Omega}}|n\rangle$ are actually independent of time. Then Eq. (6) can be integrated with the result

$$a_k(t)\approx(i\hbar\omega_{km}^2)^{-1}\langle k|\dot{\hat{\Omega}}|m\rangle(e^{i\Omega_{km}t}-1). \quad (8)$$

The criterion for the adiabatic approximation is obtained from (8) by requiring that the magnitude of $a_k(t)$ is much less than 1. Since only $\Delta(\mathbf{r},t)$ depends explicitly on time, we obtain from Eqs. (2) and (7)

$$\langle k|\dot{\hat{\Omega}}|m\rangle=\left\langle u_k\left|\frac{\partial\Delta}{\partial t}\right|v_m\right\rangle+\left\langle v_k\left|\frac{\partial\Delta^*}{\partial t}\right|u_m\right\rangle. \quad (9)$$

Using this result in Eq. (8), we obtain the condition for the adiabatic approximation in a general form

$$\frac{1}{\hbar\omega_{km}^2}\left|\left\langle u_k\left|\frac{\partial\Delta}{\partial t}\right|v_m\right\rangle+\left\langle v_k\left|\frac{\partial\Delta^*}{\partial t}\right|u_m\right\rangle\right|\ll 1. \quad (10)$$

We now examine this condition for the special case where $\Delta(\mathbf{r},t)$ corresponds to a moving vortex line. Assuming that $m_z\to\infty$, the operator H_0 defined in Eq. (3) is *z independent*. The quasistationary fermion states in the core of fluctuating vortex line are given by the solutions of Eqs. (4) with $\Delta(\mathbf{r},t)$ given by

$$\Delta(\mathbf{r},t)=|\Delta(\mathbf{r}-\mathbf{u})|\exp[i\theta(\mathbf{r}-\mathbf{u})], \quad (11)$$

where $\mathbf{u}=\mathbf{u}(z,t)$ is the two-dimensional displacement field describing the instantaneous trajectory of the vortex. The functions $|\Delta(\mathbf{r})|$ and $\theta(\mathbf{r})$ correspond to the pair potential of an isolated vortex, $\mathbf{r}$ being the radius vector in the xy plane pointing from the core center. A reasonable approximation for $|\Delta(\mathbf{r})|$ is[10]

$$|\Delta(\mathbf{r})|=\Delta_\infty\tanh\frac{r}{\xi}. \quad (12)$$

The phase θ is given by

$$\theta(\mathbf{r})=\arctan\frac{y}{x}. \quad (13)$$

We note that the Eqs. (11)–(13) are expected to hold only when the vector $\mathbf{u}(z,t)$ changes very little over the distances of the order of the coherence length. For $m_z\to\infty$ and with the pair potential (11), the Eqs. (4) depend on z only parametrically through the position $\mathbf{u}(z,t)$ of the vortex core center. Adopting the methods of Ref. 1, the solution can be written as

$$\begin{bmatrix}u_k(\mathbf{r},z)\\ v_k(\mathbf{r},z)\end{bmatrix}=N\exp\left[i\left[\mu-\frac{\sigma_z}{2}\right]\theta(\mathbf{r}-\mathbf{u})\right]\begin{bmatrix}f_+^{(k)}(|\mathbf{r}-\mathbf{u}|)\\ f_-^{(k)}(|\mathbf{r}-\mathbf{u}|)\end{bmatrix}, \quad (14)$$

where σ_z is the Pauli matrix and 2μ is an odd integer. The explicit forms for the radial functions $f_+^{(k)}$ and $f_-^{(k)}$ are given by Eqs. (5)–(8) of Ref. 1. We note that the solution (14) is obtained by setting the angle α, defined in Ref. 1, equal to $\pi/2$. We see from Eq. (14) that the quasiparticle follows closely the vortex trajectory. This, of course, is exactly true only in the limit $m_z\to\infty$. For finite value of m_z, a kind of WKB argument shows that a quasiparticle with momentum k_z can only trace vortex trajectory variations of wavelength much greater than k_z^{-1}. In what follows we shall work with the simplifying assumption $m_z\to\infty$. Following Ref. 1, the lowest bound-state eigenvalues $E_k<\Delta_\infty$ are given by

$$E_k\approx\mu_k\frac{\Delta_\infty^2}{k_F\xi}\approx\mu_k\frac{\Delta_\infty^2}{E_F}. \quad (15)$$

