# ac conduction and 1/f noise in a Cr-film lattice-percolation system

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The ac conductivity  $\sigma(\omega)$ , ac dielectric constant  $\epsilon(\omega)$ , and (1/f)-noise spectral density  $S_v(f)$  have been studied in a Cr-film lattice-percolation system generated by electron-beam lithography. A powerlaw behavior,  $\sigma(\omega) \propto \omega^x$  and  $\epsilon(\omega) \propto \omega^{-y}$ , is observed near the percolation threshold. The ac-conductivity and ac-dielectric-constant exponents x and y are found to be  $0.98\pm0.09$  and  $0.08\pm0.04$ , respectively. While these results satisfy the general scaling law x + y = 1 and are consistent with those previously obtained on Au-film continuum-percolation systems, they cannot be explained by present percolation theories applied to two-dimensional (2D) systems. The normalized (1/f)-noise spectral density  $S_v(f)/V^2$  is found to scale as  $R^w$  (where R is the sample resistance) with critical exponent  $w = 1.18\pm0.19$ . Once again, the numerical value of w is appreciably different from the predictions of present percolation theories applied to 2D systems. We discuss the discrepancy between the experimental results and percolation theories.

## I. INTRODUCTION

Extensive studies in both theory and experiment have been devoted to investigation of the critical behavior of percolation systems.<sup>1-5</sup> However, progress in understanding the behavior of percolation systems near their percolation threshold,  $p_c$ , has been hampered by a number of difficulties in experiments. These difficulties are common in experiments involving "real" composite samples such as metal-insulator powder mixtures. Some of the typical problems include lack of randomness of samples, nonreproducibility of sample characteristics, and difficulties in sample characterization.

In this paper, we report our measurements of the ac conductivity, ac dielectric constant, and 1/f noise of a thin-film lattice-percolation system. The ac-conductivity and ac-dielectric-constant measurements, and their analysis, were described in the thesis by Song<sup>6</sup> but the 1/f noise measurements, on the same set of samples, have only recently been completed. All of our samples were produced by electron-beam lithography and they belong to the family of "artificial composite systems," which has the advantage that the samples are highly reproducible, efficiently randomized, and well characterized, since all the structural information is controlled by a computer. Another experiment with artificial composites using samples with a much smaller number of sites or bonds (about three orders of magnitude less) yielded results for the ac conductivity and ac dielectric constant.<sup>7</sup> Their samples were prepared by a completely different approach.<sup>8</sup> Nonetheless, following the analysis in Song's thesis, they find critical exponents for the ac conductivity and ac dielectric constant consistent with the ones presented here.

Percolation clusters have a fractal (self-similar) geometry when the typical sample size L is in the range  $a_0 \ll L \ll \xi$ , where  $a_0$  is the unit-cell size and  $\xi$  is the correlation length.  $\xi$  diverges according to  $|p-p_c|^{-\nu}$  when the occupation probability of the conducting component, p, approaches the percolation threshold  $p_c$  from either side.<sup>4</sup> The frequency dependence of conductivity and dielectric constant becomes "anomalous" if  $L \ll \xi$  and  $\omega_{\xi} \ll \ll \ll_a$ , where  $\omega_{\xi}^{-1}$  is the time required for electrons to move a distance  $\xi$  and  $\omega_a^{-1}$  is the time to traverse a unit-cell distance. Using some general analytic properties of the effective complex dielectric constant of a random composite, Bergman and Imry<sup>9</sup> derived the following power behavior for ac conductivity  $\sigma(\omega)$  and ac dielectric constant  $\epsilon(\omega)$ :

$$\sigma(\omega) \propto \omega^x , \qquad (1)$$

$$\epsilon(\omega) \propto \omega^{-y} , \qquad (2)$$

near  $p_c$ . These critical exponents, x and y, should satisfy the following general relation:

$$x + y = 1 \tag{3}$$

if  $\sigma(\omega)$  and  $\epsilon(\omega)$  both obey scaling forms that have a single characteristic time scale.

