Universality classes for line-depinning transitions

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The universality classes and exact critical singularities for line-depinning transitions in a space of d transverse dimensions are determined using a renormalization method. Pinning potentials that fall off faster with distance than $1/r^2$ lead to nontrivial first-order phase transitions above the upper critical dimensionality $d=4$, and to second-order transitions for $d<4$. For $d=2$ the free-energy density has an essential singularity of the form $\exp(-1/\tau)$, were τ is the thermal scaling field. The next-nearest corrections to the free energy will be calculated for the case where the long-range part of the pinning potential decays faster than $1/r^2$. Pinning potentials containing an inverse square tail can give rise to a nontrivial first-order phase transition above an upper critical dimension, second-order transitions with nonuniversal exponents, or Kosterlitz-Thouless-like transitions with a multicritical point between the last two regimes, depending on the strength of the interaction. Attractive pinning potentials decaying slower than $1/r^2$ prevent depinning transitions at finite temperature, whereas repulsive ones in the presence of short-range attraction lead to first-order transitions.

I. INTRODUCTION

Our present understanding of surface depinning phenomena, namely wetting and roughening phase transitions (see reviews by Fisher[1] and Weeks,[2] correspondingly), is based mainly on the analysis of a relatively simple class of models using the solid-on-solid (SOS) approximation which accounts for the crucial surface fluctuations and ignores irrelevant bulk degrees of freedom. The SOS Hamiltonian describes a structureless interface placed in an asymmetric potential with an attractive part (for the case of wetting transition) or in a periodic potential (for the case of the roughening transition). Different theoretical techniques have been developed to analyze these Hamiltonians. The renormalization-group (RG) method is very successful in finding the critical singularities both for the roughening[2] and wetting[3] transitions in three bulk dimensions. The case of wetting phenomena in two bulk dimensions can be treated by a transfer operator technique. For short-range potentials this method gives the exact critical singularities[4,5] found first by Abraham.[4] Kroll and Lipowsky[6] and Chui and Ma[7] have used a transfer operator technique to extend the previous studies[4] to the case of pinning potentials of more general form. One of the most exciting findings of their treatment is the prediction of a wetting transition in the presence of an inverse square attraction between the wetting liquid interface and a substrate, with an essential singularity characteristic of the Kosterlitz-Thouless[8] and roughening phase transitions.[2] Discussing this curious result Fisher and Huse[3] expressed the hope that it could be rederived using a RG, but the linear functional RG worked out in Ref. 3 was not sufficiently accurate to reproduce this property. Another challenge to the theory is to explain the new wetting regimes recently discovered by Lipowsky and Nieuwenhuizen.[9]

In this paper we will demonstrate that most of the known findings concerning two-dimensional wetting can be found using a RG. In addition, new *exact* results are given concerning the singularities near the depinning transition for a linear object placed in a space of the arbitrary dimension. Some of our findings for suitable choice of pinning potentials can be obtaining using standard transfer operator technique, but we prefer to use the RG for the following reasons: it gives us an easy route to the critical singularities and makes evident which details of the pinning potential are relevant; the resulting RG equations are *exact;* and the derivation involves a novel interpretation of the RG which may be useful in other contexts. We will choose the following strategy: first we will use a standard transfer operator technique to transform our problem into one involving the Schrödinger equation, then we will analyze this equation via a RG.

II. STATEMENT OF THE PROBLEM

Let us consider a linear object (i.e., a vortex line in a type-II superconductor, a directed polymer, or an interface in the context of two-dimensional wetting) imbedded in a $d+1$-dimensional space and subject to a pinning potential $V(\mathbf{x})$, where $\mathbf{x}$ is the d-dimensional vector denoting the line position. The corresponding continuum SOS Hamiltonian has the form[1,3,6,7,9]

$$H=\int dt \left\{ \frac{m}{2}\left[\frac{d\mathbf{x}}{dt}\right]^2 + V(\mathbf{x}) \right\} , \tag{1}$$

where t is the coordinate along the line, m is the line stiffness and $\mathbf{x}(t)$ describes the line trajectory. The first term of the Hamiltonian represents the elastic energy of a distorted line, which is proportional to its excess length with respect to a straight one. The transfer operator technique[4,7,9] allows us to turn the evaluation of the partition function for (1) into an eigenvalue problem. This approach has been discussed in detail by Kogut[10] and is related to Feynman's formulation of quantum mechanics.

In the present context it gives rise to a one-particle quantum-mechanics problem in d dimensions, and the singular part F of the line free-energy density is given in the thermodynamic limit $t \to \infty$ by the lowest energy eigenvalue of the Schrödinger equation

$$\left[-\frac{T^2}{2m}\Delta_d + V(\mathbf{x})\right]\Psi_i(\mathbf{x}) = E_i\Psi_i(\mathbf{x}) , \qquad (2)$$

where the temperature T plays the role of Planck's constant and Δ_d is the d-dimensional Laplacian. Knowledge of the bound-state energy $E_0 = F$ and the wave function $\Psi_0(\mathbf{x})$ enables us to calculate the moments

$$\langle \mathbf{x}^n \rangle = \int d^d x \, \Psi_0^*(\mathbf{x})\mathbf{x}^n\Psi_0(\mathbf{x}) \Big/ \int d^d x \, \Psi_0^*(\mathbf{x})\Psi_0(\mathbf{x}) , \qquad (3)$$

where n is an integer (some of these moments could be identically zero by symmetry of the pinning potential). When there is a bound-state solution ($E_0 < 0$), the bound-state wave function is well localized, and the moments are finite. When the bound state disappears $E_0 \to 0$ and $\langle \mathbf{x}^n \rangle \to \infty$, indicating a depinning transition. If the bound-state wave function has a localization length $\xi_\perp$, the moments (3) diverge like $\xi_\perp^n$. In what follows we consider only the cases where $\xi_\perp$ is large relative to the microscopic scale, i.e., in the vicinity of the depinning transition.