Using the expressions (11), (14), and (15), the condition (10) can be applied to the case of a moving vortex trajectory. From Eq. (11) we have

$$\frac{\partial\Delta}{\partial t}=-\frac{\partial\Delta}{\partial x}\frac{\partial u_x}{\partial t}-\frac{\partial\Delta}{\partial y}\frac{\partial u_y}{\partial t}. \quad (16)$$

Using Eqs. (12) and (13) we obtain

$$\frac{\partial\Delta}{\partial x}=\Delta_\infty\left[\frac{\cos\theta(\mathbf{r}')}{\xi(\cosh r'/\xi)^2}+i\frac{\sin\theta(\mathbf{r}')}{r'}\tanh\frac{r'}{\xi}\right]e^{-i\theta(\mathbf{r}')} \quad (17)$$

and

$$\frac{\partial\Delta}{\partial x}=\Delta_\infty\left[\frac{\cos\theta(\mathbf{r}')}{\xi(\cosh r'/\xi)^2}+i\frac{\sin\theta(\mathbf{r}')}{r'}\tanh\frac{r'}{\xi}\right]e^{-i\theta(\mathbf{r}')} \quad (18)$$

where $\mathbf{r}'=\mathbf{r}-\mathbf{u}(z,t)$. Introducing Eq. (16) into Eq. (9), we have

$$\langle k|\hat{\Omega}|m\rangle = -\left[\left\langle u_k\left|\frac{\partial\Delta}{\partial x}\dot{u}_x(z,t)\right|v_m\right\rangle + \left\langle u_k\left|\frac{\partial\Delta}{\partial y}\dot{u}_y(z,t)\right|v_m\right\rangle + \left\langle v_k\left|\frac{\partial\Delta^*}{\partial x}\dot{u}_x(z,t)\right|u_m\right\rangle + \left\langle v_k\left|\frac{\partial\Delta^*}{\partial y}\dot{u}_y(z,t)\right|u_m\right\rangle\right] . \quad (19)$$

The evaluation of the matrix elements in Eq. (19) is simplified by the fact that u_k, u_m, v_k, v_m, $\partial\Delta/\partial x$, and $\partial\Delta/\partial y$ are all functions of the same two-dimensional vector $\mathbf{r}-\mathbf{u}(z,t)$. Hence, in the two-dimensional integration we can shift the coordinates to the vortex center, leading to a z-independent number,and only the functions $\dot{u}_x$ and $\dot{u}_y$ are involved in the integration over z. To illustrate this procedure we consider, in some detail, the first matrix element on the right-hand side (rhs) of Eq. (19). Using Eqs. (14) and (17), we have

$$\left\langle u_k\left|\frac{\partial\Delta}{\partial x}\dot{u}_x\right|v_m\right\rangle = N^2\int_0^D dz\,\dot{u}_x(z,t)\int_0^{2\pi} d\theta\, e^{-i(\mu_k-\mu_m)\theta}\int_0^\infty dr\, r f_+^{*(k)}(r) f_-^{(m)}(r)\left[\frac{\cos\theta}{\xi[\cosh(r/\xi)]^2} + i\frac{\sin\theta}{r}\tanh\frac{r}{\xi}\right]\Delta_\infty , \quad (20)$$

where D is the width of the specimen in the z direction. The angular integrations fix the quantum number μ_k to the value μ_m+1. To perform the radial integration we consider a simplified model for the pair potential

$$|\Delta(r)| = \begin{cases} 0, & \text{for } r \le \xi \\ \Delta_\infty, & \text{for } r > \xi . \end{cases} \quad (21)$$

The resulting matrix element (20) is

$$\left\langle u_k\left|\frac{\partial\Delta}{\partial x}\dot{u}_x\right|v_m\right\rangle = 0.27\bar{\dot{u}}_x\frac{\Delta_\infty}{\xi} , \quad (22)$$

where

$$\bar{\dot{u}}_x = \frac{1}{D}\int_0^D dz\frac{\partial u_x(z,t)}{\partial t} \quad (23)$$

is the x component of the velocity of the vortex line averaged over the thickness of the specimen. Using similar procedure we calculate the third matrix element on the rhs of Eq. (19) with the result

$$\left\langle v_k\left|\frac{\partial\Delta^*}{\partial x}\dot{u}_x\right|u_m\right\rangle = 0.08\bar{\dot{u}}_x\frac{\Delta_\infty}{\xi} . \quad (24)$$

