Optical phase conjugation via stimulated Brillouin scattering in magnetoactive doped semiconductors

M. Bose, P. Aghamkar, and P. K. Sen*
Department of Physics, Barkatullah University, Bhopal 462 026, India
(Received 8 March 1991; revised manuscript received 17 January 1992)

The occurrence of optical phase conjugation via stimulated Brillouin scattering (OPC-SBS) in bulk semiconducting crystals under the off-resonant transition regime has been investigated theoretically, the crystal being immersed in a large magnetostatic field. The model is based upon the coupled-mode approach and incorporates the effect of pump absorption through the first-order (i.e., linear) induced polarization. The linear dispersion is found not to affect the reflectivity of the phase-conjugate Stokes-shifted Brillouin mode. The reflectivity of the image radiation is dependent upon the Brillouin susceptibility and can be significantly enhanced through *n*-type doping of the crystal and the simultaneous application of a large magnetostatic field. Moreover, the threshold of the pump intensity required for the occurrence of SBS in the crystal with finite optical attenuation can be considerably diminished through a suitable choice of the excess carrier concentration and the magnetic field. Consequently, OPC-SBS becomes a possible tool in phase-conjugate optics even under not-too-high-power laser excitation by using moderately doped *n*-type semiconductors kept under the influence of large magnetostatic field. Numerical estimates made for InSb crystal at 77 K duly irradiated by nanosecond pulsed 10.6-μm CO_2 laser show that high OPC-SBS reflectivity can be achieved at pump intensities below the optical damage threshold if the crystal is used as an optical waveguide with relatively large interaction length ($L \sim 3$ mm).

I. INTRODUCTION

Optical phase conjugation (OPC) has been an active area of research in quantum electronics. The ability to couple spatially and/or temporally encoded information bearing electromagnetic or other bosonic fields using nonlinear optical techniques has led to diverse potential applications that range from adaptive optics, laser resonators, and high energy or high brightness laser systems, to optical information processing, image transmission, filtering, and ultralow noise communication schemes.

Research activities in the general area of the OPC gained much momentum with the development of the concept of "transient" or "real-time" holography by Gerritsen,[1] Woerdman[2] and Stepanov, Ivakin, and Rubanov.[3] Several techniques have been successfully employed to achieve OPC. These techniques are equivalent to the mathematical operation of complex conjugation on the spatial component of the complex field amplitude of a given beam. It is apparent that any nonlinear optical interaction occurring in virtually any state of matter, and at most wavelengths, can give rise to the desired wavefront-reversed replica.

The most promising approaches among the various mechanisms responsible for the occurrence of OPC are three- and four-wave mixing[4,5] as well as the stimulated scattering (SS) processes in the nonlinear medium. Again, OPC-SS has the added advantage that it does not need any reference radiation and is therefore known as "self-phase conjugation." Out of the large number of SS processes like stimulated Brillouin scattering (SBS), stimulated Raman scattering, stimulated Rayleigh-wing scattering and stimulated temperature scattering, OPC-SBS is mostly preferred due to the Brillouin shift in the conjugation radiation frequency being the smallest. Theoretically, it is suggested[6–8] that OPC-SBS occurs because a component of the backscattered light with a frequency downshift equal to the acoustic-phonon frequency exponentially grows at twice the rate of other random modes (viz., background noise). Under high-gain regime, this component sufficiently predominates and one may expect the OPC reflectivity to approach 100%.

OPC-SBS will give rise to significant conjugation radiation only if the input excitation intensity exceeds a threshold value. The effective threshold can be reduced by the use of a preamplifier stage in front of the SBS phase-conjugate mirror. Brusselback and Rockwell[9] have demonstrated high reflectivity phase conjugation by combining a high-gain Raman amplifier with a SBS mirror. Andreev *et al.*[10] also reported that the phase conjugation of weak laser pulses can be achieved by the process of Brillouin-enhanced four-wave mixing and demonstrated that with energies as low as 2×10^{-14} J in a 20-nsec pulse, OPC could be achieved with a signal-to-noise ratio of approximately unity and a reflection coefficient of 10^8. The theories[7,8] support the existence of a nonconjugate component along with the conjugate component in the high-gain mode. Even a small fraction of nonconjugate component in the high-gain mode can cause serious beam distortion. While taking into account the plane-wave expansion, Zel'dovich, Pilipetsky, and Shkunov[6] links the same with the intensity as well as the angular divergence of the pump wave. Hellwarth,[8] on the other hand, while considering the waveguide structures, predicts that the fidelity depends upon the distribution of pump power among the modes and not on the excitation intensity. Suni and Falk[11] attributed the origin of this disparity solely to a differing treatment of nonphase matched scattering terms. None of the above theories include pump depletion which becomes important at pump intensities much above the threshold for the onset of SBS. In a numerical study of OPC-SBS in a waveguide, Lehmberg[12] found that the pump depletion enhances the fidelity by inhibiting the small scale pulling effect. All these theories were conducted on OPC-SBS keeping in view its application in communication using optical fibers and waveguides. However, the technique of OPC-SBS in a Brillouin cell of small dimension containing a nonlinear active medium can be of potential application in achieving gain reflectivity which can be subsequently employed in processes like laser-induced fusion. Rockwell[13] discusses such phase-conjugate solid-state lasers and shows that OPC-SBS could be useful at any wavelength. Scott and Ridely[14] have reviewed the phenomena of Brillouin-enhanced four-wave mixing in a cell containing a nonlinear active medium. They discussed the difficulty in getting a suitable Brillouin-active medium in the 10-μm regime. Bespalov, Kiseljor, and Pasmanik[15] used parametric mixing technique to achieve wave-front reversal in this frequency regime although such parametric mixing is strictly restricted to the phase matching condition. Sen,[16] and Sen and Sen[17] have shown that InSb could be potential material for SBS studies in the same wavelength regime if the bulk crystal is subjected to a large magnetostatic field.

The stimulation for the present analytical investigation of OPC-SBS under pump intensity dependent absorption coefficient regime, in a Brillouin cell containing a semiconducting crystal stems from the situation as discussed above. In such a nonlinear system, the waveguide structure has been replaced by simple plane-wave formalisms. In order to examine critically the dependence of the reflectivity coefficient on the material parameters, we have considered the crystal to be n-type doped, with the doping concentration being an externally controllable parameter. We have followed a purely electromagnetic treatment where the crystal is assumed to behave like a semiconductor-plasma medium where the dynamic behavior of the plasma can be explained by the hydrodynamic model.[16,17] The third-order optical susceptibility responsible for the occurrence of OPC known as the Brillouin susceptibility (χ_B) has been obtained by following the coupled-mode approach.