Equation (2) cannot be solved for general $V(\mathbf{x})$. However, some results concerning the vicinity of the phase transition can be achieved without detailed knowledge of $V(\mathbf{x})$. Such analysis was performed by several authors[6,7,9] for $d = 1$ and the case where a half-space is not available for a particle (this imitates the presence of an impenetrable substrate in the wetting problem). We will consider another case in which the line is pinned in the bulk, i.e., the pinning potential is symmetric: $V(\mathbf{x}) = V(-\mathbf{x})$. Among the experimental realizations of our system would be the pinning of a single vortex in a type-II superconductor by a twin boundary (this problem has effective dimension $d = 1$) or by a linear pin ($d = 2$; pins of this kind have recently been created by bombarding initially clean YBCO samples with Sn ions[11]). However, we claim that our results for the symmetric potential are equally relevant to phase transitions involving depinning from the edge of a sample (like the wetting transition), since near the wetting point the distance between the interface and substrate is macroscopically large and goes to infinity at the phase-transition point; the asymmetry imposed by the edge of the system is asymptotically irrelevant in the critical region, and thus does not influence the critical singularities. The asymmetry only affects the value of the phase-transition temperature, which is nonuniversal. For example, we will see below that bulk depinning from a short-range potential is impossible at any finite temperature for $d \le 2$ (this result is an implication of standard quantum mechanics[12] and has been noted by several authors[4-6,1] for $d = 1$); nevertheless the knowledge of the localization length behavior for $T \to \infty$ (near the "phase-transition point" $T = \infty$) enables us to determine its singularity, which is the same as that for a finite temperature depinning transition which belongs to the same universality class. This is a significant advance since the one-dimensional quantum-mechanical problem in a half-space even for a special choice of the pinning potential[9] involves quite complicated mathematics. Moreover, the application of the established approach to potentials of less model form and higher space dimensionalities is extremely difficult, while the RG treatment of the bulk pinning problem is relatively simple for any space dimensionality and the corresponding results are proved to be rigorous. Using the idea of *universality* (see Ref. 1 and references therein) we can extend the results for the depinning transition to other representatives of the same universality class (the essential features characterizing the universality classes also follow from the RG).

The line-depinning transition involves two length scales, $\xi_\perp$ and $\xi_\parallel$. The latter is the longitudinal correlation length, which characterizes the correlations along the t direction.[1] In the transfer operator formalism the longitudinal correlation length is connected to the bound-state energy by the hyperscaling relation

$$\xi_\parallel = \frac{T}{|E_0|} . \qquad (4)$$

The RG, described below, allows us to calculate the bound-state wave-function localization length $\xi_\perp$ (or the transverse correlation length). To find the bound-state energy we use the scaling expression

$$E_0 = -DT^2/m\xi_\perp^2 , \qquad (5)$$

where D is a constant; this seems quite natural in the framework of the quantum mechanics. Combination of (4) and (5) implies a connection between $\xi_\parallel$ and $\xi_\perp$,

$$\xi_\perp^2 = \frac{DT}{m}\xi_\parallel , \qquad (6)$$

which is well established in the context of the two-dimensional wetting transitions.[1] Therefore, the problem has a single independent scale ($\xi_\perp$, to be definite) and all we need is to find it.

III. RG EQUATIONS AND THEIR PHYSICAL MEANING

Consider the scattering of a "particle" with zero wave vector into the final state with wave vector $\mathbf{k}$ under action of the external potential $V(\mathbf{x})$. The amplitude of this process in the first Born approximation is proportional to the Fourier component of the interaction potential $V(\mathbf{k})$.[12] Taking into account double-scattering processes requires the first-order term $V(\mathbf{k})$ to be replaced by its more exact value $V^R(\mathbf{k})$,[12]

$$V^R(\mathbf{k}) = V(\mathbf{k}) - \frac{2m}{T^2}\int^\Lambda \frac{d^d q}{(2\pi)^d}\frac{V(|\mathbf{k}-\mathbf{q}|)V(q)}{q^2} , \qquad (7)$$

where the second term is the second Born approximation for the scattering amplitude. The upper limit of the integral is a momentum cutoff set by the short-range part of the potential $V(\mathbf{x})$. For weak potentials, the second term in Eq. (7) can be considered as a small correction to the first, provided that the integral in (7) converges.

However, Eq. (7) can have a divergence at the lower limit $q=0$; for instance, this integral diverges for $d \le 2$ if $V(\mathbf{k}) \approx \text{const}$. Then we would have to include an infinite set of terms to go beyond the perturbative regime.

An equivalent and more systematic way is to use the RG.[13] The strategy is to integrate out the shortest wavelengths, giving an effective potential $\mathcal{V}$; by repetition of this process we eventually come to the observable behavior at long wavelengths (or large distances). Since we are interested in the small-$\mathbf{k}$ behavior (our final aim is to find E_0 for $E_0 \to 0$), we will expand the second-order term of Eq. (7) in $\mathbf{k}$ for $\mathbf{k} \to 0$. The leading term is just the integral of $V(q)^2/q^2$; the nonvanishing corrections in $\mathbf{k}$ (for an isotropic potential depending only on the absolute value of $\mathbf{k}$) is of order k^2 or higher and, as will be made clear below, can be omitted. So we shall have after the angular integration,

$$V^R(k) = V(k) - \frac{2mK_d}{T^2}\int_0^\Lambda dq\, q^{d-3}V^2(q)\ , \tag{8}$$

where $K_d = S_d/(2\pi)^d$ and $S_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface area of a d-dimensional unit sphere. Let us split the second integral in Eq. (8) in two parts: from zero to $\Lambda(1-dl)$ and from $\Lambda(1-dl)$ to Λ, where dl is infinitesimal, and integrate over the high-momentum slice. The result,

$$V^R(k) = V(k) - \frac{2mK_d\Lambda^{d-2}}{T^2}V^2(\Lambda)dl - \frac{2mK_d}{T^2}\int_0^{\Lambda(1-dl)} dq\, q^{d-3}V^2(q)\ , \tag{9}$$

can be used to define the renormalized potential

$$\mathcal{V}(k) = V(k) - \frac{2mK_d\Lambda^{d-2}}{T^2}V^2(\Lambda)dl\ . \tag{10}$$

This demonstrates that multiple scattering generates a short-ranged (i.e., independent of k) contribution to the external potential, even if we had initially just a k-dependent external potential. This forces us to consider a pinning potential of more general form. Suppose it is comprised of a short-range part and a long-range tail of the form V_s/r^s where V_s is the amplitude, r is the radial coordinate, and s is some exponent. The Fourier transform of this tail in leading order for $k \to 0$ must have the form

$$V_s\int_a d^dx\, e^{ikx}r^{-s} = \text{const}\, V_s\int_a^{1/k} dr\, r^{d-1-s} = AV_s\frac{k^{s-d}-a^{d-s}}{d-s}\ , \tag{11}$$

where we have replaced the oscillating factor e^{ikx} by unity for distances between the small-scale cutoff $a \equiv 1/\Lambda$ and $1/k$ and neglected the contribution from distances exceeding $1/k$. The small-scale cutoff is necessary since the integral $\int d^dx\, e^{ikx}r^{-s}$ diverges for $d \le s$ at the lower limit. The constant term in Eq. (11) depends on our cutoff procedure; however, the coefficient A does not, and is equal to

$$A = 2^{d-s+1}\pi^{d/2}\Gamma\left[\frac{d-s}{2}+1\right]/\Gamma(s/2)\ . \tag{12}$$

