Theory of electromagnetic-wave instabilities in a spatially dispersive semiconductor superlattice

B. G. Martin
Analysis Consultants, El Toro, California 92630

R. F. Wallis
Institute for Surface and Interface Science, University of California, Irvine, Irvine, California 92717
and Department of Physics, University of California, Irvine, Irvine, California 92717

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A theoretical investigation has been made of electromagnetic waves propagating in a semiconductor superlattice (SL) consisting of alternating spatially dispersive and nonspatially dispersive layers. The dispersion relations for both an infinite SL and a truncated SL are obtained by assuming the specular reflection boundary conditions of Kliewer and Fuchs and using a transfer-matrix approach. A specific type of spatial dispersion is chosen for exploration in detail, namely that caused by a dc drift current moving in alternate layers parallel to the SL layer interfaces. Amplifying instabilities are found for certain ranges of frequency and wave vector.

I. INTRODUCTION

We report on a theoretical investigation of surface electromagnetic waves propagating in a semi-infinite semiconductor superlattice (SL) consisting of alternating spatially dispersive and nonspatially dispersive layers. The dispersion relation is obtained by using the specular-reflection boundary conditions[1] and a transfer-matrix formalism.[2]

As an example of an application of this theory, we consider the spatial dispersion caused by a dc drift current moving parallel to the SL interfaces. We explore SL space-charge-wave (SCW) instabilities associated with dc drift currents. Extensive theoretical work has been done on SCW instabilities in structures simpler than the SL (for a listing of some of the pertinent papers, see Ref. 3). For example, there are so-called resistive-wall instabilities in gas plasmas that have been used for amplifying microwaves.[4] Such instabilities also occur in solid-state plasmas.[3]

Instabilities in a superlattice consisting of two-dimensional electron sheets periodically arranged in a dielectric medium have been treated by Hawrylak and Quinn.[5] In the present paper, we analyze a different superlattice system in which the current carriers occupy finite thickness slabs.

In what follows, we first obtain the transfer matrix for the SL period using the specular-reflection boundary condition. Dispersion relations are then presented for the cases of infinite and truncated superlattices. Simplified cases of two- and three-layer media are considered as a check on the theory. A specific application is made to the case where alternate SL layers carry a dc current moving parallel to the interfaces.

II. TRANSFER MATRIX FOR THE SL PERIOD

Consider the SL geometry shown in Fig. 1. Here we have alternating spatially dispersive and nonspatially dispersive layers with dielectric functions $\overleftrightarrow{\epsilon}(\mathbf{k},\omega)$ and $\overleftrightarrow{\epsilon}(\omega)$, respectively. We first consider the electromagnetic fields in region 1. The fields near the left- and right-hand interfaces, but still inside region 1, are related by the transfer matrix $\overleftrightarrow{\mathbf{M}}_1$

$$\begin{bmatrix} E_y \\ B_x \end{bmatrix}_{0^+} = \overleftrightarrow{\mathbf{M}}_1 \begin{bmatrix} E_y \\ B_x \end{bmatrix}_{a^-} , \tag{2.1}$$

where we assume p polarization for the incident radiation and where

$$\overleftrightarrow{\mathbf{M}}_1 = \begin{bmatrix} \cos(qa) & iZ_1\sin(qa) \\ iY_1\sin(qa) & \cos(qa) \end{bmatrix} . \tag{2.2}$$

Z_1 is the surface impedance of the semiconductor given by

$$Z_1 = \frac{qc}{\omega\epsilon_1(\omega)} , \tag{2.3}$$

$Y_1 = 1/Z_1$, c is the velocity of light, and ω is the frequency. The dielectric function of layer 1, $\epsilon_1(\omega)$, is given by

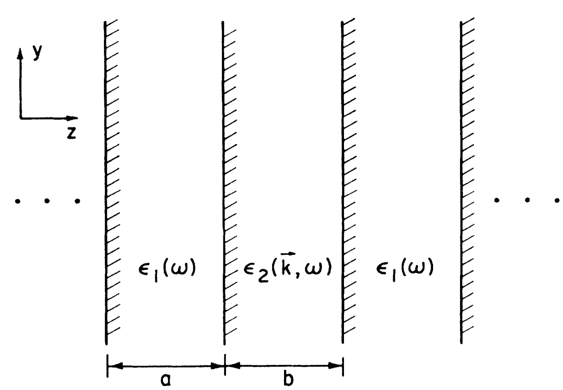

FIG. 1. Geometry of an infinite superlattice.

$$\epsilon_1(\omega)=\epsilon_\infty^{(1)}\left[1-\frac{\omega_{e1}^2}{\omega^2}\right], \tag{2.4}$$

where $\epsilon_\infty^{(1)}$ is the high-frequency background dielectric constant and ω_{e1} is the plasma frequency. The quantity q is the wave vector in the direction normal to the interfaces

$$q=\left[\frac{\omega^2}{c^2}\epsilon_1(\omega)-k_y^2\right]^{1/2}, \tag{2.5}$$

where k_y is the wave vector parallel to the interfaces.

