

Investigations of pairing in anyon systems

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(Received 31 August 1992)

We investigate pairing instabilities in the Fermi-liquid-like state of a single species of anyons. We describe the anyons as fermions interacting with a Chern-Simons gauge field and consider the weak-coupling limit where their statistics approaches that of fermions. We show that, within the conventional BCS approach, because of induced repulsive Coulomb and current-current interactions, the attractive Aharonov-Bohm interaction is not sufficient to generate a gap in the fermion spectrum.

The possibility of superconductivity in anyon systems is of general interest. The mean-field-theory approach to systems with a single species of anyons developed by Fetter, Hanna, and Laughlin and other authors¹ argued that a finite anyon density induces a background statistical magnetic field and a Landau-level picture of the ground state and the ensuing energy gap and superconductivity emerged. There are also alternative scenarios for anyon superconductivity where the background statistical magnetic field is absent, the ground state of the anyon system resembles that of a Fermi liquid, and the superconductivity is realized in the conventional BCS mode. This occurs in a system which contains *two species* of anyons of opposite charges with respect to the statistical gauge field and which therefore have opposite fractional statistics²⁻⁶ [we denote this as $(+ -)$ pairing]. There, the statistical background magnetic fields cancel and it has been found that the Aharonov-Bohm interaction is attractive enough to generate the pair formation.

Recently, it has been noted that systems with a *single species* of anyons and a Fermi-liquid-like ground state are also of interest.⁷ Anyons are described as fermions interacting with a Chern-Simons gauge field so that in the weak-coupling limit their statistics approaches that of fermions. The Fermi-liquid state is achieved when the mean background statistical magnetic field is canceled by an *external* magnetic field. A pairing instability in such systems [which we denote $(+ +)$] has been suggested as an explanation of the even denominator states in the fractional quantum Hall effect. It has been conjectured in Ref. 7 that the Aharonov-Bohm interactions (with coupling constant $1/\kappa$) which give the fermions fractional statistics are sufficient to drive a superconducting pairing instability. In this paper we shall show that this is not the case. When the back-reaction of the gap in the fermion spectrum to the interaction potential is taken into account, the BCS gap equation has only the trivial solution. We conclude that, if there is such an instability in this system, it must occur outside of the weak-coupling BCS scenario or else be driven by other interactions. It is also worth noting that in the physical systems of interest κ is small (for example, in Ref. 7 $\kappa = 1/2\pi$), and *strong-coupling* effects could lead to the gap formation (in that scenario there would be a critical κ where pairing takes place). In that case, we observe that [as was previously

found for the $(+ -)$ case⁶] the coefficient of the Chern-Simons term is renormalized as $\kappa \rightarrow \kappa - l/4\pi$, where l is the angular momentum of the Cooper pair which, because of Fermi statistics, must be an odd integer. Thus, the Hall conductivity and fractional statistics of quasiparticles and magnetic vortices are modified by the gap. Because of no renormalization beyond one-loop arguments⁸ for the Chern-Simons term we expect that the latter result is valid beyond our large- κ perturbation theory. To get an even-denominator Hall conductance, $2\pi k$ should be a half-odd integer.

It is interesting to compare the $(+ +)$ case with the $(+ -)$ case where it has been shown⁶ that Aharonov-Bohm interactions do lead to pairing and formation of an energy gap, which even proves to be parametrically larger than the simplest BCS gap. At the tree level these cases are equivalent: both contain only long-ranged Aharonov-Bohm interaction (which might be both attractive and repulsive, depending on the angular momentum of the anyon pair^{4,6}). Bare Coulomb and magnetic current-current interactions are absent, but will be generated by the radiative corrections. The difference between the $(+ -)$ and $(+ +)$ cases stems from different back-reactions of the gap formation on the effective interaction between anyons.