For the important special case $s=2$ this reduces to $A = 1/K_d$. The short-range part of the potential would add only a constant to Eq. (11). Therefore we consider a potential of the form

$$V(k) = V_0 + \frac{V_sAk^{s-d}}{d-s} \tag{13}$$

and refer to V_0 as the short-range amplitude. Since we have neglected the k^2 contributions to $V^R(k)$ in the derivation of Eq. (10), the k-dependent part of (13) is the leading term only for $s-d<2$; and we shall see below that only these values of s and d are of interest. Substitution of expression (13) into (10) gives the renormalized values of $\mathcal{V}_0$ and $\mathcal{V}_s$:

$$\mathcal{V}_0 = V_0 - \frac{2mK_d\Lambda^{d-2}}{T^2}\left[V_0 + \frac{V_sA\Lambda^{s-d}}{d-s}\right]^2 dl\ , \tag{14}$$

$$\mathcal{V}_s = V_s\ . \tag{15}$$

The original form of the Hamiltonian (1) is recovered by rescaling $\mathbf{k} \to \mathbf{k}(1-dl)$, together with $\mathbf{x} \to \mathbf{x}(1+dl)$, $t \to t(1+dl)^2$, $V_0 \to V_0(1+dl)^{d-2}$, and $V_s \to V_s(1+dl)^{s-2}$. There is an additional correction to V_0 due to the elimination of the high-momentum slice (14). This leads to the set of the RG equations

$$\frac{dV_0(l)}{dl} = (2-d)V_0(l) - \frac{2mK_d\Lambda^{d-2}}{T^2}\left[V_0(l) + \frac{V_s(l)A\Lambda^{s-d}}{d-s}\right]^2\ , \tag{16}$$

$$\frac{dV_s(l)}{dl} = (2-s)V_s(l)\ . \tag{17}$$

Multiple scattering screens the potential so that its amplitudes appear to be different at larger distances; these differential equations allow calculation of the amplitudes at distance $r = ae^l$.

Introducing the dimensionless variables

$$u(l) = \frac{2mK_d}{T^2a^{d-2}}\left[V_0(l) + \frac{V_s(l)Aa^{d-s}}{d-s}\right]\ , \tag{18}$$

$$g(l) = \frac{2mK_dA}{T^2a^{s-2}}V_s(l) \tag{19}$$

puts Eqs. (16) and (17) into a form more convenient for analysis:

$$\frac{du}{dl} = (2-d)u - u^2 + g\ , \tag{20}$$

$$\frac{dg}{dl} = (2-s)g\ . \tag{21}$$

These equations are to be solved subject to the initial conditions

$$u_0=u(l=0)=\frac{2mK_d}{T^2a^{d-2}}\left[V_0+\frac{V_s Aa^{d-s}}{d-s}\right], \quad (22)$$

$$g_0=g(l=0)=\frac{2mK_d A}{T^2a^{s-2}}V_s , \quad (23)$$

where the potential parameters V_0 and V_s in the right-hand sides correspond to the bare values. Since these conditions are extracted from the small-k part of (13) by putting $k\cong 1/a$ they are only order-of-magnitude estimates, except for $s=2$ where the initial condition for g is cutoff independent. Precise knowledge of the initial values is necessary for exact determination of the phase-transition point or other "nonuniversal" quantities. As a rule this is outside the RG, which treats model-independent properties.

Before actually applying Eqs. (20) and (21), let us discuss their range of validity. Although they arise from perturbation-theory considerations, we will argue that (20) and (21) can be used for exact determination of critical singularities. The calculation that led us to Eq. (15) demonstrated that multiple scattering can generate only analytical corrections to the renormalized potential $V^R(k)$. Therefore, for $s-d<2$ the result (21) for the amplitude for the nonanalytic part of V^R is exact in all orders in g and is just the consequence of the scaling properties of the long-range part of interaction. To realize that (20) is exact also we need to clarify the physical meaning of the variable $u(l)$.

For $r \gg a$ the radial part $R(r)$ of the wave function of the state with zero energy obeys the Schrödinger equation

$$R''+\frac{d-1}{r}R'-\frac{2mV_s}{T^2r^s}R=0 , \quad (24)$$

where we have assumed the wave function is radially symmetric (nonsymmetric wave functions will have higher energy). Let us seek R in the form

$$R=\text{const}\,\exp\int\frac{u(r)dr}{r} . \quad (25)$$

We obtain for u the differential equation

$$r\frac{du}{dr}=(2-d)u-u^2+\frac{2mV_s}{T^2r^{s-2}} . \quad (26)$$

Introducing the variables $l=\ln(r/\alpha)$ and $g(l)=(2mV_s/T^2\alpha^{s-2})e^{(2-s)l}$ (where α is some cutoff which is in general different from a) transforms Eq. (26) exactly into the RG equations (20) and (21). The proper relationship between α and a is that the dimensionless combinations $2mV_s/T^2\alpha^{s-2}$ [occurring in $g(l)$] and $(2mK_d A/T^2a^{s-2})\,V_s$ [in Eq. (23)] are equal. Hence, we have demonstrated that our RG equations (20) and (21) together with the initial condition (23) correspond to the rigorous solution of the Schrödinger equation at large distances for the state with zero energy. The corresponding wave function has to be matched with its short-range part for $r\cong a$, which in general demands the detailed knowledge of the short-ranged part of the pinning potential; in our case all this information goes into u_0. Then $u(l)$ as given by Eq. (20) via Eq. (25) gives us the spatial behavior of the wave function with $E=0$. According to the oscillation theorem[12] the number of zeros of this function tells us the number of lower ($E<0$) discrete levels. In the proximity of the disappearance of the discrete spectrum the position of the zero(s) of the $E=0$ wave function will occur at larger and larger distances from the origin; for a threshold combination of parameters of the pinning potential this zero will be at $r=\infty$, and $E=0$ will correspond to the lowest energy state. Therefore we can use a scaling argument and identify the position of the zero of the $E=0$ wave function with the localization length $\xi_\perp$ of the lowest energy level $E_0<0$ for $E_0\to 0$ ($\xi_\perp\to\infty$), and find E_0 via Eq. (5).

For some special choices of the pinning potentials these arguments can be checked by the direct solution of the Schrödinger equation. Some references will be given below.

Relationship (25) provides a physical interpretation for the different dependencies of $u(l)$. For example, if the RG equations have a stable fixed point $u_1=u(l\to\infty)$, and $u(l)$ is finite for all $l\geq 0$, we have a nonbinding pinning potential and for $r\to\infty$ the radial wave function behaves asymptotically like r^{u_1}. The case $u(l\to\infty)\to+\infty$ also corresponds to a nonbinding potential. The presence of an unstable fixed point u_2 means that depending on the relative position of u_0 (22) and u_2 we shall obtain qualitatively different behaviors of the wave function; therefore $u_0=u_2$ corresponds to the separating value between these regimes. To locate the presence of a bound state we have to find a *finite* scale l^* at which the radial wave function (25) vanishes. This evidently corresponds to $u(l\to l^*)=-\infty$. The respective spatial scale

$$\xi_\perp=a\exp l^* \quad (27)$$

can be attributed to the localization length of the $E_0<0$ wave function. More complicated situations are also possible, such as a multicritical point ($u_1=u_2$) or a periodic $u(l)$ which falls to $-\infty$ for an infinite sequence of l (in this case there are no fixed points). These cases will be discussed below. It is interesting that the simple system (20) and (21) generates this variety of behavior.