Based on the intercluster polarization effects, the following expressions were derived by many workers:<sup>10</sup>

$$x = t/(t+s), \quad y = s/(t+s)$$
, (4)

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where t and s are dc-conductivity and dc-dielectricconstant exponents, respectively. On the other hand, Gefen, Aharony, and Alexander<sup>11</sup> derived, considering the anomalous diffusion effect on the percolating clusters, the following relations:

$$x = \frac{t}{\nu(2+\theta)}, \quad y = \frac{2\nu - \beta}{\nu(2+\theta)}$$
(5)

for  $\omega \gg \omega_{\xi} \propto \xi^{-(2+\theta)}$ , where  $\nu$  is the correlation length exponent,  $\beta$  is a critical exponent that describes the probability that an arbitrary point belongs to the largest cluster of the system, and  $\theta$  is a critical exponent that describes the diffusion process for short time.  $\theta$  is related to critical exponents t,  $\beta$ , and  $\nu$  by  $\theta = (t-\beta)/\nu$ .<sup>11</sup> Equations (4) and (5) satisfy the general scaling relation, Eq. (3). The predicted numerical values for exponents x and y applied to two-dimensional (2D) and three-dimensional (3D) systems are summarized in Table I. At present, however, a unified theory of the ac conductivity and dielectric constant which contains both the intercluster polarization effect and the anomalous diffusion effect does not exist.

There have been several experimental investigations of  $\sigma(\omega)$  and  $\epsilon(\omega)$  performed on various random metalinsulator composite percolation systems. Hauser<sup>12</sup> measured  $\sigma(\omega)$  in the frequency range of 10 Hz to 1 MHz in Al-Al<sub>2</sub>O<sub>3</sub> and Sn-SnO<sub>2</sub> at 4.2 K. He was able to fit his data by the power law with  $x \approx 0.7$  and 0.8 for the above two systems, respectively. In the frequency range of 10 Hz to 13 MHz, the study of carbon-teflon composite samples by Song *et al.*<sup>13</sup> showed  $x = 0.86 \pm 0.06$  and  $y = 0.12 \pm 0.04$ . Niklasson and Granqvist<sup>14</sup> analyzed the infrared measurements on Au blacks and showed that  $x \approx 0.7$ . All three of these values obtained on 3D systems are closer to the prediction of the intercluster polarization model than that of the anomalous diffusion model. Compared to the relatively good agreement between theoretical predictions and experimental results in 3D systems, there has been considerable discrepancy between theories and experiments on 2D systems. Tick and Fehlner<sup>15</sup> reported power-law behavior of ac conductivity between 1 and 100 kHz with  $x \approx 0.9$  in a discontinuous Pd-Au composite film. Laibowitz and Gefen<sup>16</sup> showed that discontinuous Au films followed power-law behavior with  $x = 0.95 \pm 0.05$  and  $y = 0.13 \pm 0.05$  in the frequency range of 100 Hz to 10 MHz. Even though these critical exponents satisfy the general scaling relation, Eq. (3), they are significantly different from the predictions of either the intercluster polarization model or the anomalous diffusion model. All the experimental results are summarized in Table II.

TABLE I. Numerical values of exponents x and y predicted by the intercluster polarization (IP) model and anomalous diffusion model (AD) applied to 2D and 3D systems

	X		Y	
Model	IP	AD	IP	AD
2D	0.50	0.33	0.50	0.67
3D	0.72	0.58	0.28	0.42

TABLE II. Experimental results of exponents x and y obtained on various percolation systems.