In Sec. II, we have derived an expression for the OPC-SBS reflectivity $|\beta|^2$. The Brillouin susceptibility in the doped semiconductor in the presence of the external magnetic field is derived in Sec. III. The important analytical results are discussed in Sec. IV. The same section also deals with the numerical estimations of the threshold excitation intensities required for the onset of SBS as well as OPC-SBS with $|\beta|^2 \sim 1$ in InSb duly irradiated by a pulsed 10.6-μm CO_2 laser. The roles of doping as well as magnetic field in enhancing the OPC reflectivity are also examined in the same section. Section V enlists the important conclusions that can be derived from the present analytical study.

II. OPTICAL PHASE CONJUGATION REFLECTIVITY

In this section, we have obtained the analytical expression for the reflectivity of optical phase conjugation via stimulated Brillouin scattering (OPC-SBS). For determining the OPC-SBS reflectivity of the medium, we consider the irradiation of the semiconductor crystal by a slightly off-resonant laser pump. The pump laser mode undergoes stimulated scattering processes via its interactions with phonons in the medium. Here, we are dealing with the scattering due only to the acoustic phonons such that the interaction yields the scattered Brillouin mode. The propagation of these modes through the material can be represented by the generalized equation[18]

$$\frac{\partial^2 E(r_\perp x,t)}{\partial x^2}+\nabla_T^2 E(r_\perp,x,t)+\frac{\omega_L^2}{c^2}E(r_\perp,x,t) = -\mu_0\omega_L^2 P(r_\perp,x,t) , \quad (1)$$

where $\nabla_T^2=\partial^2/\partial y^2+\partial^2/\partial z^2$; $E(r_\perp,x,t)$ is the electric-field amplitude possessing both longitudinal and transverse components. The fields are assumed to vary as $\exp[i(\omega_L t-k_L x)]$, ω_L and k_L are the angular frequency and wave number of the electromagnetic mode, respectively; μ_0 is the permeability of free space. P represents the total induced polarization in the crystal and comprises linear as well as higher-order nonlinear components. The total optical susceptibility of the crystal can be obtained through the relation

$$P(r_\perp,x,t)=\epsilon_0\chi E=\epsilon_0[\chi^{(1)}+\chi^{(3)}|E(r_\perp,x)|^2+\cdots]E , \quad (2)$$

where we assume the crystal to be nearly centrosymmetric such that the even-order nonlinear optical susceptibility components like $\chi^{(2)}$, $\chi^{(4)}$, etc. are zero. In (2), the complex susceptibility $\chi^{(1)}$ accounts for the linear refraction and absorption phenomena within the crystal, while the third-order component $\chi^{(3)}$ is responsible for the active nonlinear optical effects like nonlinear refraction, nonlinear absorption, degenerate four-wave mixing, and stimulated Raman and Brillouin scatterings. In bulk semiconducting crystals, $\chi^{(1)}$ does not play any role in improving the conversion efficiency.[19] Nevertheless, it affects the spatial propagation characteristic of the conjugate beam. In the present analytical investigation of OPC-SBS, we have retained χ comprising both linear as well as the third-order components, i.e., $\chi=\chi^{(1)}+\chi^{(3)}[|E(r_\perp,x)|^2]$.

The second term on the right-hand side of (2) can be expressed in terms of a transmission function[19] proportional to $(E_P+E_S)(E_P^*+E_S^*)$, where E_P and E_S are the electric fields associated with the pump $(\omega_P,\mathbf{k}_P)$ and the scattered Stokes mode $(\omega_S,\mathbf{k}_S)$, respectively. Consequently, the third-order component of the induced polarization is obtained as

$$P^{(3)}=\epsilon_0\chi^{(3)}[|E_P|^2+|E_S|^2+E_P^*E_S+E_PE_S^*]E . \quad (3)$$

We have considered the pump wave (ω_P, k_P) to be propagating along the $-x$ direction while the Stokes mode of the scattered electromagnetic wave is assumed to propagate along $+x$ direction. Thus, one may replace k_L by $-k_P$ and k_S for the pump and the scattered mode, respectively. Making use of (1)–(3), we can obtain the corresponding electromagnetic wave equations under the slowly varying envelope approximation as

$$\frac{\partial E_P}{\partial x}-\frac{i}{2k_P}\nabla_T^2 E_P=\alpha_{IP}E_P-i\alpha_{IrP}E_P-\frac{i\omega_P^2}{2k_Pc^2}\chi_P^{(3)}|E_S|^2E_P \tag{4a}$$

and

$$\frac{\partial E_S}{\partial x}-\frac{i}{2k_S}\nabla_T^2 E_S=-\alpha_{IS}E_S+i\alpha_{IrS}E_S+\frac{i\omega_S^2}{2k_Sc^2}\chi_S^{(3)}|E_P|^2E_S\ , \tag{4b}$$

where α_{IP} (α_{IS}) is the intensity dependent absorption coefficient of the crystal at the pump frequency ω_P (Stokes frequency ω_S) and given by

$$\alpha_{IP,S}=\frac{\omega_{P,S}}{2c}[\chi_{IP,S}^{(1)}+\chi_{IP,S}^{(3)}|E_{S,P}|^2]\ . \tag{5a}$$

The parameters α_{IrP} and α_{IrS} in (4) account for the dispersive properties of the material and have their origin in the intensity dependent real part of the optical susceptibility defined as

$$\alpha_{IrP,S}=\frac{\omega_{P,S}}{2c}[\chi_{rP,S}^{(1)}+\chi_{rP,S}^{(3)}|E_{S,P}|^2]\ . \tag{5b}$$

Moreover, $\chi_{P,S}^{(3)}$ in (4) represents the complex third-order susceptibility at frequency $\omega_{P,S}$. These terms are written separately simply to bring a similarity between the equations obtainable from the coupled-mode approach and the present treatment based upon the intensity dependent optical susceptibility of the crystal.