Consider next the spatially dispersive layer whose material has a bulk dielectric function, $\epsilon_2(\mathbf{k},\omega)$, and in which a drift current is present parallel to the interface. In this layer we assume that the carriers are specularly reflected at the interfaces. To determine the field in this region, we use the specular-reflection additional boundary condition (ABC) of Kliewer and Fuchs.[1] We begin with the approach of Wallis, Castiel, and Quinn.[6] We periodically repeat layer 2 of Fig. 1 until all of the space is filled. The fields at $z=2nb$ are all identical for $n=0,\pm1,\ldots$, and the fields at $z=(2n+1)b$ are all identical for $n=0,\pm1,\ldots$. We next impose the "mirror image" ABCs as

$$E_y(nb^+)=E_y(nb^-),\quad \frac{dE_y}{dz}(nb^+)=-\frac{dE_y}{dz}(nb^-), \tag{2.6}$$

$$E_z(nb^+)=-E_z(nb^-),\quad \frac{dE_z}{dz}(nb^+)=\frac{dE_z}{dz}(nb^-), \tag{2.7}$$

where b^+ means slightly to the right of the interface at b and b^- means slightly to the left of the interface. From Maxwell's equation,

$$\nabla\times\mathbf{E}=-\frac{1}{c}\dot{\mathbf{B}}, \tag{2.8}$$

where the dot above the vector means time differentiation, we have, taking the field dependence as $\exp[i(\mathbf{k}\cdot\mathbf{r})-\omega t]$, the result

$$-i\omega B_x=\frac{dE_y}{dz}-ik_yE_z, \tag{2.9}$$

and the conditions given by Eqs. (2.6) and (2.7) lead to the result

$$B_x(nb^+)=-B_x(nb^-). \tag{2.10}$$

Now we need to find solutions to the wave equation. To this end, we use Fourier transform $\mathcal{F}$ as follows:

$$\mathcal{F}[\mathbf{E}(z)]=\mathbf{E}(k_z)=\int_{-\infty}^{\infty}dz\,e^{ik_zz}\mathbf{E}(z), \tag{2.11}$$

$$\mathcal{F}\left[\frac{d\mathbf{E}(z)}{dz}\right]=-ik_z\mathbf{E}(k_z)+\sum_{n=-\infty}^{\infty}\Delta\mathbf{E}(nb)e^{ik_znb}, \tag{2.12}$$

where

$$\Delta\mathbf{E}(nb)=\mathbf{E}(nb^-)-\mathbf{E}(nb^+). \tag{2.13}$$

We also have

$$\mathcal{F}\left[\frac{d^2\mathbf{E}(z)}{dz^2}\right]=-k_z^2\mathbf{E}(k_z)+\sum_{n=-\infty}^{\infty}\left[\frac{d\Delta\mathbf{E}(nb)}{dx}-ik_z\Delta\mathbf{E}(nb)\right]\times e^{ik_znb}. \tag{2.14}$$

Now we can proceed to the electromagnetic wave equation which we want to Fourier transform. If we combine the Maxwell equations

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \tag{2.15}$$

and

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} \tag{2.16}$$

with

$$\mathbf{D}=\overleftrightarrow{\epsilon}\mathbf{E}, \tag{2.17}$$

we can obtain the wave equation

$$\nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=\overleftrightarrow{\epsilon}\frac{\omega^2}{c^2}\mathbf{E}. \tag{2.18}$$

In component form,

$$\sum_\beta\left[\frac{\partial^2}{\partial x_\alpha\partial x_\beta}-\delta_{\alpha\beta}\nabla^2-\epsilon_{\alpha\beta}\frac{\omega^2}{c^2}\right]E_\beta=0, \tag{2.19}$$

where β is summed over components x, y, and z; $\delta_{\alpha\beta}$ is the Kronecker delta.

Fourier transforming Eq. (2.19) gives

$$\left[\frac{\omega^2}{c^2}\epsilon_{yy}-k_z^2\right]E_y(k_z)+\left[\frac{\omega^2}{c^2}\epsilon_{yz}+k_yk_z\right]E_z(k_z)=\sum_n e^{ik_znb}\left[-ik_z\Delta E_y(nb)-\frac{d\Delta E_y(nb)}{dz}+ik_y\Delta E_z(nb)\right], \tag{2.20}$$

and

$$\left[\frac{\omega^2}{c^2}\epsilon_{zy}+k_yk_z\right]E_y(k_z)+\left[\frac{\omega^2}{c^2}\epsilon_{zz}-k_y^2\right]E_z(k_z)=\sum_n e^{ik_znb}[ik_y\Delta E_y(nb)]. \tag{2.21}$$

Equations (2.20) and (2.21) can be simplified by use of the relations

$$\Delta E_y(nb)=0. \tag{2.22}$$

In addition, we can, from Eq. (2.9) write

$$-ik_y\Delta E_z(nb)+\frac{d\Delta E_y(nb)}{dz}=-\frac{i\omega}{c}\Delta B_x(nb). \tag{2.23}$$

Using these results in Eqs. (2.20) and (2.21), we have

$$\left[\frac{\omega^2}{c^2}\epsilon_{yy}-k_z^2\right]E_y(k_z)+\left[\frac{\omega^2}{c^2}\epsilon_{yz}+k_yk_z\right]E_z(k_z)$$
$$=\sum_n e^{ik_znb}\left[\frac{i\omega}{c}\Delta B_x(nb)\right], \quad (2.24)$$

and

$$\left[\frac{\omega^2}{c^2}\epsilon_{zy}+k_yk_z\right]E_y(k_z)+\left[\frac{\omega^2}{c^2}\epsilon_{zz}-k_y^2\right]E_z(k_z)=0 . \quad (2.25)$$