0	V	V	D (
System	<u>X</u>	I	Reference
Al-Al <sub>2</sub> O <sub>3</sub>	0.7		12
Sn-SnO <sub>2</sub>	0.8		12
C+ Teflon	$0.86 {\pm} 0.06$	$0.12 {\pm} 0.04$	13
Au blacks	0.7		14
Pd-Au film	0.9		15
Au film	$0.95{\pm}0.05$	$0.13 {\pm} 0.05$	16
Cr film	0.98±0.09	$0.08{\pm}0.04$	Present study

There has also been a large body of theoretic work on 1/f noise in percolation systems.<sup>17-20</sup> These theories predict a scaling law for the normalized noise spectral density  $S_R(f)/R^2$  near  $p_c$  with another critical exponent,  $\kappa$ :<sup>17-19</sup>

$$S_R(f)/R^2 \propto (p-p_c)^{-\kappa} . \tag{6}$$

In the above equation, R is the dc electric resistance of the sample, which diverges as  $p_c$  is approached from the metallic side:

$$\mathbf{R} \propto (\mathbf{p} - \mathbf{p}_c)^{-t} \quad \text{for } \mathbf{p} > \mathbf{p}_c \ . \tag{7}$$

Since it is usually difficult to obtain the exact value of  $p_c$  in experiments, one may take advantage of the two scaling laws, Eqs. (6) and (7), to compare  $S_R(f)/R^2$  with R directly:

$$S_R(f)/R^2 \propto R^w , \qquad (8)$$

where the exponent  $w = \kappa/t$ . From numerical simulations,  $\kappa$  is found to be  $1.12\pm0.03$  and  $1.56\pm0.13$  for 2D and 3D systems, respectively.<sup>19</sup> Previously reported (1/f)-noise experiments in percolation systems were all performed on random metal-insulator composite systems.<sup>21-23</sup> In mapping from these real systems onto a discrete lattice system, a distribution in the conductance of each individual bond needs to be considered. Therefore, the so-called "continuum corrections" are introduced. For the "random void" model, in which insulating holes are embedded in a conducting matrix,  $w = \kappa/t \approx 3.2$  and 2.1, for 2D and 3D systems, respectively. For the "inverted random void" model, in which conducting holes are embedded in an insulating matrix,  $w = \kappa/t \approx 0.87$  and 2.4 for 2D and 3D systems, respectively.<sup>19</sup> These theoretical results are verified to a certain degree by experiments on sand-blasted metal films, <sup>21</sup> evaporated Ag films<sup>22</sup> and Ag-Pt alloy in insulating tetrafluoroethylene.<sup>23</sup> On the other hand, there are also experiments which do not agree with the theoretical predictions. For example, the critical exponent w obtained on ion milled gold films<sup>24</sup> is  $2.0\pm0.1$ , which is in no agreement with any theoretical model of two-dimensional percolation systems. Some similar experiments were also discussed in Ref. 19. Finally we notice that there have been no direct experimental results on lattice-percolation systems to verify theoretical results for discrete systems. The numerical values of the (1/f)-noise exponents are summarized in Table III.

System	к	w	Reference
Lattice, 2D	1.12±0.03	$0.86{\pm}0.03$	19
Lattice, 3D	$1.56{\pm}0.13$	$0.82 \pm 0.13$	19
RV, 2D		3.2	19
RV, 3D		2.1	19
IRV, 2D		0.87	19
IRV, 3D		2.4	19
Sand blasted film		3.4-6	21
Evaporated Ag film		0.9	22
Ag-Pt + tetrafluoroethylene		1 and 3	23
Ion milled Au film		2.0 ±0.1	24
Cr film		1.18 ±0.19	Present study

TABLE III. Exponents  $\kappa$  and w, which describe 1/f scaling laws [see Eqs. (6) and (8) in the text], predicted theoretically for lattice, random void (RV), and inverted random void (IRV) models. Results from four previous experiments and the present study are also listed here.

