Origin of $1/f$ noise in metallic conductors and semiconductors

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(Received 6 February 1992; revised manuscript received 17 June 1992)

Presented here is a theory of bulk-generated $1/f$ noise universally applicable to all metallic conductors and semiconductors. The basic principle is that the conduction of heavy charge carriers occurs jointly with normal electronic conduction. These heavy charge carriers are termed electron-linked lattice (ELL) carriers and are mobile charge carriers with a positive electronic charge and a variable effective mass greater than a lattice atom's effective mass. They traverse through the conductor by a chain of electron-transfer interactions without the breaking of lattice bonds with one ELL carrier produced for each conduction electron generated. If a loss of acceleration of the ELL carrier occurs, which is caused during an electron transfer interaction and described by a factor β, the resulting noise spectrum is $\omega^{-(1+\beta)}$. When $\beta=1$, the ELL carriers behave as free particles and a $1/\omega^2$ spectrum results. When $\beta=0$, the ELL carriers lose acceleration during the electron transfer and a $1/\omega$ spectrum results of a magnitude that fits the Hooge empirical formula for $1/f$ noise.

INTRODUCTION

The following theory of $1/f$ noise is the result of an investigation in which the motivating ideas are that $1/f$ noise generation is a normal, definable process that fits into an ordinary view of physical phenomena. Furthermore, it is also assumed that the $1/f$ phenomenon to be described here is universal and does not depend upon secondary properties of the lattice such as defect or trap densities, which may not always be present or at least be present in a nonuniform number. The theory resulting was found to fit into the general category of experimental results of investigators in $1/f$ noise. However, the literature on $1/f$ noise clearly shows that alternative mechanisms for the production of $1/f$ noise exist and may be generated by defects,[1,2] surface effects,[3] and quantum effects.[4] However, these sources depend upon specialized assumptions and are not universally applicable. The investigation here is limited to universal, bulk-generated $1/f$ noise with an elementary viewpoint of the conduction process assumed.

HEAVY-CHARGE-CARRIER CONDUCTION

It is assumed that an electron-lattice ion pair has been formed in a metallic conductor or semiconductor as illustrated in Fig. 1 (part A) and that the positions of the particles are being averaged over their thermal motions. It is also assumed that any defects or complexes occurring are of no significance in the process to be described. If the generation of the pair is thermal, the initial velocities immediately after creation of both the free electron and the positive ion are very small on the average. Under the influence of an applied field **E**, the free electron is accelerated and continues in the normal electron conduction process. The positive ion created is accelerated in the direction of the field and is subject to the dragging action of its lattice bonds, which increases its effective mass above that of a single-lattice atom (Fig. 1, part B). Because of this lattice bonding, the ion drags the atoms surrounding it with itself. The ion may be thought of as being contained in a potential well formed by the presence of the atoms around it. As the ion moves it carries these surrounding atoms along with it. Consequently, the potential well moves with the ion. Because of this action, the ion will move even though the potential difference available from the externally applied field is much smaller than the potential needed to move the ion out of its potential well.

After causing a displacement of the ion Δd_0, which is about the same magnitude as the lattice distance a_0, recombination of the ion with a free electron occurs.

However, this electron has come from a neighboring atom, which results in an additional ion being formed. The process then repeats (Fig. 1, part C). This additional positive ion, however, has been displaced from its initial position and therefore carries a fraction of the acceleration and displacement of the original ion with it. The

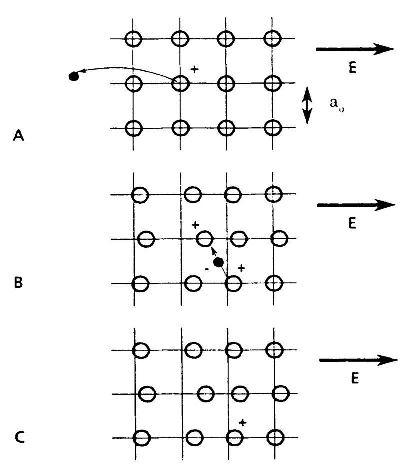

FIG. 1. Illustration of the ELL current.

resultant effective mass of the ion m_+^* during this displacement is given by

$$m_+^* = \sum_n \frac{\Delta d_n}{a_0} m_+ \ , \tag{1}$$

where Δd_n is a small displacement of the nth lattice atom and m_+ is the actual mass of a lattice atom. The name electron-linked lattice (ELL) carrier is suggested for this process. The word "charge" before "carrier" could have been used but has been dropped to make the term shorter.