Equations (2.24) and (2.25) can be written in the form

$$\begin{bmatrix} T_{yy} & T_{yz} \\ T_{zy} & T_{zz}\end{bmatrix}\begin{bmatrix} E_y(k_z) \\ E_z(k_z)\end{bmatrix}=\begin{bmatrix} A_y \\ 0\end{bmatrix}, \quad (2.26)$$

where

$$T_{yy}=\frac{\omega^2}{c^2}\epsilon_{yy}-k_z^2 , \quad (2.27)$$

$$T_{yz}=T_{zy}=\frac{\omega^2}{c^2}\epsilon_{yz}+k_yk_z , \quad (2.28)$$

$$T_{zz}=\frac{\omega^2}{c^2}\epsilon_{zz}-k_y^2 , \quad (2.29)$$

and

$$A_y=\sum_n e^{ik_znb}\left[\frac{i\omega}{c}\Delta B_x(nb)\right] . \quad (2.30)$$

We next consider Eq. (2.30). We begin with the expression

$$\Delta B_x(nb)=B_x(nb^-)-B_x(nb^+) . \quad (2.31)$$

For all n, we have

$$B_x(0^\pm)=B_x(2nb^\pm) ,$$
$$B_x(-b^\pm)=B_x[(2n+1)b^\pm] . \quad (2.32)$$

Thus, Eq. (2.30) can be written as

$$A_y=\frac{2i\omega}{c}[B_x(0^+)-e^{-ik_zb}B_x(b^-)]\sum_n e^{ik_z2bn} . \quad (2.33)$$

The summation over n in Eq. (2.33) is zero unless $2k_zb$ is a multiple of 2π, in which case the sum diverges. Consequently we have

$$\sum_n \exp[ik_z2bn]=\frac{\pi}{b}\sum_l \delta\left[k_z-\frac{l\pi}{b}\right] . \quad (2.34)$$

This, in effect, eliminates the k_z dependence, and Eq. (2.33) becomes

$$A_y=\frac{2i\omega}{c}\frac{\pi}{b}\sum_l [B_x(0^+)-(-1)^lB_x(b^-)]\delta\left[k_x-\frac{l\pi}{b}\right] . \quad (2.35)$$

Equation (2.26) can be solved for $E_y(k_z)$ as

$$E_y(k_z)=\frac{T_{zz}}{T_{yy}T_{zz}-T_{yz}^2}A_y(k_z) . \quad (2.36)$$

Inverting the Fourier transform gives

$$E_y(z)=\int\frac{dk_z}{2\pi}e^{-ik_zz}\frac{T_{zz}}{T_{yy}T_{zz}-T_{yz}^2}A_y(k_z) . \quad (2.37)$$

Because of the series of δ functions, the Fourier integral reduces to

$$E_y(z)=\frac{i\omega}{cb}\sum_{l=-\infty}^{\infty}e^{-il\pi z/b}[B_x(0^+)-(-1)^lB_x(b^-)]$$
$$\times\left[\frac{T_{zz}}{T_{yy}T_{zz}-T_{yz}^2}\right]_{k_z=l\pi/b} . \quad (2.38)$$

Just inside the boundaries of region 2, we obtain from Eq. (2.38) the following, where for the last square-bracketed form of Eq. (2.38) we use [ ]:

$$E_y(0^+)=\frac{i\omega}{cb}\sum_l [B_x(0^+)-(-1)^lB_x(b^-)][\;]_{k_z=l\pi/b} , \quad (2.39)$$

$$E_y(b^-)=\frac{i\omega}{cb}\sum_l [(-1)^lB_x(0^+)-B_x(b^-)][\;]_{k_z=l\pi/b} . \quad (2.40)$$

These expressions can be written in the form

$$E_y(0^+)=\gamma_2B_x(0^+)-\gamma_1B_x(b^-) , \quad (2.41)$$

$$E_y(b^-)=\gamma_1B_x(0^+)-\gamma_2B_x(b^-) , \quad (2.42)$$

where

$$\gamma_1=\frac{i\omega}{cb}\sum_{l=-\infty}^{\infty}(-1)^l\left[\frac{T_{zz}}{T_{yy}T_{zz}-T_{yz}^2}\right]_{k_z=l\pi/b} , \quad (2.43)$$

$$\gamma_2=\frac{i\omega}{cb}\sum_{l=-\infty}^{\infty}\left[\frac{T_{zz}}{T_{yy}T_{zz}-T_{yz}^2}\right]_{k_z=l\pi/b} . \quad (2.44)$$