The phase-matching conditions for OPC-SBS, as considered in the present study, are given by $\hbar\omega_P=\hbar\omega_S+\hbar\omega_a$ and $\hbar\mathbf{k}_P=\hbar\mathbf{k}_S+\hbar\mathbf{k}_a$ with $(\omega_a,\mathbf{k}_a)$ being the generated acoustic-phonon mode. For phase-conjugate Stokes mode, one should have $\mathbf{k}_P=-\mathbf{k}_S$ such that $\mathbf{k}_a=2\mathbf{k}_P$. In the forthcoming discussions, we have considered $|\mathbf{k}_P|=|\mathbf{k}_S|=k$ such that $|\mathbf{k}_a|=2k$ and for considerably low acoustic frequency [i.e., $\omega_P \gg \omega_a$, one may take $\omega_P\sim\omega_S=\omega$ (say)]. These assumptions enable one to take $\alpha_{IP}\sim\alpha_{IS}=\alpha_I$ (say) and $\chi_P^{(3)}\sim\chi_S^{(3)}=\chi_B$ with χ_B being the Brillouin susceptibility. For stimulated Brillouin scattering, the third-order optical susceptibility is usually an imaginary quantity for dispersionless acoustic-wave propagation[20] and hence we take $\chi_B=-i|\chi_B|$. For a single pump and Stokes mode, we have followed the well-known single-mode formalism such that the phase-conjugate Stokes mode is related to the pump via relation

$$E_S(r_\perp,x)=\beta(x)E_P^*(r_\perp,x)\ , \tag{6}$$

where $\beta(x)$ is a measure of conjugacy with $|\beta(x)|^2$ being defined as the OPC-SBS reflectivity. We have obtained the expression for $|\beta(x)|^2$ at excitation intensity above a critical value known as the threshold condition for the occurrence of SBS in an active semiconducting bulk crystal with finite attenuation and examined the suitability of such crystal in practical applications such as optical phase-conjugate mirrors.[21]

Under one-dimensional configuration, the electric field associated with the pump and the scattered mode can be obtained from (4) as

$$\frac{\partial E_P}{\partial x}=\alpha_IE_P-i\alpha_{Ir}E_p-\frac{i\omega^2}{2kc^2}\chi_B|E_S|^2E_P \tag{7a}$$

and

$$\frac{\partial E_S}{\partial x}=-\alpha_IE_S+i\alpha_{Ir}E_S+\frac{i\omega^2}{2kc^2}\chi_B|E_P|^2E_S\ , \tag{7b}$$

respectively, taking $\omega_P\sim\omega_S\sim\omega=kc$. Since $|E_S|^2$ is a generated field, one may safely assume

$$\alpha_I \gg \frac{\omega^2\chi_B|E_S|^2}{2kc^2}$$

in (7a). Consequently, one can get the solution of (7a) as

$$E_P(x)=[E_P(L)\exp\{-\alpha_I(L-x)\}]\exp\{i\alpha_{Ir}(L-x)\}\ , \tag{8}$$

where $E_P(L)$ is the pump electric field at the entrance window (i.e., at $x=L$). Equation (8) reveals the x dependence of the pump amplitude as well as the nature of phase variation of the electric field with x. From (7b) and (8), we find the electric field associated with the Stokes mode of the backscattered electromagnetic wave as

$$E_S(x)=E_S(0)e^{-\{\alpha_Ix+\mathcal{K}[1-\exp(2\alpha_Ix)]/2\alpha_I\}}\exp\{i\alpha_{Ir}x\}\ , \tag{9a}$$

where

$$\mathcal{K}=\frac{\omega^2\chi_B|E_P(L)|^2}{2kc^2}\exp(-2\alpha_IL)\ . \tag{9b}$$

In (9a), $E_S(0)$ is the electric-field amplitude at the exit window $x=0$ and can be defined as the spontaneous noise field. Now, (9a) can be rewritten in the form

$$E_S(x)=E_S'(0)[\cos(\alpha_{Ir}L)+i\sin(\alpha_{Ir}L)]\times\exp[-i\alpha_{Ir}(L-x)] \tag{10a}$$

with

$$E_S'(0)=E_S(0)e^{-\{\alpha_Ix+\mathcal{K}[1-\exp(2\alpha_Ix)]/2\alpha_I\}}\ . \tag{10b}$$

Equation (10a) manifests the occurrence of the phenomenon of optical phase conjugation in the material through the dependence of the scattered mode on the phase factor $\exp[-i\alpha_{Ir}(L-x)]$ which is backscattered. Moreover, α_{Ir} which depends upon the real part of the optical susceptibility $\chi^{(1)}$ and $\chi^{(3)}$ will not affect either the pump (E_P) or the scattered mode (E_S). As a consequence the OPC-SBS reflectivity remains unaltered irrespective of the absolute phase difference between E_P and E_S. Equation (10b) can also be employed in studying the threshold nature of the SBS phenomena responsible for the OPC processes. The backscattered Brillouin mode $E_S(x)$ possesses a gain constant given by

$$-\{\alpha_I x+\mathcal{K}[1-\exp(2\alpha_I x)]/2\alpha_I\}\ .$$

For finite gain, the condition

$$\alpha_I x+\mathcal{K}(1-e^{2\alpha_I x})/2\alpha_I<0 \tag{11a}$$

is to be achieved in the material medium.

It is clear from (11a) that the gain constant depends upon the intensity dependent absorption coefficient α_I and the parameter $\mathcal{K}$. For a bulk semiconducting crystal of mm thickness irradiated by a slightly off-resonant laser with photon energy less than the crystal band-gap energy or an optical fiber with very low loss, one may take $2\alpha_I x<1$ such that the threshold condition for Brillouin gain becomes $\mathcal{K}=\alpha_I$.

$$\mathcal{K}=\alpha_I\ . \tag{11b}$$

We have restricted our analysis to the study of OPC-SBS in the semiconducting crystals irradiated by off-resonant (below the band edge) laser sources. In this regime the third-order optical susceptibility is appreciably smaller than in the near-resonant case.[22,23] This enables one to treat α_I and α_{Ir} as the real and imaginary components of the background absorption coefficient α and α_r, respectively, at frequency ω. Using (9b) and (11) for $\alpha L<1$ at the entrance window $x=L$, we find the threshold value of the excitation intensity as

$$I_{P,\mathrm{th}}=\frac{\eta\epsilon_0 c^3 k\alpha}{\omega^2|\chi_B|}\ , \tag{12}$$

with η being the background refractive index of the crystal, ϵ_0 is the absolute permittivity, α as the background absorption coefficient at frequency ω, and $I_{P,\mathrm{th}}=\frac{1}{2}\eta\epsilon_0 c|E_{P,\mathrm{th}}|^2$. Equation (12) shows that the threshold intensity can be similar in systems with very low absorption coefficients and large Brillouin susceptibility. Again (8) and (10) give the OPC-SBS reflectivity as

$$|\beta(x)|^2=\left[\frac{|E_S(0)|}{|E_P(L)|}\right]^2 e^{2[\alpha(L-2x)-\mathcal{K}(e^{2\alpha x}-1)/2\alpha]}\ . \tag{13}$$