Recapitulating, the positive ion created after the creation of a thermal ion-electron pair has a very small mobility and undergoes a displacement due to the applied external field carrying its potential well along with the atoms surrounding it. This ion is neutralized by an electron coming from an ion created in the vicinity of the neutralized ion. The chain of positive ions behaves identically to a single particle with a positive electron charge and a mass given by Eq. (1) above, which is defined as an ELL carrier.

The relationship of the average velocities of the ELL carrier and the thermally generated electron during the first transition is given by

$$\tfrac{1}{2} m_- V_-^2 = \tfrac{1}{2} m_+^* V_+^2$$

or

$$\frac{V_+}{V_-} = \left[\frac{m_-}{m_+^*} \right]^{1/2} , \tag{2}$$

where m_- is the effective mass of the electron, and V_- and V_+ are the average velocities of the electron and ELL carrier in the time period during the first transition. In the case of the electron, the velocity V_- continues to be the average electron drift velocity throughout the lifetime of the electron. In the case of the ELL carrier, a similar condition of constancy of velocity does not occur.

As only a temporary assumption, let the ELL carrier be thought of as proceeding loss-free through the lattice. This would be the case if the neutralizing electron after each transition originated in close proximity to the ion being neutralized and, consequently, all of the acceleration of the neutralized ion gained in the transition, would be transferred to the next ion created. The ELL carrier would then behave as if it were a free positively charged atom with a large mean free path traveling through a gas. In this case, the resultant acceleration is given by

$$\frac{dV(t)}{dt} = \frac{e_- E}{m_+^*} \ . \tag{3}$$

This equation has a general solution:

$$V(t) + B = \frac{e_- E}{m_+^*}(t + A) \ , \tag{4}$$

where A and B are constants to be determined. It is intended to allow for a loss between transitions. Consequently, a factor β that is a measure of the transmission of acceleration between transitions is introduced. It is obvious that such a factor will result in a nonconstant acceleration, which means Eq. (3) cannot possibly hold. Furthermore, from Eq. (1) it is obvious that the mass of the ELL carrier is a variable. In order to allow for these observations, Eq. (4) is substituted into Eq. (3) and β is inserted in it, meaning the acceleration is linearly less than the free-particle case since, in general, $\beta < 1$. This result is

$$\frac{dV(t)}{dt} = \beta \frac{V(t) + B}{t + A} \ . \tag{5}$$

This procedure has as its objective the simplification of the equation of motion of the ELL carrier by the elimination of mass from the equation of motion, since the mass of the ELL carrier will depend upon the previous history of motion of the ELL carrier. Equation (5) has the solution

$$V(t) + B = C(t + A)^\beta \ , \tag{6}$$

where $V(t)$ is to be interpreted as the average velocity of the ELL carrier in the space during a transition. Since the acceleration loss occurs between transitions, the initial velocity and acceleration during the first transition cannot depend upon β. The average velocity in this first hop has already been given by Eq. (2) and is V_+. The acceleration at $t = 0$ is given by Eq. (3). This results in

$$A = \tau_1 \ , \tag{7}$$

$$B = V_+ \left[\frac{1 - \beta}{\beta} \right] , \tag{8}$$

and

$$C = \frac{1}{\beta} \tau_1^{-\beta} \ , \tag{9}$$

where τ_1 is given by

$$V_+ = \frac{e_- E}{m_+^*} \tau_1$$

or

$$\tau_1 = \frac{m_+^*}{e_- E} \left[\frac{m_-}{m_+^*} \right]^{1/2} V_- = (m_- m_+^*)^{1/2} \frac{\mu_-}{e_-} \ , \tag{10}$$

where μ_- is the electron mobility. The acceleration is now given by

$$\frac{dV(t)}{dt} = \frac{\beta V(t) + (1 - \beta) V_+}{t + \tau_1} \ , \tag{11}$$

with the solution

$$\frac{V(t)}{V_+} = \frac{1}{\beta} \left[\frac{t}{\tau_1} + 1 \right]^\beta + \left[1 - \frac{1}{\beta} \right] . \tag{12}$$