We can now obtain the transfer matrix for region 2 (see Fig. 1). To be consistent with Fig. 1 labeling we replace 0^+ by b^- and b^- by a^+ in Eqs. (2.41) and (2.42),

$$E_y(b^-)=\gamma_2B_x(b^-)-\gamma_1B_x(a^+) , \quad (2.45)$$

$$E_y(a^+)=\gamma_1B_x(b^-)-\gamma_2B_x(a^+) . \quad (2.46)$$

Solving Eq. (2.46) for $B_x(b^-)$ and substituting the result into Eq. (2.45) enables us to write the following matrix equation:

$$\begin{bmatrix} E_y \\ B_x\end{bmatrix}_{b^-}=\overleftrightarrow{\mathbf{M}}_2\begin{bmatrix} E_y \\ B_x\end{bmatrix}_{a^+} , \quad (2.47)$$

where $\overleftrightarrow{\mathbf{M}}_2$ is the transfer matrix given by

$$\overleftrightarrow{\mathbf{M}}_s=\begin{bmatrix} \dfrac{\gamma_2}{\gamma_1} & \dfrac{\gamma_2^2-\gamma_1^2}{\gamma_1} \\ \dfrac{1}{\gamma_1} & \dfrac{\gamma_2}{\gamma_1}\end{bmatrix} . \quad (2.48)$$

It is easily verified that the determinant of $\overleftrightarrow{M}_2$ is unity.

We can obtain the transfer matrix for layers 1 and 2 by matrix multiplication of M_1 and M_2 to give[2]

$$\overleftrightarrow{M}=\overleftrightarrow{M}_2\overleftrightarrow{M}_I=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} , \tag{2.49}$$

where

$$M_{11}=\frac{\gamma_2}{\gamma_1}\cos(qa)+\frac{\gamma_2^2-\gamma_1^2}{\gamma_1}iY_1\sin(qa) , \tag{2.50}$$

$$M_{12}=i\frac{\gamma_2}{\gamma_1}Z_1\sin(qa)+\frac{\gamma_2^2-\gamma_1^2}{\gamma_1}\cos(qa) , \tag{2.51}$$

$$M_{21}=\frac{1}{\gamma_1}\cos(qa)+i\frac{\gamma_2}{\gamma_1}Y_1\sin(qa) , \tag{2.52}$$

$$M_{22}=i\frac{Z_1}{\gamma_1}\sin(qa)+\frac{\gamma_2}{\gamma_1}\cos(qa) . \tag{2.53}$$

The quantity $\overleftrightarrow{M}$ is the transfer matrix for the SL period.

III. THE SL DISPERSION RELATION

Having obtained the required transfer-matrix elements in Sec. II, we next consider the dispersion relations for an infinite SL and a truncated SL. These expressions involve the matrix elements of $\overleftrightarrow{M}$ [Eq. (2.40)] and, consequently, are of a very general nature, i.e., appropriate for any type of spatial dispersion.

A. Infinite SL

Following the approach of Mochan, del Castillo-Mussot, and Barrera,[2] the dispersion relation for an infinite SL is

$$\cos(Qd)=\tfrac{1}{2}(M_{11}+M_{22}) , \tag{3.1}$$

where Q is the Bloch wave vector and M_{11} and M_{22} are given by Eqs. (2.50) and (2.53), respectively. The quantity $d=a+b$ is the fundamental SL period.

B. Truncated SL

The dispersion relation for a truncated SL is[2]

$$M_{11}+\frac{M_{12}}{Z}-ZM_{21}-M_{22}=0 , \tag{3.2}$$

where the matrix elements M_{11}, M_{12}, M_{21}, and M_{22} are given by Eqs. (2.50)–(2.53). For surface modes, the impedance Z is given by

$$Z=-\frac{c}{\omega}\left[\epsilon_0\frac{\omega^2}{c^2}-k_y^2\right]^{1/2} . \tag{3.3}$$

where, for a vacuum half space, $\epsilon_0=1$ so that we can write

$$Z=\frac{-ic\alpha_0}{\omega} , \tag{3.4}$$

where α_0 is the vacuum decay constant given by

$$\alpha_0=\left[k_y^2-\frac{\omega^2}{c^2}\right]^{1/2} . \tag{3.5}$$

As a simple check on the dispersion relation given by Eq. (3.2), consider the case of a semiconductor-vacuum interface. For this situation, the required matrix elements are given by Eq. (2.2). For the wave vector q we use the relation [see Eq. (2.5)]

$$q=i\left[k_y^2-\frac{\omega^2}{c^2}\epsilon_1(\omega)\right]^{1/2}=i\alpha_1 , \tag{3.6}$$

where α_1 is the decay constant for the semiconductor half space. Using Eq. (3.2), we obtain the result

$$Z^2=Z_1^2 . \tag{3.7}$$

Taking the positive square root and using Eqs. (3.4) and (2.3), we obtain

$$\epsilon_1(\omega)=-\frac{\alpha_1}{\alpha_0} , \tag{3.8}$$

which is the standard result for surface polaritons[7] at an interface between semiconductor-vacuum half spaces.