Equation (13) manifests the critical dependence of $|\beta(x)|^2$ on the pump intensity (i.e., $|E_P|^2$), the interaction path length L, and the Brillouin susceptibility of the crystal (via $\mathcal{K}$). As discussed earlier, we may consider $2\alpha x<1$. Consequently, one obtains

$$|\beta(x)|^2=\left[\frac{|E_S(0)|}{|E_P(L)|}\right]^2 e^{2[\alpha(L-2x)+\mathcal{K}x]} \tag{14}$$

with $|E_S(0)|^2$ being the noise intensity for the SBS process,[6] and its magnitude is generally taken to be about 10^{-12} to 10^{-13} times $|E_P(L)|^2$. Hence, significant phase conjugation can be achieved in the given crystal only if one finds $2[\alpha(L-2x)+\mathcal{K}x]\sim 30$ in (14) enabling oneself to have a gain $\sim e^{30}$ and the reflectivity $|\beta(x)|^2\sim 1$.

Since the phase-conjugate Brillouin mode is backscattered, it is easy to establish from (14) that at the entrance window ($x=L$) of the Brillouin cell of thickness L, one requires $[(\mathcal{K}-\alpha)L]\sim 15$ to achieve $|\beta(x=L)|^2\sim 1$. This result can be compared very well with the observations of Zel'dovich, Pilipetsky, and Shkunov[24] under the assumption of very low-loss Brillouin active media. From (11) and (12), we find that SBS with finite gain could occur even at excitation intensities as low as to satisfy the threshold condition for Brillouin gain given by $\mathcal{K}\geq\alpha$. But the above discussion makes it clear that excitation intensity $I_P(L)$ has to be much larger than the threshold value $I_{P,\mathrm{th}}$ if one aims at the attainment of significant OPC-SBS reflectivity [i.e., $|\beta(L)|^2\sim 1$] in a bulk crystal with $L=3$ mm.

It is clear from (12) that the threshold value of the excitation intensity $I_{P,\mathrm{th}}$ can be brought down by considering a semiconductor-laser interaction system yielding large Brillouin susceptibility χ_B. Furthermore, the OPC-SBS reflectivity $|\beta(L)|^2$ being directly dependent upon χ_B, an enhancement in $|\beta(L)|^2$ is also possible even at smaller sample length and not-too-high excitation intensity if one can achieve a large Brillouin susceptibility in the crystal. Thus, in Sec. III, we have examined analytically the possibility of obtaining large Brillouin susceptibility through the application of a high magnetostatic field and replacing the intrinsic semiconductor by an n-type moderately- to heavily-doped semiconducting crystal.

III. BRILLOUIN SUSCEPTIBILITY

We consider the well-known hydrodynamic model for a one-component (electron) semiconductor-plasma subjected to a pump electric field under thermal equilibrium. As the crystal is assumed to be centrosymmetric, the effects arising due to any pseudopotential have been neglected. We have employed the coupled-mode scheme to obtain the nonlinear polarization with its origin being in the finite electrostrictive strain.

The basic equations employed in the formulation of χ_B are as follows:

$$\frac{\partial^2 u}{\partial t^2}-\frac{C}{\rho}\frac{\partial^2 u}{\partial x^2}+2\Gamma_a\frac{\partial u}{\partial t}=\frac{\gamma}{2\rho}\frac{\partial}{\partial x}(E_P E_1^*)\ , \tag{15}$$

$$\frac{\partial \mathbf{V}_0}{\partial t}+\nu\mathbf{V}_0+(\mathbf{V}_0\cdot\nabla)\mathbf{V}_0=-\frac{e}{m}(\mathbf{E}_P+\mathbf{V}_0\times\mathbf{B}_0)\ , \tag{16}$$

$$\frac{\partial \mathbf{V}_1}{\partial t}+\nu\mathbf{V}_1+(\mathbf{V}_0\cdot\nabla)\mathbf{V}_1+(\mathbf{V}_1\cdot\nabla)\mathbf{V}_0=-\frac{e}{m}(\mathbf{E}_1+\mathbf{V}_1\times\mathbf{B}_0)\ , \tag{17}$$

$$V_0\frac{\partial n_1}{\partial x}+n_0\frac{\partial V_1}{\partial x}+n_1\frac{\partial V_0}{\partial x}=-\frac{\partial n_1}{\partial t}\ , \tag{18}$$

$$P_{eS}=-\gamma\frac{\partial}{\partial x}(E_P u^*)\ , \tag{19}$$

$$\frac{\partial E_1}{\partial x}=-\frac{en_1}{\epsilon}+\frac{\gamma}{\epsilon}\frac{\partial^2}{\partial x^2}(E_P u^*), \tag{20}$$

where (15) represents the motion of the lattice with u, C, ρ, Γ_a, and γ being lattice displacement, elastic constant, mass density, phenomenological acoustic damping parameter, and electrostriction coefficient of the crystal, respectively. The driving term in the right-hand side of (15) has its origin in the electrostrictive force induced by the pump electric field $\mathbf{E}_P\cos(\omega_P t)$ via the process of electrostriction. Equations (16) and (17) are the zeroth- and first-order electron momentum transfer equations, $\mathbf{V}_0$ and $\mathbf{V}_1$ being the zeroth- and first-order oscillatory fluid velocities of an effective mass m and charge e. ν is the electron collision frequency and $\mathbf{B}_0$ is the magnetostatic field applied along the z axis. Equation (18) is the continuity equation with n_0 and n_1 being the equilibrium and perturbed electron densities, respectively. Equation (19) reveals that the acoustic wave generated due to the electrostrictive strain modulates the dielectric constant, yielding a nonlinear induced polarization P_{eS}.

The space-charge field E_1 is determined by Poisson's equation (20) where ϵ is the dielectric function of the semiconductor and treated here as the lattice dielectric constant.