In this paper, we report our measurements of ac conductivity, ac dielectric constant, and 1/f noise in a thinfilm lattice-percolation system. The dc resistance critical exponent t is found to be  $1.30\pm0.15$ , in agreement with established results for 2D percolation systems. In the critical regime, we observe that the ac conductivity and dielectric constant both follow power-law behavior, Eqs. (1) and (2), with  $x = 0.98 \pm 0.09$  and  $y = 0.08 \pm 0.04$ , respectively. These results satisfy the general scaling law, Eq. (3), and they are in very close agreement with the results of the discontinuous Au-film experiment.<sup>16</sup> However, they differ significantly from the predictions of the intercluster polarization theory and anomalous diffusion theory. The critical exponent w from our 1/f noise experiments is found to be  $1.18\pm0.19$ , which also appears to differ from the numerical value of the ratio  $\kappa/t$  predicted by theories applied to 2D ( $\kappa/t = 1.12/1.3 = 0.86$ ) systems.

## **II. EXPERIMENTAL DETAILS**

### A. Sample fabrication

The samples used in this study are  $2560 \times 2560$  square lattice-percolation networks fabricated by an electronbeam lithographical process. The lattice constant is  $4 \, \mu m$ and the sample size is 1 cm  $\times$  1 cm. We started with commercially available chrome masks ("dark Chrome" coating) which were composed of  $(1300\pm200)$ -Å chrome films sputtered onto glass substrates (low expansion borosilicate). The surface resistance of the films was about 400  $\Omega/\Box$ , which was about two orders of magnitude higher than that of pure Cr films prepared by vacuum evaporation. The chrome films were coated with a layer of polymethylmethacrylate (PMMA) resist (2000 Å thick), and then they were baked at 170 °C for 60 min. Next, these films were exposed to the electron beam controlled by a scanning electron beam pattern generator (SEBPG). The SEBPG has a resolution of  $1/16 \ \mu m$ , a linewidth control of  $\pm 10\%$ , and the maximum field of view 2 mm. The sublattice was composed of  $512 \times 512$ lattice points occupying the maximum field of view of the SEBPG. The sublattice was repeated 25 times to cover

the 1 cm  $\times$  1 cm square and stitching on the boundaries was carefully performed to insure electric contact between the sublattices. The exposed films were then developed with a mixture of methy isobutyl ketone and isopropyl alcohol. The chrome etching was completed by using a solution of ceric ammonium nitrate with acetic acid and the PMMA resist was removed with methylene chloride.

In order to have good electrical contact to the sample, two contact pads,  $2 \text{ mm} \times 1.2 \text{ cm}$  each, were made along the opposing sides of the  $1 \text{ cm} \times 1 \text{ cm}$  square. The surface layer of the contact pad area was removed by plasma etching using a mechanical mask, and a silver film of about 1000 Å thickness was deposited without breaking vacuum. Electrical contact to this film was made by cold welding indium wires to the pad.

#### B. dc resistance measurements

The dc electrical resistance was measured to characterize the percolation networks of our sample system. Since the dc resistance of our samples was very large  $(R > 10^3 \Omega)$  and good electrical contacts had been made, a simple two-probe measurement was sufficient for the dc resistance determination. We also extrapolated the acconductance data to the dc limit and the results from these two methods agree very well.

### C. ac-conductance and -capacitance measurements

The ac conductance and capacitance were measured simultaneously at room temperature using a Hewlett Packard 4192A impedance analyzer with four-probe geometry. The data acquisition process was controlled by a computer. Typically, about ten measurements were repeated for each frequency point with 20 frequency points distributed in a logarithmic scale for each decade. The frequency range covered by this experiment was 10 Hz-13 MHz. The peak-to-peak voltage was 0.4 V and no heating effect was observed.