IV. SPATIAL DISPERSION CAUSES BY DRIFT CURRENT

A. Theory

The theory developed above is very general in that it did not specify the components of the dielectric tensor for a particular type of spatial dispersion. In this section, we will consider in detail spatial dispersion caused by a dc drift current flowing parallel to the layer interfaces (the y direction). We proceed by utilizing the transport equation for the motion of a charge carrier in the semiconductor,

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=\frac{-\nabla p}{m^*N}+\frac{e}{m^*}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right]-\nu\mathbf{v} , \tag{4.1}$$

and the continuity equation

$$\frac{\partial n}{\partial t}+\nabla\cdot(N\mathbf{v})=0 , \tag{4.2}$$

where $\mathbf{v}$ ν, m^* e, and N are the carrier velocity, collision frequency, effective mass, magnitude of the electron charge, and electron concentration, respectively. The term ∇P is the carrier thermal pressure gradient, and $\mathbf{E}$ and $\mathbf{B}$ are the total electric and magnetic fields, respectively. We also make use of the Maxwell equation,

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} . \tag{4.3}$$

Linearizing Eqs. (4.1)–(4.3) and proceeding as in Ref. 3, we obtain the following components of the dielectric tensor for layer 2 taking $\mathbf{k}=(0,k_y,k_z),\mathbf{v}=(0,V_{2y},0)$:

$$\epsilon_{yy}(\mathbf{k},\omega)=\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{\omega^2}\frac{\omega^2+V_{2y}^2k_z^2}{(\omega-k_yV_{2y})^2}\right] , \tag{4.4}$$

$$\epsilon_{yz}(\mathbf{k},\omega)=\epsilon_{zy}(\mathbf{k},\omega)=-\epsilon_\infty^{(2)}\frac{\omega_{e2}^2}{\omega^2}\frac{k_z V_{2y}}{\omega-k_y V_{2y}} , \quad (4.5)$$

$$\epsilon_{zz}(\mathbf{k},\omega)=\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{\omega^2}\right] , \quad (4.6)$$

where we have used the plasma frequency for layer s, namely,

$$\omega_{e2}^2=\frac{4\pi N_2 e^2}{\epsilon_\infty^{(2)} m^*} . \quad (4.7)$$

From these results we can obtain expressions for the $T_{\alpha\beta}$ of Eqs. (2.27)–(2.29) and for γ_1 and γ_2 [see Eqs. (2.43) and (2.44), respectively],

$$\gamma_1=\frac{i\omega}{c}\frac{\beta_3}{\sqrt{\beta_1\beta_2}}\text{csch}(b\sqrt{\beta_1/\beta_2}) \quad (4.8)$$

and

$$\gamma_2=\frac{i\omega}{c}\frac{\beta_3}{\sqrt{\beta_1\beta_2}}\coth(b\sqrt{\beta_1/\beta_2}) , \quad (4.9)$$

where we have used the expressions

$$T_{yy}T_{zz}-T_{yz}^2=\beta_1+k_z^2\beta_2 , \quad (4.10)$$

$$T_{zz}=\beta_3 , \quad (4.11)$$

and where

$$\beta_1=\epsilon_\infty^{(2)}\frac{\omega^2}{c^2}\left[\epsilon_\infty^{(2)}\frac{\omega^2}{c^2}\left[1-\frac{\omega_{e2}^2}{\omega^2}\right]-k_y^2\right] \times\left[1-\frac{\omega_{e2}^2}{(\omega-k_y V_{2y})^2}\right] , \quad (4.12)$$

$$\beta_2=-\epsilon_\infty^{(2)}\frac{\omega_{e2}^2}{c^2}\frac{V_{2y}^2}{(\omega-k_y V_{sy})^2}\frac{\omega^2}{c^2}\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{\omega^2}\right] -\epsilon_\infty^{(2)}\frac{\omega^2}{c^2}\left[1-\frac{\omega_{e2}^2}{(\omega-k_y V_{2y})^2}\right] -[\epsilon_\infty^{(2)}]^2\frac{\omega_{e2}^4}{c^4}\frac{V_{2y}^2}{(\omega-k_y V_{2y})^2} , \quad (4.13)$$

and

$$\beta_3=\frac{\omega^2}{c^2}\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{\omega^2}\right]-k_y^2 . \quad (4.14)$$

It is of interest to explore the nonretarded limit because retardation has little effect on amplifying instabilities of interest here.[3] Consider the expressions for β_1, β_2, and β_3 given by Eqs. (4.12)–(4.14) which enter in the expressions for γ_1 and γ_2 above. After a little algebra it can be shown that

$$\frac{\beta^3}{\sqrt{\beta_1\beta_2}}=\frac{-c^2 k_y}{\epsilon_\infty^{(2)}\omega^2\left[1-\frac{\omega_{e2}^2}{(\omega-k_y V_{2y})^2}\right]} , \quad (4.15)$$

and

$$\frac{\beta_1}{\beta_2}=k_y^2 \quad (4.16)$$

so that Eqs. (4.8) and (4.9) reduce to

$$\gamma_1=\frac{-ik_y c\,\text{csch}(k_y b)}{\epsilon_\infty^{(2)}\omega\left[1-\frac{\omega_{e2}^2}{(\omega-V_{2y}k_y)^2}\right]} , \quad (4.17)$$

and

$$\gamma_2=\frac{-ik_y c\coth(k_y b)}{\epsilon_\infty^{(2)}\omega\left[1-\frac{\omega_{e2}^2}{(\omega-V_{2y}k_y)^2}\right]} . \quad (4.18)$$