The electrostrictive force gives rise to a carrier density perturbation within the Brillouin active medium. In a doped semiconductor, this density perturbation can be obtained by using standard approach.[25] Differentiating (18), and on subsequent mathematical simplifications we get

$$\frac{\partial^2 n_1}{\partial t^2}+\nu\left[-V_0\frac{\partial n_1}{\partial x}-n_0\frac{\partial V_1}{\partial x}-n_1\frac{\partial V_0}{\partial x}\right]+V_0\frac{\partial^2 n_1}{\partial t\partial x}-V_0\frac{\partial V_0}{\partial x}\frac{\partial n_1}{\partial x}-n_0V_0\frac{\partial^2 V_1}{\partial x^2}-2n_0\frac{\partial V_1}{\partial x}\frac{\partial V_0}{\partial x}-n_0V_1\frac{\partial^2 V_0}{\partial x^2}$$
$$+\frac{\partial n_1}{\partial t}\frac{\partial V_0}{\partial x}-n_1\left[\frac{\partial V_0}{\partial x}\right]^2-n_1V_0\frac{\partial^2 V_0}{\partial x^2}-\frac{\partial}{\partial x}\left[n_1\left[\frac{e}{m}E_P+\omega_c V_{0y}\right]\right]+n_0\frac{\partial}{\partial x}\left[-\frac{e}{m}(E_1+B_0V_{1y})\right]=0\ . \tag{21}$$

Now, the fields associated with the pump, Stokes mode, and the acoustic wave are considered to be varying as $\exp[i(\omega_P t-k_P x)]$, $\exp[-i(\omega_S t-k_S x)]$, and $\exp[i(\omega_a t-k_a x)]$, respectively. These phase variations satisfy the condition to achieve optical phase conjugation via stimulated Brillouin scattering in the crystal and can be utilized to simplify (21) further, to get

$$\frac{\partial^2 n_1}{\partial t^2}+\nu'\frac{\partial n_1}{\partial t}+V_0\frac{\partial^2 n_1}{\partial t\partial x}+\omega_{Sc}^2\Omega_P^2 n_1 -\frac{\omega_{Sc}^2 n_0\gamma}{\epsilon}\frac{\partial^2}{\partial x^2}(\mathcal{E}u^*)=\omega_{Pc}^2\frac{\partial}{\partial x}(n_1\mathcal{E})\ , \tag{22}$$

where

$$\nu'=\nu+\frac{\partial V_0}{\partial x},\quad \mathcal{E}=\frac{eE_P}{m},\quad \omega_{Sc}^2=\frac{\omega_S^2}{\omega_S^2-\omega_c^2}$$
$$\omega_{Pc}^2=\frac{\omega_P^2}{\omega_P^2-\omega_c^2}\ .$$

The density perturbation n_1 will have two components given by $n_1=n_{1l}+n_{1h}$, where the low-frequency component n_{1l} is associated with the acoustic-wave vibrations at ω_a and the high-frequency component oscillates at the electromagnetic wave frequencies $(\omega_P\pm\omega_a)$. The higher-order terms with frequencies $(A\omega_P\pm\omega_a)$ for $A\geq 2$ will be off resonant and are neglected in the study of SBS mechanisms. We have considered only the Stokes component of the scattered electromagnetic wave ($\omega_S=\omega_P-\omega_a$ and $\mathbf{k}_S=\mathbf{k}_P-\mathbf{k}_a$). One may ensure oneself that the stimulated Brillouin scattering under consideration does not violate the phase-matching conditions, $\hbar\omega_P=\hbar\omega_S+\hbar\omega_a$ and $\hbar\mathbf{k}_P=\hbar\mathbf{k}_S+\hbar\mathbf{k}_a$.

Under rotating-wave approximation (RWA), (22) yields the following coupled equations:

$$\frac{\partial^2 n_{1l}}{\partial t^2}+\nu'\frac{\partial n_{1l}}{\partial t}+\omega_{Sc}^2\Omega_P^2 n_{1l}=\omega_{Pc}^2\mathcal{E}\frac{\partial n_{1h}}{\partial x}+\omega_{Pc}^2 n_{1h}\frac{\partial\mathcal{E}}{\partial x}-V_0\frac{\partial^2 n_{1h}}{\partial t\partial x} \tag{23}$$

and

$$\frac{\partial^2 n_{1h}^*}{\partial t^2}+\nu'\frac{\partial n_{1h}^*}{\partial t}+\omega_{Sc}^2\Omega_P^2 n_{1h}^*=\omega_{Pc}^2\mathcal{E}\frac{\partial n_{1l}^*}{\partial x}+\omega_{Sc}^2 n_{1l}^*\frac{\partial\mathcal{E}}{\partial x}-V_0\frac{\partial^2 n_{1l}}{\partial t\partial x}+\frac{\omega_{Sc}^2 n_0\gamma}{\epsilon}\frac{\partial^2}{\partial x^2}(\mathcal{E}u^*)\ . \tag{24}$$

Using (1), (23), and (24), we obtain

$$n_{1l}=\frac{n_0\gamma^2(k_S+k_P)(k_a-k_P)\left[\dfrac{(\delta_1^2-i\nu'\omega_S)(\delta_2^2+i\nu'\omega_a)}{k_S(k_a-k_P)\omega_{Pc}^4\mathcal{E}^2}-1\right]E_PE_S^*}{2\epsilon\rho(\omega_a^2-k_a^2V_a^2-2i\Gamma_a\omega_a)}\simeq\frac{n_0\gamma^2k^2E_PE_S^*}{\epsilon\rho(\omega_a^2-k_a^2V_a^2-2i\Gamma_a\omega_a)}\ , \tag{25}$$

where $V_a[=(C/\rho)^{1/2}]$ is the acoustic velocity of the medium. For simplicity we consider $\delta_1=[\omega_{Sc}^2\Omega_P^2-\omega_S^2]^{1/2}$ and $\delta_2=[\omega_{Sc}^2\Omega_P^2-\omega_a^2]^{1/2}$.