Since the $(+ -)$ pairs are neutral with respect to the statistical gauge field, their presence in the ground state does not lead to a Meissner effect (i.e., generation of a London mass) for the statistical gauge field. Moreover, since at zero temperature all anyons are bound in pairs, there are no free charge carriers in the system, and therefore, no Debye screening. This in turn results in the fact that radiatively induced Coulomb and magnetic current-current interactions are short ranged (topologically massive), whereas the Aharonov-Bohm interaction remains long ranged and, therefore, dominant.⁶ In the equal-charge case, on the contrary, the formation of $(+ +)$ pairs of course leads to both Debye and Meissner screening, and *all* interactions become short ranged (so that the Aharonov-Bohm interaction will no longer have the topological nature). Thus the interaction which causes anyon pairing will in turn be completely changed by this very pairing. Note the the values of the Debye and London masses are defined only by the density of the free charge carriers in the former case and that of the super-

conducting fermions in the latter one. If there is a gap in the fermionic spectrum these two are defined by the density of anyons in the system, and for zero temperature have nothing to do with the magnitude of this gap, irrespectively of how small it is. They can therefore have an important influence on the gap equation. The purpose of this paper is to consider a self-consistent picture of this feedback. We will demonstrate that for large κ ($2\pi/k$ is the statistics parameter) there is no gap in the fermionic spectrum, contrary to the result in Ref. 7. For simplicity we will consider the zero-temperature case.

We start with the Lagrangian

$$L = -\frac{\kappa}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \Psi^\dagger \left[i\partial_0 - A_0 - \epsilon \left[-i\frac{\partial}{\partial \mathbf{x}} - \mathbf{A} \right] \right] \Psi, \quad (1)$$

where the statistical coupling κ is taken positive and large, and $\epsilon(\mathbf{k})$ is the (quasi)particle (anyon) dispersion law below taken to be $\epsilon(\mathbf{k}) = \mathbf{k}^2/2m - \epsilon_F$. The tree-level potential has pure Aharonov-Bohm form and reads in the radiation gauge ($\partial_i A_i = 0$)

$$U(\mathbf{p}, \mathbf{p}') = \frac{2i}{\kappa m} \frac{\epsilon_{ij} p_i p'_j}{(\mathbf{p} - \mathbf{p}')^2} = -\frac{2i}{\kappa m} \frac{\sin\theta}{p/p' + p'/p - 2\cos\theta}, \quad (2)$$

where $\mathbf{p} = p(\cos\phi, \sin\phi)$, $\mathbf{p}' = p'(\cos\phi', \sin\phi')$, $\theta = \phi - \phi'$. This potential leads to the formation of bound pairs of anyons with nonzero angular momentum.⁴ Substituted into the standard BCS gap equation

$$\Delta_{\mathbf{p}} = -\frac{1}{2} \int \frac{d\mathbf{p}'}{(2\pi)^2} U_{\mathbf{p}\mathbf{p}'} \frac{\Delta_{\mathbf{p}'}}{\sqrt{\epsilon_{\mathbf{p}'}^2 + |\Delta_{\mathbf{p}'}|^2}}, \quad (3)$$

it gives rise to the exponentially suppressed gap, which depends nontrivially on the momentum⁶

$$\Delta(\mathbf{p}) = \Delta e^{i\phi} \left[\left[\frac{p}{p_F} \right]^l \theta(p_F - p) + \left[\frac{p_F}{p} \right]^l \theta(p - p_F) \right]. \quad (4)$$

The largest gap is achieved for P -wave pairing

$$\Delta \simeq \epsilon_F e^{-2\pi\kappa}. \quad (5)$$

Taking into account the back-reaction of the gap on the renormalization of the potential leads to drastic changes in these tree-level conclusions. In the case of $(+ -)$ pairing it removes the exponential suppression of the gap⁶

$$\Delta \sim \epsilon_F / \kappa, \quad (6)$$

which demonstrates the existence of a new universality class for the superconducting order parameter. (For small κ , which corresponds to the strong-coupling regime, one expects the gap to be of order of the Fermi energy,⁵ although no precise statements can be made in this case.)

Now we will show that in the $(+ +)$ case the result is quite the opposite: the only solution of the renormalized gap equation is $\Delta = 0$.

To find the gap improved potential we need to know the polarization operator (herewith we will be interested in the static limit, when there is no energy transfer through the gauge line, and we will restrict ourselves to the first-loop correction, as in the large- κ limit the contribution of higher loops will be suppressed). In the radiation gauge we find the following.

(1) $\Pi_{00}(\mathbf{q}) = im/2\pi$, for all $|\mathbf{q}| < 2p_F$. Indeed, in the case of the single-charge plasma the Debye screening does not depend on whether or not the fermions are bound in the pairs.