IV. DEPINNING TRANSITIONS IN THE PRESENCE OF INVERSE SQUARE INTERACTIONS

Let us start with the marginal case $s=2$. According to Eq. (21), the amplitude of the long-range part of the pinning potential is scale invariant, equal to $g=2mV_2/T^2$ [in this section we will omit the subscript introduced in (23) since here $g(l)\equiv g_0$], and plays the role of a parameter in Eq. (20). The fixed points at which the right-hand side of (20) vanishes are given by the roots of the quadratic equation

$$(2-d)u-u^2+g=0 . \quad (28)$$

This has the real solutions

$$u_1=\tfrac{1}{2}\{2-d+[(d-2)^2+4g]^{1/2}\} , \quad (29)$$

$$u_2=\tfrac{1}{2}\{2-d-[(d-2)^2+4g]^{1/2}\}\ , \tag{30}$$

whenever g satisfies the inequality

$$g\ge -\frac{(d-2)^2}{4}\ . \tag{31}$$

The solution of Eq. (20) expressed in terms of u_1 and u_2 is

$$u(l)=u_1+\frac{(u_1-u_2)(u_0-u_1)}{(u_0-u_2)\exp(u_1-u_2)l-u_0+u_1}\ , \tag{32}$$

while for $u_1=u_2=(2-d)/2$ [which occurs for $g=-(d-2)^2/4$] this reduces to

$$u(l)=\frac{2-d}{2}+\frac{u_0-(2-d)/2}{1+[u_0-(2-d)/2]l}\ . \tag{33}$$

The solutions (32) and (33) demonstrate that we obtain different asymptotic behavior of $u(l\to\infty)$ depending on the relative positions of u_0, u_1, and u_2. For $u_0>u_2$ the RG takes us to the stable fixed point u_1, and the crossover behavior depends whether u_0 is above or below u_1. The consequence of the presence of this fixed point have been discussed in detail in a different physical context.[14] Here we will restrict our attention to the set of parameters that allows the depinning transition. This transition occurs when the initial value u_0 coincides with the unstable fixed point u_2 (30), which divides the unbounded states [for which $u(l\to\infty)=u_1$] from the bound states [for which $u(l\to l^*)=-\infty$]:

$$u_0=\tfrac{1}{2}\{2-d-[(d-2)^2+4g]^{1/2}\}\ . \tag{34}$$

Interestingly, this condition for the bulk depinning transition is very similar to that obtained for the edge (wetting) depinning transition. Lipowsky and Nieuwenhuizen[9] studied in $d=1$ a model potential consisting of an impenetrable well for $x<0$, a square well for $0<x<a$, and an inverse-square interaction for $x>a$; the critical condition on μ_0, the dimensionless strength of the square well, is that $\sqrt{\mu_0}\cot\sqrt{\mu_0}$ (instead of u_0) be equal to the right-hand side of (34) evaluated for $d=1$. This occurs because both here and in Ref. 9 the right-hand side is due to the inverse-square part of the potential, and the left-hand sides are due to the specific short-range behavior.

The denominator of Eq. (32) vanishes for

$$\exp(u_1-u_2)l^*=(\xi_\perp/a)^{u_1-u_2}=\frac{u_0-u_1}{u_0-u_2}\ , \tag{35a}$$

where we have used the determination of $\xi_\perp$ (27). This equation is an analog of the matching condition at $r\cong a$ on the logarithmic derivatives of the large-distance (left-hand side) and the short-distance (right-hand side) pieces of the wave function for the state with the energy given by Eq. (5). This fact enables us to improve (35a), which determines the location of the node of the wave function (25) of the zero energy state. The short-range part of the pinning potential appears in the Schrödinger equation in the combination $E_0-V_0(\mathbf{x})$, and so this combination should appear in the matching condition. Hence a more accurate version of (35a) can be obtained by subtracting from every u_0 in (35a) the dimensionless energy (5) $-E_0ma^2/DT^2=-(a/\xi_\perp)^2$, giving instead of (35a)

$$(\xi_\perp/a)^{u_1-u_2}=\frac{u_0-u_1+(a/\xi_\perp)^2}{u_0-u_2+(a/\xi_\perp)^2}\ . \tag{35b}$$

This equation has to be solved for $u_0\to u_2$. Several different cases are possible here, depending on value of g. The possible phase diagrams are shown in Fig. 1.

A. $u_1-u_2>2$ or $g>1-(d-2)^2/4$

For $u_1-u_2>2$, g is bounded by

$$g>1-\frac{(d-2)^2}{4}\ . \tag{36}$$

This is region A of Fig. 1. The leading terms of the localization length and the free energy $F=E_0$ expansions are given by

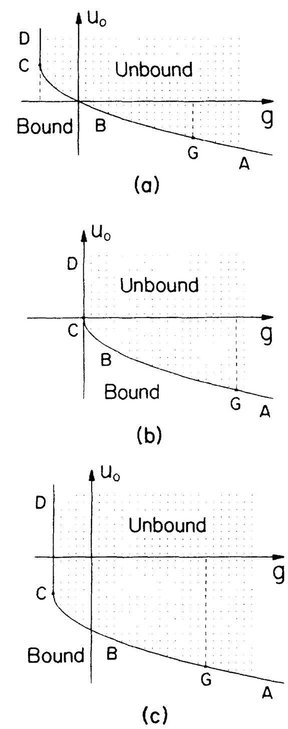

FIG. 1. The global phase diagram for a symmetric pinning potential comprised of an arbitrary proportion of a short-range part and an inverse-square tail [see Eq. (13) for $s=2$] in the dimensionless variables u_0 (22) and g (23). The phase boundary between the bound and the unbound states of the line consists of the four distinct subregimes A, B, C, and D studied in Sec. IV. The locus of transitions within subregime A is given by the part of the curve (34) which extends from the point $G(g,u_0)=[1-(d-2)^2/4,\ -d/2]$ to $(+\infty,-\infty)$; the same curve between the points G and C $[-(d-2)^2/4,(2-d)/2]$ gives the locus of transitions within subregime B; the multicritical point C divides subregime B and subregime D which extends from C to $[-(d-2)^2/4,+\infty]$ along the straight line $g=-(d-2)^2/4$: (a) $d<2$; (b) $d=2$; (c) $d>2$; here G lies at a negative value of g for $d>4$.

$$\xi_\perp = a[(u_2-u_2)^{-1/2}+\tfrac{1}{2}(u_1-u_2)(u_2-u_0)^{(u_1-u_2-3)/2}]\ , \tag{37}$$

$$F=E_0 = -\frac{DT^2}{ma^2}[u_2-u_0-(u_1-u_2)(u_2-u_0)^{(u_1-u_2)/2}]\ . \tag{38}$$

For $d=1$ this regime was discovered by Lipowsky and Nieuwenhuizen[9] with the same condition (36). For $g=0$, Eq. (36) reduces to $d>4$; therefore it is clear that this is not accessible for physical dimensionalities in the absence of the inverse-square interactions.