For the infinite SL, the dispersion relation is given by Eq. (3.1). Using the nonretarded forms of γ_1 and γ_2 given by Eqs. (4.17) and (4.18), we obtain the following dispersion relation:

$$\cos(Qd)=\tfrac{1}{2}\left[2\cosh(k_y a)\cosh(k_y b) +\frac{\epsilon_1}{\epsilon_2}\sinh(k_y a)\sinh(k_y b) +\frac{\epsilon_2}{\epsilon_1}\sinh(k_y a)\sinh(k_y b)\right] , \quad (4.19)$$

where k_y is the wave vector along the SL interface and

$$\epsilon_1=\epsilon_\infty^{(1)}\left[1-\frac{\omega_{e1}^2}{\omega^2}\right] , \quad (4.20)$$

$$\epsilon_2=\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{(\omega-V_{02}k_y)^2}\right] . \quad (4.21)$$

In these expressions ω_{ei} is the plasma frequency specified by

$$\omega_{ei}^2=\frac{4\pi N_i e^2}{m_i^*\epsilon_\infty^{(i)}} , \quad (4.22)$$

where N_i is the carrier concentration in the ith layer, e is the magnitude of the electron charge, m_i^* is the carrier effective mass, and $\epsilon_\infty^{(i)}$ is the high-frequency dielectric constant.

to take into account the presence of optical phonons in either layer type, we add the term

$$\epsilon_\infty\frac{\omega_L^2-\omega_T^2}{\omega_T^2-\omega^2}$$

to either ϵ_1 or ϵ_2, or both. In this term, ω_T is the transverse optical-phonon frequency and ω_L is the longitudinal optical-phonon frequency.

For the truncated SL, the dispersion relation is given by Eq. (3.2). Using the nonretarded forms of γ_1 and γ_2, as before, we obtain the following dispersion relation:

$$[\sinh(k_ya)+\epsilon_1\cosh(k_ya)]\sinh(k_yb)\epsilon_2^2-[(1-\epsilon_1^2)\cosh(k_yb)\sinh(k_ya)]\epsilon_2 -\epsilon_1[\epsilon_1\sinh(k_ya)+\cosh(k_ya)]\sinh(k_yb)=0 . \quad (4.23)$$

The presence of optical phonons in a truncated SL can be taken into account as indicated above for the infinite SL.

We next discuss the criterion for the existence of surface waves for the SL geometry considered here. We follow the analysis of Camley and Mills,[8] who considered a truncated SL interfacing a vacuum and with alternate layers also a vacuum. For our geometry this corresponds to taking

$$\epsilon_1=1 . \quad (4.24)$$

For this case the dispersion relation given by Eq. (4.23) reduces to

$$(1-\epsilon_2^2)[\cosh(k_ya)+\sinh(k_ya)]=0 , \quad (4.25)$$

from which we obtain the result

$$\epsilon_2=\pm 1 . \quad (4.26)$$

In their treatment of the truncated SL, Mochan, del Castillo-Mussot, and Barrera[2] give the following form for the dispersion relation:

$$Z=-\frac{M_{12}}{M_{11}-e^{iQd}}=-\frac{M_{22}-e^{iQd}}{M_{21}} , \quad (4.27)$$

where for our case, the M_{ij} are given by Eqs. (2.50)–(2.53). Equation (3.2) was obtained from eliminating e^{iQd} from Eqs. (4.27). Using the first equation of (4.27), replacing iQ by $-\beta$, we have that

$$\begin{aligned} e^{-\beta d}&=M_{11}+\frac{M_{12}}{Z} \\ &=[\sinh(k_ya)+\cosh(k_ya)] \\ &\quad\times\left[\frac{1}{\epsilon_2}\sinh(k_yb)+\cosh(k_yb)\right] . \end{aligned} \quad (4.28)$$

For $\epsilon_2=+1$, Eq. (4.28) reduces to

$$e^{-\beta d}=e^{k_yd} , \quad (4.29)$$

which gives $-\beta=k_y$. This is not a surface wave, as discussed by Camley and Mills.[8] For $\epsilon_2=-1$, we find that

$$\beta=k_y\frac{b-a}{d} , \quad (4.30)$$

which gives a surface wave as long as

$$b>a . \quad (4.31)$$

It is of interest now to look at some limiting cases of the above dispersion relations and compare them with published results.

B. Limiting cases

Regarding the infinite SL dispersion relation given by Eq. (4.19), it can easily be shown that this is equivalent to the result obtained by Camley and Mills[8] when the drift velocity $V_{02}=0$.

For truncated SL, we first consider a semiconductor-vacuum interface. To obtain this geometry, we take $a=0$ and $b=\infty$ [refer to Fig. 1]. It is straightforward to show that the dispersion relation [Eq. (3.2)] for this case reduces to

$$Z^2=\gamma_2^2 , \quad (4.32)$$

where

$$\gamma_2=\frac{ik_yc}{\omega\epsilon_\infty^{(2)}}\left[1-\frac{\omega_{e2}^2}{(\omega-V_{2y}k_y)^2}\right] . \quad (4.33)$$

Using Eq. (4.33) and the expression for Z given by Eq. (3.4), we obtain from Eq. (4.32) the result

$$\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{(\omega-V_{2y}k_y)^2}\right]=-1 , \quad (4.34)$$

which has been obtained previously[3] using a different approach.