Now we proceed to obtain the third-order optical susceptibility of the crystal. The resonant Stokes component of the nonlinear current density due to finite nonlinear induced polarization is given as

$$J_S(\omega_S)\sim -n_{1l}^{*}eV_0 \; . \tag{26a}$$

Here, we have considered only the first-order nonlinear current density which is responsible for the three-wave coupling. The preceding analysis under RWA yields the nonlinear current density as

$$J_S(\omega_S)=\frac{k^2\gamma^2\Omega_P^2\omega_S|E_P|^2E_S}{\rho(\omega_P^2-\omega_c^2)(\omega_a^2-k_a^2V_a^2-2i\Gamma_a\omega_a)} \; . \tag{26b}$$

The time integral of (26b) yields the expression for the nonlinear polarization $P_{\rm NL}$ as

$$P_{\rm NL}=\frac{k^2\gamma^2\Omega_P^2|E_P|^2E_S}{\rho(\omega_P^2-\omega_c^2)(\omega_a^2-k_a^2V_a^2-2i\Gamma_a\omega_a)} \; . \tag{27}$$

Again, the nonlinear polarization can be written in terms of the nonlinear susceptibility, known as the Brillouin susceptibility χ_B and electric field as

$$P_{\rm NL}=\epsilon_0\chi_B|E_P|^2E_S \; . \tag{28}$$

From (27) and (28), one finds the Brillouin susceptibility of the cubic nearly centrosymmetric semiconductor crystal as

$$\chi_B=\frac{k^2\gamma^2\Omega_P^2}{\epsilon_0\rho(\omega_P^2-\omega_c^2)(\omega_a^2-k_a^2V_a^2-2i\Gamma_a\omega_a)} \tag{29}$$

yielding finally

$$\chi_B=-i|\chi_B| \; , \tag{30}$$

$$|\chi_B|=\frac{k^2\gamma^2\Omega_P^2}{2\epsilon_0\rho\Gamma_a\omega_a(\omega_P^2-\omega_c^2)}$$

for dispersionless acoustic wave propagation (i.e., $\omega_a^2=k_a^2V_a^2$).

IV. RESULTS AND DISCUSSIONS

The present theoretical formulations as developed in Secs. II and III are analyzed in this section to study the nature of the dependence of OPC-SBS reflectivity $|\beta(x=L)|^2$ on system parameters like the excitation intensity, the doping concentration, and the applied magnetostatic field. For this purpose, we have chosen an n-type III-V semiconducting crystal, viz., InSb as the Brillouin active medium. In order to make the estimation compatible with requirements like off-resonant laser excitation, we consider the irradiation of n-type InSb by pulsed 10.6 μm CO_2. This enables one to assume the crystal absorption coefficient to be appreciably small. The Brillouin susceptibility χ_B of InSb is estimated for dispersionless acoustic propagation from the knowledge of the material parameters and by using (30).

The material constants are as follows:[26] $\gamma=5\times10^{-10}$ fm^{-1}, $\rho=5.8\times10^3$ kg m^{-3}, $\Gamma_a=5\times10^8$ sec^{-1}, $\omega_a=4.8\times10^9$ sec^{-1}, $k=6\times10^5$ m^{-1}, $\omega=1.778\times10^{14}$ sec^{-1}, and $\alpha=50$ m^{-1}.

The threshold value of the pump intensity required for the onset of optical phase conjugation via SBS process can be obtained by using (12) and (30) as

$$I_{P,\rm th}=\frac{2\eta\epsilon_0^2c^2\alpha\rho\Gamma_a\omega_a(\omega^2-\omega_c^2)}{\gamma^2k^2\omega\Omega_P^2} \; . \tag{31}$$

At pump intensities $I_P(L)>I_{P,\rm th}$, one can get finite OPC-SBS reflectivity. Equation (31) manifests the critical dependence of the SBS threshold on the applied magnetostatic field (via the electron cyclotron frequency $\omega_c=eB_0/m$) and the carrier concentration n_0 [via the electron plasma frequency $\Omega_P=(n_0e^2/m\epsilon_0\epsilon_L)$]. It is evident from (31) that $I_{P,\rm th}$ decreases sharply with an increase in B_0 when ω_c is in the vicinity of the pump frequency ω. Pidgeon and Brown[27] performed the experimental measurements of the interband magnetic absorption in InSb at magnetic fields as high as 9.65 T. While discussing spin-flip scattering in n-type InSb pumped by a 10.6-μm CO_2 laser, Wolff[28] considered a 0–10 T range of the magnetic field. Spector and his co-workers[29,30] studied analytically the phenomenon of free-carrier absorption in InSb crystal at 77 K duly irradiated by CO_2 and CO lasers with magnetic fields $B_0\le 20$ T. For a numerical estimation of $I_{P,\rm th}$ in InSb, we have considered magnetic field $B_0=14.1$ T, which yields the electron frequency $\omega_c=1.771\times10^{14}$ sec^{-1}. Such magnetic strengths may be achieved in most of the laboratories specialized in the study of magnetic-field effects on the optical properties of solids. Further, if one takes the doping concentration $n_0=10^{24}$ m^{-3} one gets the electron plasma frequency $\Omega_P=1.2\times10^{14}$ sec^{-1}. Using the above values, we find that the required value of the threshold intensity $I_{P,\rm th}$ from (31) as 1.44 MW cm^{-2}, such magnitudes of excitation intensity can be obtained by using a nanosecond pulsed 10.6-μm CO_2 laser.

For pump intensity $I_P(L)>I_{P,\rm th}$, one can obtain finite OPC-SBS reflectivity $|\beta(x=L)|^2$ at the entrance window by using (14) as

$$|\beta(L)|^2=\left[\frac{|E_S(0)|}{|E_P(L)|}\right]^2e^{2(\mathcal{K}-\alpha)L} \; . \tag{32}$$

At $I_P(L)>I_{P,\rm th}$, $\mathcal{K}>\alpha$, but since the ratio $|E_S(0)/E_P(L)|^2$ is usually taken to be as small as 10^{-13}, considerably large OPC-SBS reflectivity [i.e., $|\beta(L)|^2\sim 1$] can be achieved when

$$(\mathcal{K}-\alpha)L\simeq 15 \tag{33}$$

such that $e^{2(\mathcal{K}-\alpha)L}\sim 10^{13}$. One may find such large reflectivity without much difficulty in a material system that is in the form of a low-loss optical fiber or waveguide having a very large interaction path length and which is duly irradiated by a not-too-intense off-resonant laser. As is well-known in the presence of a large magnetic field, a III-V semiconductor like InSb can exhibit giant Brillouin susceptibility,[17] an interaction length in the mm range can be sufficient to get $|\beta(L)|^2 \sim 1$ at pump intensities below the optical damage threshold. Accordingly, we consider a bulk InSb crystal with $L=3$ mm subjected to off-resonant nanosecond pulsed 10.6-μm CO_2 laser excitation. The pump intensity required to achieve $|\beta(L)|^2 \sim 1$ in the n-type InSb crystal subjected to a high magnetic field is obtained by using (9b) in (33) as

$$I_{Pr} = \frac{30\eta\epsilon_0 c^2 \rho \Gamma_a \omega_a (\omega^2 - \omega_c^2)}{\gamma^2 k^2 \omega \Omega_P^2 L} . \tag{34}$$