The remaining matrix elements follow from Eqs. (22) and (24) by replacing $\bar{\dot{u}}_x$ by $\bar{\dot{u}}_y$. In this way, Eq. (19) is evaluated in the form

$$\langle k|\hat{\Omega}|m\rangle = -0.35\frac{\Delta_\infty}{\xi}(\bar{\dot{u}}_x - i\bar{\dot{u}}_y)\delta_{\mu_k,\mu_m+1} . \quad (25)$$

Expanding the displacement field $\mathbf{u}(z,t)$ in a Fourier series

$$\mathbf{u}(z,t) = \sum_{q_z}\mathbf{u}(q_z,t)e^{iq_z z} , \quad (26)$$

where

$$\mathbf{u}(q_z,t) = \frac{1}{D}\int_0^D dx\, e^{-iq_z z}\mathbf{u}(z,t) , \quad (27)$$

we have from Eq. (23)

$$\left.\begin{aligned} \bar{\dot{u}}_x &= V_x(q_z=0,t) \\ \bar{\dot{u}}_y &= V_y(q_z=0,t) \end{aligned}\right\} , \quad (28)$$

where $\mathbf{V}(q_z,t)$ is the Fourier coefficient of the vortex-line velocity field.

Assuming that the initial fermion state corresponds to the lowest eigenvalue with $\mu_m=\frac{1}{2}$, the adiabatic condition (10) can be written with the use of Eqs. (15), (25), and (28) as

$$V_\perp = V_\perp(q_z=0,t) \ll \frac{0.35\Delta_\infty}{\hbar k_F^2\xi} , \quad (29)$$

where $V_\perp^2 = V_x^2 + V_y^2$. Substituting for the coherence length the expression $\xi = a\hbar v_F/\Delta_\infty$ where a is a numerical constant of order unity,[10] we obtain from Eq. (29)

$$\hbar V_\perp \ll \frac{\Delta_\infty^2}{k_F E_F} = \Delta R\,\Delta E , \quad (30)$$

where $\Delta R = 1/k_F$ and $\Delta E = \Delta_\infty^2/E_F$ is the spacing between the lowest core bound states. The rhs of Eq. (30) is reminiscent of the condition, given by Peierls,[7] for the Born-Oppenheimer approximation in the electron-ion problem. In the context of the latter problem, ΔR is the distance over which the electronic wave function changes appreciably.[7] We note that the wave functions of the fermion bound states [see Eq. (14)] are in keeping with this interpretation. The radial functions $F_\pm^{(k)}(r)$ are Hankel functions of argument $k_F r$, multiplied by a slowly decaying (with decay length $\sim\xi$) envelope. According to Eq. (30), it is the rapid oscillations of the Hankel function that are responsible for the short distance ΔR in our case. It is interesting that the adiabatic criterion (29), derived in this work for a classical vortex motion, can be cast into a general form (30), which has been originally derived for a quantum-mechanical system of electrons coupled to the lattice.[7]

As an application of Eq. (29), we consider the problem of quantum-mechanical tunneling of a vortex, which is responsible for the quantum creep in bulk superconductors, recently observed in high-T_c material by Mota *et al.*[11,12] and Fruchter *et al.*[13] Blatter, Geshkenbein, and Vinokur[14] investigated the elementary tunnel process in which a vortex segment of average length $\sim L_c$ tunnels between pinning sites separated by a minimum characteristic distance $\sim\xi$. These authors have estimated the characteristic transit time t_c by equating the kinetic and potential energies of the vortex segment.

For an undamped vortex, the transit velocity during the tunneling event is found to be[14]

$$v_c^{(1)}=\frac{\xi}{t_c}=\frac{\xi}{L_c}\sqrt{\epsilon_l/M}\ , \tag{31}$$

where M is the vortex mass and ϵ_l is the formation energy of the vortex line

$$\epsilon_l=\left[\frac{\Phi_0}{4\pi\lambda}\right]^2\ln\left[\frac{\lambda}{\xi}\right]\ , \tag{32}$$

where Φ_0 is the flux quantum and λ is the London penetration depth.[10]