If we replace the vacuum with an insulator with dielectric constant ϵ_0, Eq. (4.34) becomes

$$\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{(\omega-V_{2y}k_y)^2}\right]=-\epsilon_0 . \quad (4.35)$$

Further, if we replace this insulator with a nondrifted solid-state plasma with dielectric function $\epsilon_\infty^{(1)}(1-\omega_{e1}^2/\omega^2)$, Eq. (4.35) becomes

$$\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{(\omega-V_{2y}k_y)^2}\right]=-\epsilon_\infty^{(1)}\left[1-\frac{\omega_{e1}^2}{\omega^2}\right] . \quad (4.36)$$

For this case, we have amplifying instabilities.[3]

We next consider a more complicated limiting case, namely, that of a two-interface system. Referring to Fig. 1, we take $a=0$ to eliminate the first layer, retain the spatially dispersive layer with dielectric function $\epsilon_2(\omega,k)$ and thickness b, and add a layer of thickness $t_3(\to\infty)$. We label this region 3. Its transfer matrix is

$$\overleftrightarrow{\mathrm{M}}_3=\begin{bmatrix}\cos(qt_3) & iZ_3\sin(qt_3)\\ iY_3\sin(qt_3) & \cos(qt_3)\end{bmatrix} . \quad (4.37)$$

Thus for our two-interface system, the transfer matrix is

$$\overleftrightarrow{\mathrm{M}}=\begin{bmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{bmatrix} , \quad (4.38)$$

where

$$m_{11}=\frac{\gamma_2}{\gamma_1}\cosh(\alpha_3t_3)-\frac{Z_3}{\gamma_1}\sinh(\alpha_3t_3) , \quad (4.39)$$

$$m_{12}=\frac{\gamma_2^2-\gamma_1^2}{\gamma_1}\cosh(\alpha_3t_3)-\frac{\gamma_2}{\gamma_1}Z_3\sinh(\alpha_3t_3) , \quad (4.40)$$

$$m_{21}=Y_3\frac{\gamma_2}{\gamma_1}\sinh(\alpha_3 t_3)+\frac{1}{\gamma_1}\cosh(\alpha_3 t_3)\ , \quad (4.41)$$

$$m_{22}=\frac{\gamma_2^2-\gamma_1^2}{\gamma_1}Y_3\sinh(\alpha_3 t_3)+\frac{\gamma_2}{\gamma_1}\cosh(\alpha_3 t_3)\ . \quad (4.42)$$

In obtaining these expressions we have used the result

$$q=i\left[k_y^2-\epsilon_3\frac{\omega^2}{c^2}\right]^{1/2}=i\alpha_3\ . \quad (4.43)$$

Putting the above matrix elements in Eq. (3.2), multiplying through by γ_1, and grouping terms, we have

$$\left[\frac{\gamma_2^2-\gamma_1^2}{Z}-Z\right]\cosh(\alpha_3 t_3)+\left[-Z_3-\gamma_2\frac{Z_3}{Z}+\gamma_2 Z Y_3+(\gamma_2^2-\gamma_1^2)Y_3\right]\times\sinh(\alpha_3 t_3)=0\ . \quad (4.44)$$

In the limit $t_3\rightarrow\infty$, the above equation can be reduced to

$$(\gamma_2-Z_3)(\gamma_2+Z)=\gamma_1^2\ . \quad (4.45)$$

Now

$$Z_3=\frac{ic\alpha_3}{\omega\epsilon_3}\ , \quad (4.46)$$

and Z is given by Eq. (3.4) so that Eq. (4.45) can be written in the form

$$\left[\gamma_2-\frac{ic\alpha_3}{\omega\epsilon_3}\right]\left[\gamma_2-\frac{ic\alpha_0}{\omega}\right]=\gamma_1^2\ . \quad (4.47)$$

For the nonretarded limit, using Eqs. (4.17) and (4.18) we can rewrite Eq. (4.47) as

$$(\epsilon_2+1)(\epsilon_2+\epsilon_3)-(\epsilon_2-1)(\epsilon_2-\epsilon_3)\exp(-2\theta)=0\ , \quad (4.48)$$

where

$$\epsilon_2=\epsilon_\infty^{(2)}\left[1-\frac{\omega_{e2}^2}{(\omega-k_y V_{2y})^2}\right] \quad (4.49)$$

and

$$\theta=k_y b\ , \quad (4.50)$$

a result equivalent to those obtained by other means for $V_{2y}=0$,[9] and for $V_{2y}\neq 0$.[10] This dispersion relation [Eq. (4.48)] exhibits amplifying instabilities.

Finally, we consider the limiting case of a truncated SL in the nonretarded limit, neglecting the drift current. The dispersion relation is given by Eq. (4.23), with ϵ_1 given by Eq. (4.20) and ϵ_2 by Eq. (4.21) with $V_{2y}=0$. This result is in agreement with those obtained by Szenics *et al.*,[11] and by Camley and Mills,[8] using a different approach.

C. Numerical results

In order to provide an explicit example of the results that can be obtained with our treatment, we have obtained numerical solutions to the dispersion relation for an infinite SL given by Eq. (4.19). We use the reduced variables ω/ω_{e1}, $k_y a$, and $V_{2y}/a\omega_{e1}$ for frequency, wave vector, and drift velocity, respectively. The values of the parameters appearing in the dispersion relation were taken to be $\omega_{e2}=\sqrt{2}\omega_{e1}$, $b=2a$, $V_{2y}/a\omega_{e1}=0.5$, and $\epsilon_\infty^{(1)}=\epsilon_\infty^{(2)}=1$, a value appropriate to GaAs.