Result (38) has a clear physical meaning. The leading term (linear in u_2-u_0) arose from the short-range part of the wave function [i.e., the right-hand side of (35b)], which would be expected to give an analytic contribution. The second term originates from the large-scale part of the wave function, and thus is in general nonanalytic and (for nonzero g) nonuniversal. Equation (38) corresponds to a first-order phase transition with a jump of the first derivative of the free energy (remember that the singular part of the free energy is zero for $u_0 \geq u_2$). This transition is very unusual since the correlation lengths (4) and (6) diverge at the phase-transition point; moreover the higher derivatives of the free energy (starting from the second) will be singular for most values of g (36). The exceptions are special points where u_1-u_2 is an even integer (4 or greater). At these special points some derivatives might have only finite jumps.

The results of the foregoing paragraph have their origin in the improvement of the initial conditions for u_0 introduced into (35b) "by hand." However, some indications of the regimes (37) and (38) are already present in (35a), as can be seen by writing it in the form

$$\xi_\perp = a\left\{\frac{u_2-u_0}{u_1-u_0}\right\}^{-\nu_\perp}, \tag{39}$$

where we have introduced the correlation length exponent $\nu_\perp$ according to standard definition[13]

$$\nu_\perp^{-1} = u_1-u_2 = [(d-2)^2+4g]^{1/2}\ . \tag{40}$$

However, the physical values of $\nu_\perp$ are frequently restricted by the inequality[13] $\nu_\perp > \frac{1}{2}$, which is opposite to Eq. (36). The bordering value $\nu_\perp = \frac{1}{2}$ then corresponds to the upper critical dimension[13] above which the correlation exponent is constant and equal to $\frac{1}{2}$, and the phase transition is described by the Landau theory.[13] Thus the values $\nu_\perp < \frac{1}{2}$ indicate to us that something may be wrong with our derivation. As was explained above, the RG equations (20) and (21) are exact, and the initial condition for g (23) is cutoff independent for $s=2$ and therefore also exact. The weak point of our derivation is the initial condition for u. Hence this inaccuracy can lead to nonphysical results if condition (36) holds, and some essential corrections to u_0 have to be introduced. However, even without these corrections we can find the leading terms of (37) and (38) by continuity arguments. Using the analogy with the theory of critical phenomena[13] we can put $\nu_\perp = \frac{1}{2}$ in Eq. (39) for all values of parameters where $u_1-u_2>2$, and via Eq. (5) reproduce the leading terms of Eq. (38). Although such a matching procedure leads to a slight difference in numerical factors, the dependence of $\xi_\perp$ (37) and E_0 (38) on u_2-u_0 will be the same.

Experience with other systems above the upper critical dimension warns us that we could expect some scaling relations to fail. At the upper critical dimensionality (point G on Fig. 1), nontrivial logarithmic corrections to the leading terms of (37) and (38) are expected. It is evident from (35b) that our method fails to produce a logarithmic correction for $u_1-u_2=2$ and a more elaborate treatment is necessary. Moreover, we do not see here any breaking of the scaling relations such as (5) and (6); the critical dependencies of E_0, $\xi_\parallel$, and $\langle \mathbf{x}^n \rangle$ can be reexpressed via the critical dependence of $\xi_\perp$. We have reproduced all these results, together with a logarithmic correction for the bordering value $u_1-u_2=2$ (the same correction was found in Ref. 9 for the two-dimensional wetting problem) by direct solution of the d-dimensional Schrödinger equation in the potential comprised of a square well and an inverse-square tail. Moreover, a part of these results was found by Lipowsky and Nieuwenhuizen[9] for the two-dimensional wetting problem. In particular, they have found a linear dependence of the free energy (38) on the vicinity to the phase transition, but also noticed a strong indication of the breaking of one-length scaling for the transverse correlations of the interface. This last result is to be expected above the upper critical dimension. The absence of a similar phenomenon in our system is due to the symmetry of the pinning potential. It is necessary to stress that the depinning transition above the upper critical dimension is not identical to the Landau theory of phase transitions:[13] although the critical dependence of the correlation length (37) in leading order is the same in both theories, the behavior of the free energy (38) is completely different for the depinning transition, and is characteristic of a first-order phase transition.

B. $0<u_1-u_2<2$ or $-[(d-2)^2/4]<g<1-[(d-2)^2/4]$

For $0<u_1-u_2<2$, g is bounded by

$$-\frac{(d-2)^2}{4} < g < 1-\frac{(d-2)^2}{4}\ . \tag{41}$$

This is region B of Fig. 1. The correction term in Eq. (35b) is irrelevant in leading order for $u_0 \to u_2$, and the localization length is given by formulas (39) and (40). The bound-state energy E_0 and the free-energy density vanish according to (5), (39), and (40) like

$$F=E_0=-\frac{DT^2}{ma^2}\left\{\frac{u_2-u_0}{u_1-u_0}\right\}^{2\nu_\perp}\ . \tag{42}$$

Before discussing the general formulas (39), (40), and (42) let us start from the important special case $g=0$, which corresponds to a short-range pinning potential. According to Eq. (41) this involves all physical dimensions less than 4 excluding $d=2$ (this case will be analyzed separately).

For $d<2$ and $g=0$ the condition (34) for a depinning transition reduces to $u_0=0$. This means that a depinning transition is impossible at any finite temperature, since we assume that V_0 is finite in Eq. (18). This conclusion is an implication of standard quantum mechanics[12] that tells us that an arbitrarily shallow well always has a bound state for $d<2$. Let us rewrite (40) and (42) for $g=0$ in a form appropriate both for $d<2$ and $d>2$:

$$\nu_{10}^{-1}=|d-2| , \tag{43}$$

$$F=E_0=-\frac{DT^2}{ma^2}\left\{\frac{|u_0|}{2-d+|u_0|}\right\}^{2/(2-d)} , \tag{44}$$

where we have written $u_0=-|u_0|$ to be explicit on signs (the bound state arises for $u_0<0$). For $d=1$, Eq. (43) reduces to the exact result of Abraham,[4] and (44) demonstrates the correct quadratic dependence near the "critical value" $u_0=0$. These conclusions are consequences of the universality principle discussed in Sec. II. For $d<2$ and $|u_0|\ll 1$, Eq. (44) reduces to

$$F=E_0=-\frac{DT^2}{ma^2}\left\{\frac{|u_0|}{2-d}\right\}^{2/(2-d)} . \tag{45}$$

This is a generalization of the one-dimensional result for the energy level in a shallow well (see Problem 1, Sec. 45 of Landau and Lifshitz;[12] recall that here the temperature plays the role of Planck's constant). Equation 44 enables us also to go to the limit $a\to 0$. Since in this case $u_0\to 0$ [see Eq. (22)] we get the a-independent expression

$$F=E_0=-\text{const}\left\{\frac{m}{T^2}\right\}^{d/(2-d)}|V_0|^{2/(2-d)} . \tag{46}$$

With the appropriate numerical factor this is the exact result for the ground-state energy level of the δ-function well.