(2) $\Pi_{ij}(\mathbf{q}) = -i\Pi(\mathbf{q})\delta_{ij}$. $\Pi(\mathbf{q}=0) = (\epsilon_F/4\pi)\delta_{ij}$. The nonzero value of $\Pi_{ij}(\mathbf{q}=0)$ indicates the appearance of the magnetic London mass of the statistical photon (Meissner effect). Note that this London mass does not depend on the value of the gap, exhibiting thus a nonanalytic behavior, as for the case of the absent gap $\Pi(\mathbf{q}=0) = 0$. This is of no surprise, since we know that the value of the London mass is determined by the density of superconducting fermions, and at zero temperature all fermions are superconducting however small the gap. When $q = |\mathbf{q}|$ increases $\Pi(q)$ decreases, and for large $q \gg \Delta/v_F$ it approximately equals $\Pi(q) \simeq (\Delta\epsilon_F/qv_F + q^2/24\pi m)\delta_{ij}$, where the last term in the parentheses is the magnetic response of the normal metal.

(3) $\Pi_{0i}(\mathbf{q}=0) = (l/4\pi)\epsilon_{ij}q_j$, where l is the angular momentum of the pair. This leads to the same renormalization of the Chern-Simons coupling as those first found⁶ in $\kappa_{\text{ren}} = \kappa - l/4\pi$, but it is worth recalling that now, after the formation of the BCS pairs, the Aharonov-Bohm interaction is short ranged, and this renormalization does not lead to a considerable enhancement of the Aharonov-Bohm attraction, and in what follows we will neglect it in the large- κ limit. Since there is no correction to the Chern-Simons term in the ordinary free plasma, for $q \gg \Delta/v_F$ Π_{0i} decreases to zero.

The full gauge propagator is in the matrix notations

$$D^{-1} = D_0^{-1} - P, \quad (7)$$

where D_0 is the bare propagator, the only nonzero component of which is

$$D_{0i} = \frac{1}{\kappa} \frac{\epsilon_{ij}q_j}{q^2}, \quad (8)$$

and P is the polarization operator. So,

$$D_{00} = iD_0, \quad D_0 = \frac{\Pi}{\kappa^2 q^2 + m\Pi/2\pi},$$

$$D_{ij} = -iD_1 \left[\delta_{ij} - \frac{q_i q_j}{q^2} \right], \quad D_1 = \frac{m/2\pi}{\kappa^2 q^2 + m\Pi/2\pi}, \quad (9)$$

$$D_{0i} = D_2 \epsilon_{ij}q_j, \quad D_2 = \frac{1}{\kappa q^2 + m\Pi/2\pi\kappa}.$$

The behavior of the functions $D_{0,1,2}$ is depicted on Fig. 1. Note, that for

$$q \lesssim q_0 \simeq p_F (\Delta/\kappa^2 \epsilon_F)^{1/3}, \quad (10)$$

the propagators $D_{1,2}$ increase with increasing q . This very unusual behavior of a propagator would be forbidden in the relativistic case, and is a net effect of the finite density of anyons, the presence of the gap in their spectrum, and the form of the bare gauge propagator (domination of the Chern-Simons term in it). It makes the most important region of the transferred momenta