The results for the dispersion relation are shown in Fig. 2 when we input a real frequency ω and calculate a value of the wave vector, which may be real or complex: $k_y=k_1\pm ik_2$. For given Q and reduced frequencies well above the value $1/\sqrt{2}$, we obtain the fast (F) and slow (S) space-charge waves with real k_y that are localized at each interface. As the reduced frequency approaches $1/\sqrt{2}$, the wave vectors for the fast and slow space-charge waves approach $-\infty$ and $+\infty$, respectively. When the reduced frequency lies below $1/\sqrt{2}$, two solutions are found to the dispersion relation with their wave vectors complex and, in fact, complex conjugates of each other. One solution, therefore, decays as it propagates, while the other solution grows as it propagates. We have verified that the growing wave solution corresponds to a convective (amplifying) instability by using a real wave vector as input into the dispersion relation and obtaining a complex conjugate pair of frequencies.[12] The unstable branch (I) starts out linearly from the origin, attains a maximum value of $|k_2|$, and then bends back to the left. This behavior is in contrast to that of two contiguous half

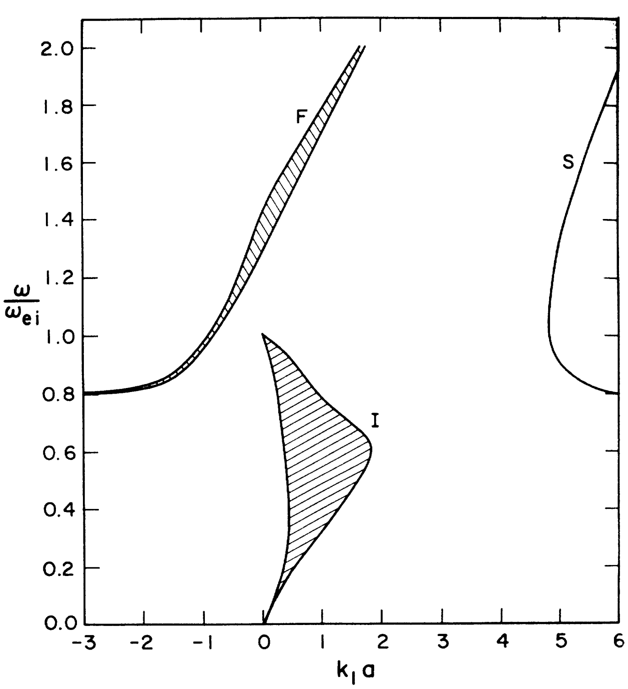

FIG. 2. Dispersion relations for fast (F), slow (S), and instability (I) branches of space-charge waves for $V_{2y}/a\omega_{e1}=0.5$, $b/a=2$, and $\omega_{e2}^2/\omega_{e1}^2=2$.

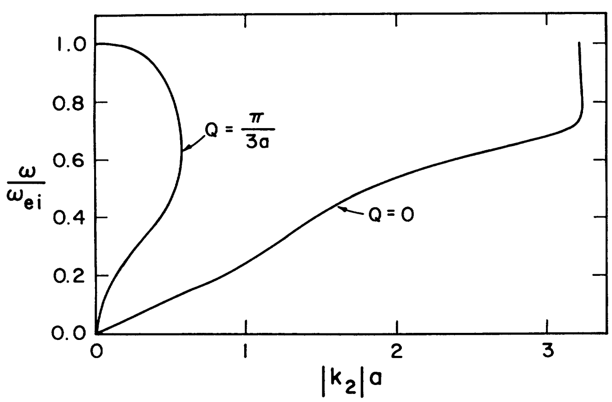

FIG. 3. Magnitude of the imaginary part of the reduced wave vector vs reduced frequency for the instability branch of Fig. 2.

spaces of dielectric constants $\epsilon_1(\omega)$ and $\epsilon_2(k_y,\omega)$ for which the unstable branch is strictly linear.[3] Another difference is that the unstable branch extends above reduced frequency $1/\sqrt{2}$ to reduced frequency unity.

The gain of the convective instability is measured by the value of $|k_2|$. In Fig. 3 we plot the reduced gain as a function of reduced frequency. The gain is seen to increase as the frequency increases, reach a maximum, and then decrease. As the SL wave vector Q is varied, the fast, slow, and unstable branches broaden into bands, although the broadening of the slow branch is not discernable in Fig. 2. Broadening of the gain curves also occurs as seen in Fig. 3.

V. DISCUSSION

We have investigated the use of the transfer-matrix approach for a semiconductor superlattice with alternating spatially dispersive and nonspatially dispersive layers. The dispersion relation for this configuration was obtained and applied to several limiting cases where the spatial dispersion is caused by a dc drift current parallel to the SL interfaces. The results for the limiting cases are in agreement with those obtained by other means.

Of particular interest, however, is the exploration of amplifying instabilities in semi-infinite SL structures. In addition to the instabilities mentioned above, there are optical-phonon amplifying instabilities.[13] The semi-infinite SL dispersion relation obtained above can be readily modified to take into account optical phonons. In addition, magnetic-field effects are of interest and possibly can be incorporated into the above theory. These and other effects, such as those due to temperature,[14] are under investigation.

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