A simple comparison between (32) and (34) enables one to relate I_{Pr} with the SBS threshold intensity as

$$I_{Pr} = \frac{15}{\alpha L} I_{P,\mathrm{th}} . \tag{35}$$

Equation (34) reveals that high OPC-SBS reflectivity of the order of unity is achievable in the Brillouin active medium at pump intensities below the optical damage threshold if one uses heavily doped n-type semiconductor waveguides with much longer interaction path lengths. In our numerical estimation, we have taken $\alpha = 0.5\ \mathrm{cm}^{-1}$ and sample length $L=3$ mm which yields $I_{Pr} = 100 I_{P,\mathrm{th}} \sim 144\ \mathrm{MW\,cm}^{-2}$ for $I_{P,\mathrm{th}} = 1.44\ \mathrm{MW\,cm}^{-2}$. Ji *et al.*[31] observed optical bistability experimentally in InSb crystal at excitation intensities as high as 1 $\mathrm{MW\,cm}^{-2}$ obtainable from 1.5- to 2-μs pulsed CO_2 lasers and did not notice any optical damage. Hence, in the present study of OPC-SBS we may safely assume that the intensity in the range of 1–140 $\mathrm{MW\,cm}^{-2}$ may be well below the crystal damage threshold under the nanosecond pulsed irradiation limit and a relatively high OPC-SBS reflectivity can be achieved in the InSb waveguide with $L=3$ mm.

The characteristic dependence of OPC-SBS reflectivity on the pump intensity, excess carrier concentration, and the magnetic field in n-type InSb at 77 K duly shined by a 10.6-μm CO_2 laser are studied by using (9b), (30), and (32) for $\alpha_I = \alpha$ and $\alpha L < 1$. The results are plotted in Figs. 1–3. Figure 1 illustrates the nature of the dependence of

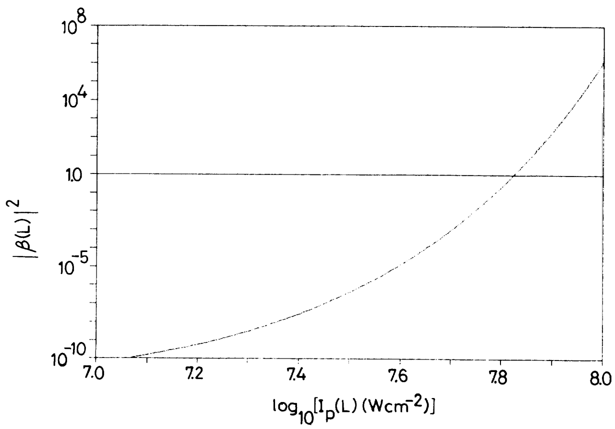

FIG. 1. Dependence of OPC-SBS reflectivity $|\beta(L)|^2$ on the pump intensity $I_P(L)$ in n-type InSb crystal at 77 K. The magnetic field is taken to be 14.1 T and the electron concentration $n_0 = 2\times10^{24}\ \mathrm{m}^{-3}$.

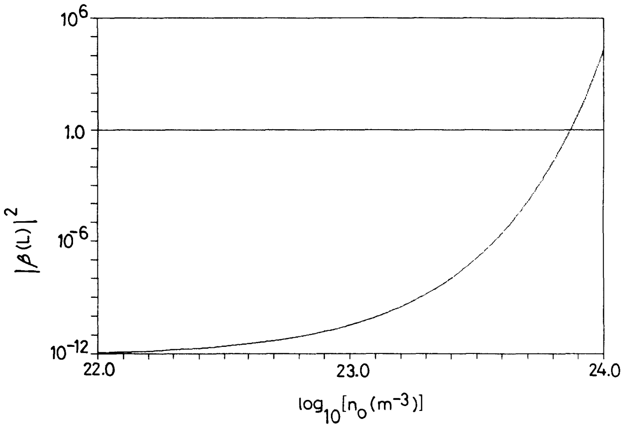

FIG. 2. Variation of OPC-SBS reflectivity $|\beta(L)|^2$ with the electron concentration n_0 in InSb crystal. The magnetic field and the pump intensity have been taken to be 14.1 T and 90 $\mathrm{MW\,cm}^{-2}$, respectively.

OPC-SBS reflectivity $|\beta(L)|^2$ on the pump intensity $I_P(L)$ at the high magnetic field. This plot shows sharp increase in $|\beta(L)|^2$ with increase in pump intensity. Thus, one can achieve $|\beta(L)|^2$ by using high excitation intensity. This is in qualitative agreement with the experimental observations in waveguides.[32] It may be recalled that $I_P(L)$ cannot be increased arbitrarily which may finally lead to the optical damage of the crystal. We have also studied the variation of the OPC-SBS reflectivity of the n-type semiconducting crystal with increment of doping concentration and the reflectivity is being plotted in Fig. 2.

This plot also shows a sharp dependence of the OPC-SBS reflectivity on the doping concentration, and one may infer that $|\beta(L)|^2$ can be appreciably high in heavily doped semiconductors even at fixed values of the magnetic field and the pump intensity.

We now address ourselves to the most important aspect of the investigation involving the role of externally applied magnetostatic field on the OPC-SBS reflectivity $|\beta(L)|^2$. The numerical estimates made for n-type InSb

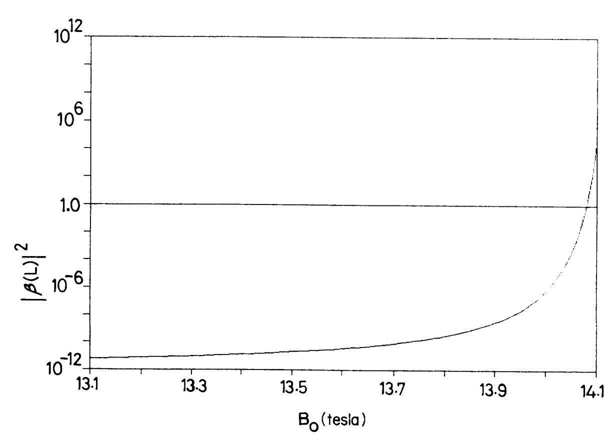

FIG. 3. Magnetic-field dependence of OPC-SBS reflectivity $|\beta(L)|^2$ in n-type InSb crystal at pump intensity $I_P(L) = 90$ MW cm^{-2} and doping concentration of $n_0 = 2\times10^{24}\ \mathrm{m}^{-3}$.

crystal show that $|\beta(L)|^2$ can be remarkably enhanced even at not-too-high excitation intensity and doping level simply by using a large magnetostatic field. This characteristic feature is depicted in Fig. 3. It is noteworthy that the enhancement is considerable only at high magnetic field when the electron cyclotron frequency becomes nearly resonant with the pump photon frequency. Thus one may infer that for high magnetic fields satisfying the condition $\omega_c \sim \omega$, the Brillouin susceptibility increases sharply and results in the sharp enhancement of the OPC-SBS reflectivity. Consequently, $|\beta(L)|^2 \sim 1$ can be obtained under off-resonant nanosecond pulsed irradiation in a mm-thick n-type crystal below the crystal damage threshold.