#### **D.** (1/f)-noise measurements

The noise power was measured from 0.1 Hz to 100 Hz by a dc four-probe cross correlation method.<sup>25</sup> A battery generated dc current (1  $\mu$ A-1 mA) was passed through the sample and a large series ballast resistor (at least ten times larger than the sample resistance). The voltage drop across the sample was dc filtered through a large capacitor (1.0 F capacitance) and then split into two parallel low-noise preamplifiers. The cross correlation voltage fluctuation spectrum  $(S_v)$  was measured by an HP3562A dynamic signal analyzer which detected signals from the two parallel preamplifiers. The electronics (except the HP3562A analyzer) and the sample were kept inside a mumetal box and batteries were used to supply power for the components inside the box. The background noise, which was essentially the amplifier noise plus the sample thermal noise, was subtracted from the data in order to obtain the excess noise (or intrinsic noise) in the samples. Care was taken to ensure that the 1/fnoise from the contacts was negligible.<sup>25</sup>

### **III. RESULTS AND DISCUSSION**

### A. Sample structure

The scanning electron microscope picture of one of our samples is shown in Fig. 1. The percolation network is a mixture of insulating bonds and conducting squares. Each tiny square occupies an area of  $3 \times 3 \ \mu m^2$ , and the spacing between these squares is  $1 \ \mu m$ . Our samples comprise a complementary system of normal bond percolation. In this system, each metallic square occupies a lattice point with probability p and an insulating bond is generated between the metallic squares with probability 1-p. In conventional lattice systems such as a bond percolation or a site percolation, the ac coupling between metallic unit cells is very small because the insulating



FIG. 1. The scanning-electron-microscope picture of Cr-film lattice system (p = 0.61 for this sample). The dark areas are occupied by a layer (1300±200 Å in thickness) of chromium film. The bright lines correspond to insulating areas and they separate metal clusters. The smallest unit in the lattice is a square occupying an area of  $3 \times 3 \ \mu\text{m}^2$ , while the spacing between the squares is  $1 \ \mu\text{m}$ . The lattice constant is  $4 \ \mu\text{m}$  and the sample size is  $1 \ \text{cm}$ .

component occupies most areas on the network. The advantage of our sample geometry is that the capacitance between elemental units is enhanced, which is very important for the ac response experiment.

Experimental studies of random metal-insulator composite systems have been hampered by difficulties in controlling and characterizing samples in such characteristics as grain size, constituent composition, finite-size effect of samples, and randomness of each component. Recent development of advanced microfabrication techniques have greatly improved these conditions. Our samples were made through electron-beam lithographical process, which allowed us to fabricate accurate lattice network with exactly known structure, to generate a large number of cells to alleviate the finite-size effect, and to achieve the maximum randomness with computer assistance. Thus these samples are random and well characterized.

## B. The percolation threshold $p_c$ and the dc-conductivity exponent t

In a 2D square lattice bond percolation system, due to the duality symmetry of the square network,<sup>26</sup> the percolation threshold  $p_c$  is exactly 0.5.<sup>27</sup> Our sample system can be considered as a normal bond percolation where lattice points are replaced by metal squares. Using  $p_c=0.5$ , we can perform a least-squares linear fitting to obtain  $t=130\pm0.15$ . The dc resistance data is plotted in Fig. 2 as a function of  $p-p_c$ . Compared with established values of the t exponent (t=1.3 and 1.9 for 2D and 3D systems, respectively),<sup>28</sup> the resistance of our samples is well described by percolation theories applied to 2D systems.

The percolation threshold  $p_c$  of our samples was also determined from the dc resistance measurement directly. Samples were considered as insulating if their dc resis-



FIG. 2. Data of dc resistance measurement as a function of  $p-p_c$  on a logarithmic scale. The straight line was obtained from the scaling relation  $R \propto (p-p_c)^{-t}$ . The parameters used in this plot are  $p_c = 0.5$  and t = 1.3.

tance was in excess of 10 M $\Omega$ . This procedure led to a  $p_c$  value of 0.490±0.05. The small difference in  $p_c$  compared to the exact theoretical result ( $p_c=0.5$ ) suggests statistical fluctuations in our sample system. Ideally, this phenomenon can be remedied by repeating a large number of statistical averages but the limited availability of the microfabrication facilities prevented us from making such an effort.<sup>29</sup>