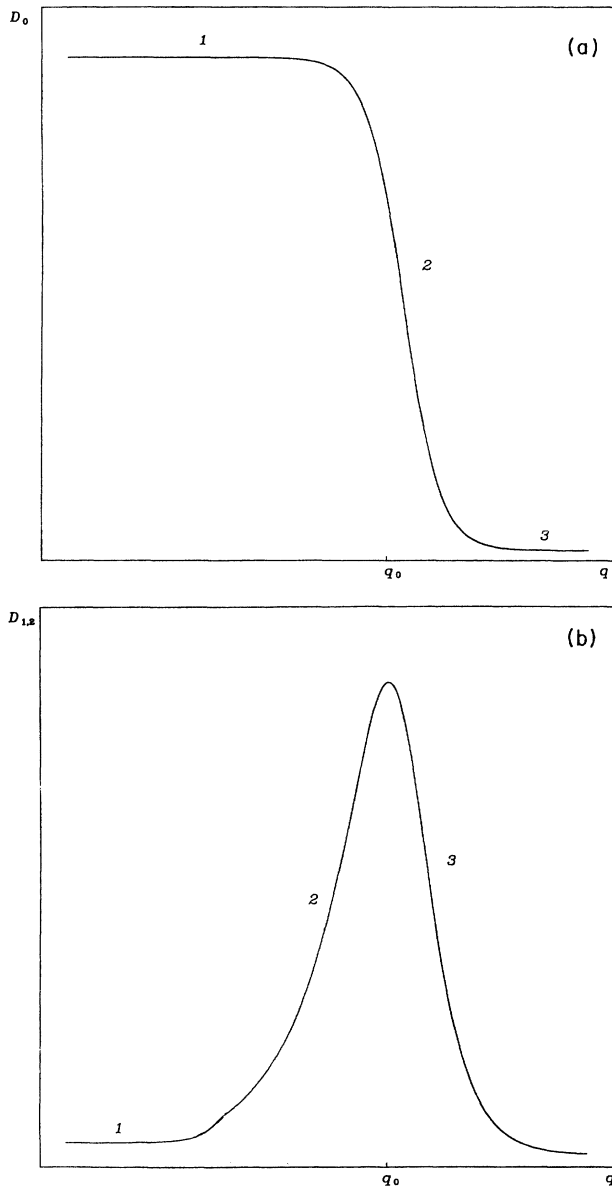


FIG. 1. (a) The behavior of the propagator $D_{00}(q) = -iD_0(q)$: (1) $q \ll q_0$, $D_0 \approx 2\pi/m$. (2) $q_0 \ll q \ll q_2$, $D_0 \approx (1/4\kappa^2 m)(\Delta/\epsilon_F)p_F^3/q^3$. (3) $q_2 \ll q \lesssim p_F$, $D_0 \approx 1/24\pi\kappa^2 m$. (b) The behavior of the propagators $D_{ij}(q) = -iD_1(q)(\delta_{ij} - q_i q_j/q^2)$, $D_{0i}(q) = D_2(q)\epsilon_{ij}q_j$: (1) $q \ll q_1$, $D_1 \approx 4\pi/\epsilon_F$, $D_2 \approx 16\pi^2/p_F^2$. (2) $q_1 \ll q \ll q_0$, $D_1 \approx (1/\epsilon_F)(2\epsilon_F/\Delta)q/p_F$, $D_2 \approx (1/p_F^2)(16\pi\kappa\epsilon_F/\Delta)q/p_F$. (3) $q_2 \ll q \lesssim p_F$, $D_1 \approx (1/4\pi\kappa^2\epsilon_F)p_F^2/q^2$, $D_2 \approx (1/\kappa p_F^2)p_F^2/q^2$. The maximal values are $D_1(q_0) \sim (1/\epsilon_F)(\epsilon_F/\kappa\Delta)^{2/3}$, $D_2(q_0) \sim (1/p_F^2)\kappa(\epsilon_F/\kappa\Delta)^{2/3}$. The values of the crossover momenta are $q_0 \sim (\Delta/\kappa^2\epsilon_F)^{1/3}p_F$, $q_1 \sim (\Delta/\epsilon_F)p_F$, and $q_2 \sim (\Delta/\epsilon_F)^{1/3}p_F$.

$q^2 = (\mathbf{p} - \mathbf{p}')^2$ in the gap equation (3) to be $q \approx q_0$, as opposed to the standard case where this region was $q \approx (\Delta/\epsilon_F)p_F \ll q_0$.

Note that despite the fact that all our interactions are short ranged in the sense that all D_i are finite (and nonzero) at $q=0$, the magnetic interaction still contains the factor $1/q^2$ from the tensor structure $D_{ij} = iD_1(\delta_{ij} - q_i q_j/q^2)$ [this is reminiscent of the gauge symmetry of the original Lagrangian (1)]. Therefore, the corresponding potential for the current-current interaction in the l -wave pairing [cf. (2) and (4)]

$$U_M(\mathbf{p}, \mathbf{p}') = \frac{i}{m^2} p_i p_j D_{ij} \cos l\theta = \frac{1}{m^2} D_1 \frac{p^2 p'^2 \sin^2 \theta \cos l\theta}{p^2 + p'^2 - 2pp' \cos \theta}, \quad (11)$$

is finite when $\mathbf{p}' \rightarrow \mathbf{p}$ ($p' \rightarrow p$, $\theta \rightarrow 0$).