For an overdamped vortex the transit velocity is[14]

$$v_c^{(2)}=\frac{\epsilon_l\xi}{L_c^2\eta}\ , \tag{33}$$

where η is the viscous drag coefficient. We now make orientational estimates of the inequality (29) for the Y-Ba-Cu-O sample. To estimate the rhs of (29), we take

$$\Delta_\infty\simeq 10k_BT_c\simeq 10^3\ \mathrm{K}\ ,$$

$$E_F=0.28\ \mathrm{eV}\ ,\quad m^*=10m_e$$

(where m_e is the electron mass) and $\xi=\xi_{ab}=3\times10^{-7}$ cm.[15] The latter choice of ξ is consistent with the assumption of a vortex segment perpendicular to the *ab* plane. In this way, we obtain from Eq. (29) a critical "adiabatic velocity" v_a given by

$$v_a=\frac{0.35\Delta_\infty}{\hbar k_F^2\xi}=\frac{0.17\hbar\Delta_\infty}{m^*\xi E_F}\simeq 2\times10^4\ \mathrm{cm/sec}\ . \tag{34}$$

For the condition (29) to hold, the velocities $v_c^{(1)}$ and $v_c^{(2)}$ must be much smaller than v_a.

To estimate $v_c^{(1)}$, we use in Eq. (31) the core inertial mass derived by Suhl[16]

$$M=\mu_{\mathrm{core}}=\frac{m^*H_c^2\xi^2}{E_F}\ , \tag{35}$$

where $H_c=2.7\times10^4$ G.[15] With use of the above parameters, Eq. (35) yields $\mu_{\mathrm{core}}=2.3\times10^8 m_e/\mathrm{cm}$. Using this value, taking $\lambda=2.6\times10^{-6}$ cm, and assuming $\xi/L_c=0.1$,[14] we obtain from Eqs. (31) and (32) $v_c^{(1)}=1.9\times10^6$ cm/sec, which is *two orders of magnitude faster* than the adiabatic critical velocity v_a of Eq. (34). We note that M may be possibly an order of magnitude larger than the expression (35), owing to the contribution to the inertial mass caused by lattice strains.[4] Even if this correction is taken into account, the inequality (29) remains to be violated by at least one order of magnitude.

Next we turn to the case of a strongly overdamped vortex. We assume that the core resistivity ρ_n is that of a normal metal and has a value of $25\times10^{-6}\ \Omega\,\mathrm{cm}$.[17] According to the Bardeen and Stephen model, the viscous drag coefficient is given by[3]

$$\eta=\frac{\pi\hbar^2}{4e^2\rho_n\xi^2}\ . \tag{36}$$

Introducing this expression into Eq. (33), we obtain (taking $L_c\simeq10\xi$) a vortex transit velocity $v_c\simeq5\times10^5$ cm/sec, which exceeds the velocity v_a by one order of magnitude. The above estimates lead us to conclude that during the tunneling event the vortex tends to shake off its normal core. This inability of the core bound states to follow the potential well, set up by the gap function (11), has deep consequences for models in which the concept of normal core is applied to a moving vortex. Recent calculation of the inertial mass of a vortex due to lattice strains depends in an essential way on the assumption of a normal vortex core, rigidly bound to the gap well. It has been pointed out recently by Duan and Leggett[18] that for a vortex velocity larger than the speed of sound, the vortex is expected to "shed" its lattice deformation cloud, so that the large inertial mass contribution calculated in Ref. 4 should not be included under such conditions. Since the sound velocity in Y-Ba-Cu-O is of order 10^5 cm/sec, their argument is confirmed for tunneling especially in the undamped case, where $v_c^{(1)}$ is an order of magnitude larger than the speed of sound. The present calculation finds that the lattice contribution to vortex mass may cease to be effective at a vortex velocity well below the speed of sound, owing to the breakdown of the adiabatic approximation for the core states. The latter should also play a role in the mechanism of inertial mass proposed by Suhl,[16] since the gap function of the moving vortex (11) is self-consistently determined by the quasiparticle core bound states.[19] Moreover, when these states fail to follow the motion of the vortex line we expect relaxation processes to take place that may contribute to the viscous drag coefficient η. This may be of importance in high-T_c materials, where the energy gap Δ_∞/E_F may block the core conductivity rendering the Bardeen-Stephen mechanism ineffective. We note that the assumption of an *overdamped* vortex has been made for vortex tunneling[14] and in the interpretation of microwave absorption in high-T_c superconductors.[20]

In summary, a criterion for the adiabatic approximation for fermion bound states in the core of a moving vortex is derived. Estimates, made for a high-T_c superconductor, show that this criterion is violated for processes involving quantum tunneling of vortices. Consequently, models for vortex mass and viscosity utilizing the concept of a normal vortex core anchored to the moving vortex line should be reexamined when the adiabatic approximation ceases to hold.

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