For the case of $\beta = 0$, Eq. (11) may be solved again with the result

$$\frac{V(t)}{V_+} = 1 + \ln \left[\frac{t}{\tau_1} + 1 \right] . \tag{13}$$

Recapitulating, the assumptions have been as follows: (1) β is an assumed linear differential-loss factor <1; (2) for $\beta=1$, Eq. (3) applies; (3) the velocities $V(t)$ and $dV(t)/dt$ cannot depend upon β during the first transition. Therefore, $V(0)=V_+$ and Eq. (3) again applies for $dV(t)/dt$ independently of β at $t=0$. The above equations meet these conditions, and there are no other assumptions needed to derive an expression for bulk $1/f$ noise. In the remainder of this paper this expression is derived and an interpretation of the significance of the variable mass will be discussed.

DERIVATION OF THE 1/f NOISE SPECTRUM

The autocorrelation function of the electron portion of the current will be derived following well-known procedures.[5] Since the ELL current is very small compared with the electron current, the ELL carrier's autocorrelation function will then be derived by a perturbation of the electron's autocorrelation function.

A conductor has N_0 mobile conduction electrons and a current I_0. In general, the observed current fluctuation $\Delta i_-(t)$ in a subensemble of charges is related to the thermal-electron charge fluctuation $\Delta N_-(t)$ by

$$\Delta i_-(t)=\frac{I_0}{N_0}\Delta N_-(t) \ . \tag{14}$$

The charge fluctuations behave as

$$\Delta N_-(s)=\Delta N_-(0)e^{-s/\tau_-} \ , \tag{15}$$

where τ_- is the mobile electron lifetime and s is the temporal displacement. The autocorrelation function of the electrons is, then,

$$\langle \Delta i_-(t)\Delta i_-(t+s)\rangle=\langle \Delta i_-(s)\Delta i_-(0)\rangle =\frac{I_0^2}{N_0}e^{-s/\tau_-} \ , \tag{16}$$

where $\Delta N_-(0)=N_0^{1/2}$ at room temperature. If $\Delta i_+(t)$ is the ELL current, the total current fluctuation $\Delta i(t)$ is the sum of the electron and ELL carrier fluctuations[6] since both currents are totally correlated with each other:

$$\Delta i(t)=\Delta i_-(t)+\Delta i_+(t) \ . \tag{17}$$

The current pulse of the ELL carrier is

$$\Delta i_+(t)=\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{V(t)}{V_+}\Delta i_-(t) \tag{18}$$

by Eq. (2) and the relation $\Delta i=e_-L^{-1}\Delta V$ where L is the mean free path. This results in a total current fluctuation of

$$\Delta i(s)=\frac{I_0^2}{N_0}\left[1+\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{V(s)}{V_+}\right]e^{-s/\tau_+} \ . \tag{19}$$

Since $(m_-/m_+^*)^{1/2}\sim10^{-3}$, this can be considered a perturbation of small order on the electron current. Applying this perturbation to Eq. (16) gives the autocorrelation function for ELL carriers and electrons:

$$\langle \Delta i(t)\Delta i(t+s)\rangle=\frac{I_0^2}{N_0}\left[1+\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{V(s)}{V_+}\right]\times e^{-s/\tau_+} \ . \tag{20}$$

A physical interpretation of the above result will now be attempted. The derivation of the electron noise current requires a sorting procedure which is illustrated in Fig. 2. Here, a selected number of $g-r$ pulses that are nonzero at $s=0$ are cut at $s=0$. They are then rearranged so that the severed pulses for $s<0$ are added in order of decreasing length and a similar procedure follows for the pulses $s>0$. The result of such a sorting is the autocorrelation function of Eq. (15).

A similar method can be applied to the ELL current as shown in Fig. 3. The pulses shown are the pulses of the previous figure with the ELL carrier pulses added. It is assumed here that these pulses represent the average pulse that occurs at a given starting time so the durations may be taken as equal. The magnitude of the ELL current pulse is exaggerated. These pulses are similarly severed, rearranged, and the resultant autocorrelation function illustrated. It is noted that if τ_- is made unboundedly large, all the pulses shown forming the autocorrelation function approach a state of being identical in shape. This is what Eq. (20) states. Another way of looking at it, is that if τ_- is made large and the observation period is small, then the only transitions generating noise are the pulses that are just ending or just starting in the observation period. This, again, is Eq. (20).