The two others interactions, Coulomb,

$$U_C = -iD_{00} \cos l\theta = D_0 \cos l\theta,$$

and Aharonov-Bohm,

$$U_{AB} = -\frac{2ip_i}{m} D_{0i} i \sin l\theta = -\frac{2}{m} D_2 \frac{pp' \sin \theta \sin l\theta}{p^2 + p'^2 - 2pp' \cos \theta},$$

vanish in this limit, and are, therefore, subdominant with respect to the current-current interaction.

Let us stress that taking into account the back-reaction of pairing onto the potential between anyons results in weakening of the attractive part of the potential and in the appearance of the strong repulsive part. So we conclude that the gap should be suppressed even stronger than it was at the tree level [see (5)], and the ratio Δ/ϵ_F is the smallest parameter in the theory.

It can be seen that parametrically the random-phase approximation improved gap equation reads⁹

$$\Delta = \left\{ -\frac{\Delta^{2/3} \epsilon_F^{1/3}}{\kappa^{4/3}} - \frac{\Delta^{4/3}}{\kappa^{4/3} \epsilon_F^{1/3}} + \frac{\Delta}{\kappa} \right\} \ln \left(\frac{\epsilon_F}{\Delta} \right), \quad (12)$$

where the first term in the curly brackets is the contribution of the magnetic current-current interaction, the second one is that of the Coulomb interaction [these two come from the small momentum transfer in the gap equation (3), $|\mathbf{p} - \mathbf{p}'| \sim q_0$], and the third one is the contribution of the Aharonov-Bohm interaction (it comes from the large momentum transfer in the gap equation, $|\mathbf{p} - \mathbf{p}'| \gg q_0$ for which it has essentially the bare form). We see that only the trivial solution $\Delta = 0$ satisfies (12), provided the constraint $\Delta \lesssim \epsilon_F e^{-2\pi\kappa}$ is fulfilled. Therefore we conclude that within the simplest BCS picture there is no pairing in the Fermi liquid of anyons of the same charge. Note also that any additional short-ranged attraction between anyons will give the contribution to the gap equation of the type of the third term in (12), and therefore will not change this picture. Taking into account the electromagnetic interaction between the anyons gives an additional repulsion and does not change this general picture.

It seems unlikely that for pairing at large angular momenta l the system might become superconducting be-

cause of the Kohn-Luttinger effect.¹⁰ Indeed, as we showed, the most important region of the transferred momentum in the gap equation (3) is $q \sim q_0$, where the net interaction is repulsive, which makes the improved gap equation (12) have no nontrivial solutions. To smear out this repulsion one needs very large angular momenta,

$$l \gtrsim (\kappa^2 \epsilon_F / \Delta)^{1/3}, \quad (13)$$

which look implausible. We do not expect the gapless superconductivity¹¹ for the anyon system either. Indeed, for this scenario one in general needs the attraction to vary in the vicinity of the Fermi surface much faster than the repulsion, whereas in our case the attraction (Aharonov-Bohm interaction) and the repulsion (current-current interaction) have essentially the same behavior near the Fermi surface.

The arguments presented above, of course, do not rule out the general possibility of the superconductivity in these systems. As we mentioned above, for real physical systems the value of the statistics parameter κ is rather

small, so these systems correspond to the strong-coupling regime. For example, for $\kappa \sim 2\pi$ the statistics of anyons is close to that of bosons, and the system of equally charged anyons in this case will behave like the XY model, for which the long-range order exists.¹² All these should be a subject of further investigations.

Note added. B. Halperin, P. Lee, and N. Read¹³ have recently applied the usual methods of theory of the Fermi liquid for anyon systems in the absence of the background magnetic field. These authors found a divergence in the effective mass of the fermions. Our results support their conjecture that this problem can be cured without drastic changes in the Fermi-liquid-like description of this system.

The authors are grateful to D. Eliezer, S. Yu. Khlebnikov, P. C. E. Stamp, and N. Weiss for numerous valuable discussions. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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