For $d>2$ and $g=0$ the condition for the depinning transition (34) reduces to $u_0=2-d$ which tells us, according to quantum mechanics, that an energy level arises if the potential well is sufficiently deep. This result means that there is a finite temperature depinning transition. For $d=3$ Eq. (43) and the exponent of Eq. (44) agree with the explicit calculation for the square-well potential (see Problem 1, Sec. 33 of Landau and Lifshitz[12]), and the critical depth of the well $|u_0|=1$ has the same order of magnitude as found in the same problem. We already have noted that we have confirmed these conclusions for general d by direct solution of the Schrödinger equation for a model potential comprised of the square well and inverse-square tail. For general d and $g=0$ the results (43)–(45) and their analogs for $d>4$ [see Eqs. (37) and (38)] have been found by Grosberg and Shakhnovich[15] in the closely related physical context of the collapse transition for a Gaussian polymer in an external attractive potential. The same singularities have been obtained more recently by Lipowsky[16] for an unbinding transition from a short-range *asymmetric* potential by direct solution of the Schrödinger equation.

For the case of nonzero g Eqs. (39), (40), and (42) tell us that the critical exponents depend upon g and thus are *nonuniversal*. For $d=1$ these results were obtained in Ref. 9.

C. Multicritical point: $g=-(d-2)^2/4$, $u_0=(2-d)/2$

This case is the point labeled C on Fig. 1. For the marginal case $u_1=u_2$ or $g=-(d-2)^2/4$ we have to use Eq. (33) to find the scale l^* at which $u(l\to l^*)=-\infty$:

$$l^*=\frac{1}{(2-d)/2-u_0} . \tag{47}$$

The expressions for the localization length and the free energy follow from (47), (27), and (5):

$$\xi_\perp=a\exp\left\{\frac{1}{(2-d)/2-u_0}\right\} , \tag{48}$$

$$F=E_0=-\frac{DT^2}{ma^2}\exp\left\{-\frac{2}{(2-d)/2-u_0}\right\} . \tag{49}$$

The condition for the phase transition (34) reduces now to $u_0=(2-d)/2$, and the localized region is given by $u_0<(2-d)/2$. Using the determination of g (23) for $s=2$ we obtain the following *exact* expression for the multicritical temperature:

$$T_c=\frac{2(2m|V_2|)^{1/2}}{|d-2|} . \tag{50}$$

Since g is temperature dependent ($g=2mV_2/T^2$) we can go through the multicritical point holding g fixed at its critical value $-(d-2)^2/4$ only by changing u_0 while keeping the temperature constant [for example, by changing V_0 (22)]. The same qualitative result and Eqs. (48)–(50) were obtained by Lipowsky and Nieuwenhuizen[9] for $d=1$.

In the special case $d=2$ ($g=0$) Eqs. (48) and (49) reduce to

$$\xi_\perp=a\exp(2\pi T^2/m|V_0|) , \tag{51}$$

$$F=E_0=-\frac{DT^2}{ma^2}\exp(-4\pi T^2/m|V_0|) , \tag{52}$$

where we have used the determination of u_0 (22). These expressions tell us that a phase transition is impossible for any finite temperature just as a quantum particle in an arbitrarily shallow two-dimensional potential well has a bound state.[12] Note that (52) represents the familiar expression for this energy level (see Problem 2, Sec. 45 of Landau and Lifshitz[12]). Moreover we can conclude from (52) and the universality principle that any finite-temperature depinning transition for $d=2$ in the absence of the long-range forces must have an essential singularity of the free-energy-like $\exp(-1/\tau)$, where τ is a dimensionless variable denoting the vicinity to the phase-transition point. For the case of the edge-depinning transition this result has been obtained by Vallade and Lajzerowicz.[5]

D. Kosterlitz-Thouless-like transition

Up to now we have analyzed the cases where the RG equation (20) has fixed points. For

$$g < -\frac{(d-2)^2}{4}, \tag{53}$$

the roots (29) and (30) are complex and we shall obtain, instead of Eqs. (32) and (33),

$$u(l) = \frac{2-d}{2} + \sqrt{\lambda}\tan\left\{\arctan\frac{u_0-(2-d)/2}{\sqrt{\lambda}} - \sqrt{\lambda}l\right\}, \tag{54}$$

$$\lambda = -\frac{(d-2)^2}{4} - g. \tag{55}$$

Equation (54) has an infinite periodic sequence of values of l^* at which $u(l \to l^*) = -\infty$. This corresponds to the presence of an infinite number of nodes in the radial function R [Eq. (25)], implying the presence of an infinite number of energy levels having negative energy. For small λ the asymptotic behavior of l^* is found to be

$$l^* = \begin{cases} \pi n/\sqrt{\lambda} & \text{for } u_0 \neq \frac{2-d}{2} & (56)\\ \pi(n-1/2)/\sqrt{\lambda} & \text{for } u_0 = \frac{2-d}{2}, & (57)\end{cases}$$

where n is an integer. The negative n are unphysical since $l \geq 0$ by definition. Thus the energy levels can be found by the combination of Eqs. (27) and (5) with (56) for non-negative integers n, or (57) for positive integers n. The lowest energy level, which gives us the line free energy, must correspond either to $n=0$ or $n=1$. If it corresponds to $n=0$ we have from (56) $l^*=0$ and from (5) and (27) $E_0 \to -DT^2/ma^2$ as $\lambda \to 0$. This applies to $u_0 < 2-d/2$ for which a bound state exists even for $g > -(d-2)^2/4$. If the ground-state energy is given by $n=1$, one has $l^* \to \infty$ as $\lambda \to 0$, which applies to $u_0 \geq 2-d/2$. In the latter case, one finds from (56), (57), (27), and (5)

$$\xi_\perp = a\exp(\pi/\sqrt{\lambda}), \tag{58}$$

$$F = E_0 = -\frac{DT^2}{ma^2}\exp(-2\pi/\sqrt{\lambda}) \tag{59}$$

for $u_0 > (2-d)/2$, and

$$\xi_\perp = a\exp(\pi/2\sqrt{\lambda}), \tag{60}$$

$$F = E_0 = -\frac{DT^2}{ma^2}\exp(-\pi/\sqrt{\lambda}) \tag{61}$$

for $u_0 = (2-d)/2$. The latter equations reflect the thermodynamic singularities for passage through the multicritical point (see previous subsection) at fixed $u_0 = (2-d)/2$. Previously [see Eqs. (48) and (49)] we have found the critical behavior if one passes through the multicritical point for fixed $g = -(d-2)^2/4$. Therefore we can rewrite Eqs. (48) and (60) in more generic form in the terms of $\Delta u = (2-d)/2 - u_0$ and λ (55) as follows:

$$\xi_\perp = a\exp\left\{\frac{1}{\Delta u}\Omega(\lambda/\Delta u^2)\right\}, \tag{62}$$

where the shape function Ω goes as

$$\Omega(0) = 1 \tag{63}$$

and

$$\Omega(y) = \pi/\sqrt{y} \quad \text{for } y \to \infty. \tag{64}$$

The Kosterlitz-Thouless-like[8] singularities (58) and (59) have been found first by Kroll and Lipowsky[6] and Chui and Ma[7] in the context of the two-dimensional wetting transitions. The results (58)–(64) have been obtained by Lipowsky and Nieuwenhuizen[9] for a more general model potential by direct solution of corresponding one-dimensional Schrödinger equation.