It is noted that Eq. (20) is not exactly symmetric about $s=0$. This indicates that a nonstationary component exists in the perturbation term. It should become obvious, however, that over a long period of time stationarity will occur.

The noise spectrum of Eq. (20) may be evaluated using the Wiener-Khintchine theorem:

$$S(\omega)=4\int_0^\infty \langle \Delta i(t)\Delta i(t+s)\rangle \cos(\omega s)ds \ .$$

Applying this to the first term of Eq. (20) results in an or-

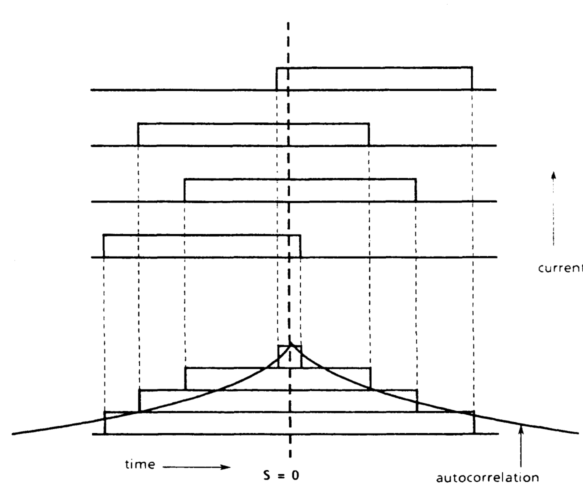

FIG. 2. Current pulses for electrons.

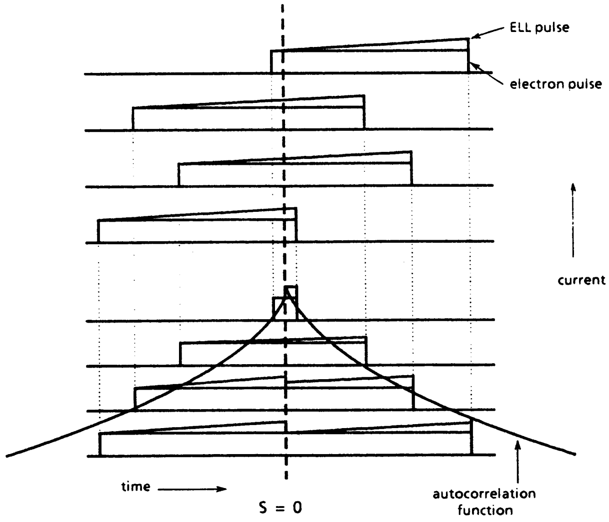

FIG. 3. Current pulses for ELL carriers and electrons.

dinary Lorentzian noise spectrum for generation-recombination noise. Applying this theorem to the second term, for the case of large τ_+ and small τ_1, results in

$$S_+(\omega)=4\left[\frac{\pi}{2}\right]^{1-\beta}\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{I_0^2}{N_0}\frac{1}{(\omega\tau_1)^\beta}\frac{1}{\omega} . \quad (21)$$

This is the equation for $1/f$ noise. Use has been made of the function

$$\int_0^\infty \frac{\sin x}{x^n}dx=\frac{\pi^{1/2}}{2^n}\frac{\Gamma((2-n)/2)}{\Gamma((1+n)/2)}\approx\left[\frac{\pi}{2}\right]^n \quad (22)$$

with the approximation good to 1% for $0<\beta<1$.

The noise of the neutralizing electron after each transition is not significant, since it would be of Lorentzian form with a very short time constant.