V. CONCLUSIONS

The present paper deals with the analytical investigation of optical phase conjugation via stimulated Brillouin scattering in nearly cubic crystal like n-type III-V semiconductors GaAs and InSb subjected to large magnetostatic field. The following important conclusions can be drawn on the basis of the above study.

(i) We have followed the simple electromagnetic coupled-mode approach considering the hydrodynamic model of the semiconductor plasma. The total induced polarization is assumed to have the origin in the nonlinear current density. The electrons are taken to be highly mobile while the holes are at rest. The free-electron hole plasma generation occurs due to relatively high excitation intensity when the weakly bound Wannier-Mott excitation structure of the absorption edge disappears completely.

(ii) The crystals are irradiated by an off-resonant nanosecond pulsed laser with pump photon energy below the band-gap energy such that the possibility of band-to-band photoinduced electronic transitions can be ruled out and the background absorption coefficient is low enough to satisfy the condition $\alpha L < 1$ for sample thickness in the millimeter regime.

(iii) n-type doping of the semiconductor can yield SBS with a high-gain constant at lower excitation intensity. Doping enhances the OPC reflectivity as well.

(iv) The application of a large magnetic field reduces the threshold value of the pump intensity quite significantly. For magnetic fields at which electron-cyclotron frequency becomes comparable to the pump frequency, OPC reflectivity is enhanced by a few orders of magnitude.

(v) Numerical estimates made for the InSb–CO_2-pulsed-laser system establishes the possibility of phase conjugating infrared radiation in the 10-μm region with very high reflectivity by using n-type crystal and large magnetic field.

ACKNOWLEDGMENTS

The authors are thankful to Dr. Pratima Sen for fruitful discussions. Financial assistance from the Council of Scientific and Industrial Research, India is gratefully acknowledged.

---

*Present address: Department of Applied Physics, Shri G. S. Institute of Technology and Science, 23 Park Road, Indore 452003, India.

[1]J. J. Gerritsen, Appl. Phys. Lett. **10**, 237 (1967).

[2]P. Woerdman, Opt. Commun. **2**, 212 (1971).

[3]B. I. Stepanov, E. V. Ivakin, and A. S Rubanov, Sov. Phys. Dokl. Tech. Phys. **15**, 46 (1971).

[4]A. Yariv and D. M. Pepper, Opt. Lett. **1**, 16 (1977).

[5]R. W. Hellwarth, J. Opt. Soc. Am. **67**, 1 (1977).

[6]B. Ya. Zel'dovich, N. F. Pilipetsky, and V. V. Shkunov, *Principles of Phase Conjugation* (Springer, New York, 1985), pp. 1–65.

[7]B. Ya. Zel'dovich, V. I. Popovichev, V. V. Ragulsky, and F. S. Faizullov, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 160 (1972) [JETP Lett. **15**, 109 (1972)].

[8]R. W. Hellwarth, J. Opt. Soc. Am. **68**, 1050 (1978).

[9]H. Brusselbach and D. Rockwell (private communication).

[10]N. F. Andreev, V. I. Bespalov, M. A. Dvoretsky, E. V. Katin, A. Z. Matreev, and G. A. Pasmanik, Rev. Romain Phys. **32**, 951 (1986).

[11]P. Suni and J. Falk, J. Opt. Soc. Am. B **13**, 1681 (1986).

[12]R. H. Lehmberg, J. Opt. Soc. Am. **73**, 558 (1983).

[13]D. A. Rockwell, IEEE J. Quantum Electron. **QE-24**, 1124 (1988).

[14]A. M. Scott and K. D. Ridley, IEEE J. Quantum Electron. **QE-25**, 438 (1989).

[15]V. I. Bespalov, A. M. Kiseljor, and G. A. Pasmanik, SPIE Sympos. Opt. **84**, 286 (1984).

[16]P. K. Sen, Phys. Rev. B **21**, 3604 (1980).

[17]P. K. Sen and P. Sen, Phys. Rev. B **25**, 7741 (1982).

[18]D. M. Pepper, Opt. Eng. **21**, 155 (1982).

[19]P. K. Sen and P. Sen, J. Appl. Phys. **63**, 4637 (1988).

[20]P. Yeh, IEEE J. Quantum Electron. **QE-25**, 484 (1989).

[21]D. M. Pepper, in *Laser Handbook*, edited by M. L. Stitch and M. Bass (Elsevier, Amsterdam, 1985), Vol. 4, pp. 333–485.

[22]P. K. Sen, Solid State Commun. **43**, 141 (1982).

[23]P. K. Sen, Phys. Status Solidi B **124**, 117 (1984).

[24]B. Ya. Zel'dovich, N. F. Pilipetsky, and V. V. Shkunov, in *Optical Phase Conjugation*, edited by R. A. Fisher (Academic, New York, 1983), pp. 135–167.

[25]S. Guha and P. K. Sen, Phys. Lett. **70A**, 61 (1979).

[26]P. Sen and P. K. Sen, Phys. Rev. B **31**, 1034 (1985).

[27]C. R. Pidgeon and R. N. Brown, Phys. Rev. **146**, 575 (1966).

[28]P. A. Wolff, in *Nonlinear Optics*, edited by P. G. Harper and B. S. Wherrett (Academic, London, 1977), pp. 169–212.

[29]E. R. Generazio and H. N. Spector, Phys. Rev. B **20**, 5162 (1979).

[30]T. M. Rynne and H. N. Spector, J. Phys. Chem. Solids **42**, 121 (1981).

[31]W. Ji, A. K. Kar, U. Keller, J. G. H. Mathew, and A. C. Walker, in *Optical Bistability III*, edited by H. M. Gibbs, P. Mandel, N. Peyghambarian, and S. D. Smith (Springer-Verlag, Berlin, 1986), pp. 35–38.

[32]J. T. Remillard, H. Wang, M. D. Webb, and D. G. Steel, IEEE J. Quantum Electron. **QE-25**, 408 (1989).