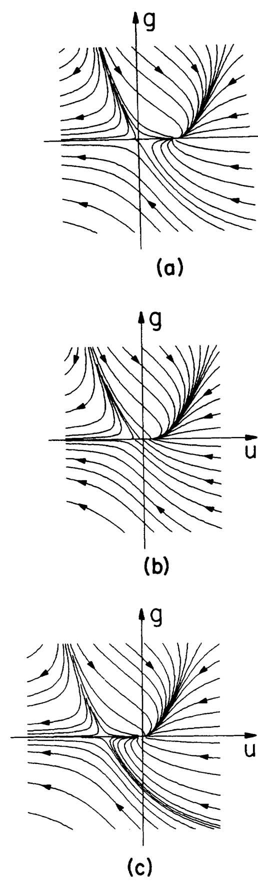

FIG. 2. Flow diagrams for the RG equations (20) and (21) displaying a second-order depinning transition for $s > 2$. The arrows show the direction of the RG flow. Values of u and g which are carried by the RG transformation to the stable fixed point $u = u_1$ (29) correspond to the unbound state of the line. Potential parameters which are carried by the RG to $u = -\infty$ correspond to the pinned state of the line. The separatrix between these two regimes gives the locus of the phase-transition points (a) $d < 2$; (b) $d = 2$ [this case is degenerate since the positions of the stable u_1 (29) and the unstable u_2 (30) fixed points coincide]; and (c) $d > 2$.

V. LONG-RANGE TAILS DECAYING FASTER THAN $1/r^2$

If the long-range part of the pinning potential has a tail decaying faster than $1/r^2$, the variable g (21) is irrelevant in the RG sense.[13] Thus the critical singularities in leading order are given by the results of the previous section for $g=0$. However, the presence of a tail leads to a shift in the locus of phase transitions represented by the separatrixes in Figs. 2. Note that Figs. 2 clearly show that, in the presence of long-range forces, a finite-temperature bulk depinning transition always occurs, even for cases (such as $d\le 2$) for which depinning from a purely short-range potential is impossible.

We will now demonstrate how to find the correction to the free-energy density in the presence of an irrelevant (in the RG sense) tail. The simplest way is to use the general ideas for calculating of corrections to scaling.[13] We will restrict ourselves to the case $d<4$. To simplify the formulas we will use a thermal scaling field τ to denote the (dimensionless) proximity to the phase-transition point (vicinity to the corresponding separatrix in Figs. 2), and omit all dimensional and irrelevant numerical factors, keeping only the relative signs of the leading- and next-order term.

Consider first the case $d\neq 2$. Here the leading term of the free-energy expansion is given by $F\sim\tau^{2\nu_{\perp 0}}$ with $\nu_{\perp 0}$ from Eq. (43). The value of $g(l)=g_0\exp(2-s)l$ (21) evaluated at the correlation length scale $e^{l^*}=\tau^{-\nu_{\perp 0}}$ is equal to $g^*=g_0\tau^{\nu_{\perp 0}(s-2)}$ and is small compared with unity for $\tau\ll 1$. Therefore we can seek the free-energy singularity in the form $F\sim\tau^{2\nu_{\perp 0}}f(g_0\tau^{\nu_{\perp 0}(s-2)})$ where $f(x)$ is an analytic function behaving for $x\ll 1$ like $f(x)=1-x$. This leads to the desired expansion of the free energy

$$F\sim\tau^{2\nu_{\perp 0}}-g_0\tau^{\nu_{\perp 0}s}\,. \tag{65}$$

A very similar result has been obtained by Kroll and Lipowsky[6] for $d=1$, where $\nu_{\perp 0}=1$. They also found for integer s a logarithmic term in the correction. This is in agreement with the results of the correction-to-scaling theory (see Ref. 13 and references therein), according to which logarithmic corrections are always to be expected for integer exponents.

For $d=2$ in the absence of a long-range perturbation the correlation length and the free energy have singularities $\exp(1/\tau)$ and $\exp(-2/\tau)$, respectively [see (51) and (52)]. The value of $g(l)=g_0\exp(2-s)l$ evaluated at the correlation length scale $e^{l^*}=\exp(1/\tau)$ is given by $g^*=g_0\exp[(2-s)/\tau]$. The scaling argument now leads to the expression

$$F\sim\exp(-1/\tau)\left[1-g_0\exp\left[\frac{2-s}{\tau}\right]\right]. \tag{66}$$

VI. LONG-RANGE TAILS DECAYING SLOWER THAN $1/r^2$

If the long-range part of the pinning potential falls off slower than $1/r^2$, it is relevant in the RG sense and grows under rescaling like $g(l)=g_0\exp(2-s)l$ (21), leading to completely new physical picture. The outcome depends on the sign of the long-range tail of the pinning potential. For the case of a repulsive tail ($g_0>0$) and an attractive short-range well a first-order depinning transition is possible. In this case the $E=0$ wave function has a finite localization length; the moments (3) are finite at the phase-transition point and jump to infinity above the phase transition. The flow picture of the RG equations (20) and (21) corresponding to this case is displayed in Figs. 3 for different space dimensionalities. A separatrix in the $g>0$ region of Figs. 3 gives us the locus of bare values which correspond to first-order transition points. The separatrixes end at the critical end points which have coordinates $(u,g)=(0,0)$ for $d\le 2$ and $(2-d,0)$ for $d\ge 2$. Comparison of Figs. 3 and 2 clearly demonstrates the difference between the second- and the first-order transitions in terms of the renormalization flow; in the latter case we have no fixed points corresponding to either phase, and only a separatrix between distinct regimes of the runaway of the RG trajectories: $u\to-\infty$ (bound state) and $u\to+\infty$ (unbound state). On the other hand, the RG flow describing a second-order transition (Fig. 2)

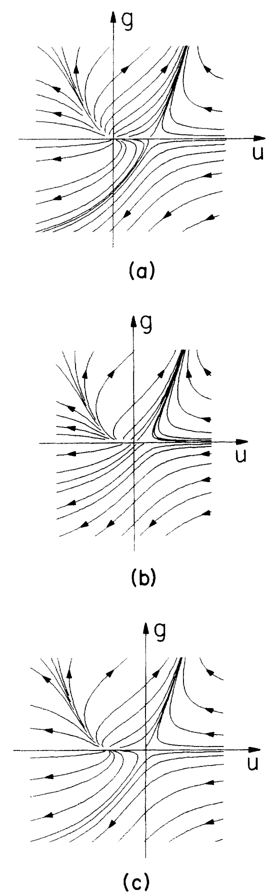

FIG. 3. Flow diagrams for the RG equations (20) and (21) displaying a first-order depinning transition for $s<2$. Here the unbound and bound states are characterized by the runaway of the RG trajectories to $u=+\infty$ and $-\infty$, respectively, depending on the relative position of the initial values with respect to the separatrix: (a) $d<2$; (b) $d=2$; (c) $d>2$.

contains a separatrix dividing the bound state with $u=-\infty$ from an unbound one which corresponds to a *finite stable fixed point* u^*.