APPLICATION TO SEMICONDUCTORS

The above applies, in general, to metallic conduction. In the case of n-type semiconductors, the time taken by the electron while it is moving τ_-, and the ELL carrier lifetime τ_+, are not equal. In this case, the current pulse of the electron is a series of n hops of average duration τ_- with recombination occurring at an average time equal to τ_+ after creation. If N_- is the actual number of ion pairs, which is the sum of the mobile current carriers N_0 and the trapped ion pairs, then

$$N_0=N_-\frac{n\tau_-}{\tau_+} , \quad (23)$$

and the observed electron current is

$$I_0=I_-\frac{n\tau_-}{\tau_+} . \quad (24)$$

In the case of metallic conduction, the factor in Eq. (18) of the ELL component is

$$\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{I_0^2}{N_0}=\frac{I_-I_+}{N_0}=\frac{I_-I_+}{N_-N_+}\Delta N_-^2(0) , \quad (25)$$

where I_- and I_+ are the electron and ELL currents and N_- and N_+ are the respective carrier densities. For the metallic case, $N_+=N_-$. In the case of a semiconductor at room temperature, this quantity, which has the same form in any material, becomes

$$\frac{I_-I_+}{N_-N_+}\Delta N_-^2(0)=\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{(\tau_+/n\tau_-)I_0^2}{(\tau_+/n\tau_-)N_0^2}$$
$$=\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{I_0^2}{N_0} , \quad (26)$$

since $\Delta N_-=\Delta N_+$ and $I_+=(m_-/m_+^*)^{1/2}I_-$. This shows that Eqs. (20) and (21) apply to both the metallic and semiconductor case. A similar argument applies in the case of p-type semiconductor material.

COMPARISON WITH EXPERIMENTAL RESULTS

For $\beta=0$ in Eq. (21) or using Eq. (13) in Eq. (20) and using the Wiener-Khintchine theorem results in

$$S_{1/f}(\omega)=\left[\frac{m_-}{m_+^*}\right]^{1/2}\frac{I_0^2}{N_0}\frac{1}{f} . \quad (27)$$

This is almost the Hooge[7] empirical formula, which is

$$S_H(\omega)=2\times10^{-3}\frac{I_0^2}{N_0}\frac{1}{f} . \quad (28)$$

Using the mass of a single silicon atom, $(m_-/m_+^*)^{1/2}=4.4\times10^{-3}$ instead of the empirical value of 2×10^{-3} obtained by Hooge. Agreement can be obtained by allowing an effective mass $m_+^*=5m_+$, which means each silicon ELL carrier starts with the equivalent mass of about five lattice atoms with it during its motion.

Other values have been reported of a magnitude considerably different than the value given by the Hooge formula.[8] One possibility is that this is the result of the attenuation factor in Eq. (23) of $(\omega\tau_1)^{-\beta}$. The effect of this

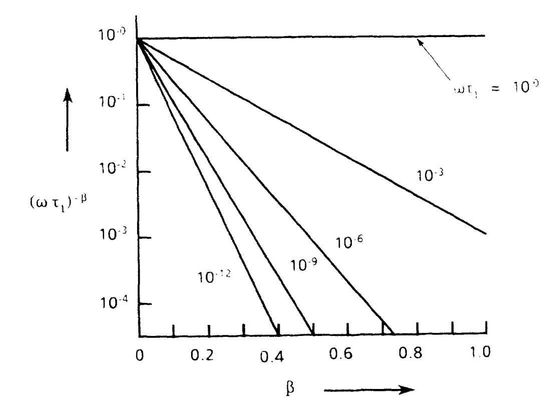

FIG. 4. Plot of attenuating coefficient β vs $(\omega\tau_1)^{-\beta}$ for various values of $\omega\tau_1$.

factor is shown in Fig. 4. Generally, if $V_- \sim 10^{-2}$ cm/sec for a typical conductor, then $V_+ \sim 10^{-5}$ cm/sec. With a lattice spacing of about 10^{-7} cm, this gives $\tau_1 \sim 10^{-12}$ sec. From the figure, an attenuation factor of 10^{-3} is easily obtained with $\beta \sim 0.25$, which is about the magnitude of the observed departure from the Hooge value.

Another possibility is that the assumption of Eq. (2) may have to be replaced by an equal-momentum condition in certain circumstances. In this case, the $(m_-/m_+^*)^{1/2}$ factor could be replaced by m_-/m_+^*, which places the magnitude of the noise a factor of 10^{-3} below that predicted by the Hooge formula. This could happen when high-energy photons activate the electron-ELL pair.