## C. ac-conductance exponent x and ac-dielectric-constant exponent y

In Fig. 3, the ac-conductance data of our samples are plotted on a logarithmic scale as a function of frequency. For samples relatively far from  $p_c$ , they show a flat frequency response which can be understood from the Drude model for metals:<sup>30</sup>

$$\sigma(\omega) = \frac{\sigma(0)}{1 - i\omega\tau} , \qquad (9)$$

where  $\sigma(0)$  is the dc conductivity and  $\tau$  is the relaxation time for the collision of free electrons with phonons, lattice imperfections, and impurities. For good metals,  $1/\tau$ can be as high as  $10^{13}$  Hz, therefore the conductivity of a highly metallic sample is expected to be independent of frequency up to the far infrared region.<sup>30</sup> When approaching  $p_c$ , however, the correlation length  $\xi$  becomes very large, and the samples have a self-similar geometry. The anomalous diffusion on percolation clusters and the intercluster polarization effect both make contributions to the conduction process. In the frequency range  $\omega_{\xi} \ll \omega \ll \omega_a$ , the ac conductivity is expected to vary as  $\sigma(\omega) \propto \omega^x$ . In our samples, this behavior is observed in samples with p = 50.0% and 50.2% for frequencies above 100 kHz, with the exponent  $x = 0.98 \pm 0.09$ .

Figure 4 is a logarithmic plot of the capacitance data. A weak frequency dependence is observed. Since the ac dielectric constant  $\epsilon(\omega)$  is proportional to the capaci-



FIG. 3. ac-conductance data of the Cr-film system as a function of frequency. All samples are on the conducting side. Power-law behavior is observed from samples with p=0.500and 0.502 at frequencies above 100 kHz. The ac-conductivity exponent x is found to be  $0.98\pm0.09$ .



FIG. 4. ac capacitance data of the Cr-film percolation system as a function of frequency. The ac-dielectric-constant exponent is determined to be  $y = 0.08 \pm 0.04$ .

tance, we may find the ac dielectric constant exponent y defined in Eq. (2) from the capacitance data. From Fig. 4, the numerical value of y is determined to be  $0.08\pm0.04$ . It is clear that the x and y exponents derived from the ac-conductance and capacitance data  $(x+y=1.06\pm0.13)$  satisfy the general scaling relation x+y=1.

Several interesting observations can be made from our ac-conductance and dielectric-constant experiments.

(1) There is a very close agreement between our experiments and the experiments performed on Au films by Laibowitz and Gefen (see Table II), although the samples are very different. The sample geometries are totally different in these two cases since our Cr-film samples belong to the square lattice percolation system while the Au-film samples belong to the continuum-percolation system. The close agreement between the two independent experiments removes possible errors related to experimental details. Furthermore, it suggests that for acconduction processes in 2D percolation systems, there is no observable difference between discrete systems and continuum systems. This is in noticeable contrast with the recognition that the behavior of transport critical exponents in continuum-percolation systems is generally nonuniversal.<sup>31</sup>

(2) The discrepancy between the experimental results and the theoretical predictions for the ac conductivity and dielectric constant may be interpreted as evidence of inadequacy of present theoretical studies of percolation phenomena. Even for 3D systems, the experimental results (Table II) do not agree completely with the predictions of the two available models (Table I). For 2D percolation systems, the discrepancy is seen to be even larger. Physically, the anomalous diffusion model neglects the capactive coupling between the percolation clusters, therefore it is a noninteracting cluster model. The intercluster polarization model, on the other hand, does not include the consequence of the fractal nature of each individual cluster, namely, the anomalous diffusion on percolation clusters. Therefore, neither of these two models is complete by itself.