For the case of an attractive tail ($g_0<0$), the $g<0$ part of Fig. 3 shows that the RG trajectories run away to $u=-\infty$ for any value of the short-range part of the pinning potential. This demonstrates that the line is pinned at all finite temperatures. We will consider this case in more detail to show how to calculate the ground-state energy (and the line free-energy density) in the limit $|g_0|\ll 1$. We need to know the spatial scale of the ground-state wave function, which evidently is given by $g(l^*)\cong 1$. We can, however, give a more accurate condition. The inequality $|g_0|\ll 1$ implies that our picture of the depinning transition holds for intermediate scales less than the spatial scale e^{l^*} imposed by the presence of the long-range tail. For small $|g(l)|$ and $d\neq 2$ the equation of state can be described by the effective scale-dependent exponent (40). It is clear that the maximal scale at which such description is still meaningful is given by the zero of the expression in the square brackets of Eq. (40): $(d-2)^2-4|g_0|\exp(2-s)l^*=0$. Therefore the localization length of the wave function and the ground-state energy are given by [see Eqs. (27) and (5)]

$$\xi_\perp \cong a\left\{\frac{(d-2)^2}{|g_0|}\right\}^{1/(2-s)}, \tag{67}$$

$$F=E_0\sim\left\{\frac{|g_0|}{(d-2)^2}\right\}^{2/(2-s)}. \tag{68}$$

For $d=1$ these functional dependencies have been found by Kroll and Lipowsky.[6] They also noticed that the last expression has an important special case. Let us imagine the presence of an additional term in the Hamiltonian (1) imitating an external field term which localizes the line and destroys the phase transition. For the edge-depinning transition this term is proportional to $\mathbf{x}^1$. Therefore it corresponds to a relevant long-range tail with $s=-1$. Then we obtain from (68) the free-energy density above the depinning transition in an external field $|g_0|=h$ for $h\to+0$ in the form

$$F\sim h^{2/3}. \tag{69}$$

The exponent of this expression is the known exact result[4–6] for $d=1$. Hence we can conclude that it holds for all physical dimensionalities $d<4$ excepting $d=2$, where a logarithmic correction is expected (see below). We can combine (69) with $F\sim\tau^{2\nu_{\perp 0}}$ [$\nu_{\perp 0}$ is from (43)] for $h=0$ and rewrite F both for nonzero τ and h in the form

$$F=\tau^{2\nu_{\perp 0}}\Omega(h/\tau^{3\nu_{\perp 0}}), \tag{70}$$

where the scaling function Ω has the properties $\Omega(0)=\text{const}$ and $\Omega(y)\to y^{2/3}$ as $y\to\infty$. We will not write down the corresponding formulas for the symmetric field ($\sim\mathbf{x}^2$) since they follow from (68) for $s=-2$ in the same way.

Consider now the case $d=2$. Here the prefactor of Eq. (68) diverges for $d\to 2$ but the exponent of g_0 is dimensionality independent and well determined. This implies that for $d=2$ we have to obtain the same functional dependence for F that Eq. (68) does, with a logarithmic correction going to infinity for $g_0\to 0$ instead of the divergent combination $1/(d-2)^2$. Let us demonstrate how to get this correction. For $d=2$ and intermediate scales less than e^{l^*} our system can be described by the theory of subsection IVD. Therefore the scale at which this description fails is given by Eq. (56) for $n=1$ and $\lambda=|g_0|\exp(2-s)l^*$ [see Eq. (55) for $d=2$]. Hence we obtain

$$l^*=\frac{\pi}{\sqrt{|g_0|}}\exp\left[\frac{s-2}{2}l^*\right].$$

The solution of this equation for $|g_0|\to 0$ to the necessary accuracy leads to the following expressions replacing Eqs. (67) and (68) for $d=2$:

$$\xi_\perp\cong a[|g_0|\ln(1/|g_0|)]^{1/(s-2)}, \tag{71}$$

$$F=E_0\sim[|g_0|\ln(1/|g_0|)]^{2/(2-s)}. \tag{72}$$

So for the case of the edge-depinning transition we have the same dependence (69) on the external field h with an additional logarithmic term $\ln^{2/3}(1/h)$. The analog of Eq. (70) looks like

$$F=[h\ln(1/h)]^{2/3}\Omega\left\{\frac{\exp(-1/\tau)}{[h\ln(1/h)]^{2/3}}\right\}, \tag{73}$$

where the scaling function Ω has the properties $\Omega(0)=\text{const}$ and $\Omega(y)\to y$ as $y\to\infty$.

VII. CONCLUSIONS

Using the SOS Hamiltonian (1) for a linear object in a $d+1$-dimensional space we have given a complete classification of the possible depinning transitions via an exact RG analysis of the related Schrödinger equation (2). For the special case $d=1$ we have recovered most of the known results for the two-dimensional wetting transition. For pinning potentials falling off faster than $1/r^2$ the leading-order free-energy singularity is given by $\tau^{2\nu_{\perp 0}}$ (43) for $d<4$ and $d\neq 2$, $\exp(-1/\tau)$ for $d=2$, and τ for $d>4$. At the upper critical dimension $d=4$ the free energy vanishes like τ with a logarithmic correction. The behavior above the upper critical dimension is nontrivial: from one side the free-energy singularity reflects the first-order phase transition, while the correlation lengths diverge at the phase-transition point.

The presence of long-range tails falling off faster than $1/r^2$ leads to corrections nonanalytic in τ. We have calculated these corrections for $d<4$.

The presence of an inverse-square tail in the pinning potential leads to a rich phase diagram with regions of a nontrivial first-order phase transition above the upper critical dimensionality, second-order phase transitions with nonuniversal exponents, or Kosterlitz-Thouless-like transitions with a multicritical point between the two last regimes.

Attractive pinning potentials decaying slower than $1/r^2$ prevent line depinning at any finite temperature.

For this case the free-energy density and the related behavior in an external transition-destroying field have been calculated. Repulsive pinning potentials in the presence of a short-range attraction can lead to first-order depinning transitions.

The system we have studied is interesting since it allows us to perform rigorous RG treatment while exhibiting most of the known types of the phase transitions, including the Kosterlitz-Thouless type, second-order phase transitions both with universal and nonuniversal exponents, and first-order phase transitions. On the border between the latter regimes unusual phase transitions are found which have features characteristic both of first- and second-order transitions.

ACKNOWLEDGMENT

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