THE VARIABLE MASS OF THE ELL CARRIER

Let Eq. (11) be rewritten using Eq. (10) as

$$\frac{dV(t)}{dt} = \frac{e_- E}{m_+^*} \frac{1+\beta[(V(t)/V_+)-1]}{(t/\tau_1)+1} . \tag{29}$$

A variable mass $m_+^*(V)$ is now defined using Eq. (3):

$$\frac{dV(t)}{dt} = \frac{e_- E}{m_+^*(V)} . \tag{30}$$

Equating Eqs. (29) and (30) defines the variable mass:

$$\frac{m_+^*(V)}{m_+^*} = \left[1+\beta \left[\frac{V(t)}{V_+} - 1 \right] \right]^{(1-\beta)/\beta} , \tag{31}$$

which is a solution of the differential equation

$$\frac{dm_+^*(V)}{m_+^*(V)} = \frac{dV(t)}{(\beta/(1-\beta))V(t)+V_+} . \tag{32}$$

This is a statement that the mass of the ELL carrier increases with velocity. The mass of the ELL carrier consists of a certain number of atoms defined as those atoms carrying a significant portion of the total ELL momentum. In equilibrium, the number of atoms entering the ELL carrier is proportional to velocity. The number of atoms per unit time being left behind is equal to the number taken in but lags behind this number if a change in velocity occurs. This action is due to the contraction of the lattice bonds before the ELL carrier so that atoms pile up in a density wave and contribute their mass to the ELL carrier in much the same way that snow piles up before a plow.

The procedure followed to derive Eq. (5) can now be justified. Because of the variation in mass with velocity, as illustrated above, it was necessary to eliminate mass from the equation of motion. Had this not been done, the equation of motion would have been Eq. (31) substituted into Eq. (3), a very difficult result to obtain or hypothesize at the start of the theory. Furthermore, β has the significance that it represents the transmission of acceleration between transitions of the ELL carrier. With $\beta=1$ the neutralizing electron is in very close proximity to the ion forming the ELL, so that the total acceleration is transferred from ion to ion. With $\beta=0$, the neutralizing electron originates a considerable distance from the ion, consequently losing the acceleration gained during a transition. Generally, the more perfect and defect-free the lattice, the closer β is to zero. If granularity limits the average distance of the neutralizing electron, then β is greater than zero.

LOW-FREQUENCY CUTOFF

The current pulse of Eq. (12) has no cutoff. This raises the question of whether or not the $1/f$ spectrum continues to increase in amplitude unboundedly as $\omega \to 0$ if the carrier lifetime is infinite. Assume that $\beta=0$ and an ELL carrier proceeds down an infinitely long wire. Eventually, the mass of the carrier increases until the diameter of the ELL carrier equals the diameter of the wire and then $\beta=1$, since no more mass will be available for the carrier to consume. This would be the theoretical maximum cutoff for $1/f$ noise. However, one has a long wait. For the case of $\beta=0$, then,

$$\frac{m_+^*(V)}{m_+^*} = \frac{t}{\tau_1} + 1 . \tag{33}$$

If, as before, $\tau_1 \sim 10^{-12}$ and the number of atoms in a cubic millimeter is $\sim 10^{20}$, then it would take $\sim 10^{8}$ sec, or about a few years, for an ELL carrier to fill a cubic millimeter. In an actual sample, the low-frequency cutoff would occur when the carrier leaves the area of measurement in the sample or the carriers reach the end of their lifetime.

SUGGESTED EXPERIMENTAL VERIFICATION

It is proposed that experimental verification of Eq. (12) could be obtained by applying a field to a photoconductor and illuminating the positive electrode, which has to be transparent, with a very small photon density to prevent space-charge effects. The photon energy should be as close to the band gap as possible to insure a low initial velocity. This would create ELL carriers which then would produce a current pulse described by Eq. (12).

CONCLUSION

A universal source of bulk $1/f$ noise has been shown to be due to the mass increase of an ELL carrier with velocity as lattice atoms traversing the location of the ELL carrier pile up in a density wave before it. However, as stated at the beginning, other sources of $1/f$ noise exist although not in the universal sense. Perhaps the most remarkable thing about the above theory is that only an elementary viewpoint of the conduction process was needed.

ACKNOWLEDGMENTS

Appreciation is owed to M. B. Weissman of the University of Illinois, R. F. Voss of the IBM Watson Research Center, and A. van der Ziel of the University of Minnesota, for correspondence, direction, and encouragement during the long evolution of the ideas of this paper. Thanks are also owed to S. Vanderbroek for suggested improvements in the manuscript.

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