(3) The discrepancy between experiments performed on thin-metal-film systems and percolation theories may also suggest that there exist some intrinsic problems for such thin-metal-film systems. For instance, the conduction process should mainly proceed in the planar space defined by the thin films, in order to meet the condition of two-dimensional geometry. In our dc resistance measurement, this requirement is satisfied and we indeed observe 2D behavior in the dc conduction. In the acconduction process, the total current, including both conduction current and displacement current, should completely flow into the metallic films. However, the fringe effect takes place near the corners of unit cells that are separated by insulating bonds. If the fringe effect is severe enough, it may cause serious "leakage" into the three-dimensional space and we should not expect to observe a truly 2D behavior in the ac-conduction process.<sup>32</sup> It is interesting to observe, from Table I and Table II, that the numerical values of exponents x and y obtained on thin metal films are actually closer to theoretical predictions on 3D systems rather than on 2D systems, even though they do not agree completely with the 3D predictions.

We also notice that the Al sheet samples used by Yoon and Lee<sup>7</sup> have poorer quality and they observe finite-size effects. By comparison, our samples are infinite in size being a factor of  $25^2$  larger in number of cells. Their quoted accuracy of the exponents is better than ours since we only have data on one sample at each value of pwhereas they have seven in some cases. However, better accuracy for an exponent (if true) is on marginal value if the first significant figure is uncertain due to finite-size effects.



FIG. 5. The (1/f)-noise spectra density  $S_v(f)$  (in units of  $V^2/\text{Hz}$ ) measured at f=10 Hz as a function of  $V^2$  (in units of  $V^2$ ), where V is the mean dc voltage across the sample. The sample's metal fill fraction is p=0.51. The solid line is a guide to the eye and a clear  $S_v(f) \propto V^2$  dependence is observed.



FIG. 6. The normalized (1/f)-noise spectral density  $S_v(f)/V^2$  plotted as a function of the sample resistance R. The critical exponent w is found from the data to be  $1.18\pm0.19$ .

# D. 1/f noise

In Fig. 5, we plot the (1/f)-noise spectral density  $S_v(f)$  measured at different values of dc voltage V for one sample. The data reveals a clear  $S_v(f) \propto V^2$  dependence. This  $V^2$  dependence is expected for 1/f noise generated by resistance fluctuations probed by a constant current.

The normalized (1/f)-noise spectral density  $S_n(f)/V^2$ is plotted in Fig. 6 versus the sample resistance R. Here V is the mean dc voltage across the sample. For samples with  $R < 3 \text{ k} \Omega$   $(p - p_c > 0.1)$ , their (1/f)-noise level was too low to measure. It can be seen from this figure that the data of  $S_n(f)/V^2$  can be fitted to a power of R. This behavior is described by Eq. (8) with critical exponent  $w = \kappa/t$ , since  $S_R(f)/R^2 = S_v(f)/V^2$ .<sup>33</sup> The critical exponent w is found to be  $1.18\pm0.19$  from our data. This value is noticeably larger than the numerical value of the ratio  $\kappa/t$  predicted by percolation theories<sup>17-19</sup> applied to 2D ( $\kappa/t = 1.12/1.3 = 0.86$ ) lattice systems. Finally, we notice that the w exponent derived from our data is different from previous results of 1/f noise experiments performed on various continuum-percolation systems (see Table III). To our best knowledge, the 1/f noise theories have not been directly tested on lattice-percolation systems experimentally.

## **IV. CONCLUSIONS**

In summary, we have measured the ac conductivity, ac dielectric constant, and 1/f noise on a Cr-film latticepercolation system generated by electron-beam photolithography. This system demonstrates a twodimensional behavior in dc-conduction process with critical exponent  $t=1.30\pm0.15$ . The ac conductivity and dielectric constant both exhibit the power-law behavior, with critical exponents  $x=0.98\pm0.09$  and  $y = 0.08 \pm 0.04$ , respectively. These results are consistent with the general scaling law x + y = 1 and they agree very closely with previous experiments on Au-films samples.<sup>16</sup> However, these experimental results cannot be explained by present percolation theories applied to 2D systems. Our (1/f)-noise experiments lead to a critical exponent  $w = 1.18 \pm 0.19$ , which again cannot be explained by percolation theories applied to 2D systems. Our acconductance and dielectric-constant data and (1/f)noise data indicate that a consistent discrepancy exists between experimental results obtained on thin-film per-

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